Magnetic Sensor Design For NCSX

Ed Lazarus

Oak Ridge National Laboratory

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This work is an initial phase of a project to build a 3D equilibrium reconstruction code (V3FIT). Collaborators are J. D. Hanson (Auburn University), S. P. Hirshman (Oak Ridge National Laboratory), S. F. Knowlton (Auburn University), L. L. Lao (General Atomics)

Separation of Problem

- Geometric Part - time-consuming calculation (Biot Savart) - precompute and tabulate for a specific coil set and diagnostic set.
- Plasma Part - integration over plasma for specific equilibrium

This separation is made possible by the reciprocity of the flux expressions (discussed later).

Magnetic Flux

- Magnetic field (labeled j) gives flux through a closed loop (labeled i) $\mathbf{c}_i(\lambda)$

$$\Phi_{ij} \equiv \int_{S_i} \mathbf{B}_j \cdot \hat{\mathbf{n}}\, d^2 a_i = \oint_{\partial S_i} \mathbf{A}_j \cdot d\vec{l}_i$$

$$\partial S_i = \mathbf{c}_i(\lambda) \qquad d\vec{l}_i = \mathbf{c}_i'(\lambda)\, d\lambda \qquad \mathbf{A}_j = \mathbf{A}_j(\mathbf{c}_i(\lambda))$$

º Diagnostic coil (sensor) i: flux loop, saddle coil, magnetic probe, Rogowski coil, etc.

º Source of field j: plasma, a magnetic field coil, currents in the vacuum vessel, etc.

º Change from flux integral to line integral: Stoke's Theorem

- Biot-Savart form for the Vector Potential (Coulomb gauge):

$$\mathbf{A}_j(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_{V_j} \frac{\mathbf{J}_j(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\, d^3\mathbf{x}' = \frac{\mu_0 I_j}{4\pi} \oint_{\mathbf{c}_j} \frac{\mathbf{c}_j'(\lambda)}{|\mathbf{x} - \mathbf{c}_j(\lambda)|}\, d\lambda$$

Second relation - special case when current j is restricted to a wire.

Magnetic Flux and Vector Potential

- Substitute expression for Vector Potential into the flux expression:

$$\Phi_{ij} = \frac{\mu_0}{4\pi} \oint_{\mathbf{c}_i} \int_{V_j} \frac{\mathbf{J}_j(\mathbf{x}') \cdot \mathbf{c}_i(\lambda)}{|\mathbf{c}_i(\lambda) - \mathbf{x}'|} \, d^3\mathbf{x}' \, d\lambda$$

- Rearrange, recognizing the expression for the Vector Potential $\mathbf{A}_i$ due to the ***diagnostic coil (sensor) i:***

$$\Phi_{ij} = \int_{V_j} \mathbf{J}_j(\mathbf{x}') \cdot \left[\frac{\mathbf{A}_i(\mathbf{x}')}{I_i} \right] d^3\mathbf{x}'$$

- To calculate the magnetic flux through a loop i, due to a current distributed through the region j, we compute the volume integral of the dot product of the current density in region j with the vector potential (per unit current) of the loop i.

Signals and Responses

$$S_i = \sum_j R_{ij} I_j + \sum (R^{plasma})(J_{plasma})$$

S_i Signal in the ith diagnostic coil (sensor)
R_{ij} Coil response function
I_j current in the jth field coil

- Second summation - plasma response function. Details to follow.
- If S_i is a magnetic flux, then the R_{ij} is just a mutual inductance.
- Coil response function details:

$$R_{ij} = \oint_{\partial S_i} \left[\frac{\mathbf{A}_j}{I_j} \right] \bullet d\vec{\ell}_i = \frac{\mu_0}{4\pi} \oint_{\mathbf{c}_i} \oint_{\mathbf{c}_j} \frac{\mathbf{c}_i(\alpha) \bullet \mathbf{c}_j(\beta)}{\left| \mathbf{c}_i(\alpha) - \mathbf{c}_j(\beta) \right|} d\alpha\, d\beta$$

Note $R_{ji}=R_{ij}$, the relationship is reciprocal as expected for a mutual inductance.

