Resonant Errors, Reconstruction, Nescoil, and all that

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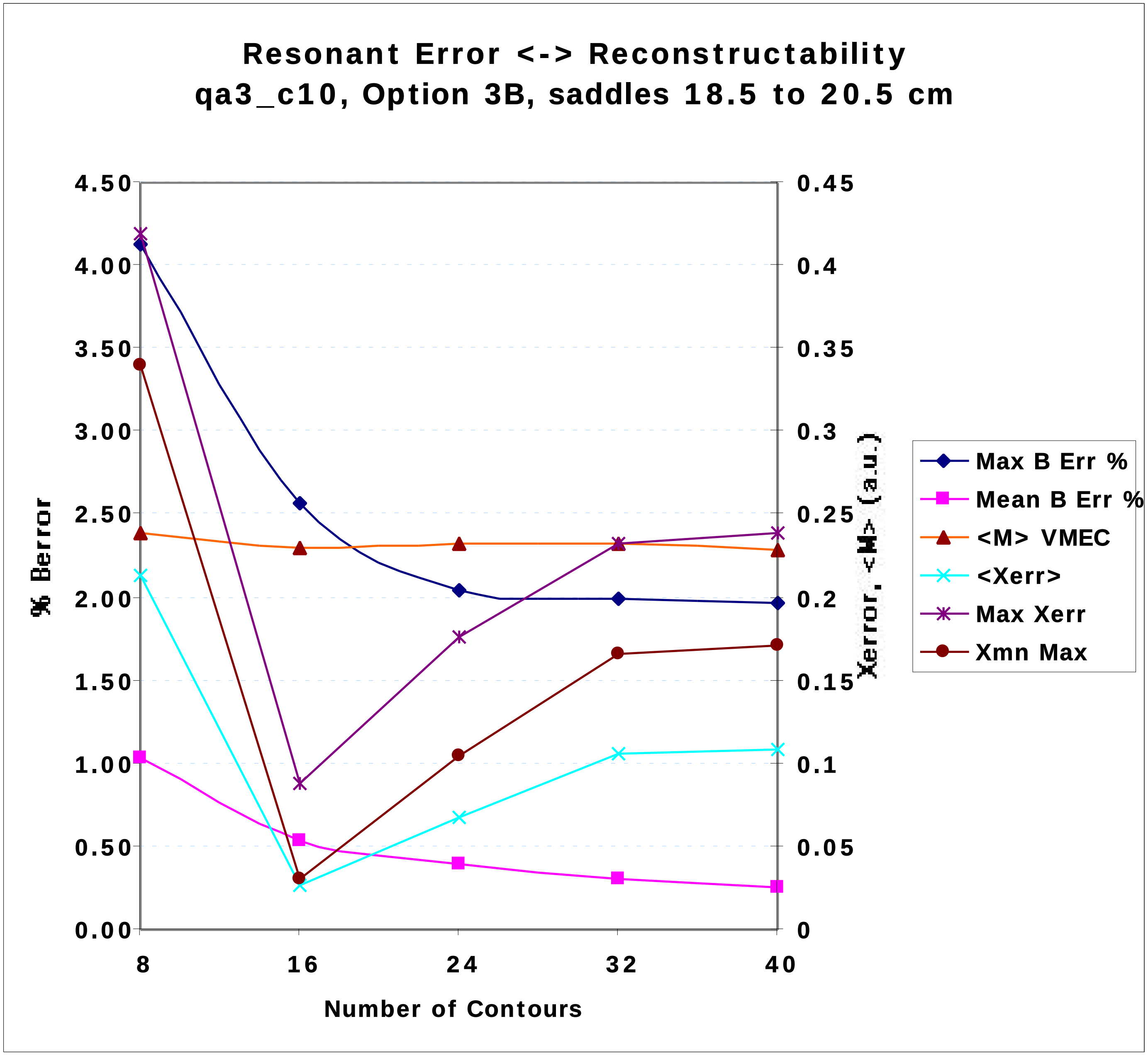
- **Encouraging results:**

  1. Find good correlation between Xerror and reconstructability for the qa3_c10 case.
  2. Method to do Xerror calculation by going to straight-line coordinates and using fourier transforms seems to work well – Xerror satisfies original equation.
  3. This calculation can be performed within Nescoil so that we can target Xerror for minimization <u>within</u> Nescoil's Green's function method. This has been implemented but not used yet, will be ready for use in a few days.


