VMEC2000 and TERPSICHORE Convergence Studies for NCSX-c82

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Content

- **Errors in equilibrium quantities and numerical ballooning modes;**
- **VMEC2000 equilibrium convergence study;**
- **Terpsichore reconstructed equilibirum;**
- **Ballooning coefficients study;**
- **Summary**

Numerical ballooning modes

Ballooning equation:

$$\partial(C_2 \partial\chi/\partial\theta)/\partial\theta + C_1\chi = 0 \ , \tag{1}$$

The bending term must be negative:

$$C_2 = C_p + C_s(\theta - \theta_0) + C_q(\theta - \theta_0)^2 \tag{2}$$

$B_i = \{B_s,\ J,\ F\}$ are covariant components in Boozer coordinates.

Δ_i can characterize the equilibrium errors:

$$B_s + \Delta_s = \Psi' \, (\, g_{s\theta} \quad + \quad q g_{s\zeta} \,) \, / \sqrt{g}, \tag{3}$$

$$J + \Delta_\theta = \Psi' \, (\, g_{\theta\theta} \quad + \quad q g_{\theta\zeta} \,) \, / \sqrt{g}, \tag{4}$$

$$F + \Delta_\zeta = \Psi' \, (\, g_{\theta\zeta} \quad + \quad q g_{\zeta\zeta} \,) \, / \sqrt{g}, \tag{5}$$

With Δ_i:

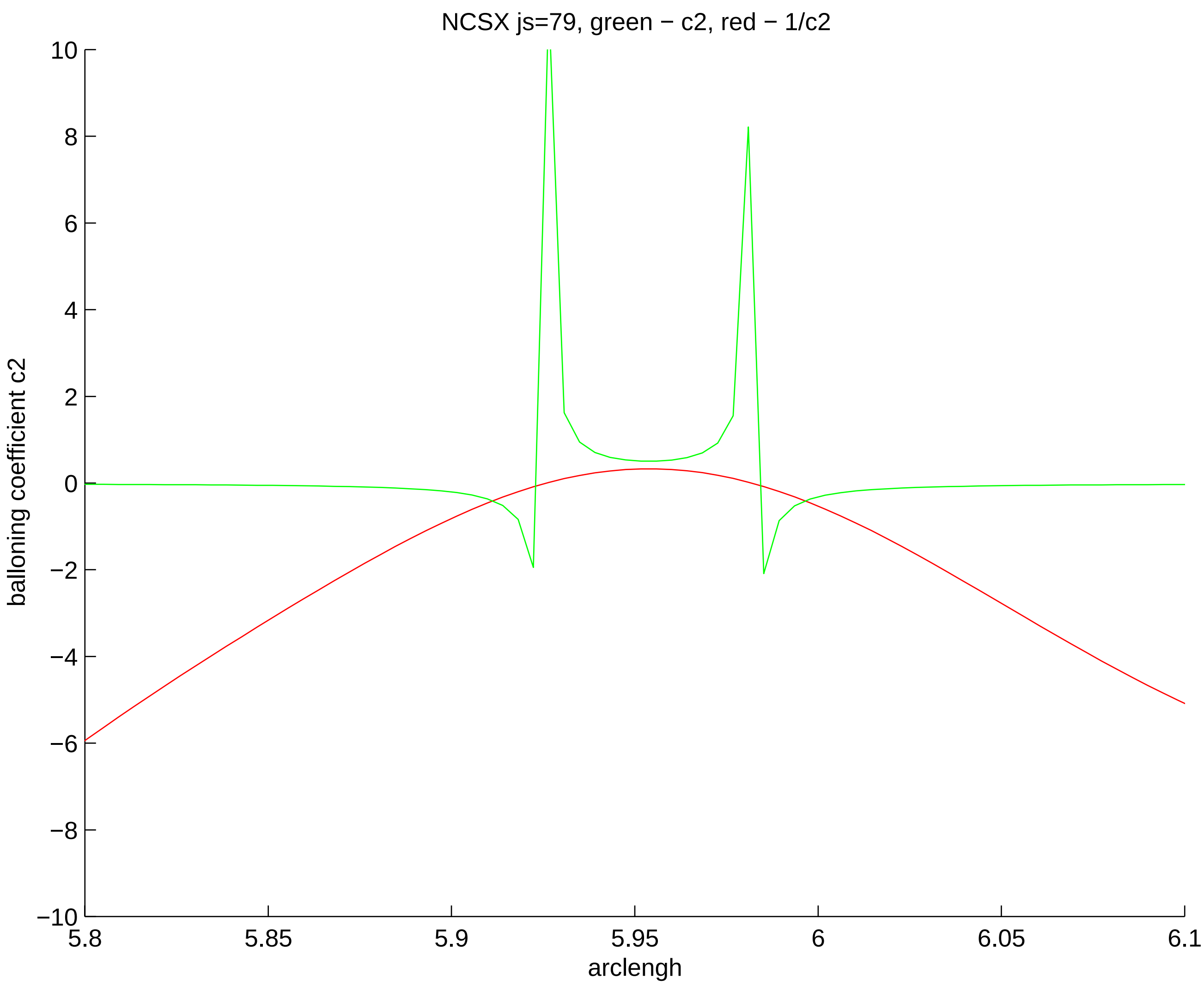
$$C_p = \frac{g_{ss}}{\sqrt{g}} - \frac{(B_s + \Delta_s)^2}{B^2\sqrt{g}}, \tag{6}$$

$$C_s = \frac{2q'\Psi'}{\Phi'}\left(\frac{JB_s}{B^2\sqrt{g}} - \frac{g_{s\theta}}{\sqrt{g}} + B_s\Delta_\theta + B_\theta\Delta_s\right), \tag{7}$$

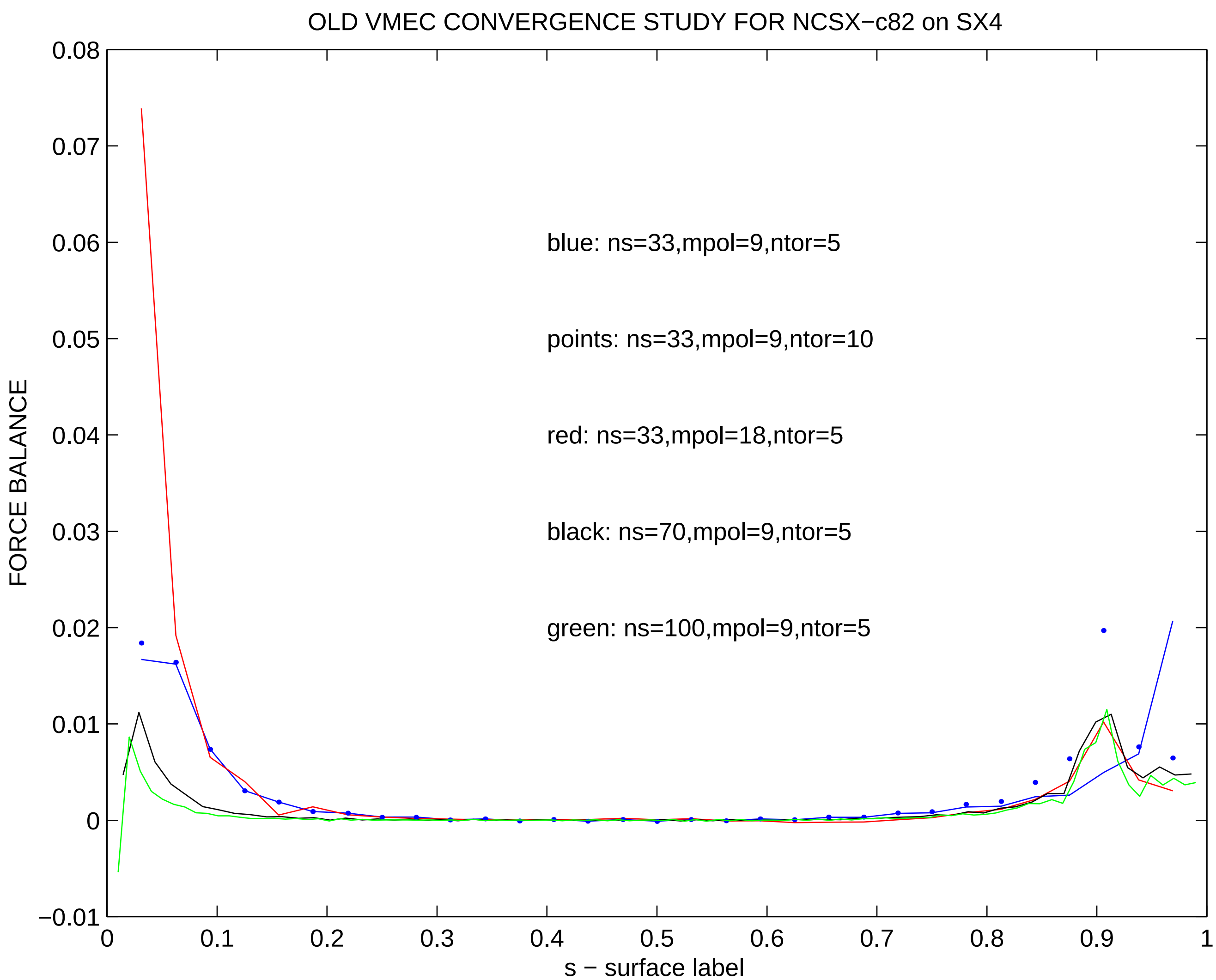
$$C_q = \frac{q'^2}{q^2}\left(\frac{g_{\theta\theta}}{\sqrt{g}} - \frac{(B_\theta + \Delta_\theta)^2}{B^2\sqrt{g}}\right) \tag{8}$$

