Kink Stabilization using Furth-Hartman Coils / Use of SVD for Improving Coil Calculations

National Compact Stellarator Experiment Project Meeting

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1. Improvement in Kink Stability of QAS Configurations using Furth-Hartman Coils

- Improvement in the kink stability of QAS configurations has been achieved by corrugating the plasma boundary with a low M/N helical deformation localized to the outboard midplane region of the plasma.
- The corrugated QAS preserves its quasisymmetry.
- Tilted window-pane coils ("Furth-Hartman" coils) can produce the surface corrugation. A simple estimate yields the requirement $I_{FH} \sim 100kA - t$ (per pane). This estimate is supported by NESCOIL calculations.

- A QAS configuration is calculated by VMEC $\Rightarrow$ plasma boundary $R(\theta, \phi), Z(\theta, \phi)$.
- Re-express the plasma boundary in terms of a quasi-polar radius $\rho_0(\theta, \phi)$ such that

$$R(\theta, \phi) = R_0 + \rho_0 \cos\theta$$
$$Z(\theta, \phi) = Z_0 + \rho_0 \sin\theta.$$

  Then $\rho_0(\theta, \phi)$, together with R_0 and Z_0 defines the plasma boundary.

- Now corrugate the boundary by perturbing ρ_0:

$$R(\theta, \phi) = R_0 + (\rho_0 + \tilde{\rho}) \cos\theta$$
$$Z(\theta, \phi) = Z_0 + (\rho_0 + \tilde{\rho}) \sin\theta,$$

  where $\rho(\tilde{\theta}, \phi)$ has the chosen form

$$\tilde{\rho}/\rho_0 = Ae^{-\theta^2/W^2} \cos(M\theta - N\phi).$$

  The parameters A, W, M, and N are varied and the stability of the corrugated configuration is tested.

