Initial DKES Results for C82

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References:

W. I. van Rij and S. P. Hirshman, Phys. Fluids B, Vol. 1, March 1989.

S. P. Hirshman, K. C. Shaing, et al., Phys. Fluids, Vol. 29, September, 1986

DKES Notation and Normalizations

At this point, results will only be given in terms of the Γ_{ij}'s

$$I_i = \begin{bmatrix} \vec{\Gamma}\cdot\vec{s} \\ \frac{1}{T}\vec{Q}\cdot\vec{s} \\ n\left\langle(\vec{u}-\vec{u}_s)\cdot\vec{B}\right\rangle \end{bmatrix} = -\sum_{j=1}^{3} L_{ij}A_j \qquad A_j = \begin{bmatrix} \frac{n'}{n} - \frac{3}{2}\frac{T'}{T} - \frac{eE_r}{T} \\ \frac{T'}{T} \\ -\left(\frac{e}{T}\right)\left(\frac{\langle\vec{E}\cdot\vec{B}\rangle}{\langle B^2\rangle}\right) \end{bmatrix}$$

$$L_{ij} = n\frac{2}{\sqrt{\pi}}\int_0^\infty dK\sqrt{K}\; e^{-K} g_i g_j D_{ij}$$

where $g_1 = g_3 = 1,\; g_2 = K, \quad K = \left(\frac{v}{v_{th}}\right)^2$

$$D_{11} = D_{12} = D_{21} = D_{22} = -\frac{v_{th}}{2}\left[\frac{Bv_{th}}{\Omega}\left(\frac{d\rho}{dr}\right)^{-1}\right]^2 K\sqrt{K}\;\Gamma_{11}$$

$$D_{31} = D_{32} = -D_{13} = -D_{23} = -\frac{v_{th}}{2}\left[\frac{Bv_{th}}{\Omega}\left(\frac{d\rho}{dr}\right)^{-1}\right] K\;\Gamma_{31}$$

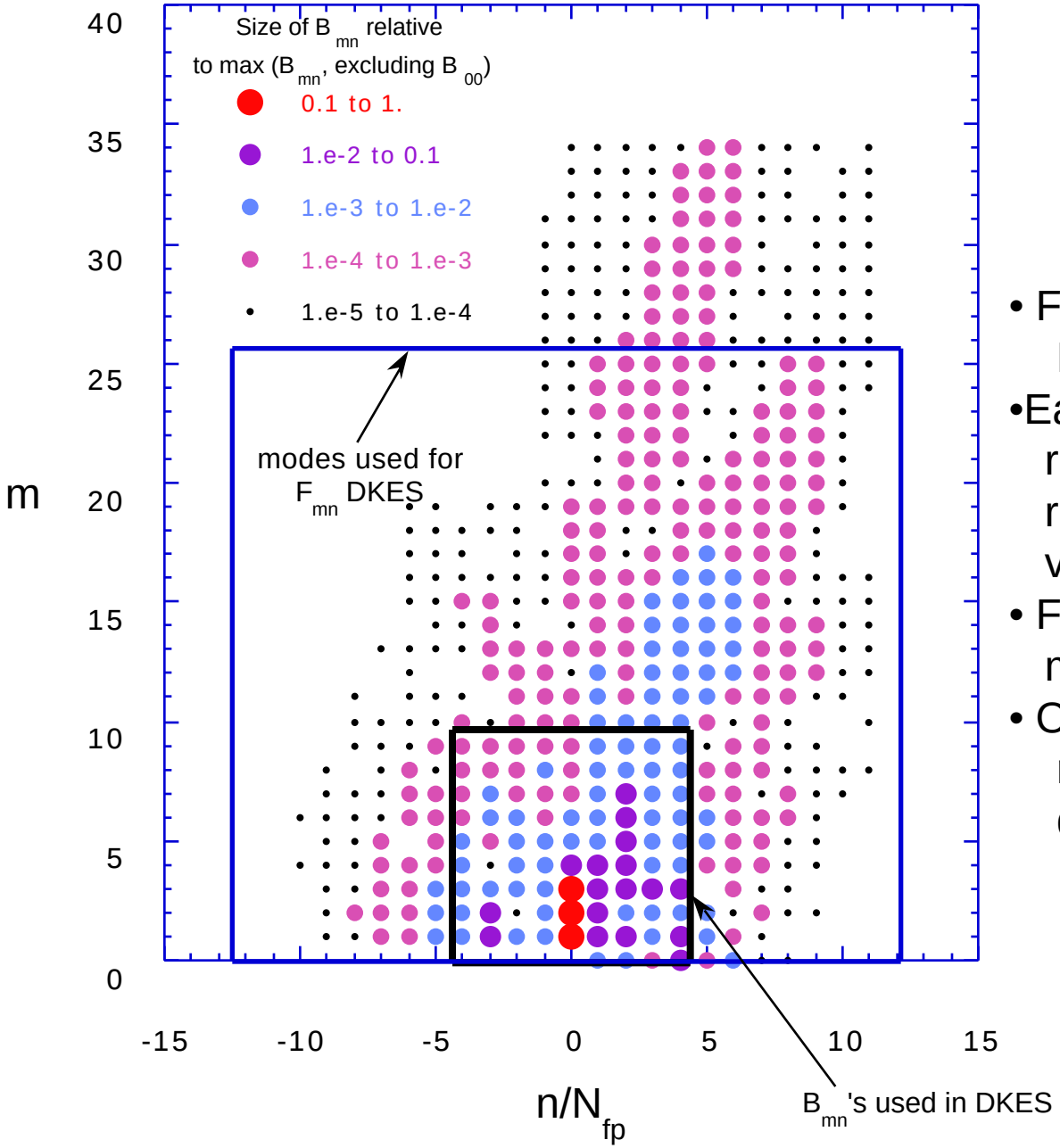
$$D_{33} = -\frac{v_{th}}{2}\sqrt{K}\;\Gamma_{33}$$

