Control Matrix Approach for QAS Transport and Stability

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8/11/99

- Question posed by Hutch Neilson: In light of "success" of showing improvements in C82 kink stability by "tweaking" selected coil currents, can we do same for C10?

- The motivation for trying this is that the original C10 coil set presented by Art appears to be much more benign than the C82 or C93 coil sets. If we can modify just one or two coil currents to improve the kink stability to the level of C82 (presumably by inducing outboard indentation) we may end up with a much more attractive system.

STEPS LEADING UP TO FREE-BOUNDARY RECONSTRUCTIONS

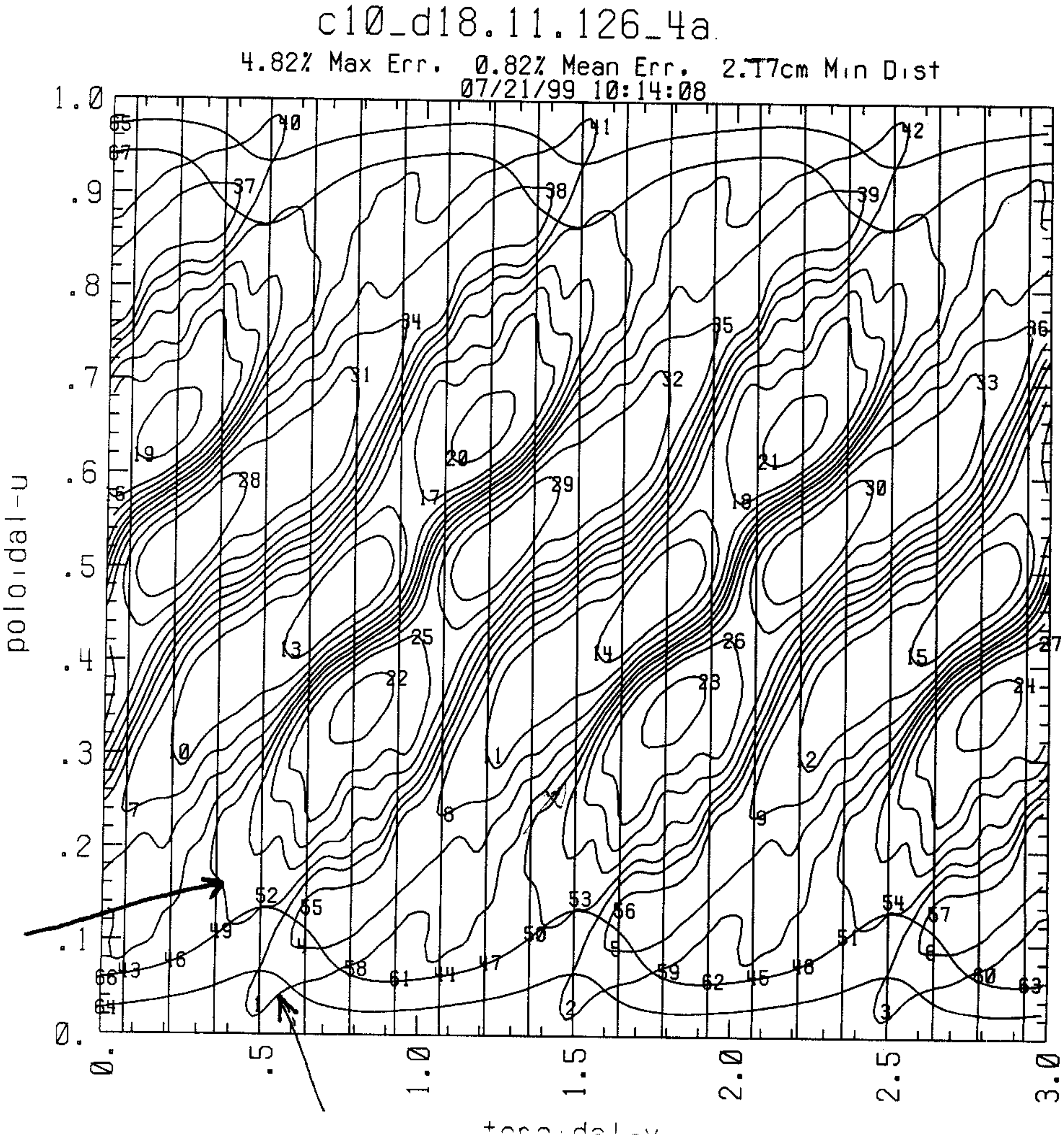
- As first step, a "true" 18 cm coil surface was generated for C10 (recall that original C10 calculations were made with "faulty" coil surface).
- NESVD was run (Prashant Valanju) to obtain a current sheet solution (c10-d18.11.126).
- The Genetic Algorithm code was run to obtain a coil set with 7 coils per 1/2 period (c10-d18.11.126-4a). Max/Mean B-errors were 4.82%/0.82%. Min coil-to-coil separation = 2.17cm

  The $J_{max} = 7.9kA/cm^2$!!! The coil contours in u-v space are shown in Figure 1.