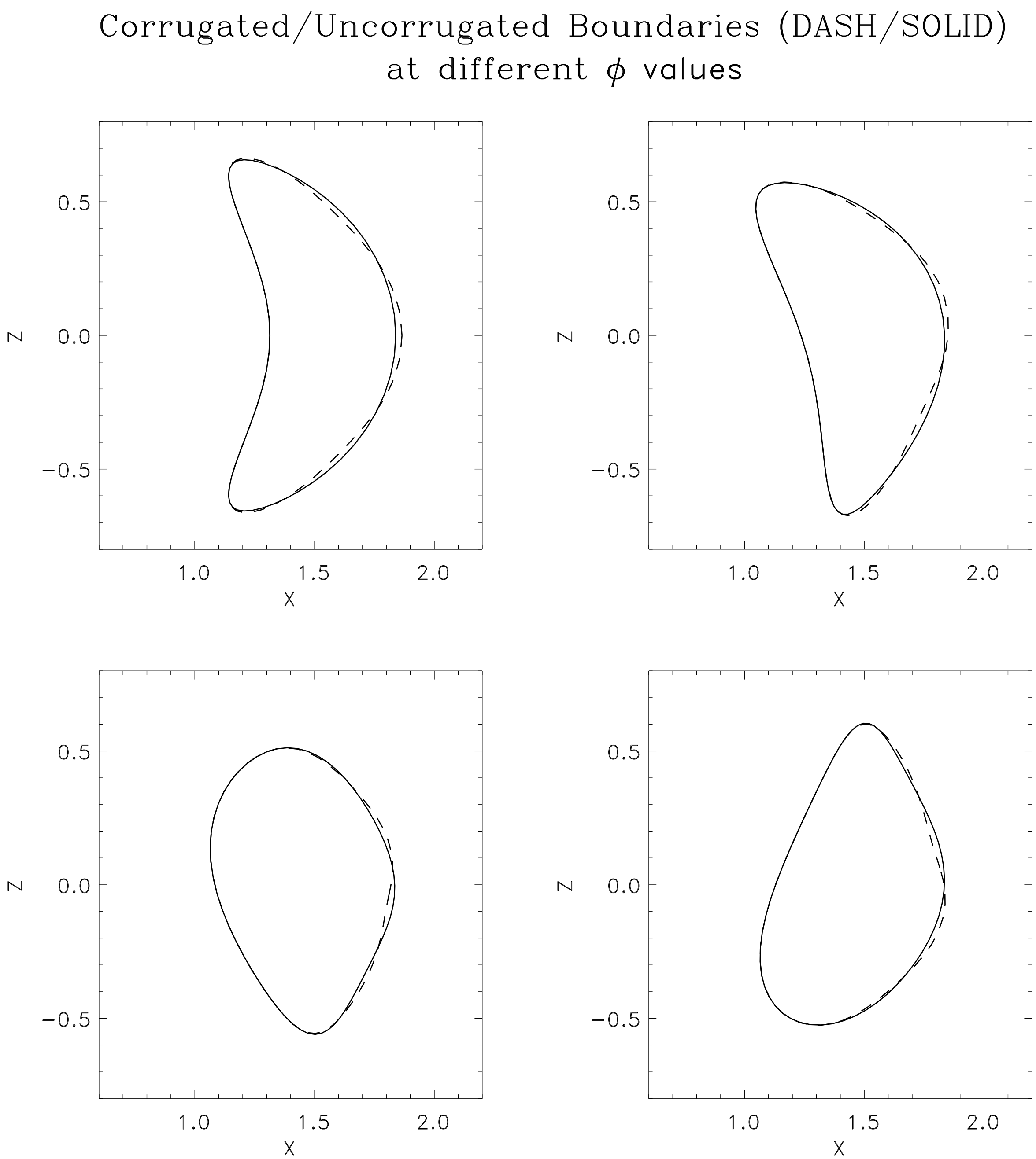


Figure 1: QAS3-g2: $M = 4, N = 1, A = 0.10, W = 1.0$

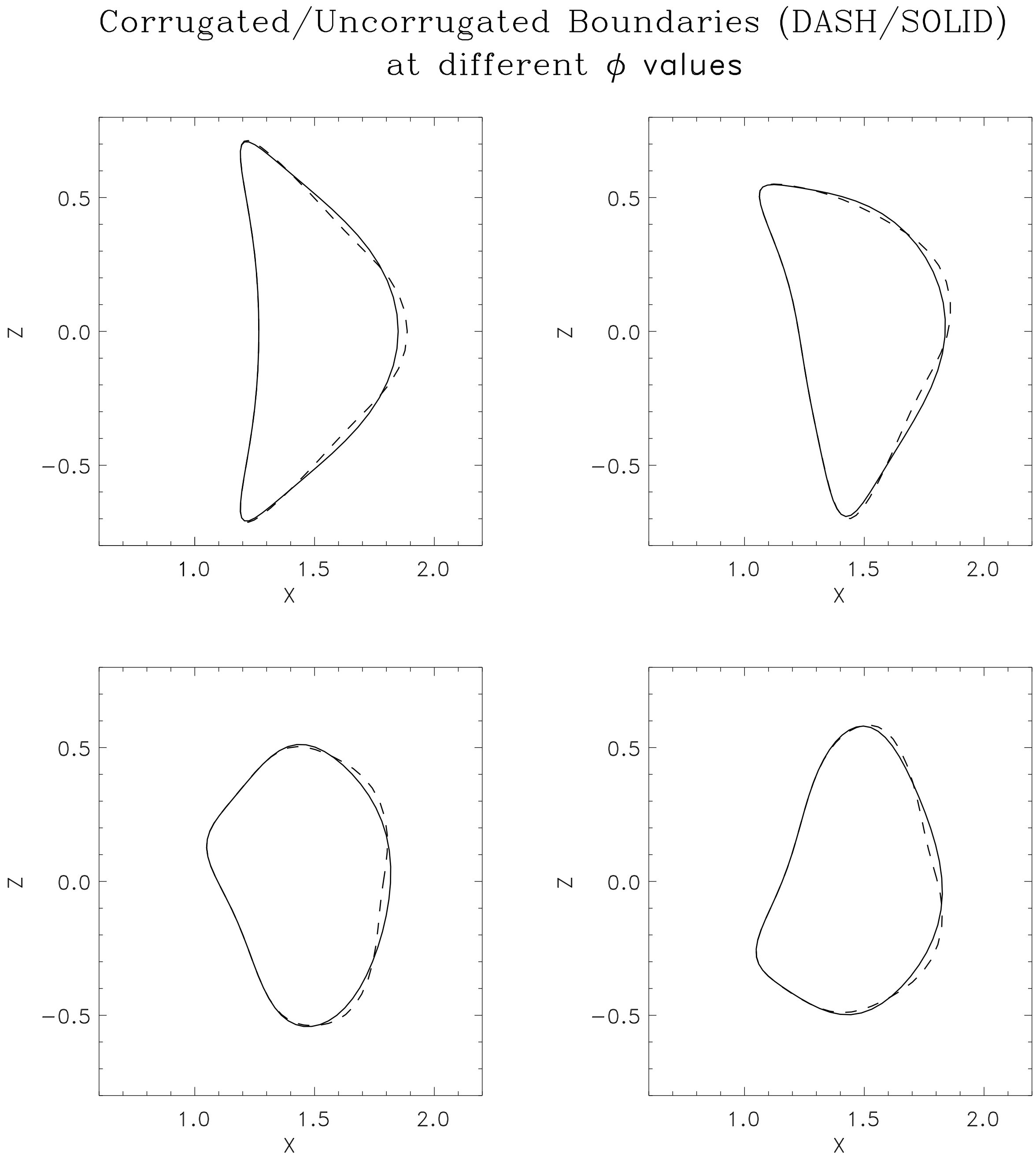


Figure 2: QAS3-h3a: $M = 4, N = 1, A = 0.13, W = 1.0$

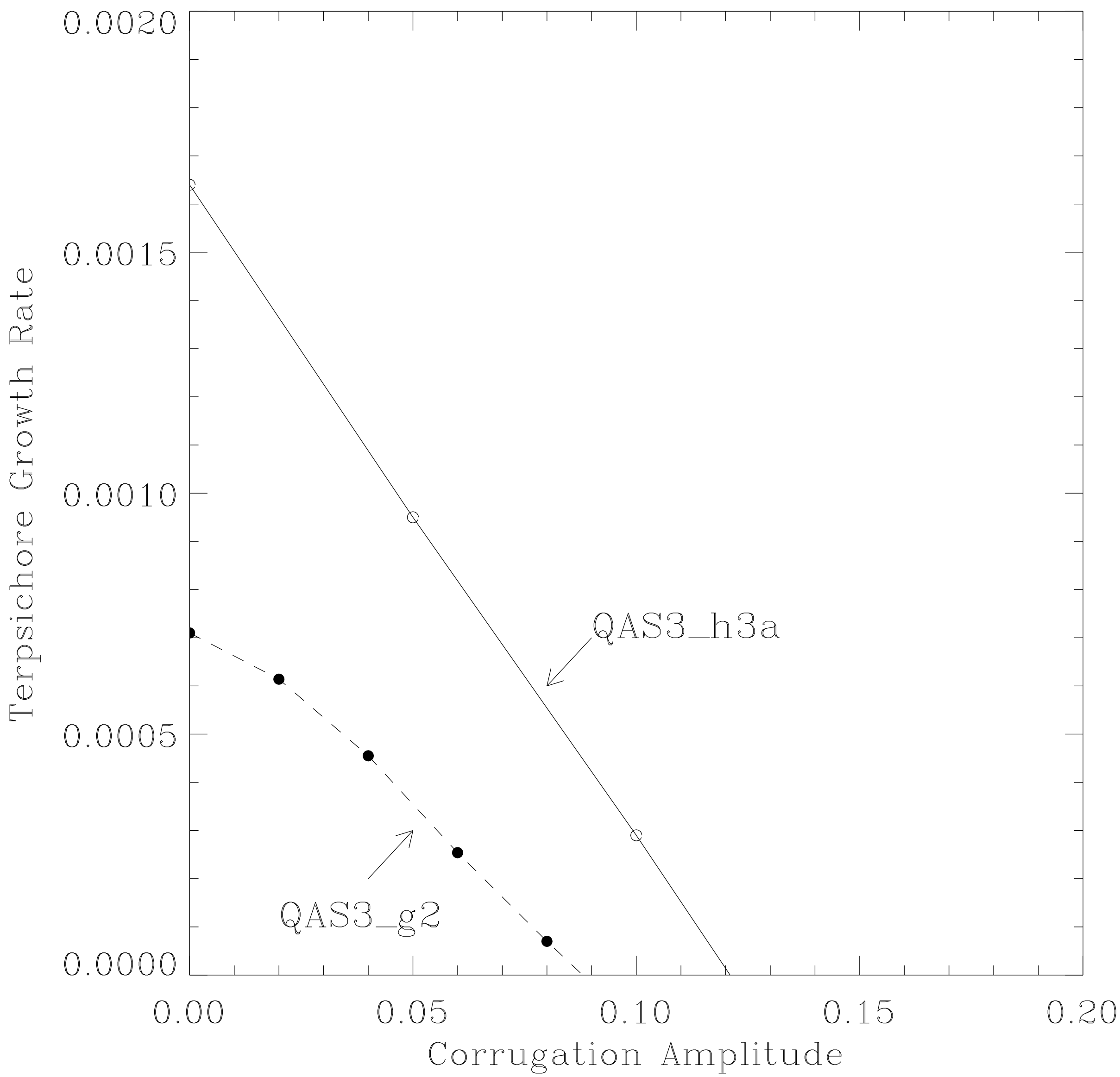


Figure 3: Growth rates versus corrugation amplitudes, A for QAS3-h3a and QAS3-g2 configurations.