The condition that the bending term does not change the sign can be formulated as follows:

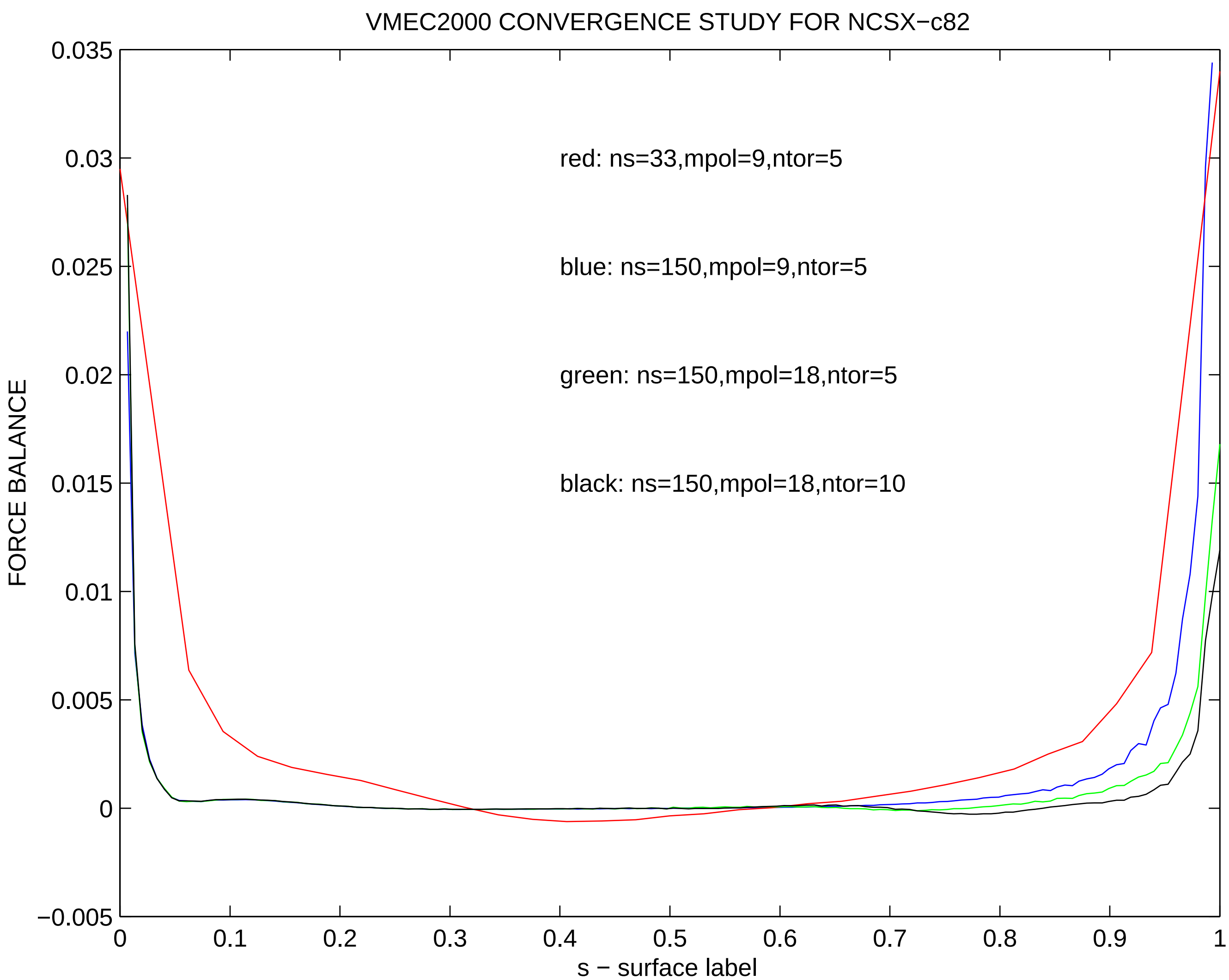
$$D = C_s^2 - 4C_pC_q < 0 \tag{9}$$

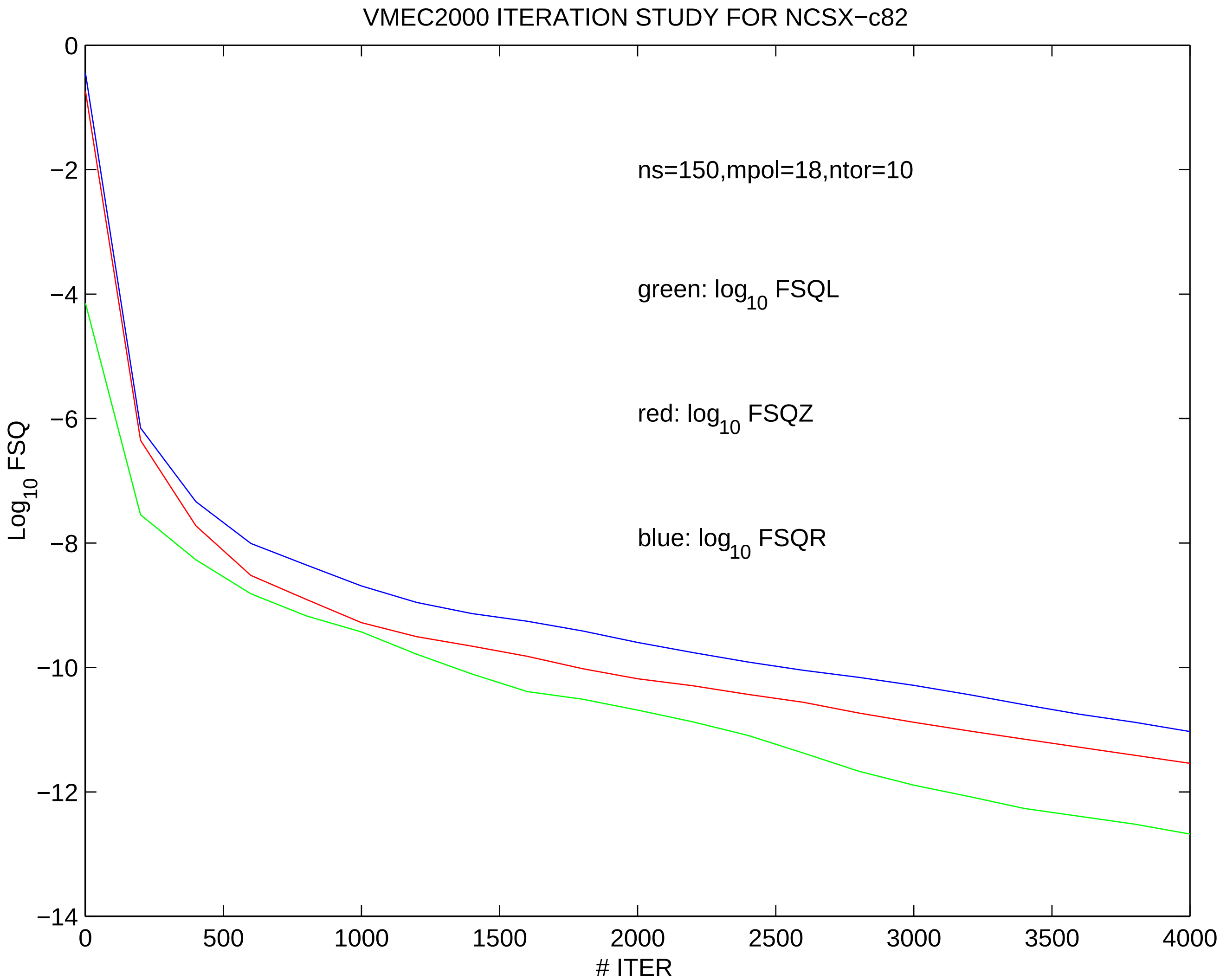


NCSX js=79, green − c2, red − 1/c2
balloning coefficient c2
arclengh
10
8
6
4
2
0
−2
−4
−6
−8
−10
5.8
5.85
5.9
5.95
6
6.05
6.1