Magnetic Flux Reciprocity

The flux from the plasma through the sensor can be written so as to remove the plasma field and involve only the vector potential of the sensor. We will use the second expression.

$$\Phi_{sensor} = \oint_{\partial S} \mathbf{A}_{plasma} \cdot \mathbf{c}(\ell)\, d\ell =$$

$$\frac{\mu_0}{4\pi} \int_{V_{plasma}} \mathbf{J}_{plasma}(\mathbf{x}) \cdot \left[\frac{\mathbf{A}(x')}{I} \right]_{sensor} d^3\mathbf{x}$$

Magnetic flux through the ith (sensor) due to the plasma current $\mathbf{J}_j$:

$$\Phi_{ij} = \int_{V_j} \mathbf{J}_j(\mathbf{x}) \cdot \left[\frac{\mathbf{A}_i(\mathbf{x})}{I_i} \right] d^3\mathbf{x}$$

Plasma Response Details

- Plasma current $\mathbf{J}_j$ most convenient in VMEC coordinates (s,θ,ζ)
- Vector potential $\mathbf{A}_i$ most convenient in cylindrical coordinates

$$\Phi_{ij} = \int_0^{2\pi/N_p} d\zeta_p \int_0^{2\pi} d\theta \int_0^1 ds\sqrt{g}\,[J^\theta\left(R_\theta \tilde{A}_{iR} + Z_\theta \tilde{A}_{iZ}\right) + J^\zeta\left(R_\zeta \tilde{A}_{iR} + R\tilde{A}_{i\zeta} + Z_\zeta \tilde{A}_{iZ}\right)]$$

$$\tilde{A}_{iR} = \frac{1}{I_i}\sum_{k=1,N_p}\left[\cos\zeta_k A_{ix}(\mathbf{x}') + \sin\zeta_k A_{iy}(\mathbf{x}')\right] \qquad = \frac{1}{I_i}\sum_{k=1,N_p} A_{iR}(\mathbf{x}')$$

$$\tilde{A}_{i\zeta} = \frac{1}{I_i}\sum_{k=1,N_p}\left[-\sin\zeta_k A_{ix}(\mathbf{x}') + \cos\zeta_k A_{iy}(\mathbf{x}')\right] \qquad = \frac{1}{I_i}\sum_{k=1,N_p} A_{i\zeta}(\mathbf{x}')$$

$$\tilde{A}_{iZ} = \frac{1}{I_i}\sum_{k=1,N_p} A_{iz}(\mathbf{x}')$$

$$\mathbf{x}' = R(s,\theta,\zeta_p)\cos\zeta_k\,\mathbf{e}_x + R(s,\theta,\zeta_p)\sin\zeta_k\,\mathbf{e}_y + Z(s,\theta,\zeta_p)\mathbf{e}_z$$

$$\zeta_k = \zeta_p + 2\pi(k-1)/N_p$$

Plasma Response Details cont.

- Used field period symmetry to reduce the integration volume.
- Precompute and store $(\tilde{A}_{iR}, \tilde{A}_{i\phi}, \tilde{A}_{iZ})$ on an (R, ϕ, Z) grid, for each sensor i.
- When computing integral, use summation over VMEC (s,ϕ) grid at each toroidal plane.
- Interpolate $(\tilde{A}_{iR}, \tilde{A}_{i\phi}, \tilde{A}_{iZ})$ onto VMEC (s,ϕ) grid.

- Factors $(R_\phi, Z_\phi, R_\phi, R, Z_\phi)$ take care of geometry of dot product.
- $J^s = 0$.
- We call $(\tilde{A}_{iR}, \tilde{A}_{i\phi}, \tilde{A}_{iZ})$ the plasma response function of the ith sensor.