$M = 4, N = 1, W = 1.0$

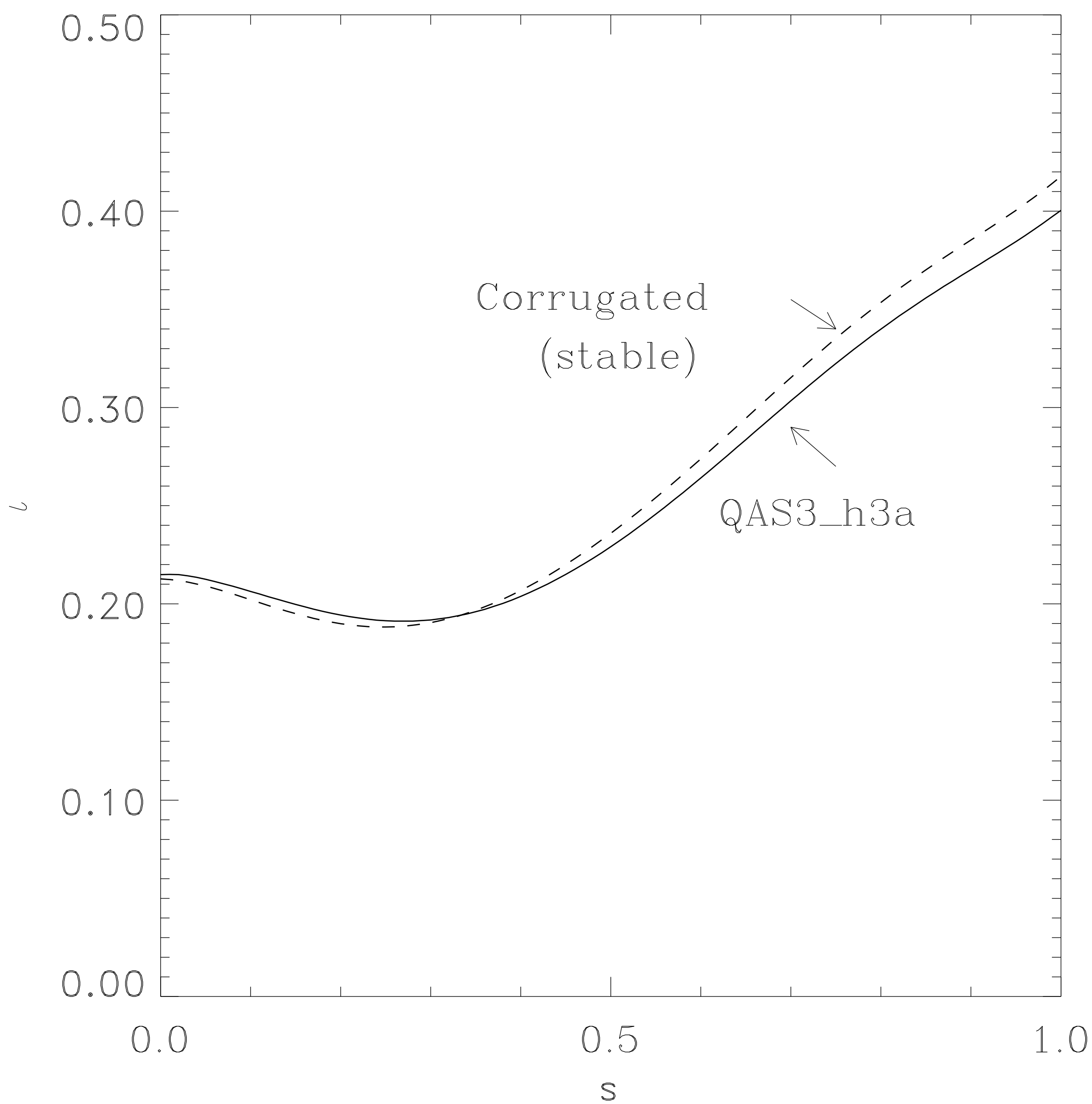


Figure 4: Comparison of iota profiles for QAS3-h3a corrugated and uncorrugated configurations. The primary reason for stabilization is the change in shape, not in ι or the shear.

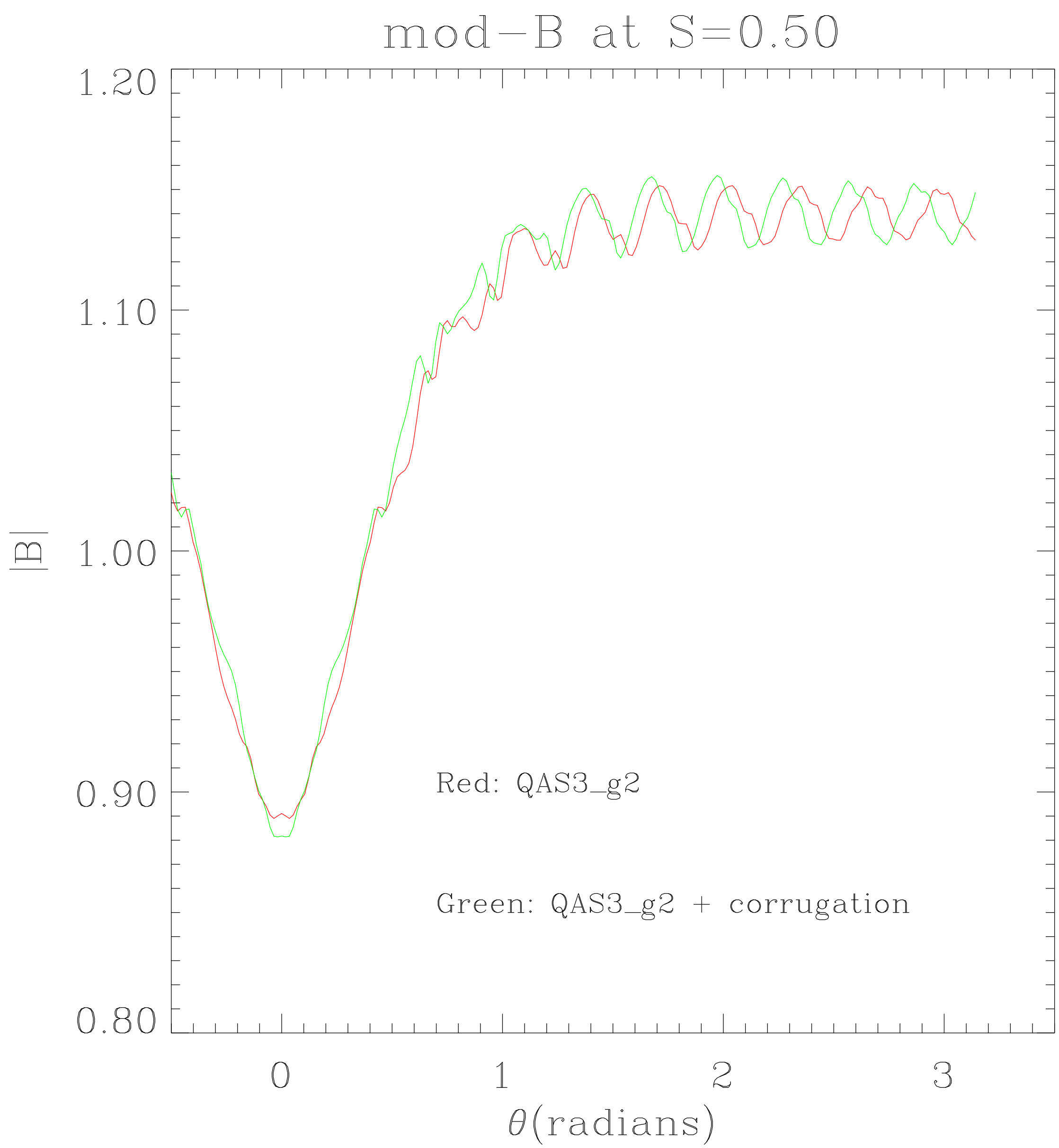


Figure 5: Comparison of ModB along field line for corrugated and uncorrugated QAS3-g2 configurations. The Quasi-symmetry is preserved at the level of corrugation amplitude required for stabilization.