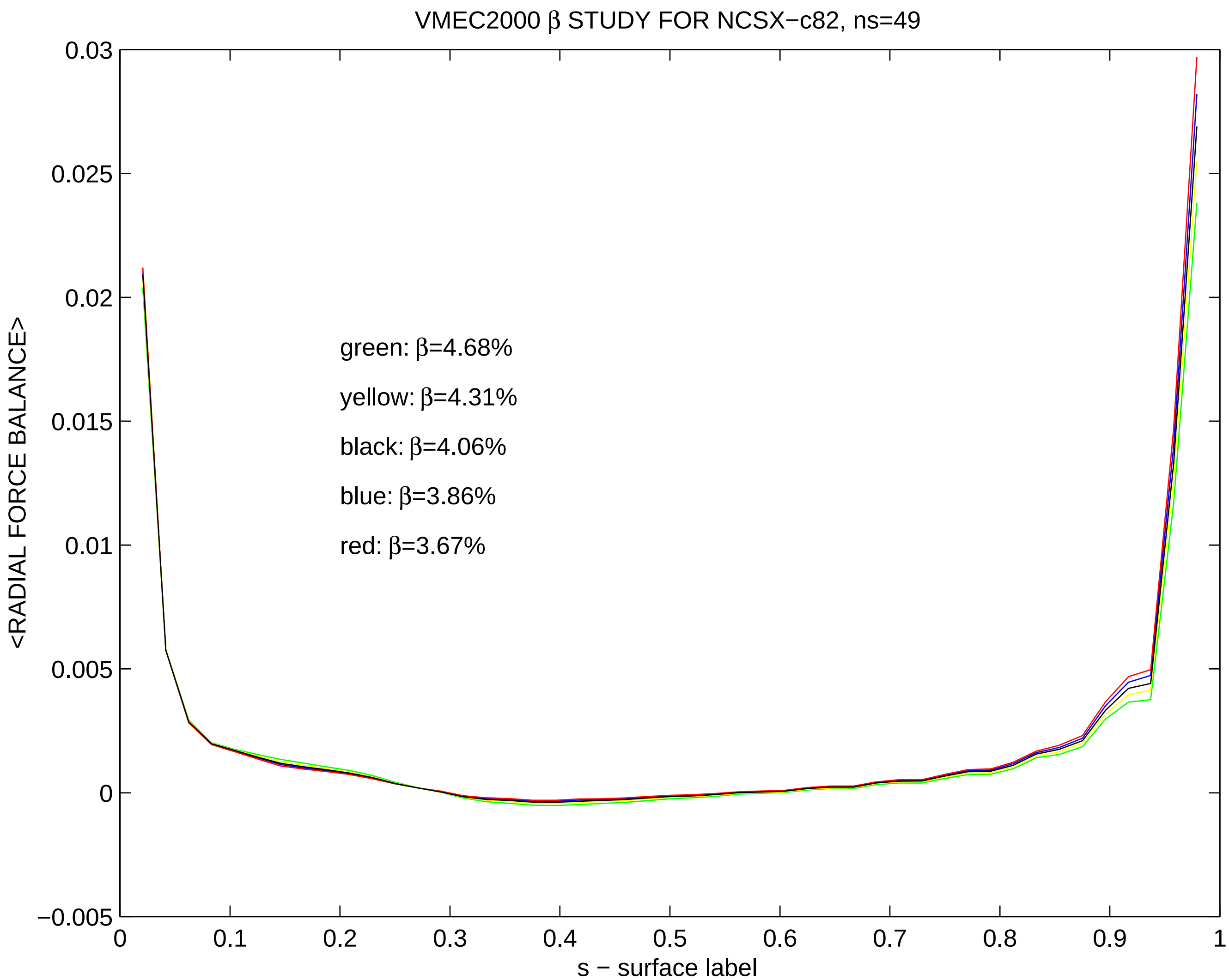


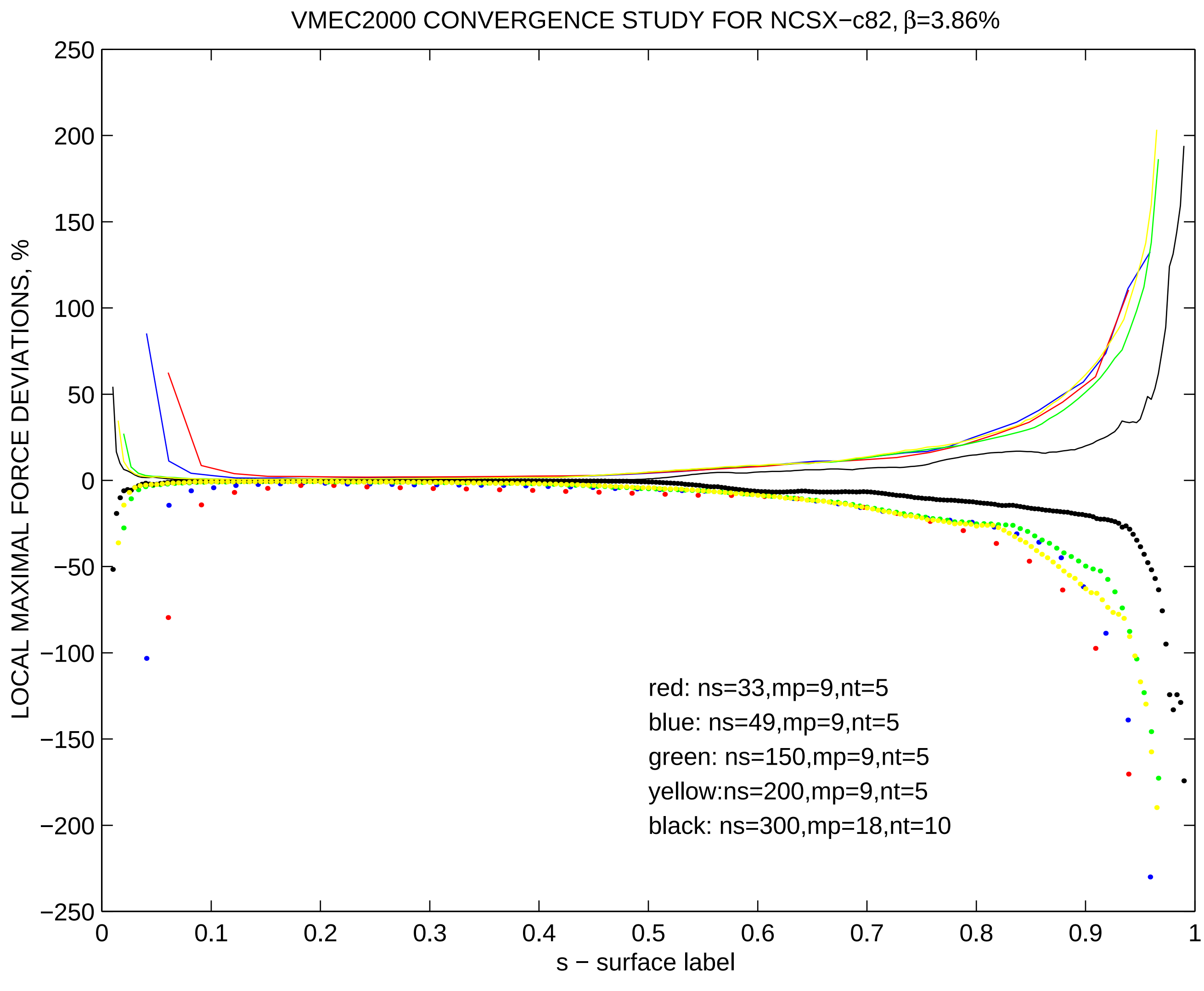
OLD VMEC CONVERGENCE STUDY FOR NCSX−c82 on SX4
FORCE BALANCE
s − surface label
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
blue: ns=33,mpol=9,ntor=5
points: ns=33,mpol=9,ntor=10
red: ns=33,mpol=18,ntor=5
black: ns=70,mpol=9,ntor=5
green: ns=100,mpol=9,ntor=5

