Issues in Stellarator Design

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I. Requirement for Success

Present budget for magnetic fusion can only be justified by exciting new experiments.

Program vibrancy measured by ability to couple resources to highest priority issues.

Success of NCSX design will impact the future of entire magnetic fusion program.

II. Coil Design

a. Desirable coils require an iteration between plasma shape and coil complexity.

Consider straight z-symmetric system.

elliptical plasma requires ∞ number of coils.

b. Non-resonant perturbations $(n-\iota m)^2 >> 1$ represent design freedom,

$$\mathbf{J}\,\mathbf{B}\ \ \Upsilon = \sum B_{mn} \sin(n\phi - m\theta).$$

c. Coil requirements for equilibrium control

κ_c current potential of coils $\mathbf{j_c} = \delta(r-r_o)\ \Upsilon r X \Upsilon \kappa_c(\theta,\phi)$.

Hope $\kappa_c = \kappa_v(\theta,\phi) + \beta\ \kappa_\beta(\theta,\phi) + I\ \kappa_I(\theta,\phi)$.

d. Plasma flexibility implies:

i. coils sufficiently far back

ii. $\kappa_c = \kappa_o(\theta,\phi) + s_1\kappa_1(\theta,\phi) + s_2\kappa_2(\theta,\phi) + \ldots$

s_1, s_2, ... shape parameters

How many are needed? In toroidal symmetry there are 4: (m=0) minor radius; (m=1) major radius; (m=2) ellipticity; (m=3) triangularity.

III. Finding $\kappa_c(\theta,\phi)$ in Presence of Plasma

Need to determine:

1. κ_p surface equivalent of plasma current.
2. I toroidal plasma current.
3. G poloidal coil current.
4. κ_c current potential of coil current.

Unique κ_c determined if one specifies:

1. plasma shape $\mathbf{X}(r_o,\theta,\phi)$ with $(\mathbf{B}\ \ \nabla)_{ro} = 0$.
2. toroidal flux in plasma $\psi_t = \int \mathbf{B}\ \ d\mathbf{a}_\phi$.
3. $\mathbf{J}\ \mathbf{B}\ \ \nabla\theta = \mathbf{B}\left[\frac{\partial \mathbf{X}}{\partial \phi} \times \frac{\partial \mathbf{X}}{\partial r}\right]$ at $r = r_o$.

These three pieces of data allow the construction of $\mathbf{B} = (\mathbf{B}\ \ \nabla\theta)\ \ \partial\mathbf{X}/\partial\theta + (\mathbf{B}\ \ \nabla\theta)\ \ \partial\mathbf{X}/\partial\theta$.

IV. Dipole Method for Finding $\kappa(\theta,\phi)$

Dipoles normal to surface give current potential (Bialek & Boozer 12/97 and Pomphrey and Bialek).

Dipole moment of ith dipole related to surface potential an area of grid cells, a_o, by

$$\mathbf{m_i} = - a_o\, \kappa(\theta_i,\phi_i)\, \hat{\mathbf{n}} \qquad \text{with } \hat{\mathbf{n}} = \nabla r/|\nabla r|.$$

Coils are a terrace approximation to $\kappa(\theta,\phi)$.

Dipoles relatively simply represent:

1. saddle coils.
2. holes in current distribution for ports.
3. volumetric current.

   $\mathbf{j} = - \nabla \times \mathbf{M}$ with $\mathbf{M}$ $\mathbf{m}/v_o$ and v_o volume of cells (dipoles are corners of cells).

V. Physics Design

Issues

1. quasisymmetry (or J-optimized)

2. poloidal flux

For quasi-axisymmetry $P_\phi = Rmv_{||} - e\psi_p/2\pi$ constant.

For highly passing beam particles of energy E in eV and mass μm_p, constancy of P_ϕ implies

$$\delta\psi_p/2\pi = 10^{-4} (\mu E)^{1/2} \delta R \qquad \text{(MKS units)}$$

δR is change in major radius over magnetic surface.

δR = 1m and μE = 40 keV, then $\delta\psi_p/2\pi = 0.02$ V s
$\psi_p = \langle\iota\rangle\psi_t$ but $\psi_t/2\pi = (1T)(.16m^2)/2 = 0.08$ V s

Need $\langle\iota\rangle > 1/4$.

3. kink stability and *vertical stability*

4. ballooning stability

5. bootstrap consistency (axisymmetric)

6. simplicity of coils

VI. Iteration between Physics and Coil Design

Critical Elements

1. Resonant Fourier terms $(n-\iota m)^2 < 1$.

   Lack of control of these terms prevents a direct design from coils.

   Probably need coils to control resonant terms produced by: (a) configuration changes and (b) construction errors.

2. Ability to hold plasma shape with changing β and plasma current I.

   $\kappa_c(\theta,\phi) = \kappa_v + \beta\kappa_\beta + I\kappa_I$; need to know form of these κ's.

3. Ability to change most important shape parameters.

   $\kappa_c(\theta,\phi) = \kappa_o + \sum s_i\, \kappa_i$; need to know number and form of the important κ_i's.

   The κ_I and κ_β may coincide with each other or with some shape parameters.