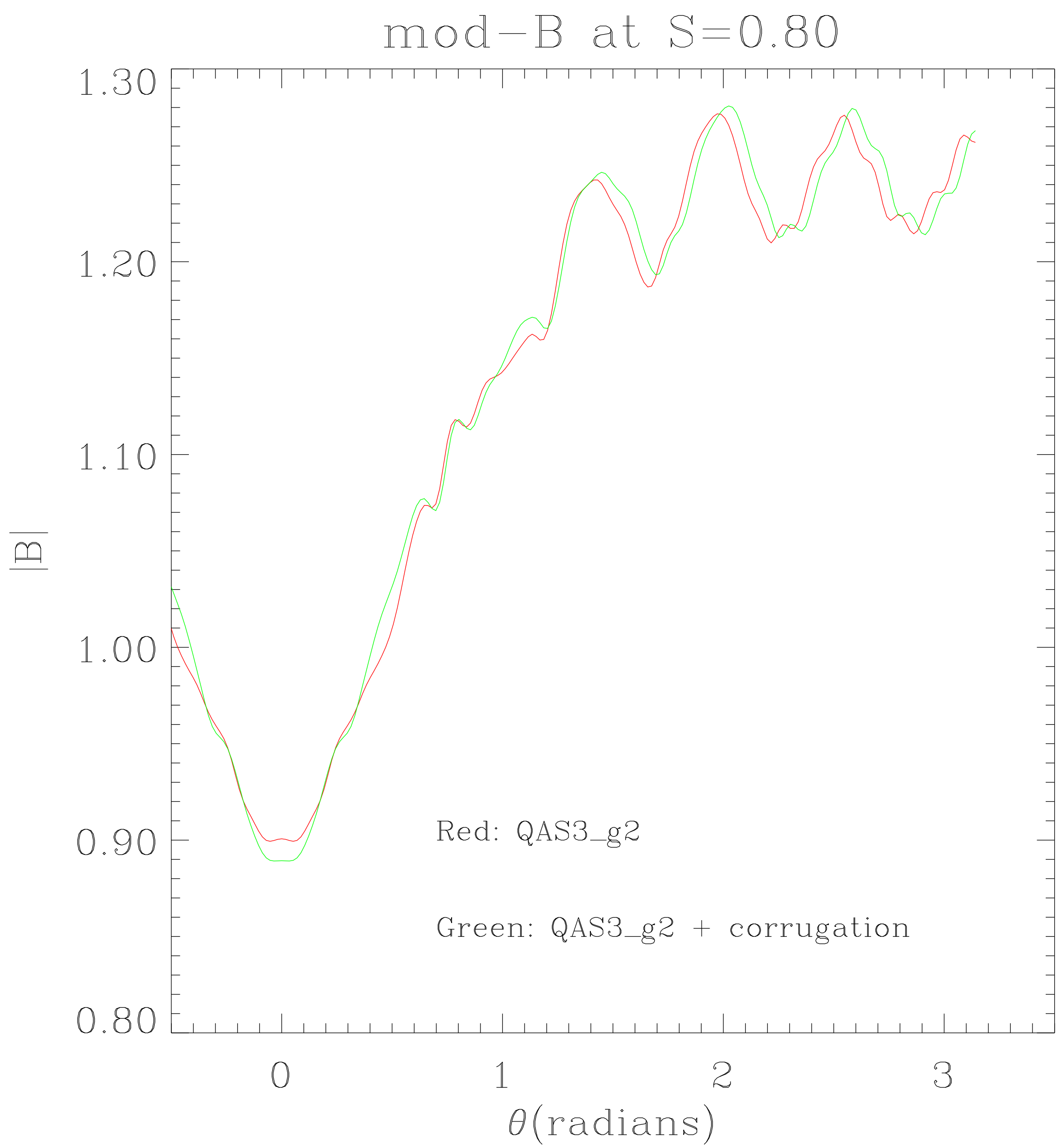

mod-B at S=0.80
1.30
1.20
1.10
1.00
0.90
0.80
|B|
0
1
2
3
θ(radians)
Red: QAS3_g2
Green: QAS3_g2 + corrugation

Figure 6:

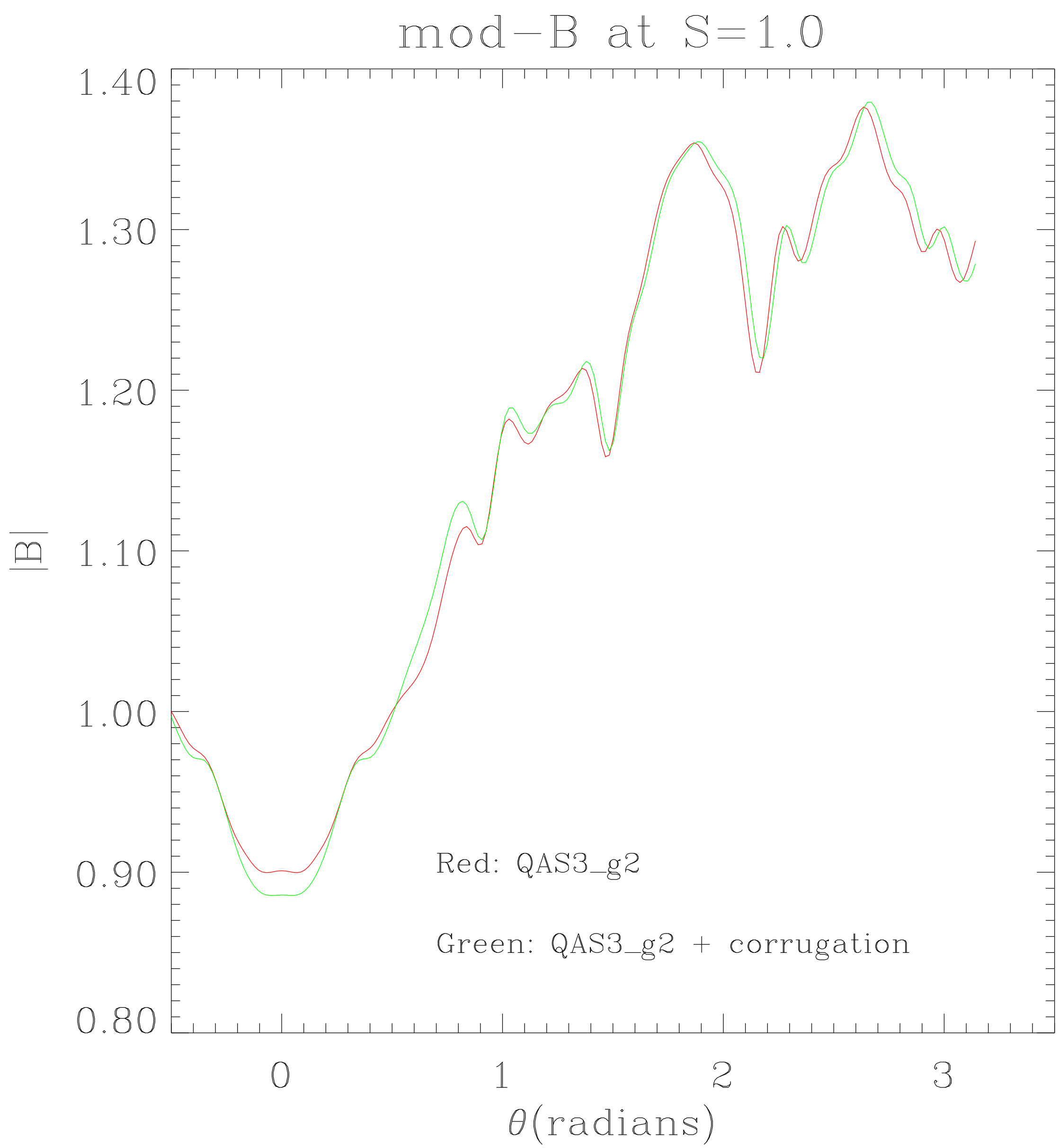

mod−B at S=1.0
1.40
1.30
1.20
1.10
1.00
0.90
0.80
|B|
0
1
2
3
θ(radians)
Red: QAS3_g2
Green: QAS3_g2 + corrugation

Figure 7:

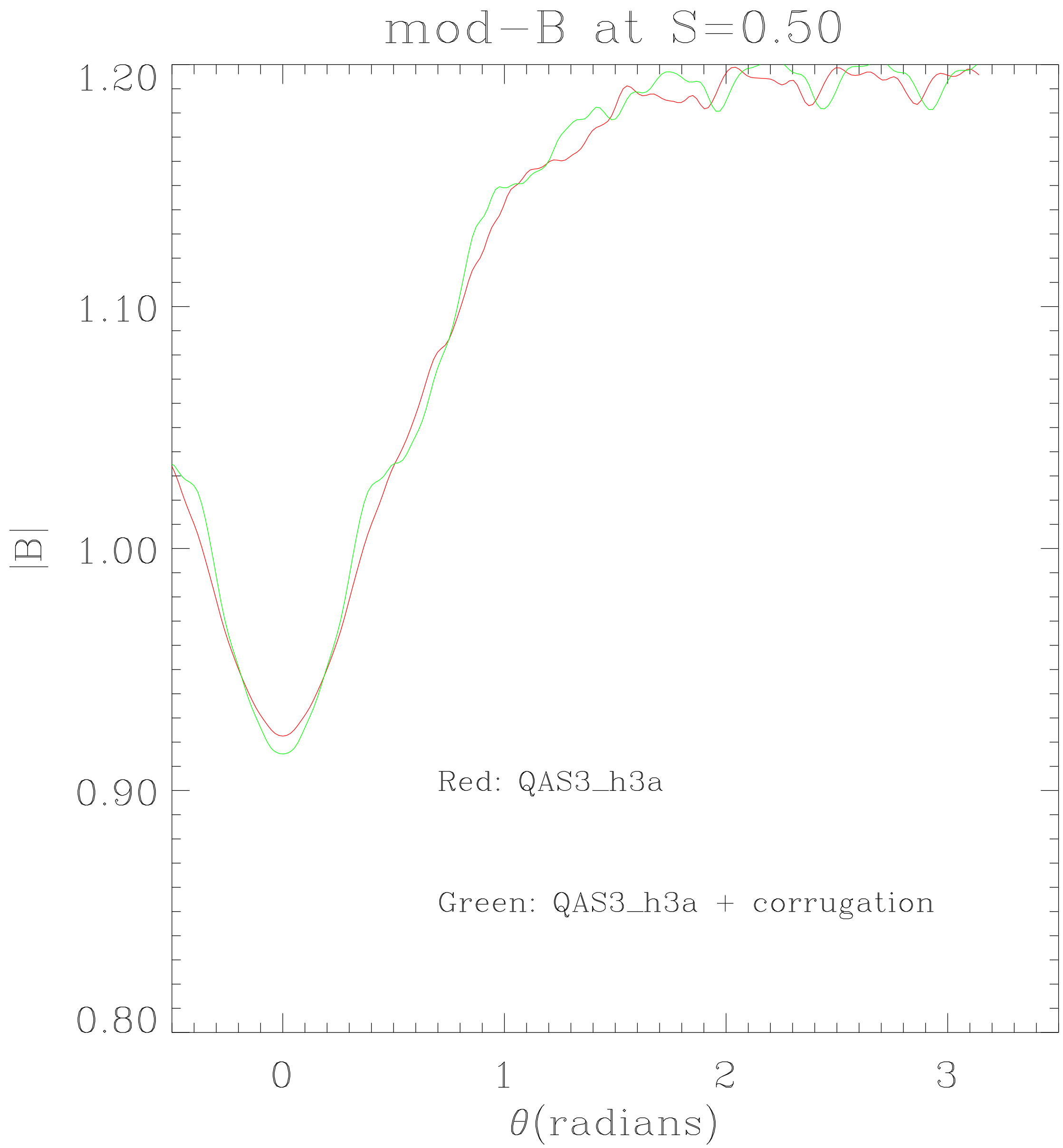
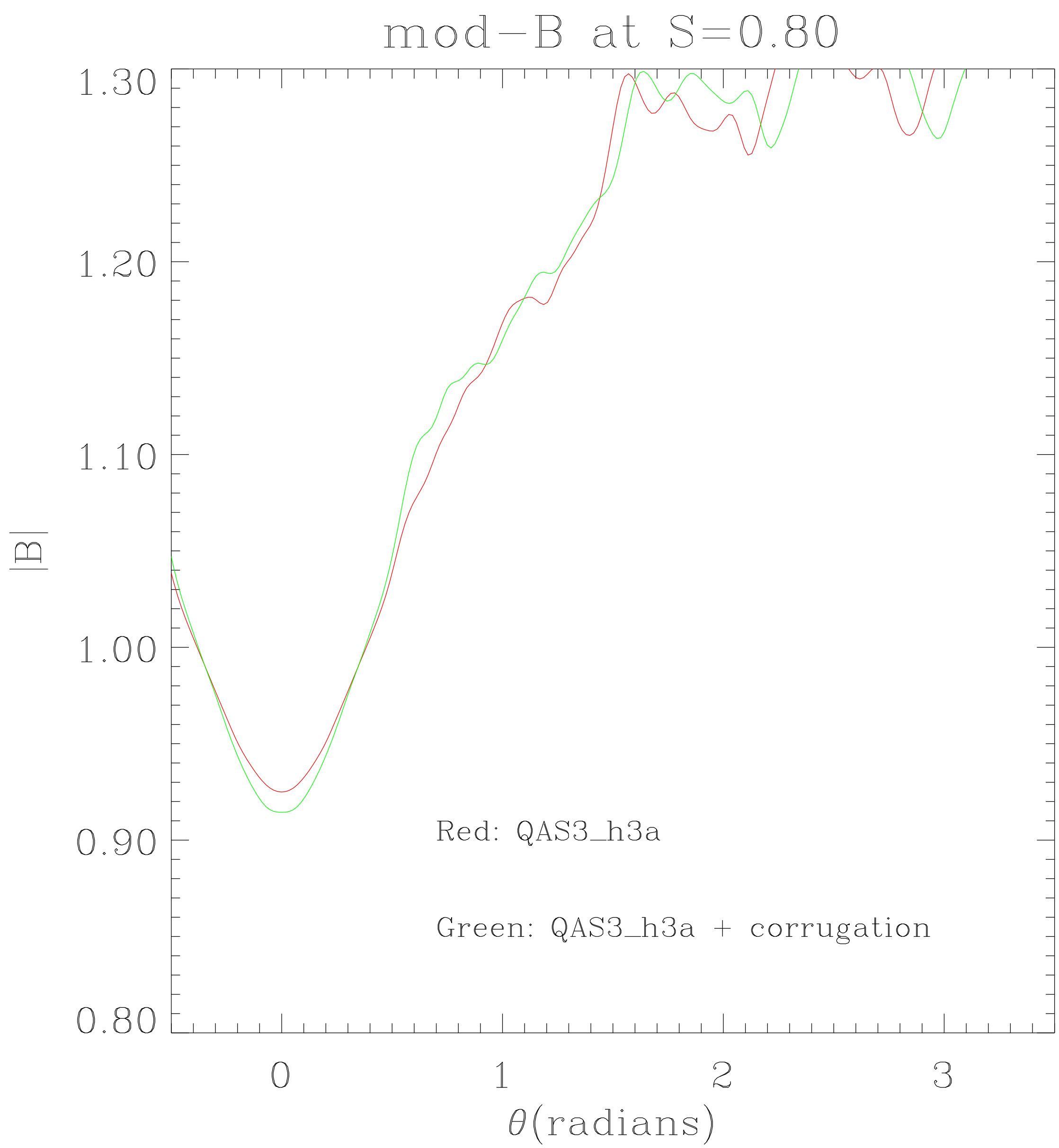