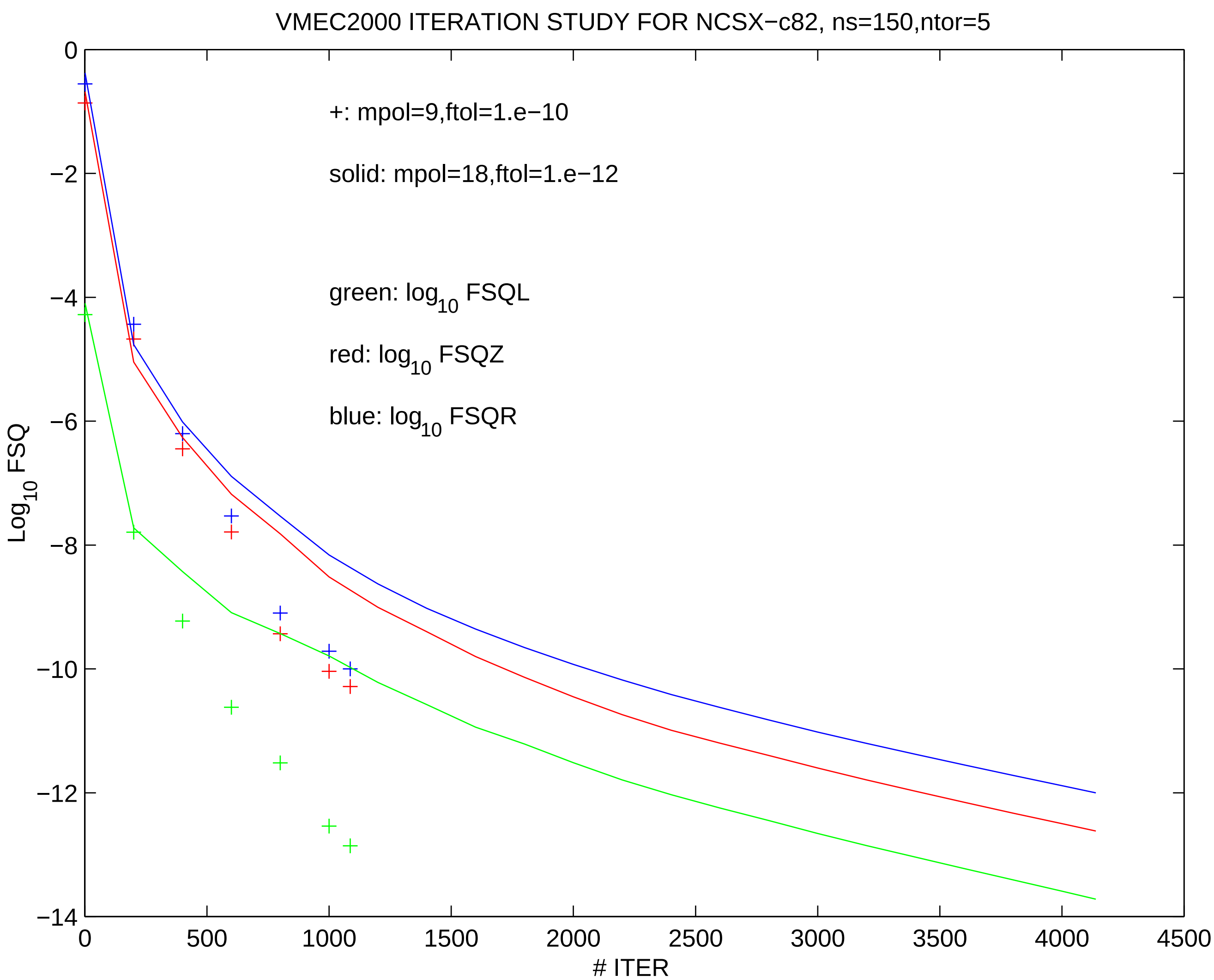

VMEC2000 CONVERGENCE STUDY FOR NCSX−c82
red: ns=33,mpol=9,ntor=5
blue: ns=150,mpol=9,ntor=5
green: ns=150,mpol=18,ntor=5
black: ns=150,mpol=18,ntor=10
FORCE BALANCE
s − surface label
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
−0.005
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



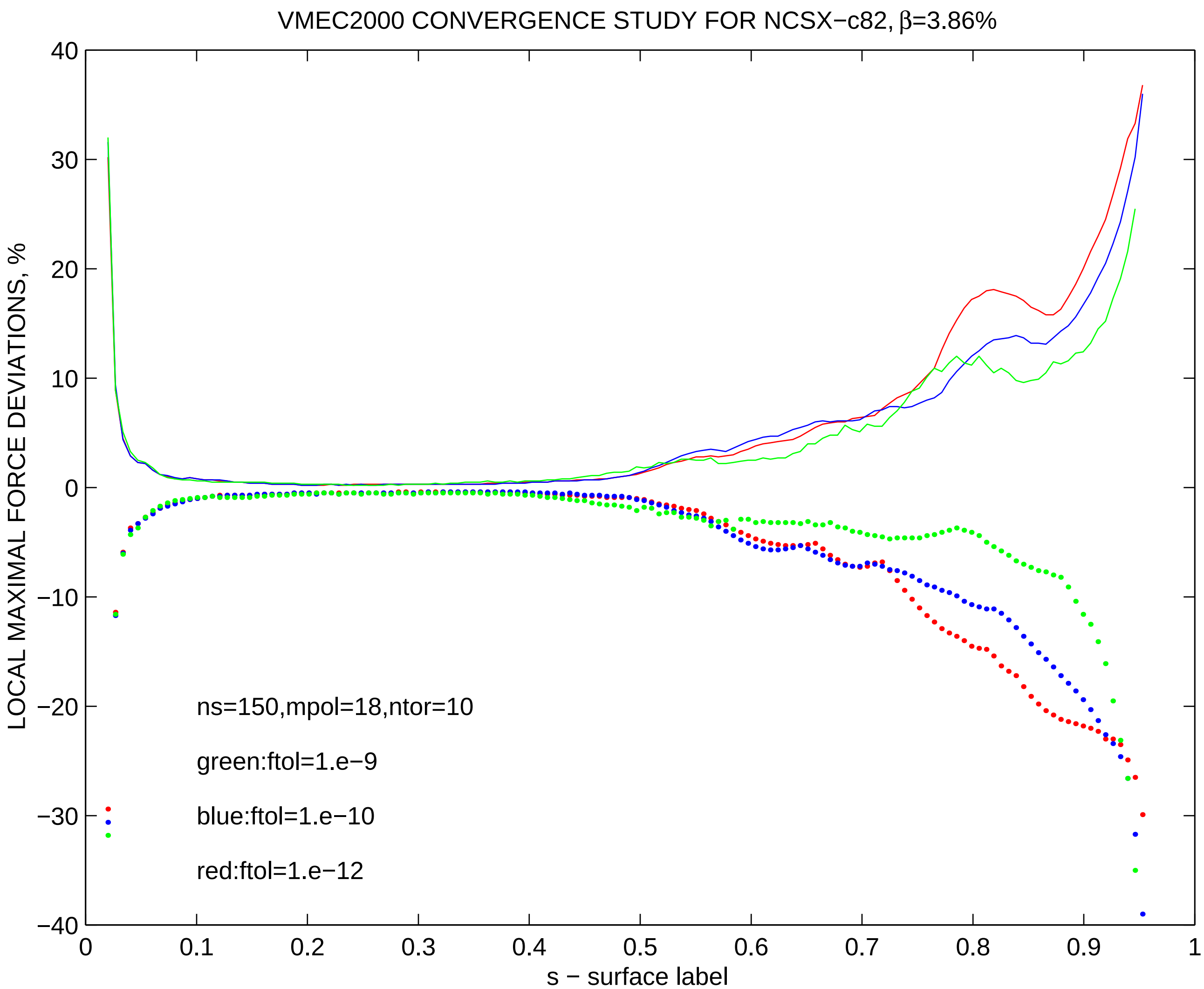
VMEC2000 ITERATION STUDY FOR NCSX−c82
ns=150,mpol=18,ntor=10
green: log10 FSQL
red: log10 FSQZ
blue: log10 FSQR
Log10 FSQ
ITER
0
−2
−4
−6
−8
−10
−12
−14
0
500
1000
1500
2000
2500
3000
3500
4000