Code Descriptions

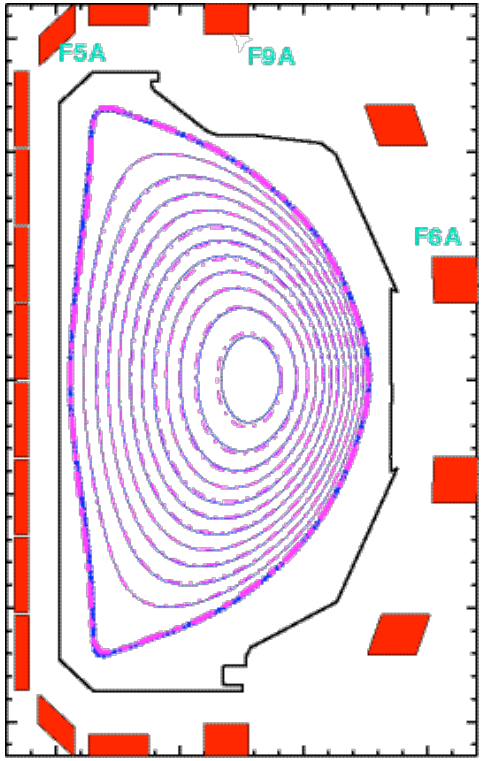
- V3RFUN calculates and tabulates response functions
  - Both coil and plasma response functions.
  - Do Biot-Savart integrals over the diagnostic loops.
  - Writes response functions to files (netCDF).

- V3POST calculates signals
  - Reads in response functions tables calculated by V3RFUN.
  - Reads in equilibrium data from a VMEC “wout” file.
  - Writes signals to file (netCDF).

BENCHMARKING in 2D AGAINST DIII-D

- 18 shaping coils plus Ohmic solenoid
  - single filament representations

	PSF5A (Wb)	*PSI6A (Wb)*	*PSF9A (Wb)*	*MPI6FA322 (Tesla)*
EFIT	0.72719	-3.4350	-0.87030	0.22893
V3POST	0.73885	-3.4426	-0.89424	0.22720
Difference(%)	1.6034	0.22054	2.7510	0.75602

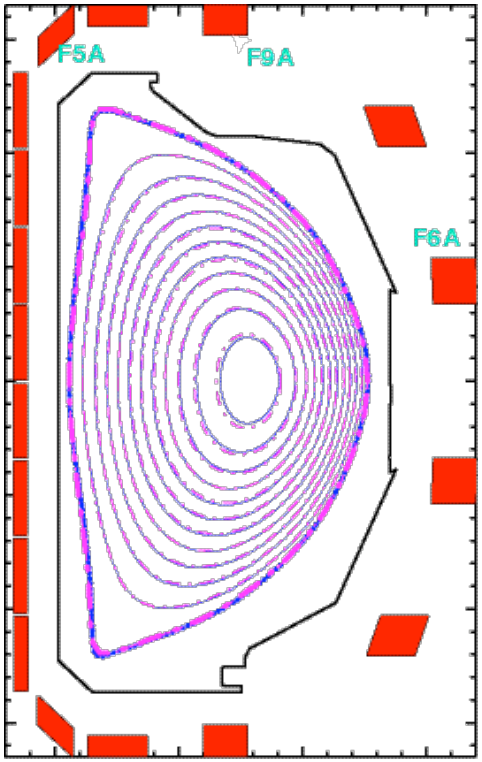


V3POST EXTERNAL COIL CONTRIBUTIONS AGREE WITH EFIT RESULTS

BENCHMARKING in 2D AGAINST DIII-D

EFIT 1.1963 MA
VMEC 1.1963 MA
Fixed Boundary VMEC

PLASMA	*PSF5A (Wb)*	*PSI6A (Wb)*	*PSF9A (Wb)*	*MPI6FA322 (Tesla)*
EFIT	0.62708	3.1785	1.3081	0.12635
V3POST	0.62868	3.1786	1.3106	0.12439
Difference(%)	0.25449	0.00259	0.19151	1.5578