Figure 8: Comparison of ModB along field line for corrugated and uncorrugated QAS3-h3a configurations. The Quasi-symmetry is preserved at the level of corrugation amplitude required for stabilization.


mod−B at S=0.80
1.30
1.20
1.10
1.00
0.90
0.80
|B|
0
1
2
3
θ(radians)
Red: QAS3_h3a
Green: QAS3_h3a + corrugation
Figure 9:

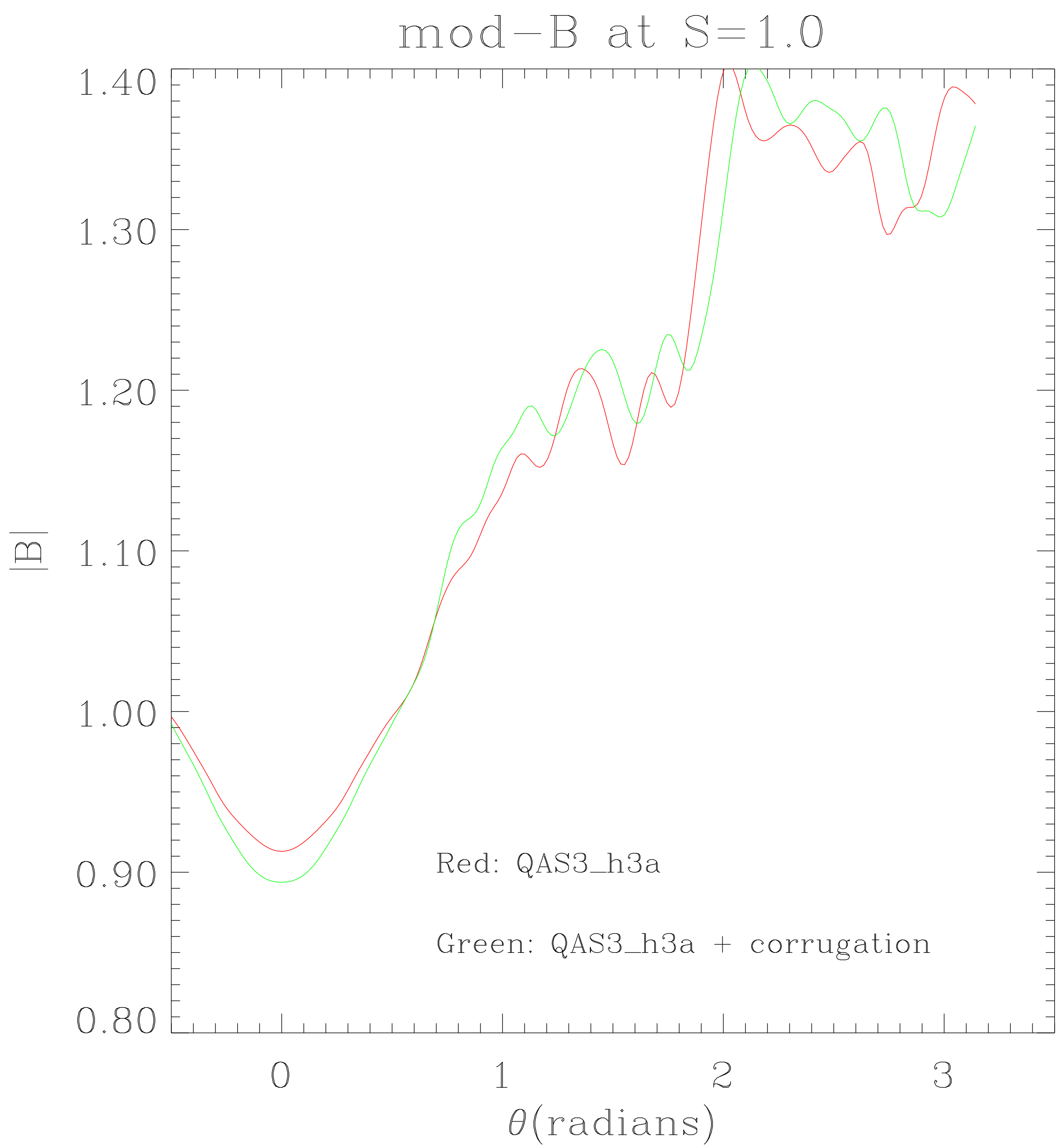

mod−B at S=1.0
1.40
1.30
1.20
1.10
1.00
0.90
0.80
|B|
0
1
2
3
θ(radians)
Red: QAS3_h3a
Green: QAS3_h3a + corrugation

Figure 10:

We see that given a surface corrugation can be stabilizing for the kink mode. The question is, what size coils (Furth-Hartman coils) are needed to produce a given corrugation.