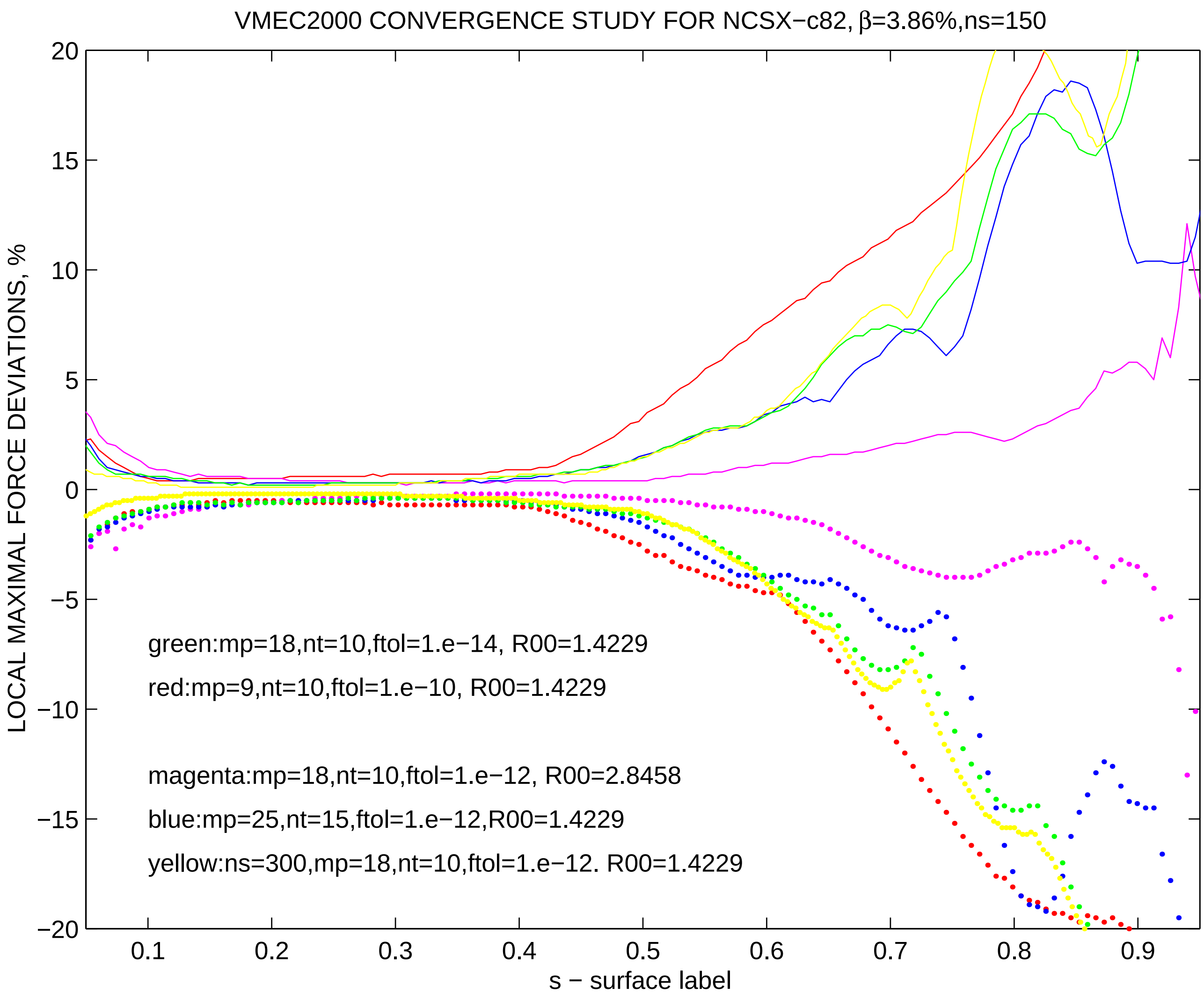

VMEC2000 β STUDY FOR NCSX−c82, ns=49
<RADIAL FORCE BALANCE>
s − surface label
green: β=4.68%
yellow: β=4.31%
black: β=4.06%
blue: β=3.86%
red: β=3.67%
0.03
0.025
0.02
0.015
0.01
0.005
0
−0.005
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


VMEC2000 CONVERGENCE STUDY FOR NCSX−c82, β=3.86%
LOCAL MAXIMAL FORCE DEVIATIONS, %
s − surface label
250
200
150
100
50
0
−50
−100
−150
−200
−250
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
red: ns=33,mp=9,nt=5
blue: ns=49,mp=9,nt=5
green: ns=150,mp=9,nt=5
yellow:ns=200,mp=9,nt=5
black: ns=300,mp=18,nt=10

