Chapter 5 -- Resistive Stability

The NCSX stellarator will have a substantial amount, ≈ 100 - 200 kA, of plasma current driven inductively or by the bootstrap effect. The presence of plasma current provides a potential source of free energy, which can then drive MHD instabilities such as tearing modes, [5.1, 5.2] degrading confinement and performance. Calculations of the non-linear evolution of tearing modes, including neoclassical effects [5.3] (bootstrap current) in the full 3-D geometry of NCSX is beyond the capability of present MHD codes. In the tokamak community, qualitative modeling of tearing modes has been successfully done using a simple, quasi-cylindrical, low beta model as described below [5.4-5.10]. The validity of the application of this model to stellarators is supported by experiments on W7-A and W7-AS where reasonable agreement between experiment and modeling was found [5.11, 5.12]. The calculations presented below suggest that the start-up, equilibrium, and high beta phases of the baseline NCSX plasma should be stable to internally driven tearing modes. Neoclassical effects are predicted to further enhance this stability.

5.1 Description of resistive model used for stability analysis

The Δ' formalism used in the following analysis is derived in a zero beta, straight circular cylindrical geometry. Somewhat more refined formalisms (PEST III) allow for finite beta and shaping. However they are still constrained to axisymmetric equilibria and do not easily allow decoupling of ι (= 1/q) and J, nor do they calculate $\Delta'(w)$. Codes such as PIES or M3D can do a much more complete analysis, but are prohibitively expensive in terms of time to run. Tearing mode stability results found by application of the Δ' formalism to shaped, finite beta and toroidally asymmetric plasmas must be viewed with some skepticism.

The Δ' code used in the following calculations separates the $\iota(r)$ and J(r) profiles, necessary even in circular tokamaks such as TFTR, and particularly so in stellarators where a substantial fraction of the transform is not from the plasma current. The $\iota(r)$ and J(r) profiles are used in the standard differential equation governing the perturbed helical flux function [5.1]

$$[\partial^2/\partial r^2 + 1/r\ \partial/\partial r - m^2/r^2 - (\partial J_0/\partial r)/(\partial\psi_0/\partial r)]\ \psi_{m,n} = 0 \qquad (5.1.1)$$

where ψ_0 is defined from $\iota(r)$ by

$$\psi_0(r) = B_0/R_0 \int_0^r (\iota(r) - m/n)\ r\ dr\ . \qquad (5.1.2)$$

The J(r) includes the bootstrap, beam driven and inductively driven currents. Equation 5.1.1 has a pole at the mode rational surface where $\iota(r)$ = n/m. In the boundary layer region near this surface a full fourth order differential equation must be used; however it has been shown that the mode stability is determined by matching the external solution across the boundary layer using the "constant – ψ" approximation. The matching condition yields a discontinuity in the first derivative, which is quantified in Δ'. A positive value for Δ' represents an unstable tearing mode, a negative eigenvalue; stability.

The growth of the island changes from exponential to linear at very small island size. This linear regime of growth rate is referred to as the Rutherford regime [5.2]. The equation describing the island width evolution in this regime is the Rutherford equation,

$$dw/dt = 1.22\ \eta / \mu\ [\ \Delta'(w)]. \qquad (5.1.3)$$

and the time dependent island width evolution is calculated by numerically integrating this equation. Here η is the resistivity and μ is the magnetic permeability. The $\Delta'(w)$ is calculated numerically using the constant-ψ approximation [5.2].

The density and temperature gradient driven current in neoclassical theory, the bootstrap current, is affected by the presence of the magnetic island. The island flattens the temperature and density profile, locally reducing the bootstrap current and generating a helical current perturbation. The effect of this perturbed current on the island is modeled with an additional term in the Rutherford equation [5.4-5.10]

$$dw/dt = 1.22\, \eta / \mu\, [\, \Delta'(w) + \Delta_{nc}\,]. \tag{5.1.4}$$

In the model used below, Δ_{nc} is evaluated by using parameters calculated by TRANSP in the equation [5.6, 5.7]

$$\Delta_{nc} = (16\, \pi / 5)\ k_1\, R_0\, J_{bs} / (s\, \iota\, B_0\, w\,). \tag{5.1.5}$$

Here J_{bs} is the local bootstrap current density, s is the local shear and w is the island width. The constant k_1 accounts for approximations made in deriving the effective perturbation in the bootstrap current due to the island. For the simulations shown here, the same $k_1 \approx 1$ was used as had been used to fit TFTR experimental data.

5.2 Previous comparisons of extended Rutherford model with experiments.

This approach has been well studied and used extensively to analyze experimental tearing mode data in tokamak experiments. It provides both a basis for translating the measured external magnetic fluctuation levels into a measure of the island size as well as predictions of mode stability, growth rate and saturated island widths. The inclusion of neoclassical effects, i.e., the modeling of the effect of the island on the bootstrap current density and the concomitant effect of the perturbed bootstrap current on the island, has very successfully reproduced some of the observed characteristics of tearing modes in normal shear high beta, low collisionality plasmas. This extensive experimental database [*e.g.*, 5.4-5.10] gives some credence to the neoclassical tearing mode model. However, neoclassical theory (applied to tearing modes) in the context of reversed shear plasmas has not been extensively tested.

