Update on coil-healing NCSX 3/14/2002.

(Dated: March 14, 2002)

I. COIL HEALING

The method can be considered as removing the constraint of nested flux surfaces and allowing the VMEC initialization to relax into an equilibrium, potentially with broken flux-surfaces, ie. islands, while making adjustments to the coil set to remove selected islands as they develop.

OUTLINE

1. **Initialization** with VMEC/COILOPT
2. **PIES calculation**
3. **Variation of Coil Geometry** to remove resonances
4. **Including Engineering Constraints**
5. **Including Physics Constraints** to preserve stability
6. **Application to M45**
7. **Comments**

II. INITIALIZATION

1. Solving the equilibrium equation and the adjustment of a coil design to eliminate selected magnetic islands proceeds using an iterative approach with initialization given

$$\mathbf{B} = \mathbf{B}_P^n + \mathbf{B}_C(\boldsymbol{\xi}^n). \tag{1}$$

   The total magnetic field is the sum of the magnetic field produced by the plasma, $\mathbf{B}_P$, and the magnetic field produced by the confining coils, $\mathbf{B}_C$, which is a function of a set of Fourier harmonics, $\boldsymbol{\xi}$, which describe the coil geometry, at the nth iteration.

2. The initial plasma state is provided by the VMEC code, and the initial coil geometry is provided by the COILOPT code.

III. CALCULATION OF RESONANCES

1. The PIES iterations solve for the plasma current $\mathbf{J}$ given $\mathbf{B}$ and pressure profile p

$$\nabla p = \mathbf{J} \times \mathbf{B}. \tag{2}$$

2. The plasma magnetic field is then solved given $\mathbf{J}$

$$\mathbf{J} = \boldsymbol{\nabla} \times \mathbf{B}_P. \tag{3}$$

3. Blending is used to provide numerical stability to the Picard method

$$\mathbf{B}_P^{n+1} = \alpha \mathbf{B}_P^n + (1-\alpha)\mathbf{B}_P. \tag{4}$$

4. The total magnetic field $\bar{\mathbf{B}}$ at this stage is written

$$\bar{\mathbf{B}} = \mathbf{B}_P^{n+1} + \mathbf{B}_C(\boldsymbol{\xi}^n). \tag{5}$$

5. The amplitude of each selected resonant field harmonic is calculated by Fourier decomposing the magnetic field normal to a quadratic-flux-minimizing surface.

6. The selected set of resonant harmonic amplitudes thus calculated is denoted $\{\bar{B}_i : i = 1, N\}$.

IV. VARIATION OF COIL GEOMETRY

1. A set of COILOPT coil harmonics $\{\xi_j : j = 1, M\}$ are systematically varied to set $\bar{B}_i = 0$ using a Newton method.

2. The coupling matrix, ∇B^n_{Cij}, is defined and calculated using finite-differences.

3. $$-\bar{B}_i = \nabla B^n_{Cij} \cdot \delta\xi^n_j. \tag{6}$$

4. This equation is solved for the $\delta\xi_j$ in a few iterations by inverting the $N \times M$ matrix ∇B^n_{Cij} using singular-value decomposition.

5. The new coil set is obtained

$$\xi^{n+1}_j = \xi^n_j + \delta\xi^n_j, \tag{7}$$

V. INCORPORATION OF ENGINEERING CONSTRAINTS

1. The healing algorithm is modified to preserve the minimum curvature and coil-coil separation by adding to the set of resonant fields to be eliminated the (appropriately weighted) differences in minimum curvature and coil-separation of the nth coil set, described by $\boldsymbol{\xi}^n$, from the initial coil set.

2. This constrains the island-eliminating coil variations to lie in the nullspace of these measures of engineering acceptability.