7 coils / ½ period

Figure 1

7.9 kA/cm²



THE RECONSTRUCTIONS FOR C10 (7 COILS PER 1/2 PERIOD)

- With no tweaking of coil currents, the reconstruction of c10 is pretty good. Max/Mean separations between the target and reconstructed plasma are 3.0/0.7 cm (see Figure 2).

- The reconstruction can be improved by decreasing the vertical field to 98% of the original value. This obtains 2.2/0.4cm (see Figure 3). Note that the v=0.5 cross section which seems to be so important for the kink stability differs from the target c10 in a way that looks beneficial to stability, namely, on the outboard edge the plasma boundary is straight vertical (like c10), but more triangular (ie the straight section is taller). Long-Poe has analysed the transport and stability of the current sheet and finite coil C10 solutions and finds the following:

Case	λ_{kink}	$\chi^2(s=0.3)$	$\chi^2(s=0.5)$	$\chi^2(s=0.8)$
c10-sheet	0.96	1.01	1.03	1.01
c10-4a1	0.91	2.15	1.43	1.15

(Values in Table are normalized to those of the target fixed boundary C10).

- The kink stability is, indeed, slightly better. However we are looking for about a factor of 10 decrease in λ to get it down to the C82 level (which someone, somewhere has declared acceptable whereas the C10 value is not (can we revisit the scientific basis for this?))

- Note the degradation in the χ^2 values for transport. Without more detailed analysis, we cannot say that this degradation is significant, For the moment, ignore this issue.