$$\Gamma_{ij} = \Gamma_{ij}\left(\frac{v'}{v}, \frac{E_r}{v}\right)$$

(i.e., to carry out the above integrals, one will need to generate a 2-D matrix of Γ's vs. these parameters for each flux surface)

Spectrum of B_{mn}'s and F_{mn}'s used in DKES for C82

(color coding and size of dots represents strength of B_{mn}'s)



- For C82 we used 99 B_{mn}'s and 575 F_{mn}'s
- Early version of DKES required rectangular region of B_{mn}'s (current version does not)
- F_{mn}'s must include m,n's used for B
- Optimal mode selection rules for F_{mn}'s under development

Upper and Lower Bound Estimates

- DKES is variational with respect to overall entropy production rate:

  $\dot{S}(F^{+}) < \dot{S}^{*} < \dot{S}(F^{-})$

  where $F^{+/-}$ = components of distr. fcn. which are symmetric/antisymmetric w/r to time reversal

  $\dot{S}$ = entropy production rate

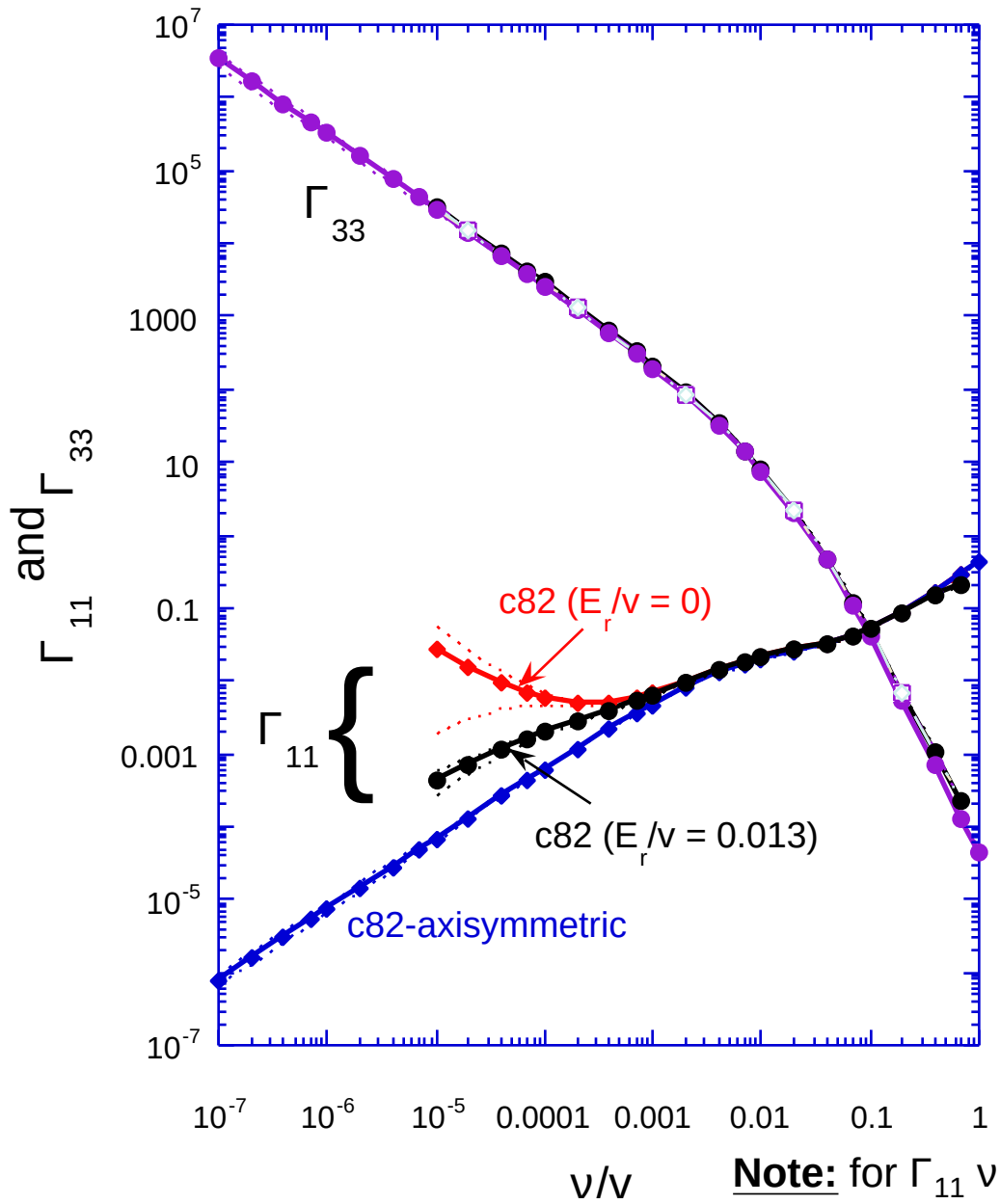