VI. INCORPORATION OF PHYSICS CONSTRAINTS (STABILITY)

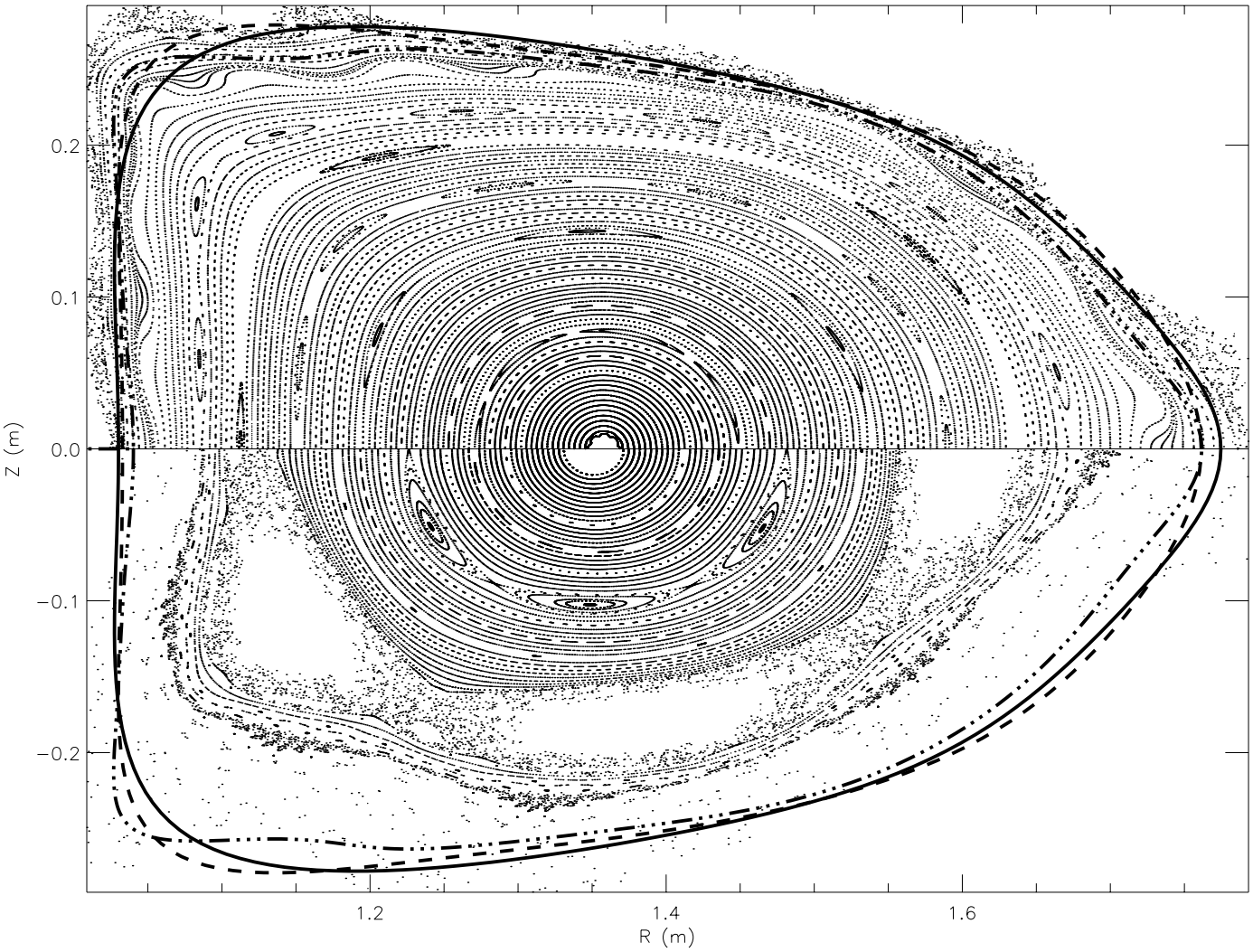
1. In a similar manner, the algorithm is extended to preserve kink stability.

2. The VMEC initialization is stable with respect to kink modes.

3. Kink and ballooning stability are calculated using TERPSICHORE and COBRA (via xstellopt) on the free-boundary VMEC calculation for each trial coil set.

VII. APPLICATION TO M45 COILS

1. Upper half : almost converged PIES Poincaré plot.
2. Lower half : PIES Poincaré plot for original coils.
3. Solid : Initialization boundary.
4. Dashed : Free-boundary VMEC for 'healed' coils.
5. Dash-Dot-Dot-Dot : PIES boundary.



VIII. ANALYSIS

1. almost converged = little change after 150+ PIES iterations

2. Coil Changes

	pt-pt	pt-line
coil 1	2.1 cm	1.9 cm
coil 2	2.7 cm	2.4 cm
coil 3	2.7 cm	2.4 cm

3. Stability

boundary	kink	ballooning
solid	9.884 E-06	7.211 E-03 (1)
dashed	9.957 E-06	1.897 E-01 (5)

4. Quasi-symmetry

16	0.970 E-04
26	0.241 E-03
40	0.112 E-02

5. Engineering Constraints

coil-coil	0.1599	0.1599	0.1732
min rad. curve	0.1054	0.1053	0.1076

IX. COMMENTS

1. Ballooning stability has been included – presently running.

2. Quasi-symmetry thus far preserved – hopefully do not need to include.

3. Capacity to target given edge included – presently running.

4. Method is slow ! At each PIES iteration requires multiple free-boundary VMEC, COBRA, TERPSICHORE evaluations ...

5. Essentially, a coil-plasma optimizer has been successfully built into the PIES iterative scheme.

X. CONCERNS

1. There is some slight discrepancy between PIES boundary and VMEC-free boundary for the 'healed' coils :
    1. PIES/VMEC not completely converged ?
    2. Existence of small m=10,11,12,14 islands ?
    3. Required to benchmark PIES/VMEC ?
2. Though this difference is small, is it sufficient to alter the kink and ballooning stability ??
3. Perhaps it is required to use the PIES boundary (and perform fixed boundary VMEC etc.) for stability calculations ?