- Analytic estimates based on a "straight stellarator" approximation have been given (see my presentation of 6/18/98):

$$\tilde{\rho} = \Sigma\, \tilde{\rho}_{m,n} \cos\left(m\theta - n\phi\right)$$

$$\sum_{m,n} (n - \bar{\iota}_0 m)\tilde{\rho}_{m,n} \sin\left(m\theta - n\phi\right) = \frac{\tilde{B}^{\rho}}{B_0^{\phi}}.$$

is relation of $\tilde{\rho}$ to $\tilde{B}^{\rho}$ at the plasma surface.

After expanding model currugation form of $\tilde{\rho}$ and calculating sheet current giving rise to $\tilde{B}^{\rho}$ (making "straight stellarator" approximation, obtain

$$I_{FH} = \frac{AW}{\sqrt{\pi}} \frac{B_0 R}{\mu_0} \frac{r_p}{r_c} \sum_m \frac{\frac{m}{N^2}(\frac{N}{m} - \bar{\iota}_0) e^{-(L-l)^2 W^2/4}}{K'_m(N\frac{r_c}{R}) I'_m(N\frac{r_p}{R})}$$

- For calculations, assume a minimum separation distance between the inner edge of the F-H coil and the plasma edge of 12cm (5cm SOL + 3.5cm Tiles/Liner + 3.5cm gap) and a max coil current density of $J = 3.0kA/cm^2$. The calculations indicate $I_{F-H} \sim 100kA$ and are backed up by NESCOIL calculations where current potentials are compared (subtracted) for corrugated and uncorrugated plasmas.

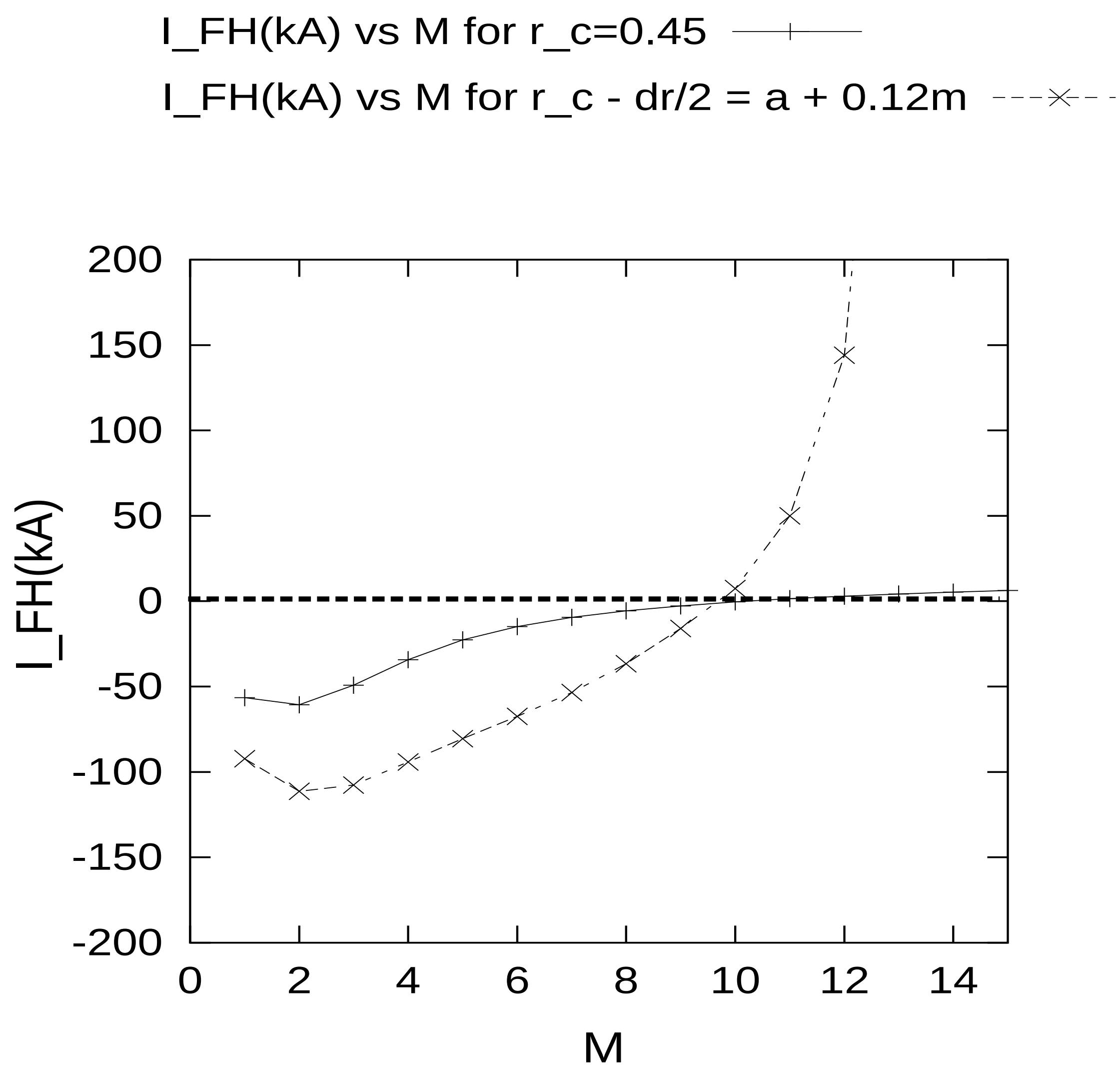


Figure 11: Furth-Hartman coil current estimate for QAS3 configuration. $A = 0.10, W = 1.0, N = 1$, various M.