- However, one needs to know how these bounds translate down to the L_{ij}'s (or Γ_{ij}'s) (see section IV.C of van Rij, Hirshman)
  - simple for diagonal coefficients (e.g., for L_{11} set A_2=A_3=0)

    $L^{-}_{ii} < L^{*}_{ii} < L^{+}_{ii}$
  - less simple for non-diagonal coefficients (for L_{31} set A_2=0, but must keep A_1 = A_3 = 0)

$$\frac{1}{2}\left(L^{+}_{31} - L^{-}_{31}\right) - \frac{1}{2}\sqrt{\left(L^{-}_{11} - L^{+}_{11}\right)\left(L^{-}_{33} - L^{+}_{33}\right)} < L^{*}_{31} < \frac{1}{2}\left(L^{+}_{31} - L^{-}_{31}\right) + \frac{1}{2}\sqrt{\left(L^{-}_{11} - L^{+}_{11}\right)\left(L^{-}_{33} - L^{+}_{33}\right)}$$

- Note: the above are sufficient, but not necessary for convergence
- Problem for L_{31}: L_{33}^{-} - L_{33}^{+} involves difference of large nearly equal numbers
- Convergence of L_{31} bounds is difficult - see Fig. 2 of ref. 1: ~ 200 F_{mn}'s required for only 3 equilibrium B_{mn}'s