NO TWEAKING

Fig. 2

dmax= 3.0 cm

d mean = 0.7cm

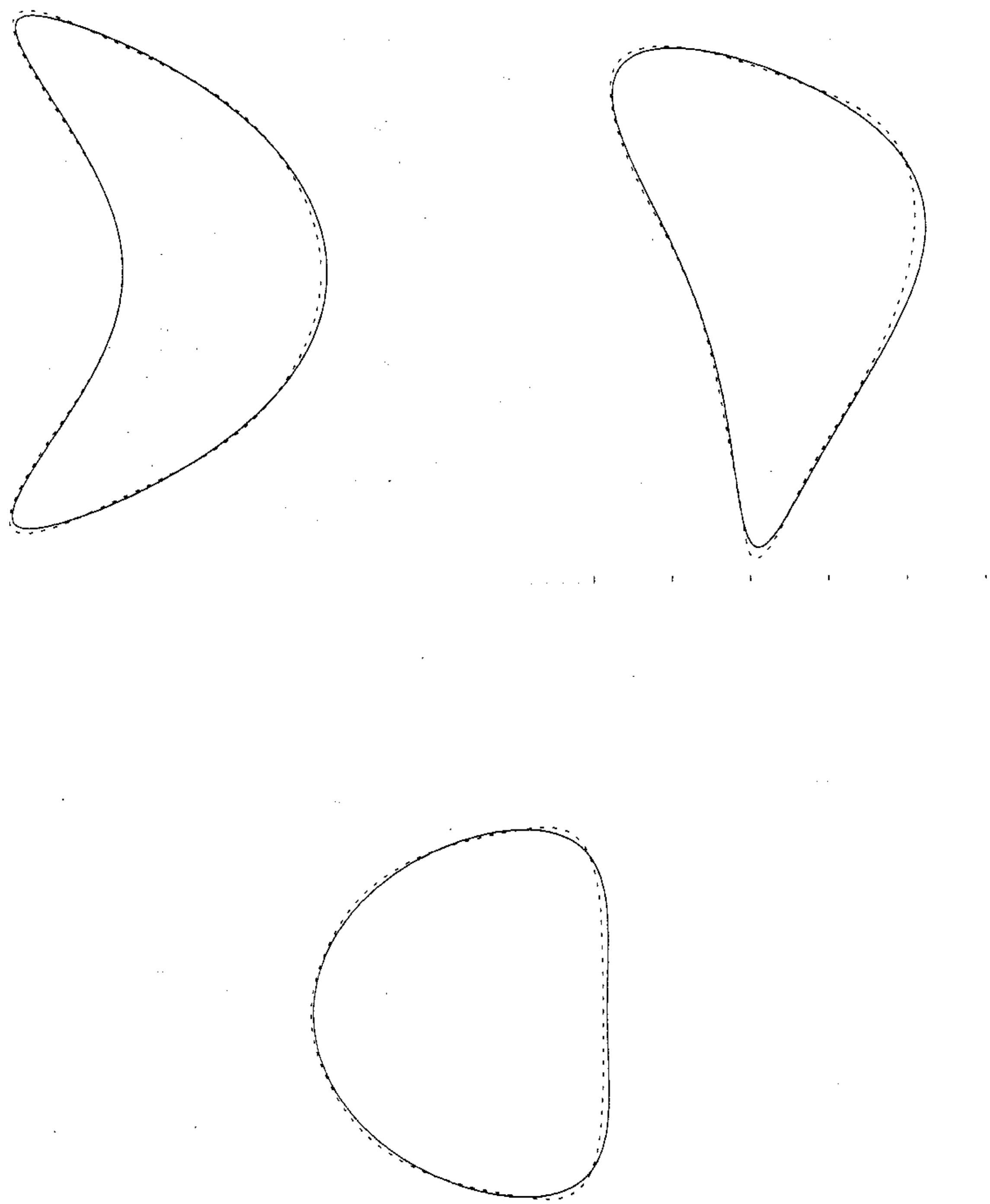
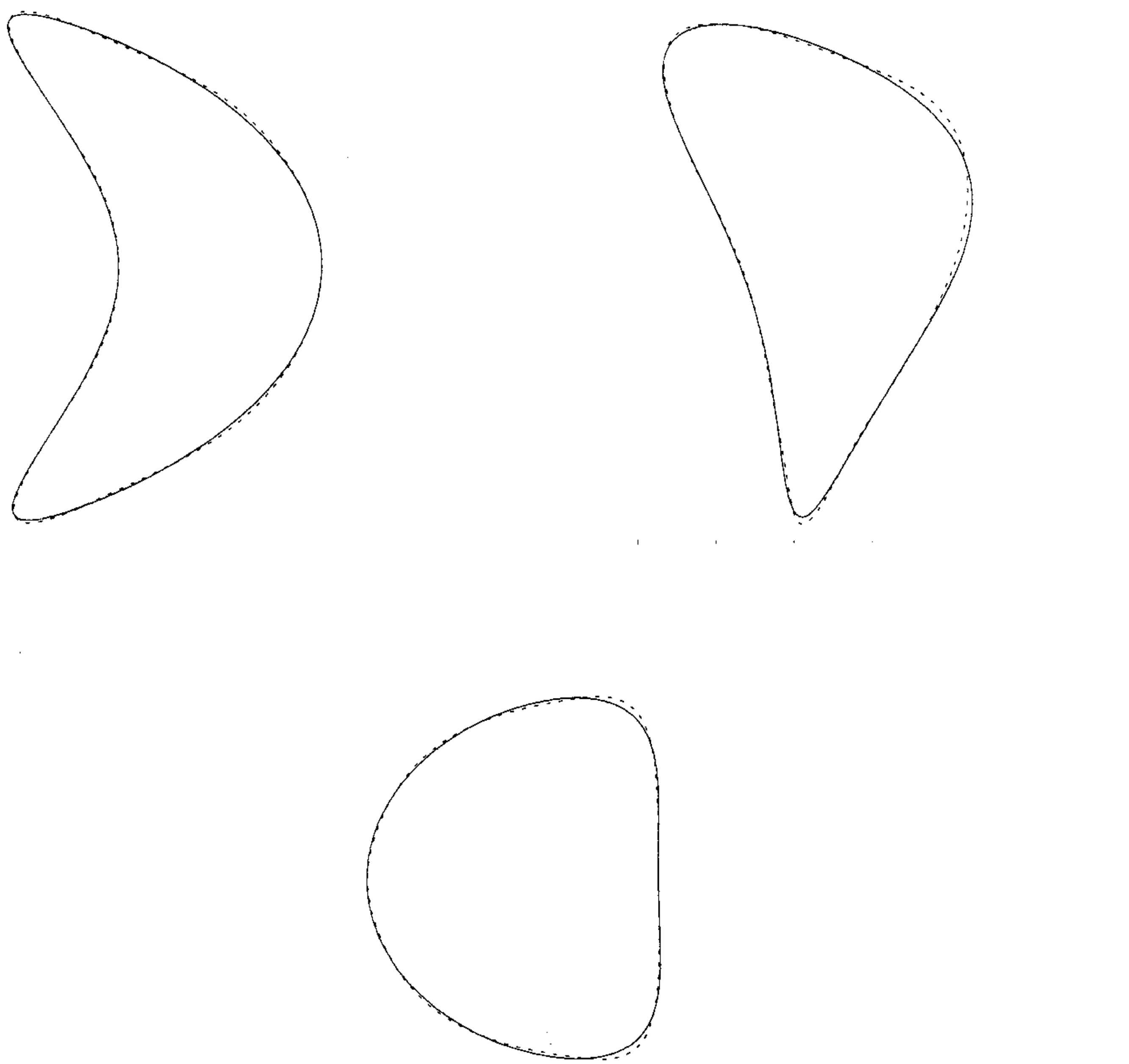