V3POST MAGNETIC SIGNALS FROM THE PLASMA CONTRIBUTION AGREE WITH EFIT RESULTS

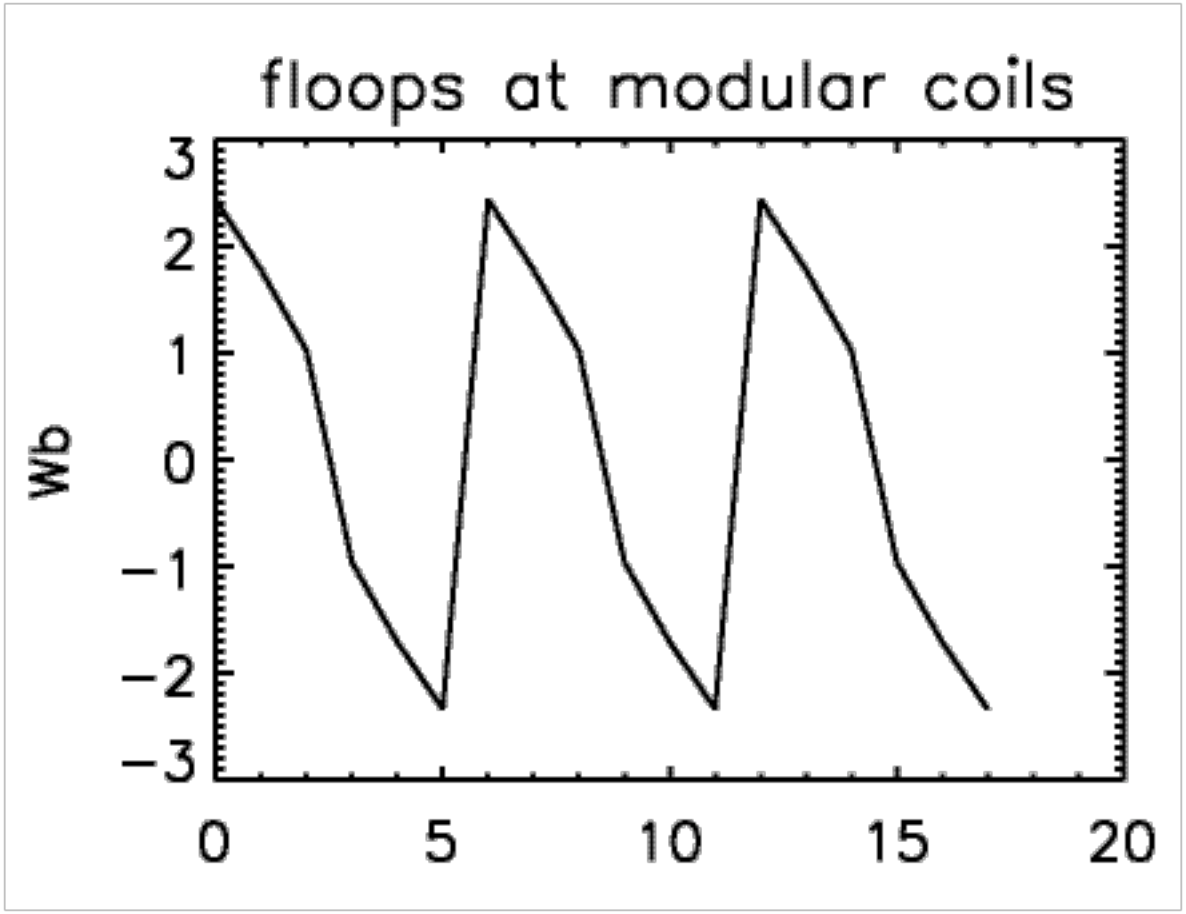
Stellarator Test

- Having determined that the axisymmetric results are correct, we can do a test for non-axisymmetric systems still using D3D. Non-axisymmetry only enters the problem explicitly through the vector potential terms $(\tilde{A}_{iR}, \tilde{A}_{i\square}, \tilde{A}_{iZ})$, via the sum over field periods. We can take a D3D case, then calculate the response tables and equilibrium asserting multiple field periods (N_P=3). The computed signals are identical to the values for N_P=1 demonstrating that this summation is done correctly. Non-axisymmetry also enters the problem through the toroidal variation of **A** and the geometry. We plan to also benchmark against DIAGNO.