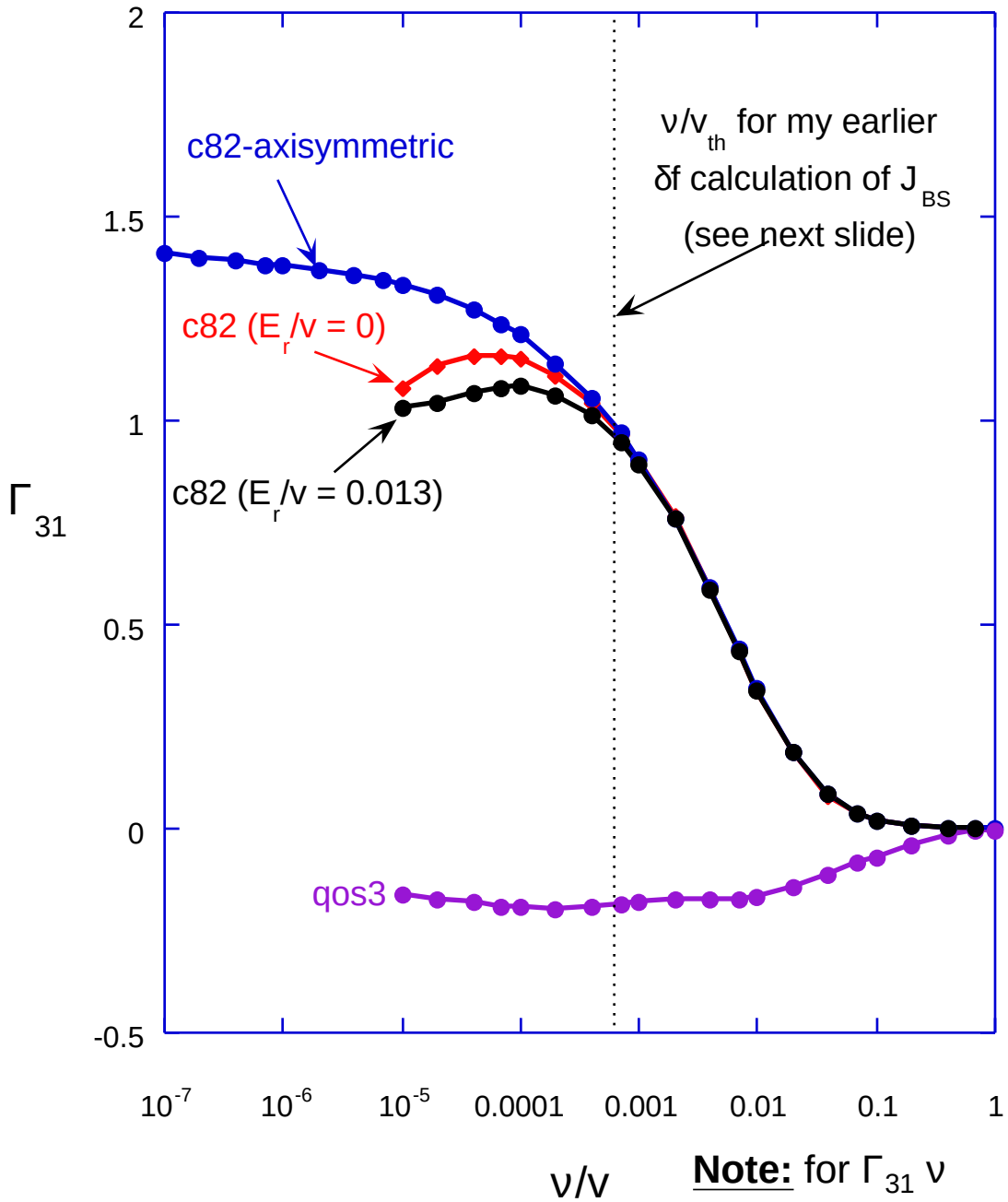
Γ_{11} and Γ_{33} (diffusion/resistivity enhancement) coefficients vs. collisionality



(E_r/v = 0.013 e.g., corresponds to a 1 keV ion with $e\phi/kT_{ion}$ = 1, based on a constant electric field)