Resonant Error <-> Reconstructability
qa3_c10, Option 3B, saddles 18.5 to 20.5 cm
% Berror
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Xerror, <M> (a.u.)
8
16
24
32
40
Number of Contours
Max B Err %
Mean B Err %
<M> VMEC
<Xerr>
Max Xerr
Xmn Max

- **Discussion of these results:**

  1. Starting from current sheet, Art used 8, 16, 24, 32, and 40 current filament contours (= 6, 12, 18, 24, 29 coils/period) for the 5 cases.
  2. The quality of reconstruction is indicated by the <M> VMEC (smaller is better).
  3. Nescoil accuracy target was <Berr> (~ Berr_max) (smaller is better).
  4. Going from 8 to 40 contours (thus approaching current sheet case), the average and max Berr monotonically went down, but <M>, i.e. the reconstructability, was not monotonic. It reconstructed best at 16. **Why?**
  5. Displacements <Xerr>, <Xerr_max_u_v>, and Max of X_m_n all have minima at 16. Even with smaller Berr, 18, and 24 cases give worse reconstruction than 16.

Modes near resonance with 1/(m*iota-n*nfp) > 1, with mf=nf=32, eps=0, iota=0.46888

8 Contours: (bad reconstruction)

<Berr>=1.03%, <Xerr>=0.21

i,	m,	n :	bmn,	1/(m*I-n*nfp),	xmn
1	32	5 :	0.7608E-05	238.7	0.3404
2	7	1 :	-0.1450E-02	3.544	0.2935E-01
3	2	0 :	0.2860E-01	1.066	-0.2076E-01
4	13	2 :	0.3397E-02	10.48	-0.1776E-01
5	6	1 :	0.7477E-02	-5.356	-0.1726E-01
6	5	1 :	0.1340E-02	-1.525	-0.1726E-01
10	12	2 :	-0.1321E-02	-2.678	0.7263E-02
14	19	3 :	-0.1660E-03	-10.96	0.3940E-02
15	18	3 :	0.4107E-04	-1.785	0.3357E-02
17	14	2 :	0.1325E-02	1.772	-0.2861E-02
19	20	3 :	-0.3472E-04	2.648	0.1982E-02
21	31	5 :	-0.7682E-05	-2.152	0.1891E-02
24	1	0 :	0.5512E-02	2.133	0.1677E-02
44	24	4 :	-0.4839E-04	-1.339	-0.7074E-03
45	11	2 :	-0.4275E-03	-1.187	0.6616E-03
49	8	1 :	0.8490E-02	1.331	-0.5600E-03
55	21	3 :	-0.1067E-03	1.181	0.4623E-03
57	27	4 :	0.6987E-05	1.516	-0.4534E-03
58	26	4 :	0.6558E-05	5.238	-0.4031E-03
66	30	5 :	-0.9314E-05	-1.071	-0.3565E-03
171	25	4 :	0.2299E-04	-3.597	-0.9726E-04

