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Initial Results from Control-Matrix Approach to NCSX Design

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- Formulation :

$$\underset{\approx}{G}_{M\times N} \cdot \underset{\sim}{X}_N = \underset{\sim}{P}_M , \quad N \equiv N_x \sim 78, \quad M \equiv M_p \sim 5$$

$$\underset{\sim}{X} = (\delta R_{mn_1}, \delta Z_{mn_1}, \delta R_{mn_2}, \dots \delta Z_{mn_{\frac{N}{2}}}) = \text{boundary deformations}$$

$$\underset{\sim}{P} = (\underbrace{\chi_1^2, \chi_2^2, W_1, W_2}_{\text{from JMC, BFLD}}, \underbrace{\lambda_{-Kink}}_{\text{from TERPSICHORE}})$$

- Use SVD theorem to invert $\underset{\approx}{G}$.
$$\underset{\approx}{G}_{M\times N} = \underset{\approx}{U}_{M\times N} \cdot \underset{\approx}{W}_{N\times N} \cdot \underset{\approx}{V}^T_{N\times N}$$

- For basis set for $\underset{\sim}{P}$'s $\hat{\Pi}^{(i)} = (0, .., \underset{\substack{\uparrow \\ i^{th}\text{-position}}}{1}, 0, ..), (i = 1, \dots M)$

compute $\underset{\sim}{\xi}^{(i)}$'s :

$$\underset{\sim}{\xi}^{(i)} \equiv \underset{\approx}{G}^{-1} \cdot \underset{\sim}{\Pi}^{(i)} = \text{deformation needed to change } P_i \text{ by } 1.$$

$$\Rightarrow \underset{\sim}{\xi}^{(i)}(\theta, \zeta) \equiv \hat{R}\, \xi_R^{(i)}(\theta,\zeta) + \hat{z}\, \xi_z^{(i)}(\theta,\zeta)$$

$$\xi_R^{(i)}(\theta,\zeta) \equiv \sum_{j=1}^{N/2} \underbrace{\xi_{2j-1}^{(i)}}_{=\delta R_{mn_j}} \cos\eta_{mn_j}, \quad \xi_z^{(i)}(\theta,\zeta) \equiv \sum_{j=1}^{N/2} \underbrace{\xi_{2j}^{(i)}}_{=\delta Z_{mn_j}} \sin\eta_{mn_j},$$

with $\eta_{mn} \equiv m\theta - n\zeta$ (VMEC-convention).

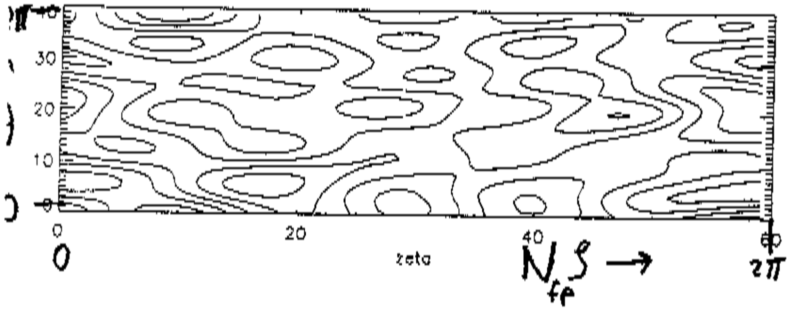
- Begin by looking at 'range' of $\underset{\approx}{G}$. Study of vectors $\tilde{\underset{\sim}{\xi}}$ in nullspace also important.

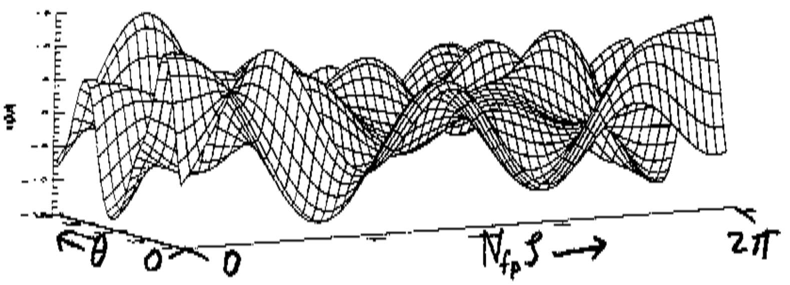
• Preliminary results:

(A) 'More-constrained' case ($M_p = 5$):

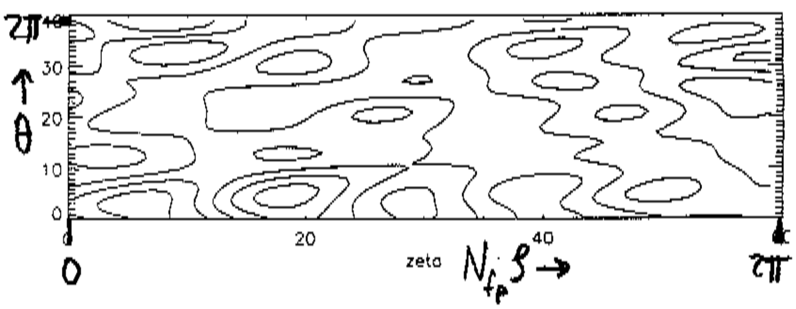
• Plot $\xi^{(i)}(\theta, \zeta) \equiv \xi_R^{(i)} + \xi_Z^{(i)}$:

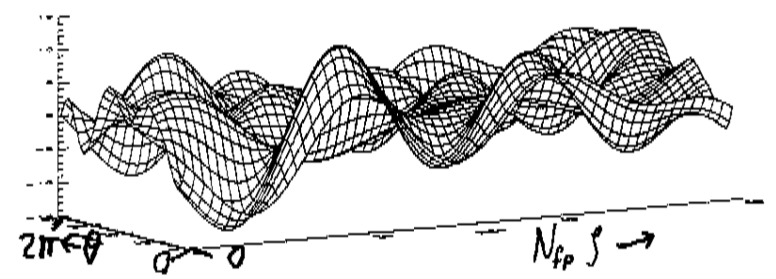
$i=1$ (χ_1^2)



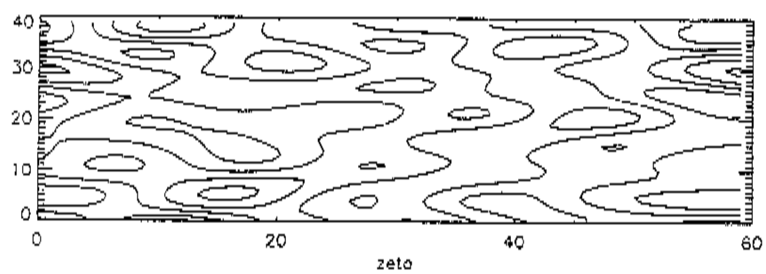


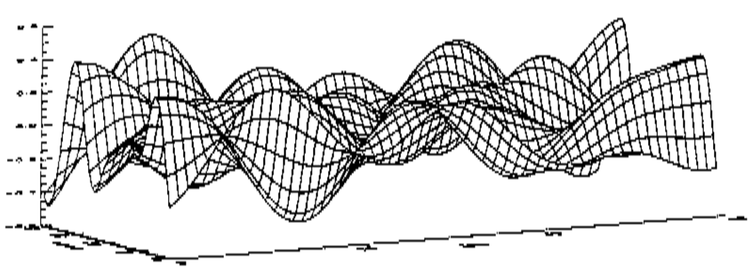
$i=2$ (χ_2^2)