In the circular cross-section TFTR tokamak this model found very good agreement between island widths predicted from edge magnetic fluctuation levels and island widths measured with the electron cyclotron emission temperature profile diagnostic [5.4,5.5]. The code used here to model tearing mode behavior in the simulated NCSX plasmas was extensively benchmarked on these TFTR data [5.6,5.7].

A study of double tearing modes in reversed shear plasmas in the TFTR tokamak found no evidence for neoclassical modifications to the tearing mode stability in the negative shear regions [5.6]. However, in this case the analysis of double tearing modes was sufficiently unique that it is quite possible that the physics of the coupling in the double tearing modes was not adequately represented, leading to uncertainty in the conclusions. Further, single tearing modes were not observed in the reversed shear region of TFTR plasmas, consistent with the prediction of the neoclassical model that the bootstrap term is stabilizing in reversed shear.

Tearing modes have also been observed in stellarators such as the W7-AS and W7–A when net current is present. Simulations of the linear stability and non-linear evolution of the islands has been done, primarily with simple cylindrical Δ' models such as the one used here. In W7–A the analysis was able to predict reasonably well the observed magnetic fluctuation level, *i.e.*, the saturated island width [5.11]. In the W7-AS experiment, these predictions were within an order of magnitude for the external magnetic fluctuation level and in reasonable agreement with the tomographically determined island size [5.12] without the inclusion of neoclassical effects. Whether the neoclassical terms would have qualitatively changed the results is not clear. The conclusion of the authors was that, "…, *so far no direct evidence of neoclassical effects on the stability has been found*." This statement could be interpreted as meaning there is no evidence either for or against the validity of the neoclassical theory of tearing modes.

Evidence for healing of (vacuum) magnetic islands by low collisionality, moderate β plasmas with negative shear (in the tokamak sense) has been seen on the Large Helical Device [5.13]. In these experiments, the presence of magnetic islands was inferred from Thomson Scattering measurements of the electron temperature and density profiles. It was found that the vacuum islands were present in colder, collisional plasmas, but healed in hotter, higher β plasmas. The results were felt to be consistent with neoclassical theoretical predictions, however more detailed analysis was not presented.

5.3 Δ' Analysis

Simulations of the start-up phase of the target NCSX plasma with the TRANSP code are described in detail in Chapter 9. These simulations predict the evolution of the ohmic, beam driven and bootstrap current profiles through the start-up phase to the target equilibrium. These time-dependent profiles have been analyzed for resistive stability to the lowest order tearing modes, the (2/1), (7/3) and (7/4) modes. The startup scenario has reversed shear (in the tokamak sense) and begins with $\iota(a) < 0.5$ [$q(a) > 2$]. As the plasma evolves the $\iota(a)$ rises until an m = 2, n = 1 rational surface enters the plasma from the edge at about 0.05 s.

The results of a tearing mode stability analysis for the 2/1 mode are shown in Figure 5.1. In the time period between about 0.08 and 0.19 s, during the startup, there is a flattish region in the iota profile around $\iota \approx 0.5$ or r/a of about 0.6 to 0.9. During this time period, the flatspot results in unphysical eigenfunctions. In simpler cylindrical or toroidal geometry, this would be indicative of approaching an ideal stability limit. (However, as shown in Chapt. 9, these plasmas are stable.) Thus, the analysis of the 2/1 stability with this code is not meaningful during this period.