Some NCSX Comparisons

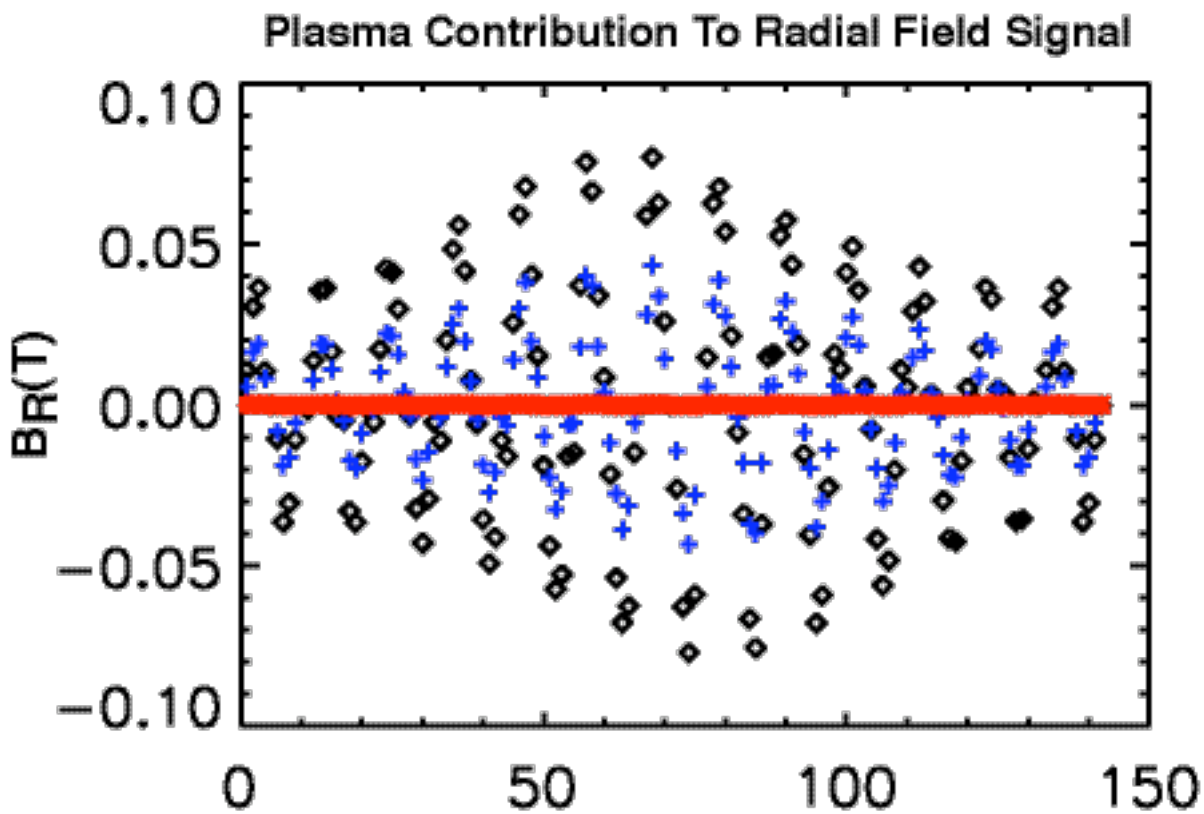
- Define an array of B-probes on the vacuum vessel, over a single field period the array is uniform in (θ,ζ) and each is a triplet with orientations (R,ζ,Z). The probes are 1 turn, 1 cm diameter.
- Define an array of floops at the inner centerline of each modular coils. A floop is a non-planar flux loop that make a complete circuit around the plasma. (Not a saddle coil.)
- Examine 3 NCSX equilibria
  - A: $I_P = 0.5$ kA, $\beta = 0.3\%$ (red)
  - B: $I_P = 77$ kA, $\beta = 2.3\%$ (blue)
  - C: $I_P = 145$ kA, $\beta = 5.6\%$ (black)

Each would be the recipient of the control signal that drives the coil.