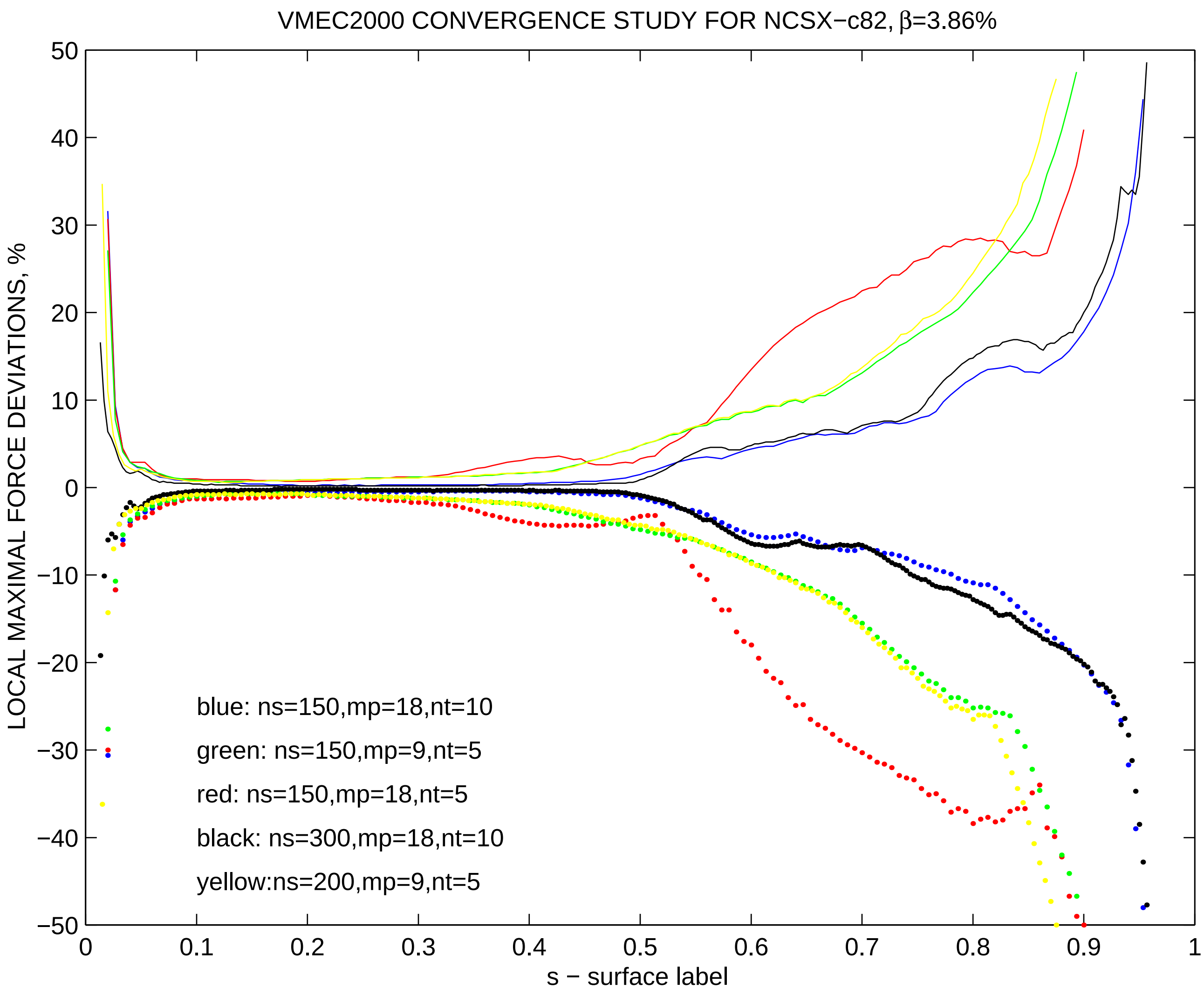


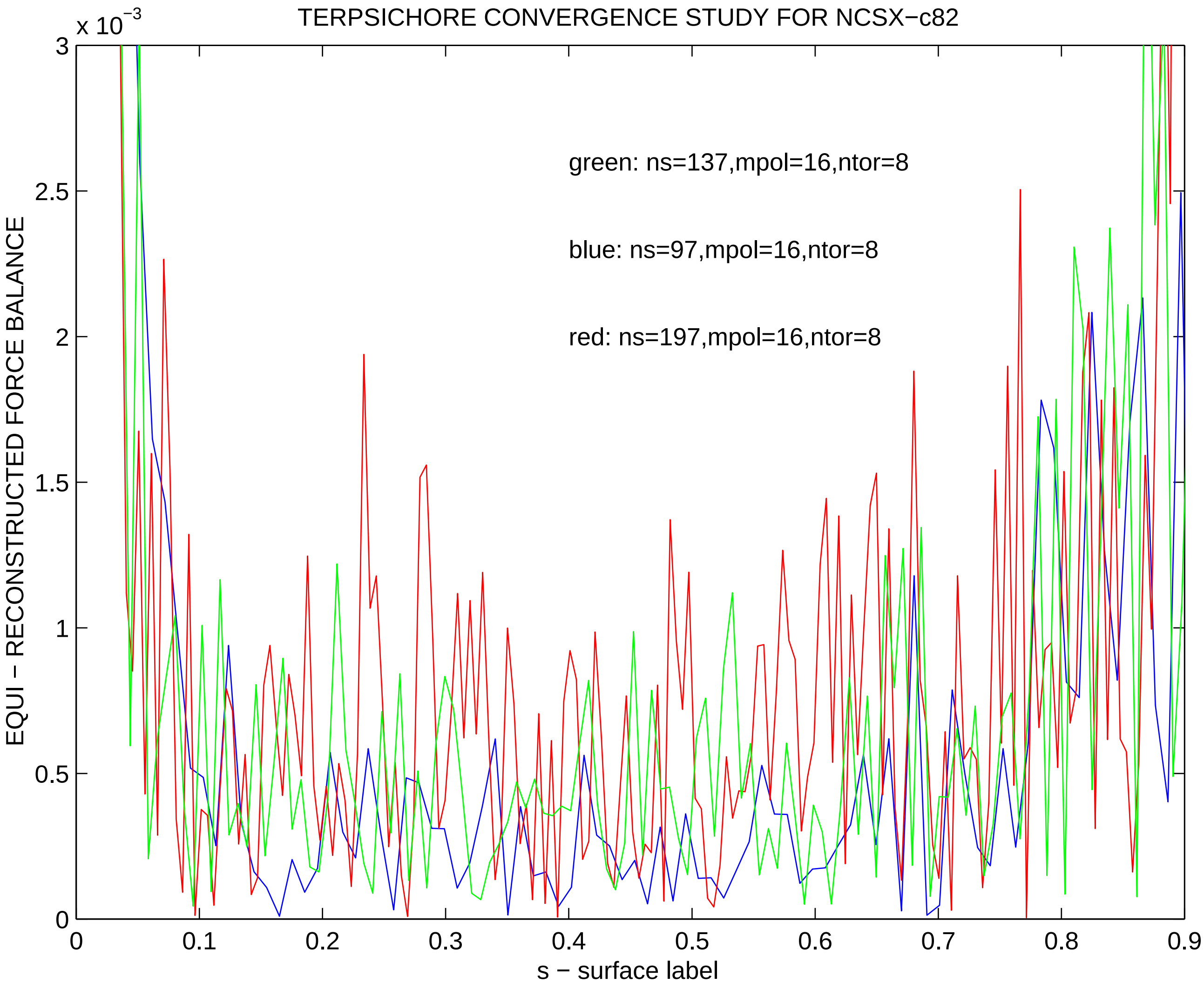
VMEC2000 ITERATION STUDY FOR NCSX−c82, ns=150,ntor=5
+: mpol=9,ftol=1.e−10
solid: mpol=18,ftol=1.e−12
green: log10 FSQL
red: log10 FSQZ
blue: log10 FSQR
Log10 FSQ
ITER
0
−2
−4
−6
−8
−10
−12
−14
0
500
1000
1500
2000
2500
3000
3500
4000
4500


VMEC2000 CONVERGENCE STUDY FOR NCSX−c82, β=3.86%
LOCAL MAXIMAL FORCE DEVIATIONS, %
s − surface label
ns=150,mpol=18,ntor=10
green:ftol=1.e−9
blue:ftol=1.e−10
red:ftol=1.e−12
40
30
20
10
0
−10
−20
−30
−40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