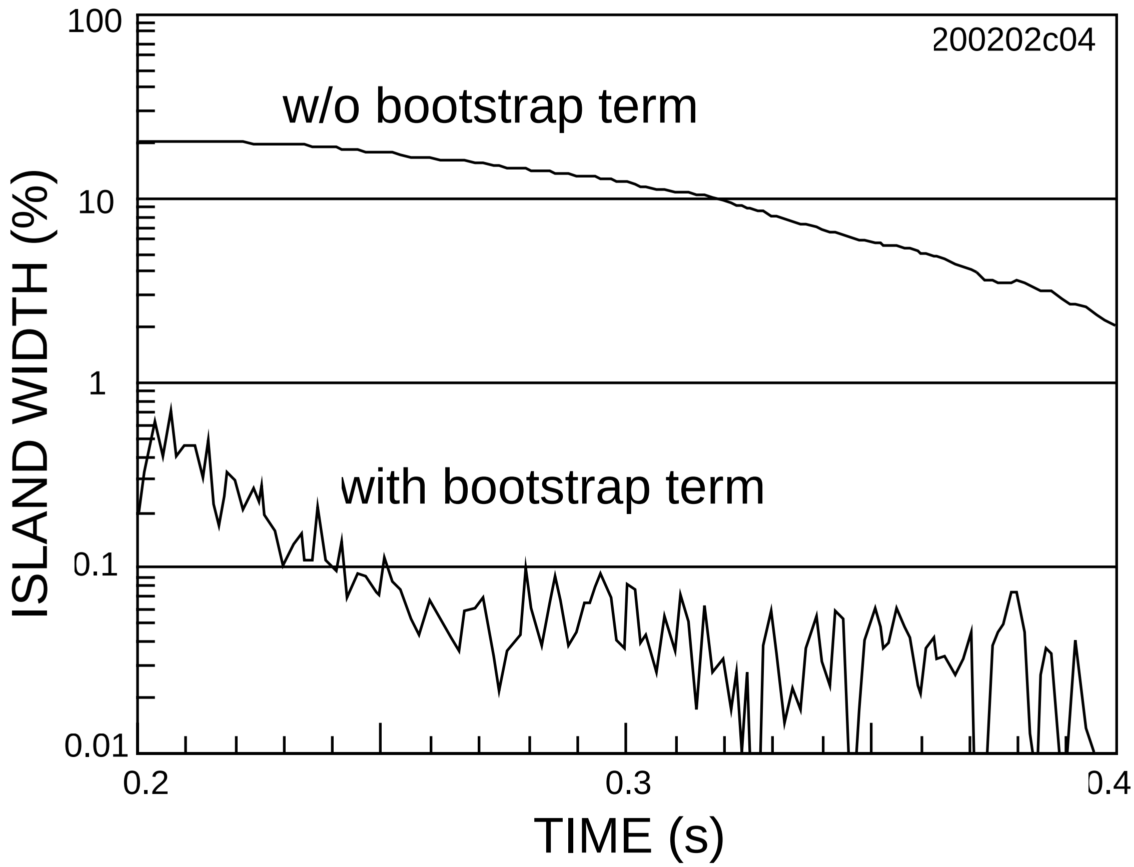
The predicted width of the 2/1 island is inconsequential, of order 2% of the plasma minor radius, after $\approx$ 0.3s, i.e., in the "steady state" phase of the simulation. Inclusion of the neoclassical term, which is stabilizing, reduces the island size even further, as discussed in Section 3.7. As the stability calculation fails during the time period from 0.08 to 0.19 s, it is not possible to do a full, time-dependent island evolution calculation over the whole start-up. In Fig. 5.1 are thus shown two curves for the time period beginning at 0.2 s and extending until the end of the shot. The first, solid, curve shows the evolution of the width of an island starting at 0.2 s with the predicted saturated width at that time. The actual island evolution would depend on it's width at 0.2s. The second curve shows the saturated island width including the (stabilizing) bootstrap current term. In this case the predicted island widths are less than 0.1%; inclusion of either Glasser-Greene-Johnson or the polarization-drift terms would completely stabilize the islands.

The next lowest order modes are the 7/3, and 7/4 modes. The 7/3 mode was calculated to be robustly stable, apart from a short period during which the core iota locally exceeded 3/7. The stability calculation for the 7/4 mode was problematic. For this mode, located near the plasma boundary, the relatively large local edge current density introduces strong curvature in the radial eigenmode structure. The appearance of the eigenfunction shape suggests that this formalism is not applicable. The failure could either result from the mapping of non-axisymmetric, finite beta and shaped equilibria to a circular cross-section, quasi-cylindrical, zero beta model or might indicate that the plasma was nearing the ideal stability marginal point (known to result in similar problems even in the simpler tokamak axisymmetric geometry).

The TRANSP time-dependent simulation analyzed above used profiles from transport simulations. There are some differences between these profiles and those for the reference configuration. A Δ' analysis was also done of the single time point li383 equilibrium. In this analysis the saturated island width reaches about 3.6 % of the minor radius without inclusion of neoclassical effects. With neoclassical effects the saturated island width is inconsequential ($\approx$ 0.2 %).

5.4 Summary

The simulation of the NCSX start-up described in Chapter 9 has been analyzed for stability to tearing modes driven by ohmic, beam and bootstrap driven currents. The analysis has been done with a simple quasi-cylindrical Δ' code of the type used successfully in the analysis of tokamak plasmas. The plasmas are found to be stable to the low order tearing instabilities (7/3 mode) and somewhat unstable to the 2/1 mode in the growth phase. The inclusion of neoclassical effects is generally believed to be stabilizing for plasmas with negative shear ($d\iota/dr > 0$, or $dq/dr < 0$), and the calculations suggest that the neoclassical terms result in a robustly stable 2/1 mode. The simple quasi-cylindrical stability calculations for the 7/4 mode located between $r/a \approx 0.85$ and the plasma edge did not give reasonable results, possibly indicating problems with the Δ' formulation or with the high local current density near the plasma edge. Likewise, the calculation for the 2/1 mode stability



200202c04
w/o bootstrap term
with bootstrap term
ISLAND WIDTH (%)
100
10
1
0.1
0.01
0.2
0.3
0.4
TIME (s)

Figure 5-1

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