Fig 3.

V.F. TWEAKING (* 0.98)

$d_{max} = 2.2$ cm

$d_{mean} = 0.4$ cm



ATTEMPTS TO IMPROVE C10 KINK STABILITY BY TWEAKING COIL CURRENTS

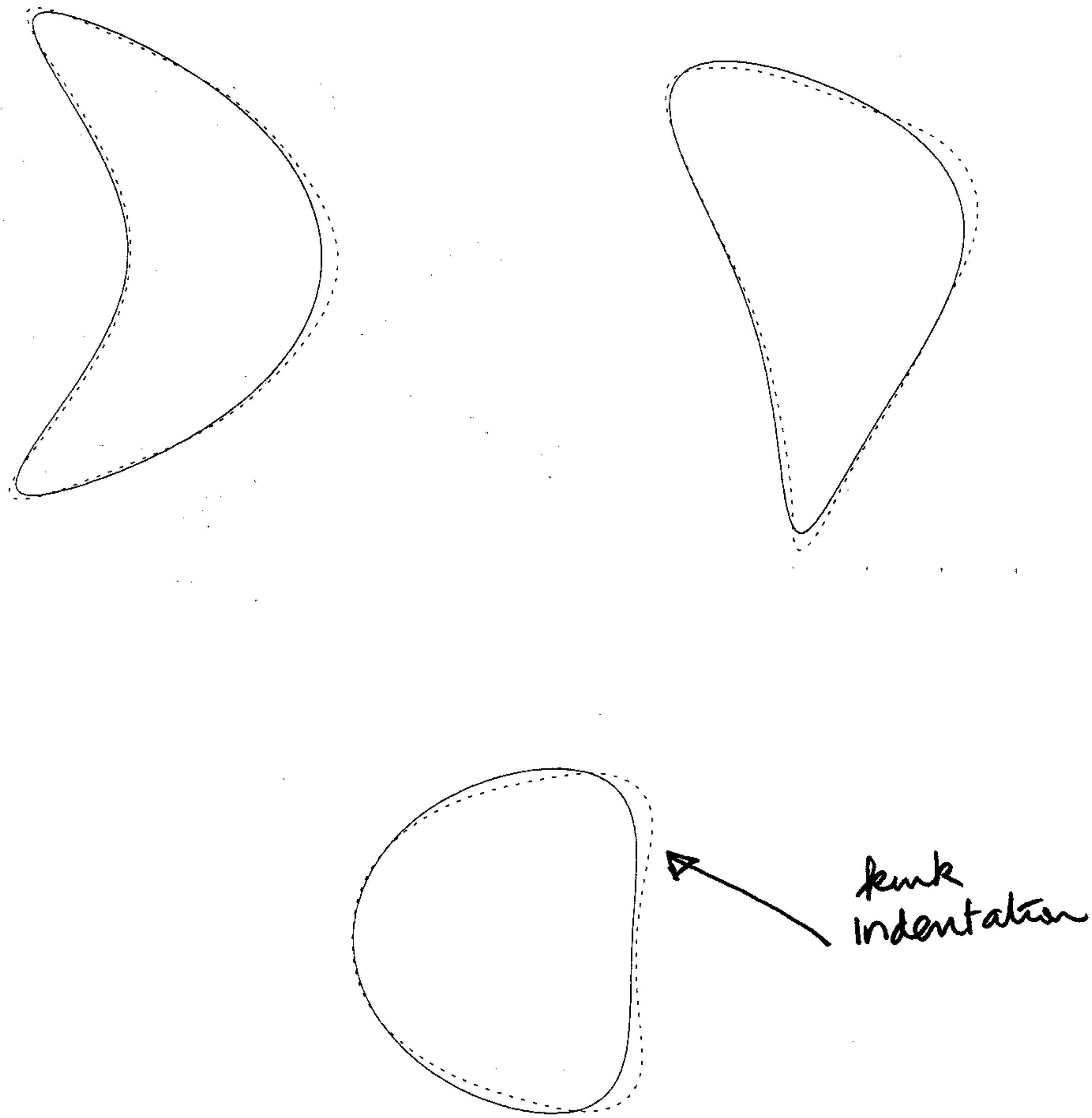
- As reported at previous working group meetings, coils which intersect the outboard region of the $v = 0.5$ plane have been shown to be effective for controlling outboard indentation of C82 plasmas. Similar coils can be identified in C10 (see arrows designating these coils in Figure 1). The effect of increasing the current by 30% in these "primary indentation coils" is shown in Figure 4.