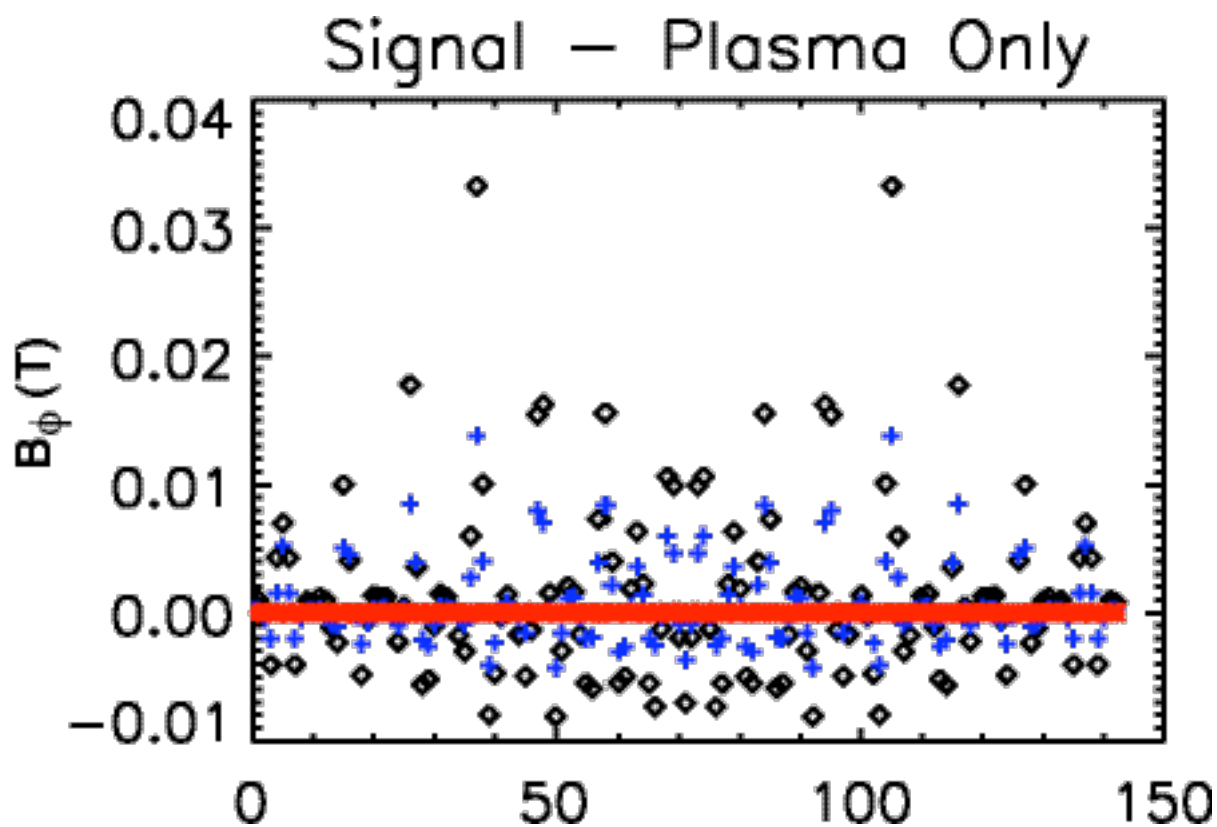
The contribution of the plasma to these floops is negligible.