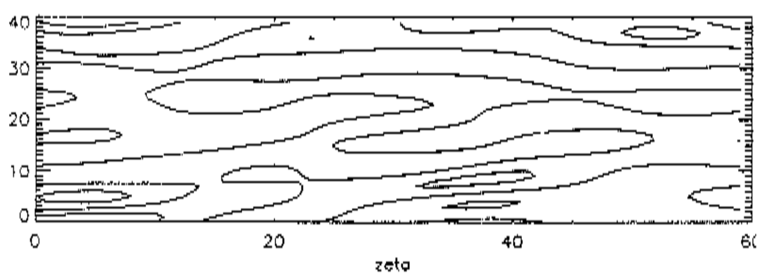


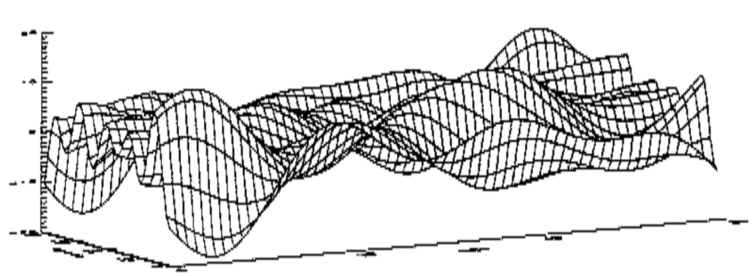
$i=4$ (W_2)





$i=5$ ($\lambda_{\text{-kink}}$)

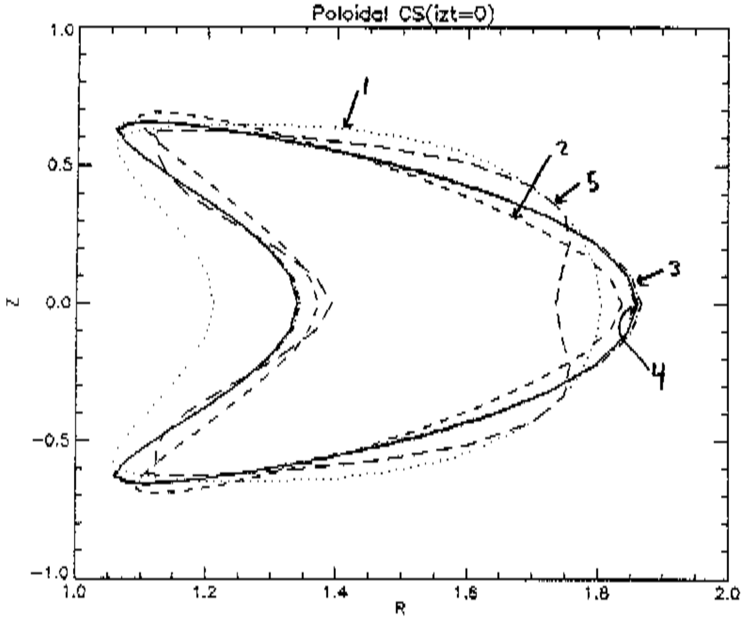




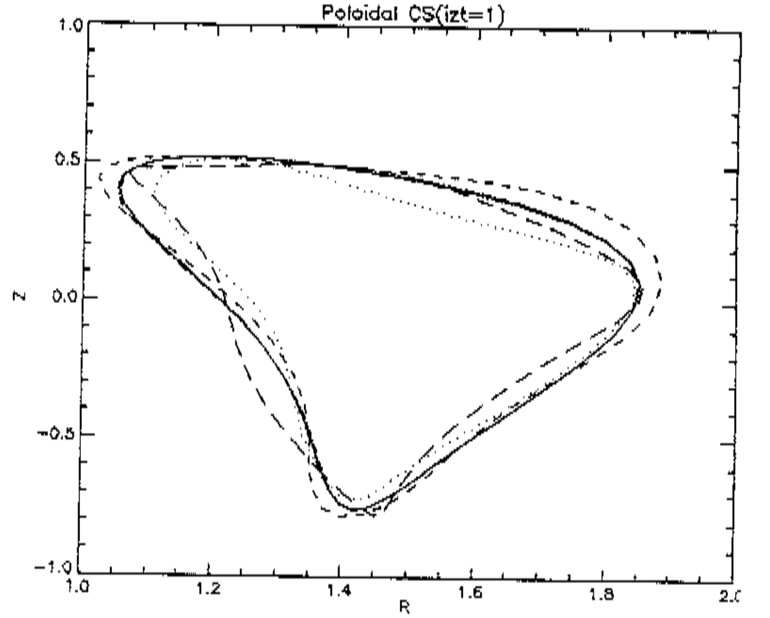
(A), cont:

- Surface deformations:

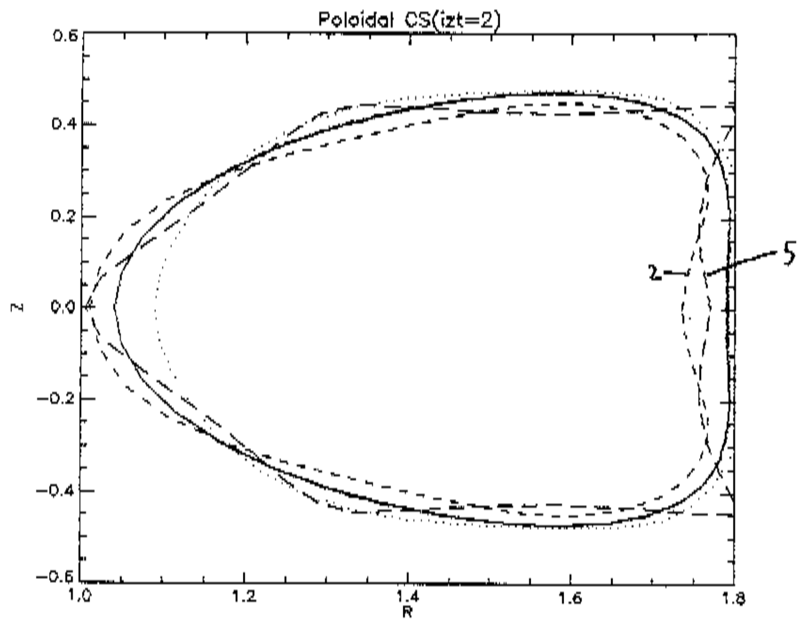
$N_{fp}\zeta = 0$:



$N_{fp}\zeta = \pi/2$:



$N_{fp}\zeta = \pi$:



i

unpert = 0 (c10)

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2

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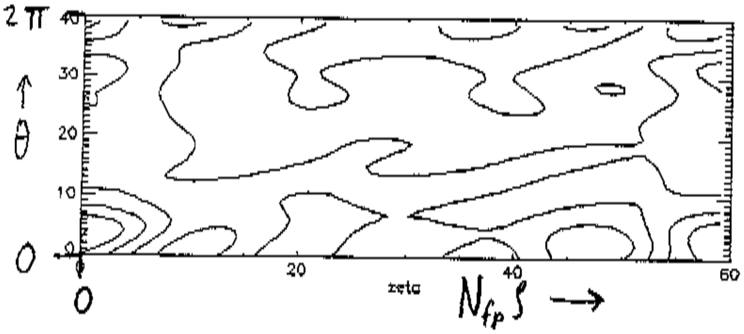
4

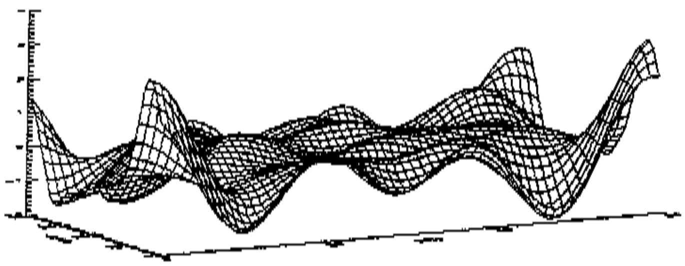
5

(B) 'Less-constrained' case ($M_p = 2$):

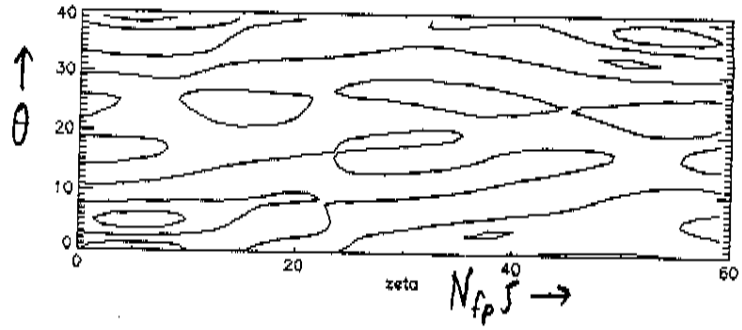
• Plot $\xi^{(i)}(\theta,\zeta)$:

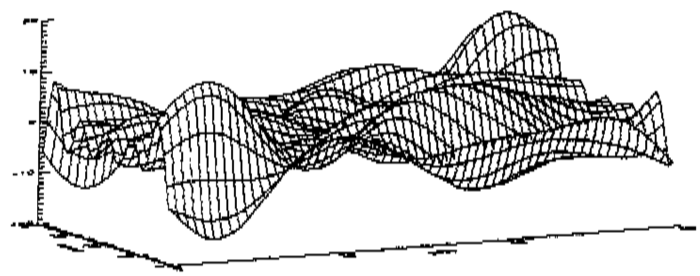
$i=1$ (χ_1^2)





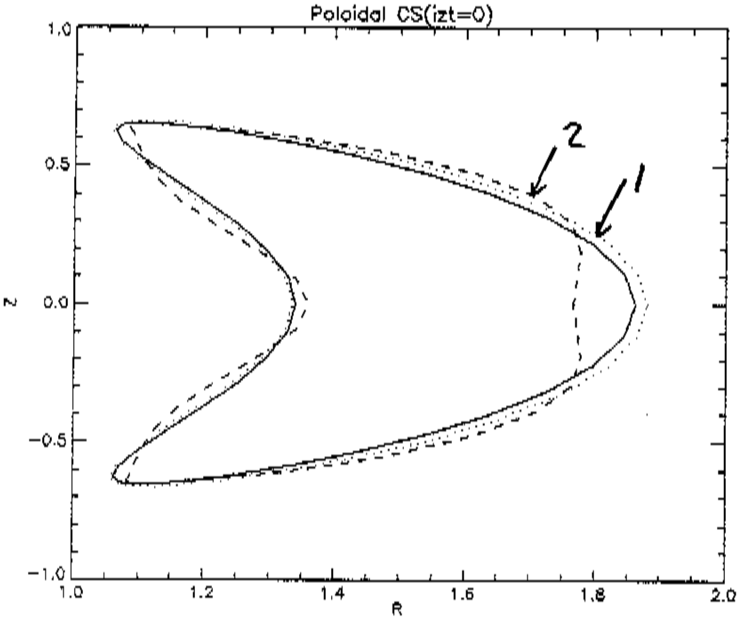
$i=2$ (λ_{kink})



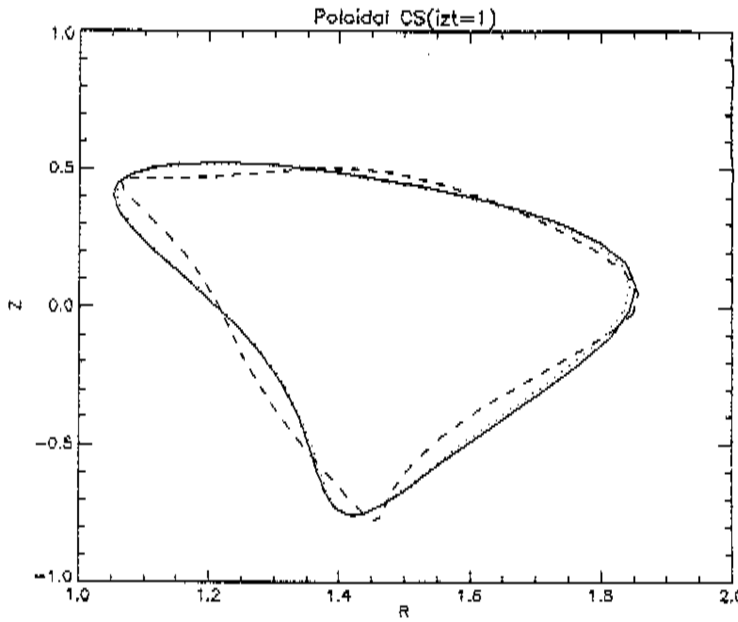


(B), cont:

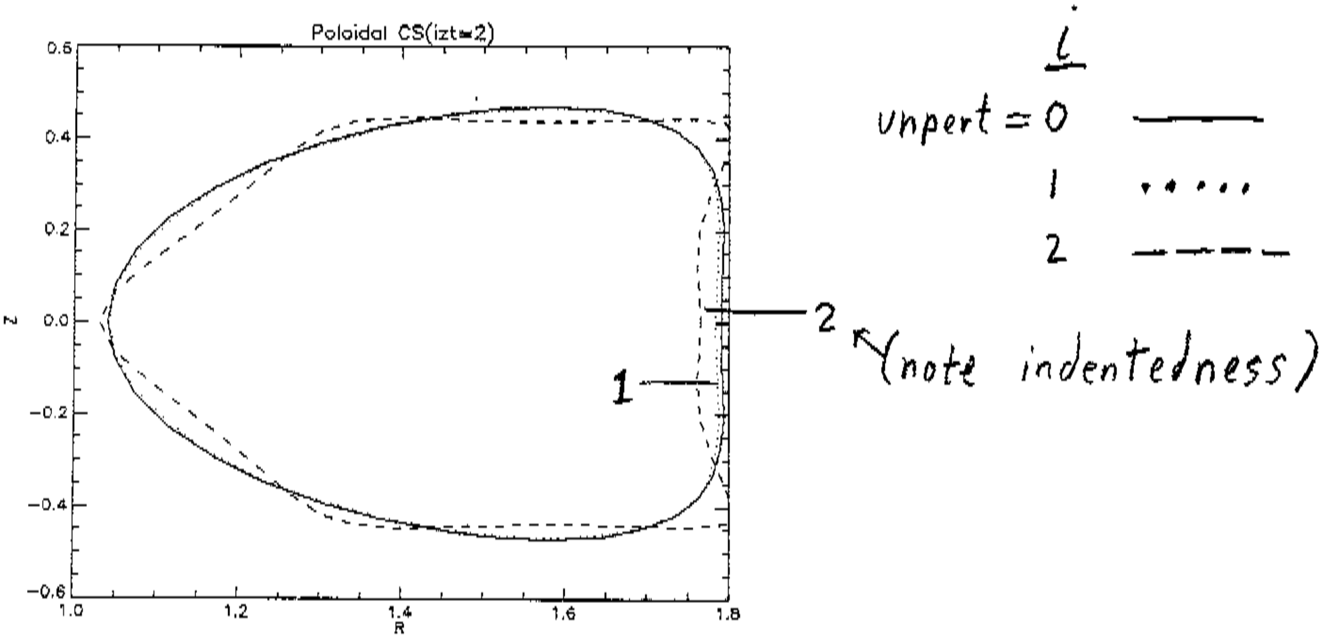
$N_{fp}\zeta = 0$:



$N_{fp}\zeta = \pi/2$:



$N_{fp}\zeta = \pi$:



ι		
unpert = 0	———	(c10)
1	······	
2	– – – –	

∘Summary:

- We have set up most of the machinery needed to do the indicated control–matrix analysis of NCSX, and are now getting our 1st results.
- The 4 different transport figures of merit produce boundary displacements $\boldsymbol{\xi}^i(\theta, \zeta)$ similar in appearance. However, the G-matrix eigenvalues w_i show these are NOT nearly collinear: They are different by independent vectors in the nullspace.
- The $\boldsymbol{\xi}^i$ for kink stability differs in appearance from those for transport, and 'tries' to provide the outboard indentation previously seen to stabilize the kink.
- The same approach could also be applied to seeing how a given set of coils (with perturbations $\delta\mathbf{I}$) could produce a range of physics behavior $\boldsymbol{\pi}$ for experimental flexibility:
  $\mathbf{G} \cdot \boldsymbol{\xi} = \boldsymbol{\pi}$ (as above),
  $\mathbf{H} \cdot \delta\mathbf{I} = \boldsymbol{\xi}$ (from free-boundary runs),

  $\Rightarrow \mathbf{G_2} \cdot \delta\mathbf{I} = \boldsymbol{\pi}$, with $\mathbf{G_2} \equiv \mathbf{G} \cdot \mathbf{H}$.