- The desired effect on indentation has been achieved, and the λ_{kink} has been reduced to ?. However, the shape has changed non-locally (ie far from the $v = 0.5$ cross section), with a consequent degradation of the transport χ^2.

- The "primary indentation coils" are long coils so the fact that they produce extensive shape changes is not surprising. I have done some preliminary calculations where the long saddle is split into a pair of saddles, one member of which is a short saddle that passes through $v = 0.5$. (See Figure

Fig. 4 TWEAK SADDLE "INDENTATION COIL"

*1.30



5). Now we can vary the current in the short (local) saddle and keep the current in the remainder of the long saddle fixed. Preliminary results (surprisingly?) lack promise, but I may have screwed up! More work is needed here (later).

Fig. 5
split saddle

c10_d18.11.126_4a_split
4.82% Max Err. 0.82% Mean Err. 2.17cm Min Dist
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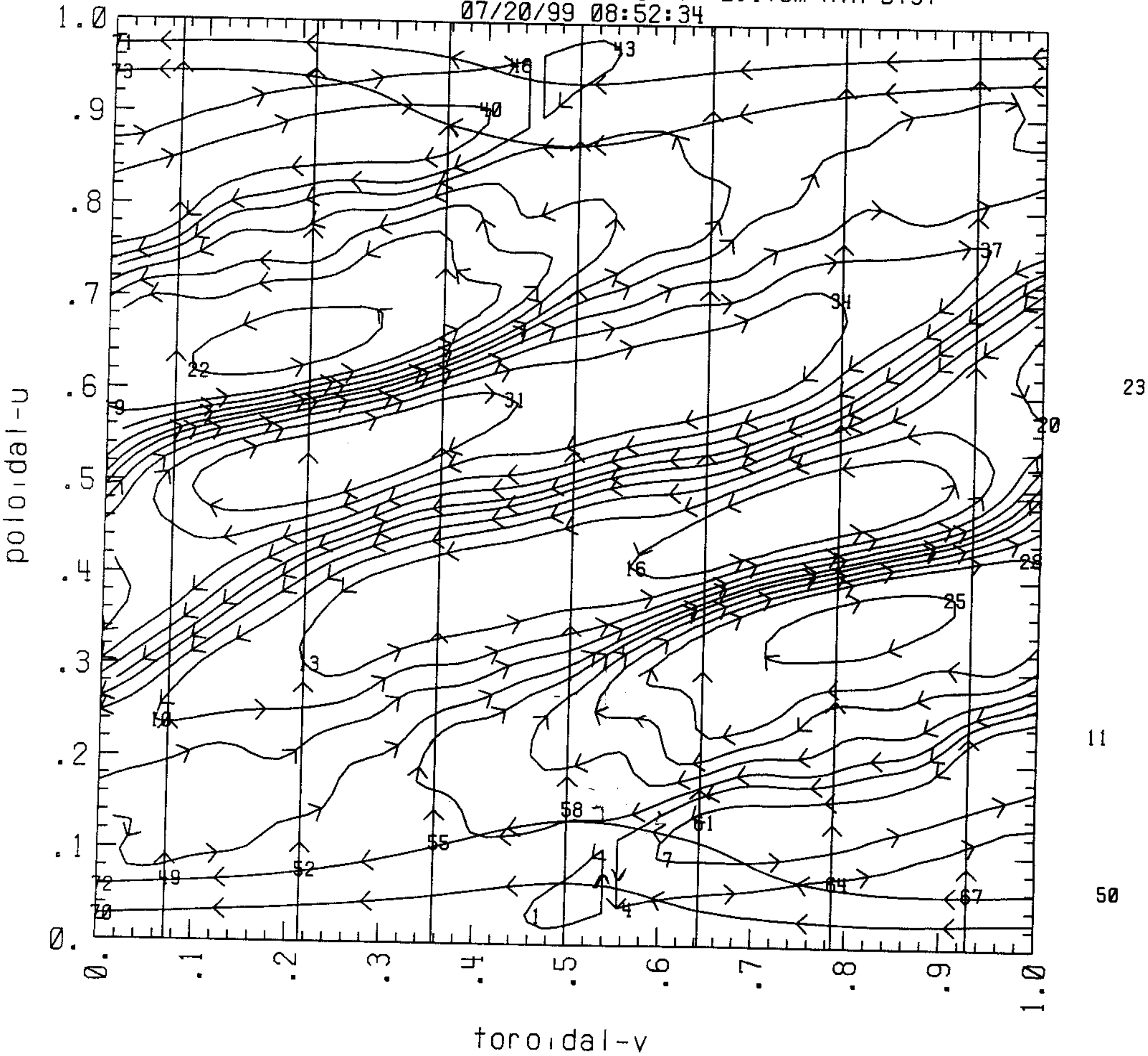