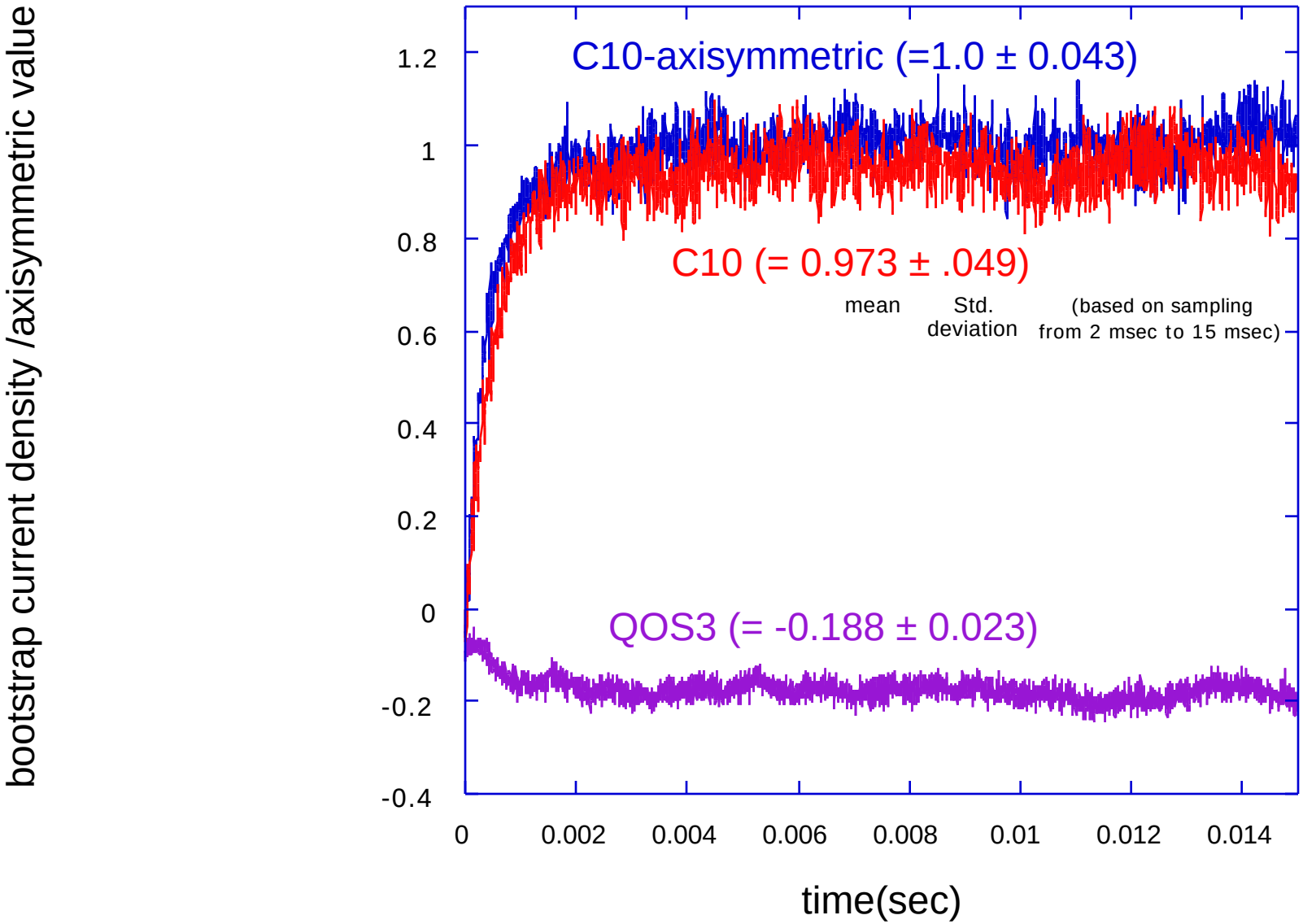
Note: for Γ_{11} ν 63(ν/v)