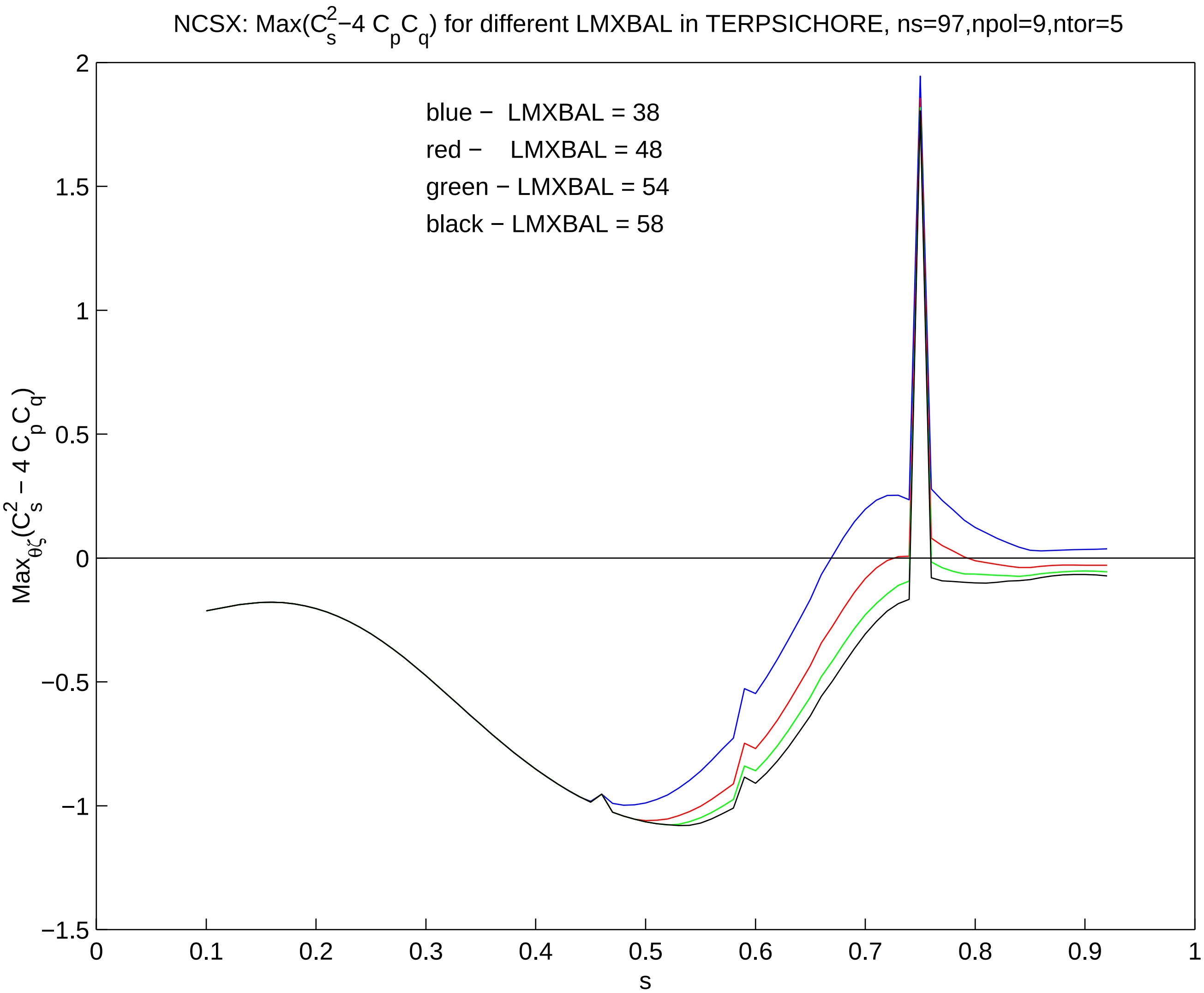


VMEC2000 CONVERGENCE STUDY FOR NCSX−c82, β=3.86%,ns=150
LOCAL MAXIMAL FORCE DEVIATIONS, %
s − surface label
green:mp=18,nt=10,ftol=1.e−14, R00=1.4229
red:mp=9,nt=10,ftol=1.e−10, R00=1.4229
magenta:mp=18,nt=10,ftol=1.e−12, R00=2.8458
blue:mp=25,nt=15,ftol=1.e−12,R00=1.4229
yellow:ns=300,mp=18,nt=10,ftol=1.e−12. R00=1.4229

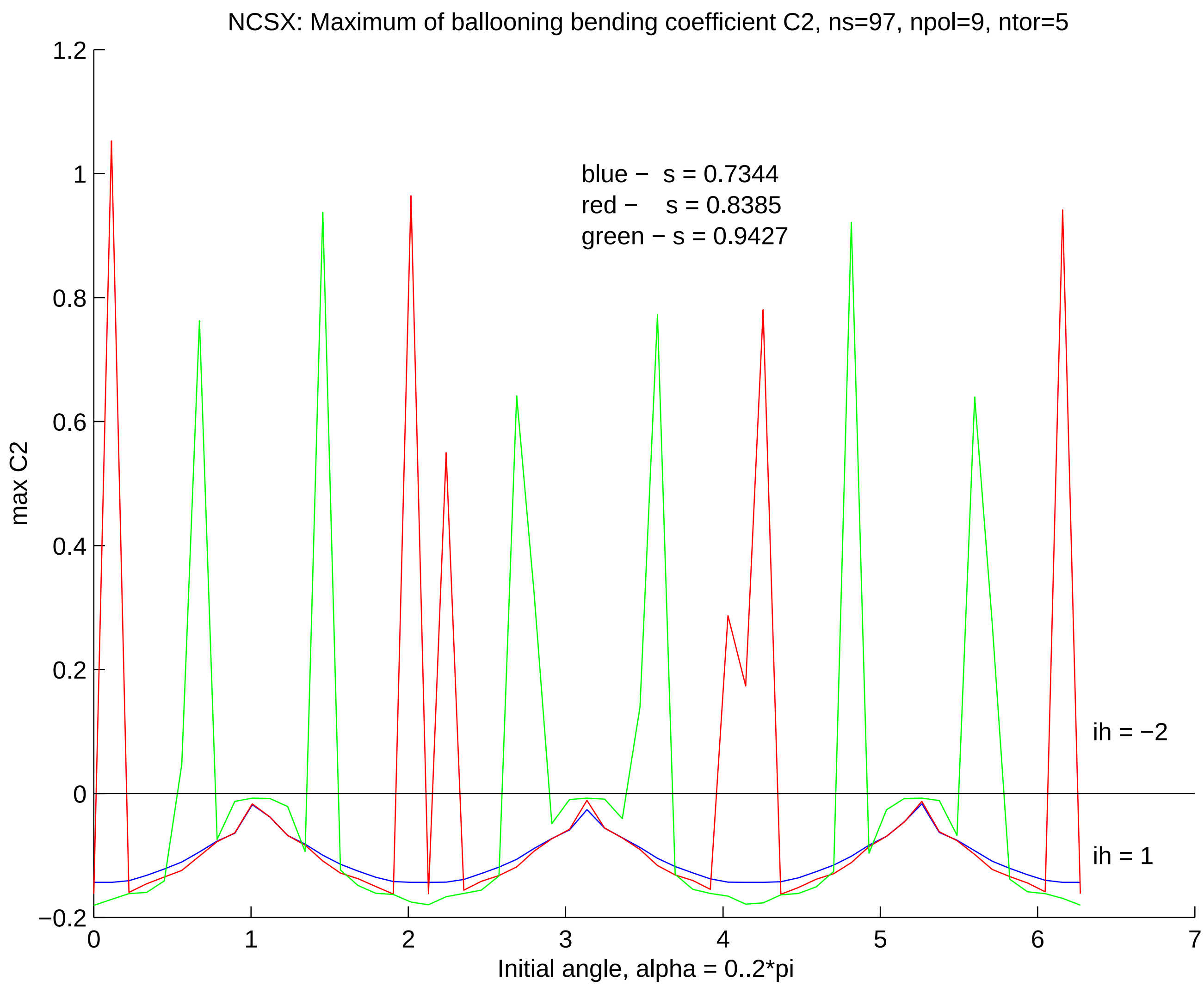

VMEC2000 CONVERGENCE STUDY FOR NCSX−c82, β=3.86%
LOCAL MAXIMAL FORCE DEVIATIONS, %
s − surface label
50
40
30
20
10
0
−10
−20
−30
−40
−50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
blue: ns=150,mp=18,nt=10
green: ns=150,mp=9,nt=5
red: ns=150,mp=18,nt=5
black: ns=300,mp=18,nt=10
yellow:ns=200,mp=9,nt=5