FINALLY, THE CONTROL MATRIX APPROACH

- In every case where we have tried to improve kink stability by tweaking a single coil current, we have met with success.
- However this has always been at the expense of degrading transport χ^2 values.
- What we really need to do is understand how to (or whether we can) tweak currents in such a way that stability is enhanced (or degraded) without changing the transport.
- We also need to understand how to (or whether we can) tweak such that the transport is enhanced (or degraded) without changing the stability.
- This is where a control matrix approach is useful:

Consider a plasma configuration, $\mathbf{Z}$, which corresponds to a set of physics parameters, $\mathbf{P}$. For example, in the contest of the present VMEC optimzation code, $\mathbf{Z}$ is the set of plasma boundary fourier coefficients R_{mn}, Z_{mn}. The physics parameters can be whatever you like that depends on the $\mathbf{Z}$, such as iota, $\chi^2_{transport}$, λ_{kink}, $\lambda_{ballooning}$, etc. The relationship between $\mathbf{P}$ and $\mathbf{Z}$ can be represented as a matrix equation:

$$\mathbf{GZ} = \mathbf{P} \tag{1}$$

Now change the configuration in some way so that $\mathbf{Z} \rightarrow \mathbf{Z} + \xi$. Then the physics parameters change to the new values $\mathbf{P} + \pi$. We can write

$$\tilde{\mathbf{G}}\xi = \pi \tag{2}$$

$\tilde{\mathbf{G}}$ is an "influence matrix" that relates the changes in shape to the consequent changes in physics (Note: I do <u>not</u> have to think of Eq 2 as being derived <u>perturbatively</u> from Eq. 1).

Let N_z denote the number of parameters that describe the shape, and N_p be the number of parameters that describe the interesting physics. Typically, $N_z \gg N_p$ (eg., Long-Poe uses $N_z \approx 30$ and we might have $N_p =$ a few.

The influence matrix elements can be determined by a sequence of N_z step response calculations (where individual ξ vector elements are excited and VMEC, +JMC +TERPSICHORE are run to determine the resulting π values.

We can invert Eq. 2 to obtain

$$\xi = \tilde{\mathbf{G}}^{-1}\pi \tag{3}$$

where $\tilde{\mathbf{G}}^{-1}$ is a generalized inverse of $\tilde{\mathbf{G}}$. It is found by Singular Value Decomposition.

The final step is to determine the N_p "empirical orthogonal functions" $\xi^{(k)}$, obtained by inserting π vectors on the RHS of Eq 3 where, for the kth function, the kth element is finite and all other elements are zero. The resulting $\xi^{(k)}$ vectors tell us how to change the shape in such a way as to excite a pure change in only one of the physics parameters.

HOW DOES THIS CONNECT WITH COILS?

Given a set of coils, one requirement for their flexibility should be that through a prescribed variation in the coil currents we can independently control the desired physics parameters. The $\xi^{(\mathbf{k})}$ vectors tell us which shape changes the coils need to access.

See Harry