Γ_{13} (bootstrap current) coefficient vs. collisionality

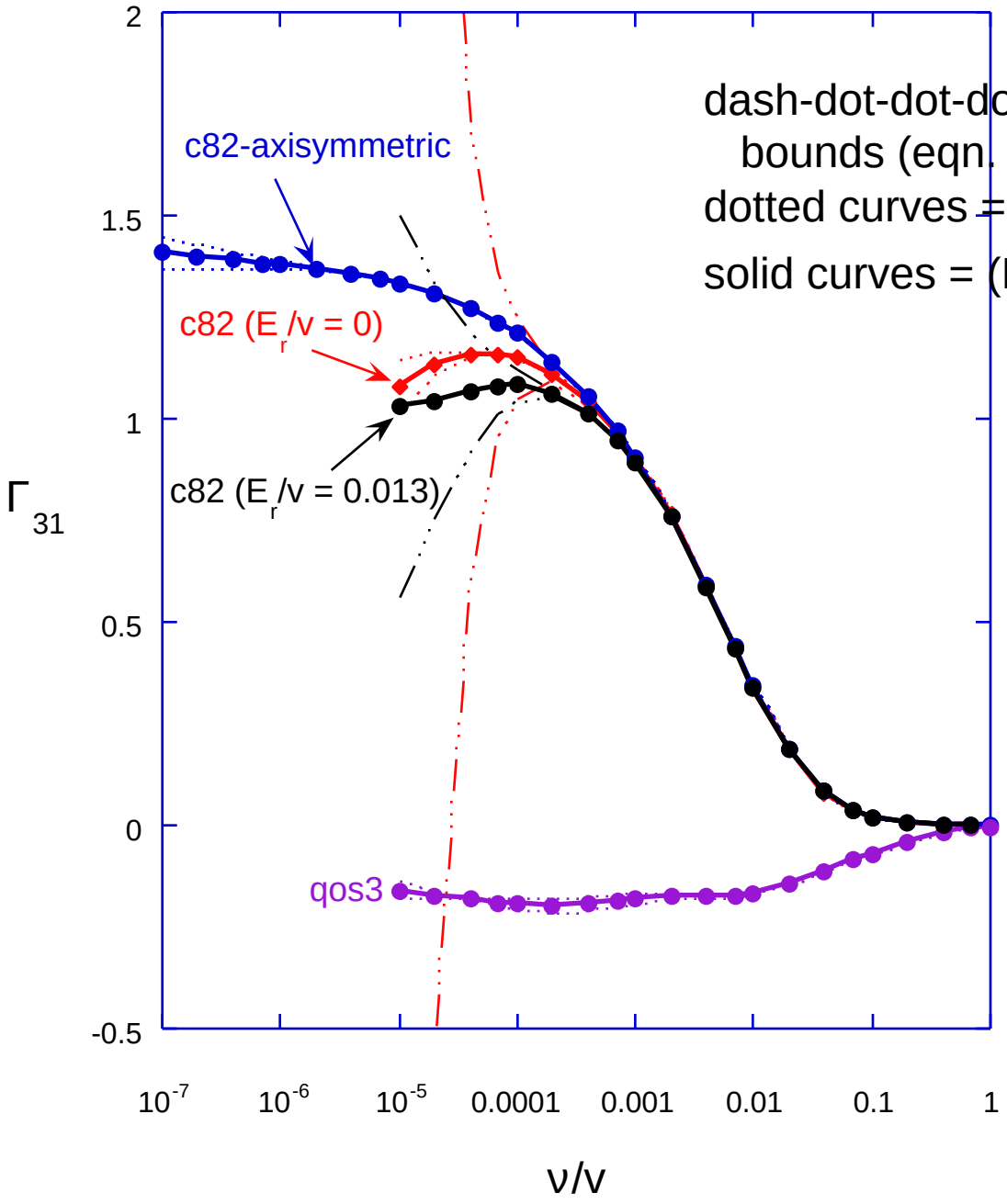


Note: for Γ_{31} ν 41(ν/)

In the following bootstrap currents (normalized to the axisymmetric value) are compared between quasi-axisymmetric (c10) and quasi-omnigenous (QOS3) devices)



Bootstrap current coefficient with upper and lower bounds



dash-dot-dot-dot curves = rigourous bounds (eqn. at bottom of slide 4)

dotted curves = Γ_{31}^- and Γ_{31}^+

solid curves = $(\Gamma_{31}^+ + \Gamma_{31}^-)/2$