16 Contours: (good reconstruction)

<Berr>= 0.54%, <Xerr>=0.027

i,	m,	n :	bmn,	1/(m*I-n*nfp),	xmn
1	32	5 :	-0.8359E-05	238.7	0.3108E-01
2	13	2 :	0.2354E-02	10.48	-0.2823E-01
3	7	1 :	0.1912E-02	3.544	0.1512E-01
4	5	1 :	-0.1382E-02	-1.525	-0.6833E-02
5	19	3 :	-0.3759E-04	-10.96	0.5968E-02
6	14	2 :	0.8102E-03	1.772	-0.4466E-02
7	1	0 :	0.1739E-02	2.133	-0.3360E-02
11	12	2 :	0.1405E-02	-2.678	0.2566E-02
12	18	3 :	0.1928E-03	-1.785	0.2523E-02
14	8	1 :	0.8789E-02	1.331	0.2441E-02
18	26	4 :	-0.1723E-04	5.238	0.1532E-02
19	2	0 :	0.2232E-02	1.066	-0.1529E-02
23	24	4 :	-0.7424E-06	-1.339	-0.1181E-02
26	25	4 :	-0.9203E-05	-3.597	-0.1134E-02
41	6	1 :	0.4025E-02	-5.356	0.6963E-03
45	30	5 :	0.9284E-05	-1.071	0.5832E-03
47	20	3 :	-0.1192E-03	2.648	0.5501E-03
50	11	2 :	0.2819E-02	-1.187	0.5197E-03
57	27	4 :	-0.7117E-05	1.516	-0.3954E-03
63	21	3 :	-0.1898E-04	1.181	-0.3577E-03
142	31	5 :	-0.1836E-05	-2.152	0.8498E-04

Modes near resonance with 1/(m*iota-n*nfp) > 1, with mf=nf=32, eps=0, iota=0.46888

24 Contours: (bad reconstruction)

<Berr>= 0.39%, <Xerr>=0.068

i,	m,	n :	bmn,	1./(m*I-n*nfp),	xmn
1	32	5 :	-0.4889E-05	238.7	0.1045
2	13	2 :	0.1799E-02	10.48	-0.2935E-01
3	6	1 :	0.1971E-02	-5.356	0.2181E-01
4	14	2 :	0.1144E-02	1.772	-0.5643E-02
5	7	1 :	-0.1351E-02	3.544	0.5568E-02
6	1	0 :	0.2395E-02	2.133	-0.4572E-02
7	5	1 :	-0.1494E-02	-1.525	-0.3376E-02
9	12	2 :	-0.1743E-03	-2.678	0.2362E-02
10	20	3 :	0.4688E-04	2.648	0.2241E-02
11	18	3 :	0.1807E-03	-1.785	0.1858E-02
27	24	4 :	0.7932E-05	-1.339	-0.8823E-03
30	26	4 :	0.3042E-06	5.238	0.7818E-03
38	31	5 :	0.2949E-05	-2.152	0.6086E-03
42	25	4 :	0.2383E-04	-3.597	-0.4983E-03
44	27	4 :	-0.2306E-05	1.516	-0.4748E-03
49	30	5 :	0.9901E-05	-1.071	0.3930E-03
53	8	1 :	0.6933E-02	1.331	-0.3668E-03
59	11	2 :	0.1386E-02	-1.187	0.2834E-03
70	21	3 :	0.4074E-04	1.181	0.2325E-03
112	2	0 :	0.1586E-03	1.066	-0.8776E-04
209	19	3 :	-0.9110E-05	-10.96	0.3834E-04

32 Contours: (bad reconstruction)

<Berr>= 0.31%, <Xerr>=0.11

i,	m,	n :	bmn,	1./(m*I-n*nfp),	xmn
1	32	5 :	-0.5548E-05	238.7	0.1668
2	6	1 :	0.2906E-02	-5.356	0.2705E-01
3	13	2 :	0.1165E-02	10.48	-0.1660E-01
4	1	0 :	0.2528E-02	2.133	-0.5388E-02
5	14	2 :	0.6030E-03	1.772	-0.4311E-02
6	5	1 :	-0.8635E-03	-1.525	-0.2760E-02
7	18	3 :	0.6315E-04	-1.785	0.2649E-02
8	20	3 :	0.1163E-04	2.648	0.2594E-02
9	8	1 :	0.5171E-02	1.331	-0.2436E-02
10	7	1 :	-0.2017E-02	3.544	0.2384E-02
11	11	2 :	0.7687E-03	-1.187	-0.1799E-02
15	12	2 :	-0.9627E-03	-2.678	-0.1576E-02
16	19	3 :	-0.4146E-04	-10.96	0.1569E-02
20	27	4 :	-0.5959E-05	1.516	-0.1038E-02
21	26	4 :	-0.8582E-05	5.238	-0.9329E-03
25	2	0 :	-0.1131E-02	1.066	0.8801E-03
27	24	4 :	-0.1306E-04	-1.339	-0.8348E-03
33	21	3 :	0.6555E-05	1.181	0.6698E-03
78	30	5 :	0.5228E-05	-1.071	0.1951E-03
82	31	5 :	0.2950E-05	-2.152	0.1774E-03
89	25	4 :	-0.8321E-08	-3.597	0.1593E-03