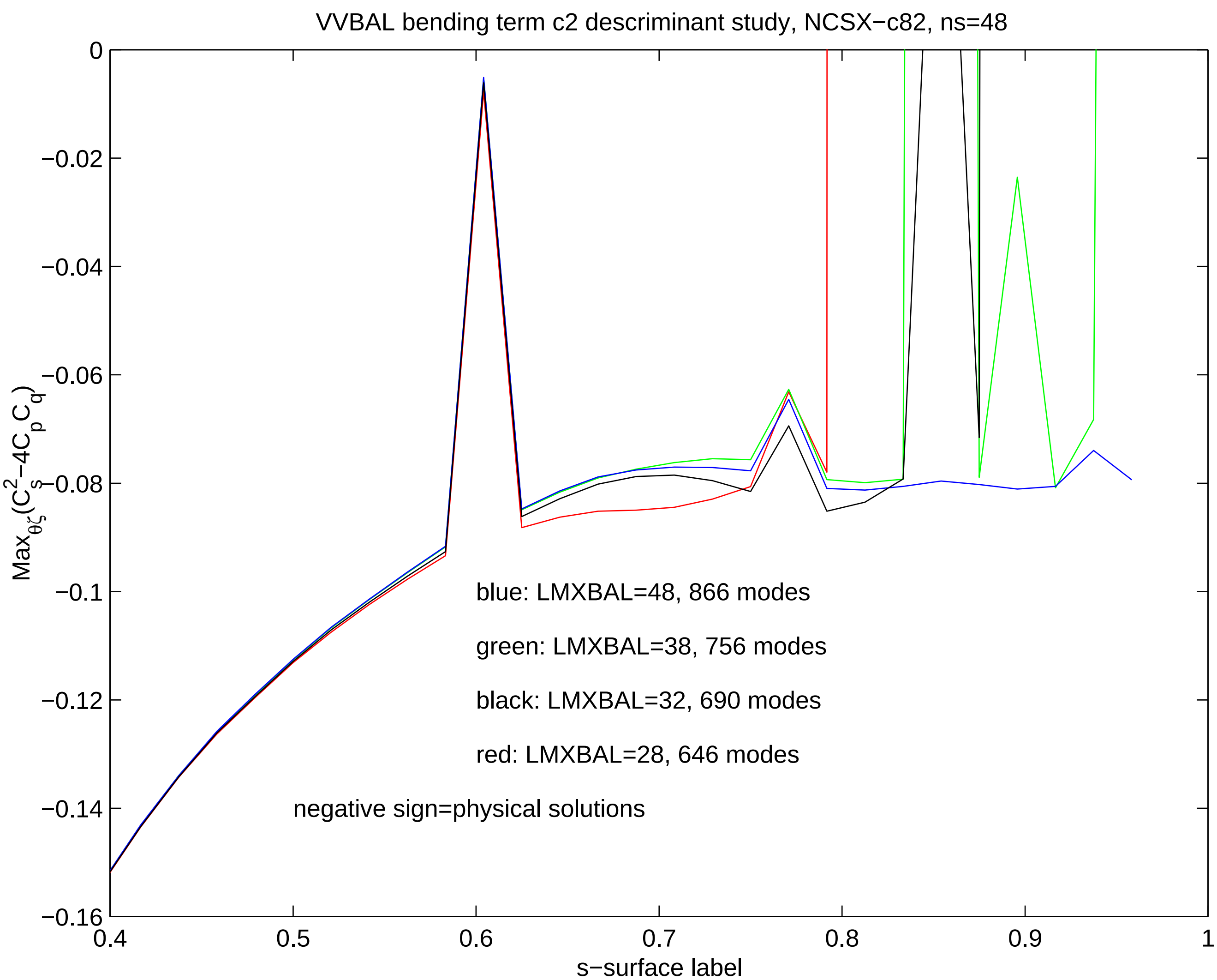
TERPSICHORE CONVERGENCE STUDY FOR NCSX−c82
x 10^−3
green: ns=137,mpol=16,ntor=8
blue: ns=97,mpol=16,ntor=8
red: ns=197,mpol=16,ntor=8
EQUI − RECONSTRUCTED FORCE BALANCE
s − surface label

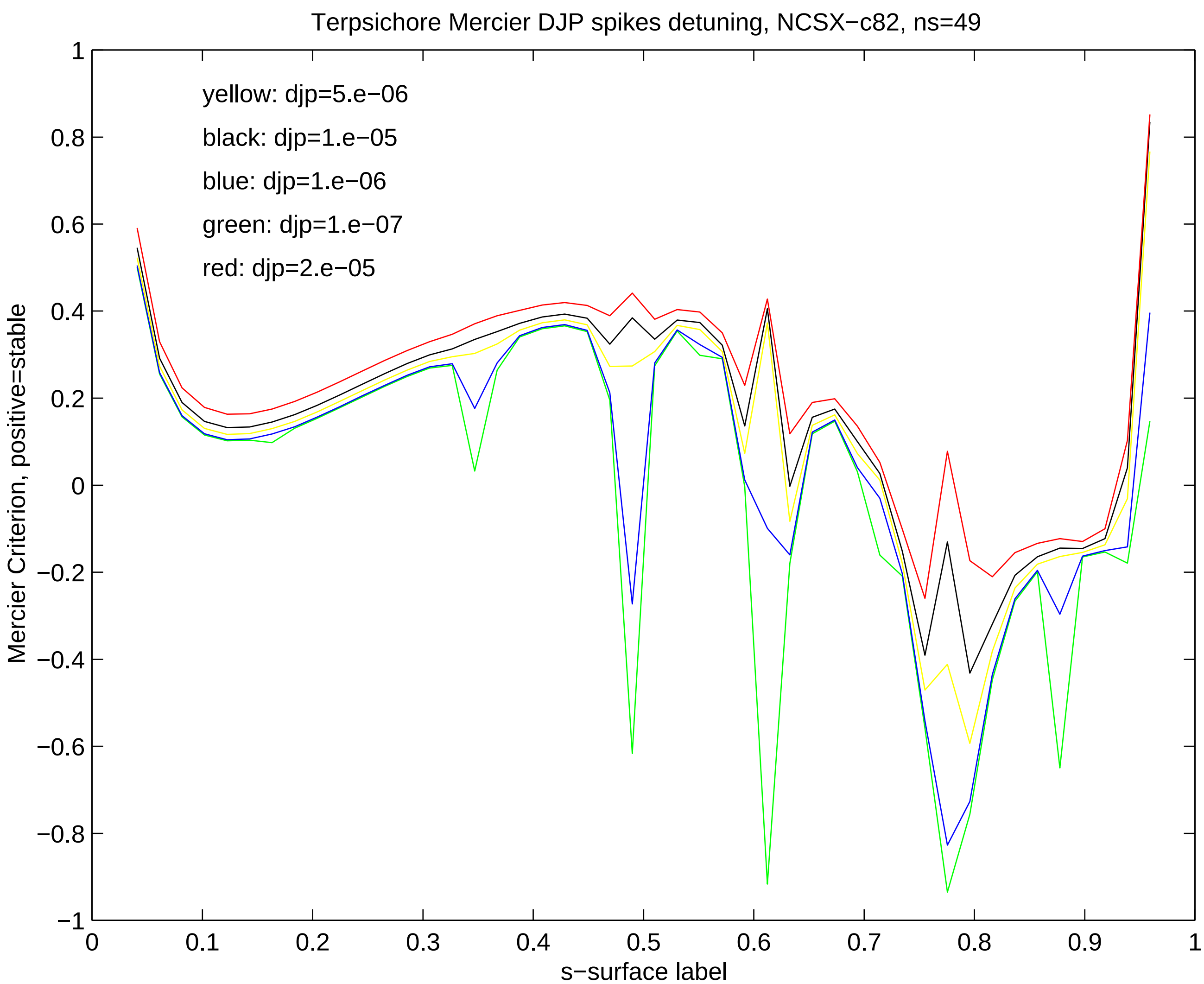


NCSX: Max(C_s^2−4 C_pC_q) for different LMXBAL in TERPSICHORE, ns=97,npol=9,ntor=5
blue − LMXBAL = 38
red − LMXBAL = 48
green − LMXBAL = 54
black − LMXBAL = 58
$Max_{\theta\zeta}(C_s^2 - 4\ C_pC_q)$
s
2
1.5
1
0.5
0
−0.5
−1
−1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



NCSX: Maximum of ballooning bending coefficient C2, ns=97, npol=9, ntor=5
max C2
Initial angle, alpha = 0..2*pi
blue − s = 0.7344
red − s = 0.8385
green − s = 0.9427
ih = −2
ih = 1
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
1
2
3
4
5
6
7


VVBAL bending term c2 descriminant study, NCSX−c82, ns=48
$Max_{\theta\zeta}(C_s^2-4C_pC_q)$
s−surface label
0
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
−0.14
−0.16
0.4
0.5
0.6
0.7
0.8
0.9
1
blue: LMXBAL=48, 866 modes
green: LMXBAL=38, 756 modes
black: LMXBAL=32, 690 modes
red: LMXBAL=28, 646 modes
negative sign=physical solutions


Terpsichore Mercier DJP spikes detuning, NCSX−c82, ns=49
yellow: djp=5.e−06
black: djp=1.e−05
blue: djp=1.e−06
green: djp=1.e−07
red: djp=2.e−05
Mercier Criterion, positive=stable
s−surface label
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Summary

1. VMEC2000 with large numbers of radial grid points, poloidal and toroidal modes has better equilibrium force balance.

2. VVBAL ballooning code convergence depends on Boozer reconstructed equilibrium force balance and extra-ballooning modes numbers. We have shown that for small numbers of radial points (49) it is possible to eliminate numerical modes.

3. For large number of radial points the reconstructed equilibrium force balance deteriorates probably due to the increase the number of singular surfaces. This deterioration requires a larger number of extra ballooning modes.

4. Convergence study for the large number of radial points is limited by hardware (memory, CPU limits) and software (xminv failure) .