Plasma Signals Discriminate Equilibrium Changes to Varying Degree

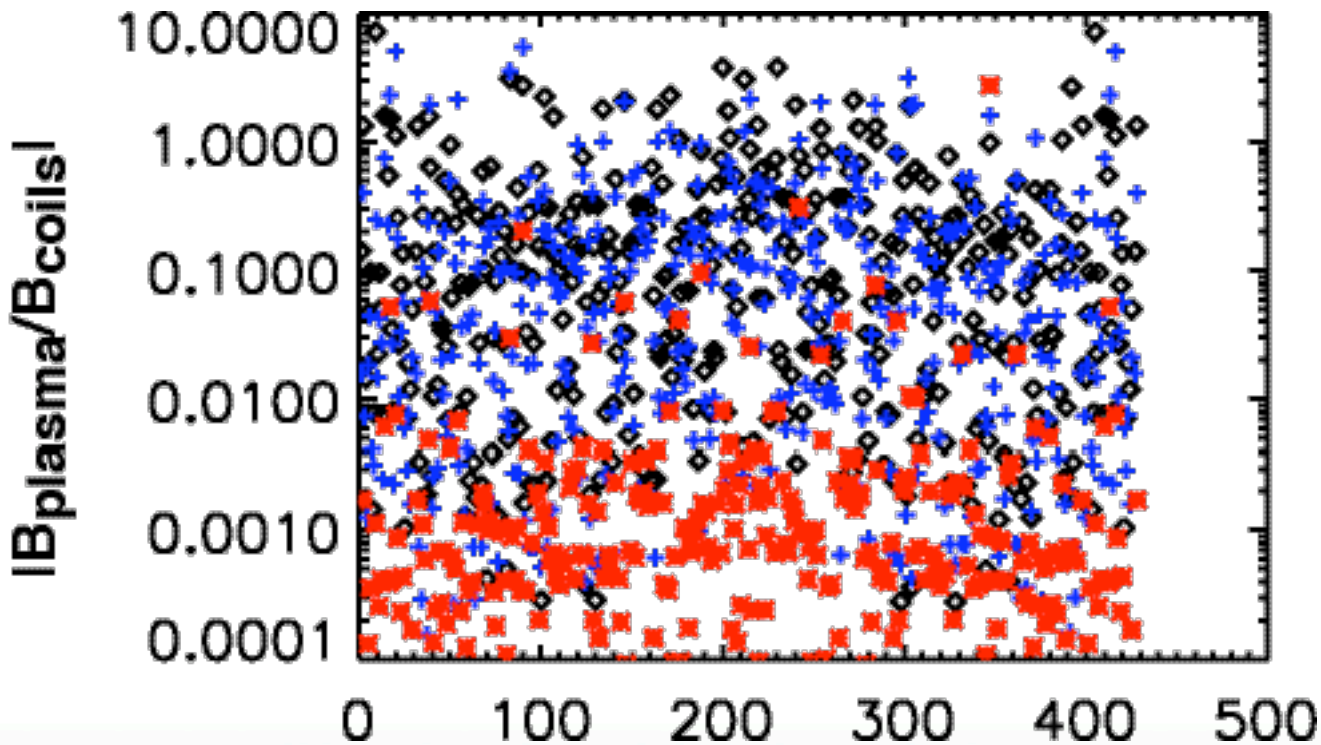


As expected, the plasma field changes with the equilibrium, and is virtually non existent for case A

Results for the other **B** components are similar to B_R but different probes show more sensitivity to equilibrium changes. This is not unexpected.



For plasma analysis and control, the discrimination of plasma field from external field is important. We can plot the ratio for this set of probes. Clearly some will be of more interest than others.



Design of Magnetic Sensor Set

We have completed work on V3RFUN and V3POST and now can calculate signals observed by magnetics for stellarator equilibria.

Certainly we could not contemplate the design of sensors without this capability. Were this a tokamak, we would calculate signals for a large set of equilibria and test whether we could reconstruct the plasma equilibrium we started with sufficient accuracy. (An accurate boundary shape, plasma current and plasma beta would probably be sufficient.) A set of sensors that allowed this over a sufficient range of equilibria would be adequate. Note, this does not say how we would use these sensors for plasma control, only that it is possible.

Design of Magnetic Sensor Set, cont'd

Unfortunately, for the stellarator there is no such reconstruction code. That is a task for the next few years, but we need to commit to a design for the magnetic diagnostic set in the next few months.

We will need to characterize equilibria and then determine whether the information content of the test diagnostic set is adequate to discriminate the characteristics.

Design of Magnetic Sensor Set, cont'd

For example, we might choose as a characterization the Fourier components of the outer boundary shape along with β and ℓ_i.

Then there would be two thing to determine:

1. Is this characterization adequate? Namely, how large an effect is there on the plasma boundary shape at fixed coil currents if higher order moments of the pressure and current profiles are varied.
2. Does our trial set of magnetics contain the necessary information. It is likely that the relationship will not be linear, making this determination non-trivial.

Design of Magnetic Sensor Set, cont'd

Another issue is defining classes of equilibria. For example, we might include in our equilibrium database both Ohmic and beam-heated plasmas. These have very different current profile. It is possible we would not find predictive relationships in our sensor set, but if this were separated we would find such predictive relationships for each class separately.

However we define classes, for each class we can make random variations in the coil currents to alter the plasma shape within a class.

Summary

We have completed the development of V3RFUN and V3POST and can calculate magnetic sensor response for free or fixed boundary VMEC equilibria.

The codes have been tested against EFIT for DIII-D cases.

Some test cases have been done for NCSX and results appear quite reasonable.

We are ready to begin the design of the equilibrium magnetic diagnostic set for NCSX

Other

- We are working to compare our codes to the W7AS DIAGNO code [Gardner, Nucl. Fusion 30(1990)1417].
- We have not yet implemented Rogowski coils.
- A self-inductance calculation is not yet implemented in V3RFUN but we plan to do so.k