• Fourier method of calculating Xerr by going to straight-line coordinates:

In order to calculate the Xerror displacement, we wish to solve

$(B\cdot\quad)X = (B^{\theta}\partial_{\theta} + B^{\varphi}\partial_{\varphi})X(\theta,\varphi) = B\quad(\theta,\varphi)$ with given error field B•n.

Using $\lambda(\theta,\varphi)$ to transform to straight-line coordinates $u = \theta + \lambda(\theta,\varphi)$, and $v = \varphi$, we get

$$\frac{\Phi'}{\sqrt{g}}\left[\left(\iota - \lambda_{\varphi}\right)\partial_{\theta} + \left(1 + \lambda_{\theta}\right)\partial_{\varphi}\right]X(\theta,\varphi) = B\quad(\theta,\varphi),\ \left(\iota\partial_{u} + \partial_{v}\right)x(u,v) = b(u,v) \equiv \left[\frac{B\quad\sqrt{g}}{\Phi'\left(1+\lambda_{\theta}\right)}\right].$$

Fourier transforming in (u,v) space, we get $x(m,n) = -\dfrac{b(m,n)}{m\cdot\iota + n}$, with

$$b(m,n) = \frac{\delta_{mn}}{4\pi^2}\iint du\,dv\,\sin(mu + nv)\,b(u,v) = \frac{\delta_{mn}}{4\pi^2}\iint d\theta\,d\varphi\,\sin\left(m\left[\theta + \lambda\right] + n\varphi\right)\frac{\sqrt{g}}{\Phi'}B\quad(\theta,\varphi).$$

• **Implementation within Nescoil :**

1. Nescoil tries to solve (by least-square minimization or direct SVD inversion)

$B(\theta,\varphi) = H(\theta,\varphi) + \sum \Phi_{mn} G_{mn}(\theta,\varphi) \Rightarrow 0$ to find the current potential Φ_{mn}.

2. Since the Bnormal error is linear in Φ_{mn}, and since its conversion to Xerr is a linear operation, we can transform the Green's functions H and G to write Xerr(m,n) as

$x_{m,n} = h_{m,n} + \sum g_{mn,m'n'} \Phi_{m',n'} \Rightarrow 0$, and solve these by direct SVD inversion, either in the (m,n) fourier space or in the real (θ,φ) space. Here the Xerr Green's functions are:

$$g_{mn,m'n'} = \frac{\delta_{mn}}{4\pi^2(m\iota + n)} \iint d\theta d\varphi \sin\left(m\left[\theta + \lambda(\theta,\varphi)\right] + n\varphi\right) \frac{\sqrt{g}}{\Phi'} G_{m'n'}(\theta,\varphi), \text{ and}$$

$$h_{mn} = \frac{\delta_{mn}}{4\pi^2(m\iota + n)} \iint d\theta d\varphi \sin\left(m\left[\theta + \lambda(\theta,\varphi)\right] + n\varphi\right) \frac{\sqrt{g}}{\Phi'} H(\theta,\varphi) \ .$$

• **Codes generated so far for this task:**

1. Postprocessor to turn Berr(θ,φ) into Xerr(m,n) and Xerr(θ,φ). This can be used with Berr generated from any source (Nescoil, Whitson's code, Onset, etc.). It runs fast (2-4 sec on k.nersc), so it can be used inside an external optimizer loop.
2. Test version of post-processor to check if everything is ok:
   a. Does calculated Xerr(θ,φ) satisfy original equation at each (θ,φ) point?
   b. Is the number of fourier modes used adequate to represent Berr(θ,φ)?
   c. Is data obtained from VMEC wout file adequate to represent $\lambda(\theta,\varphi)$ and g(θ,φ)?
3. All optimized SVD and Fourier routines needed in this and SVD calculation.
4. Modified version of Nescoil with both SVD and Resonance modifications.