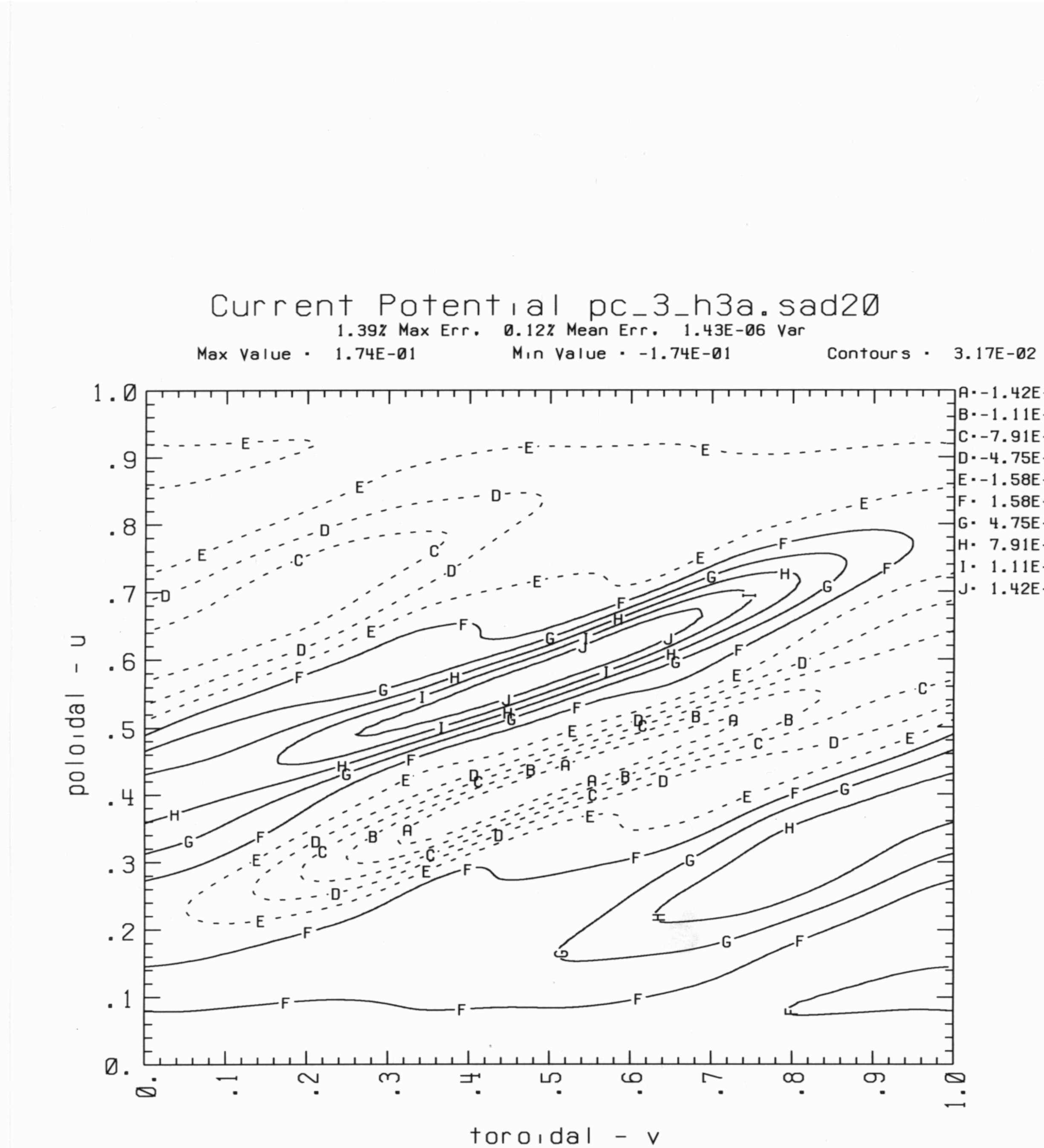


Figure 12: Current potential for uncorrugated qas3-h3a configuration

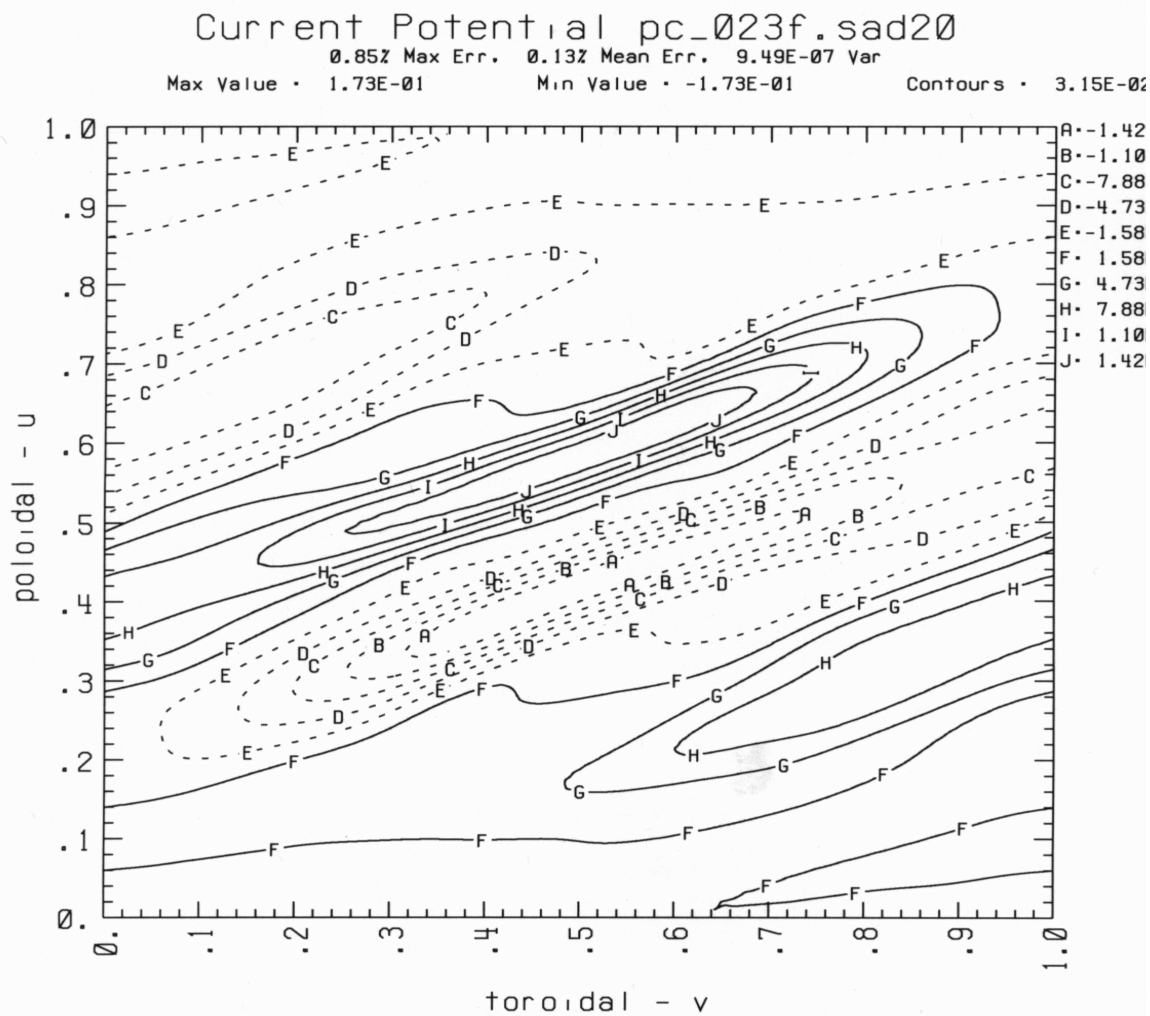


Figure 13: Current potential for corrugated qas3-h3a configuration

78.5 KA

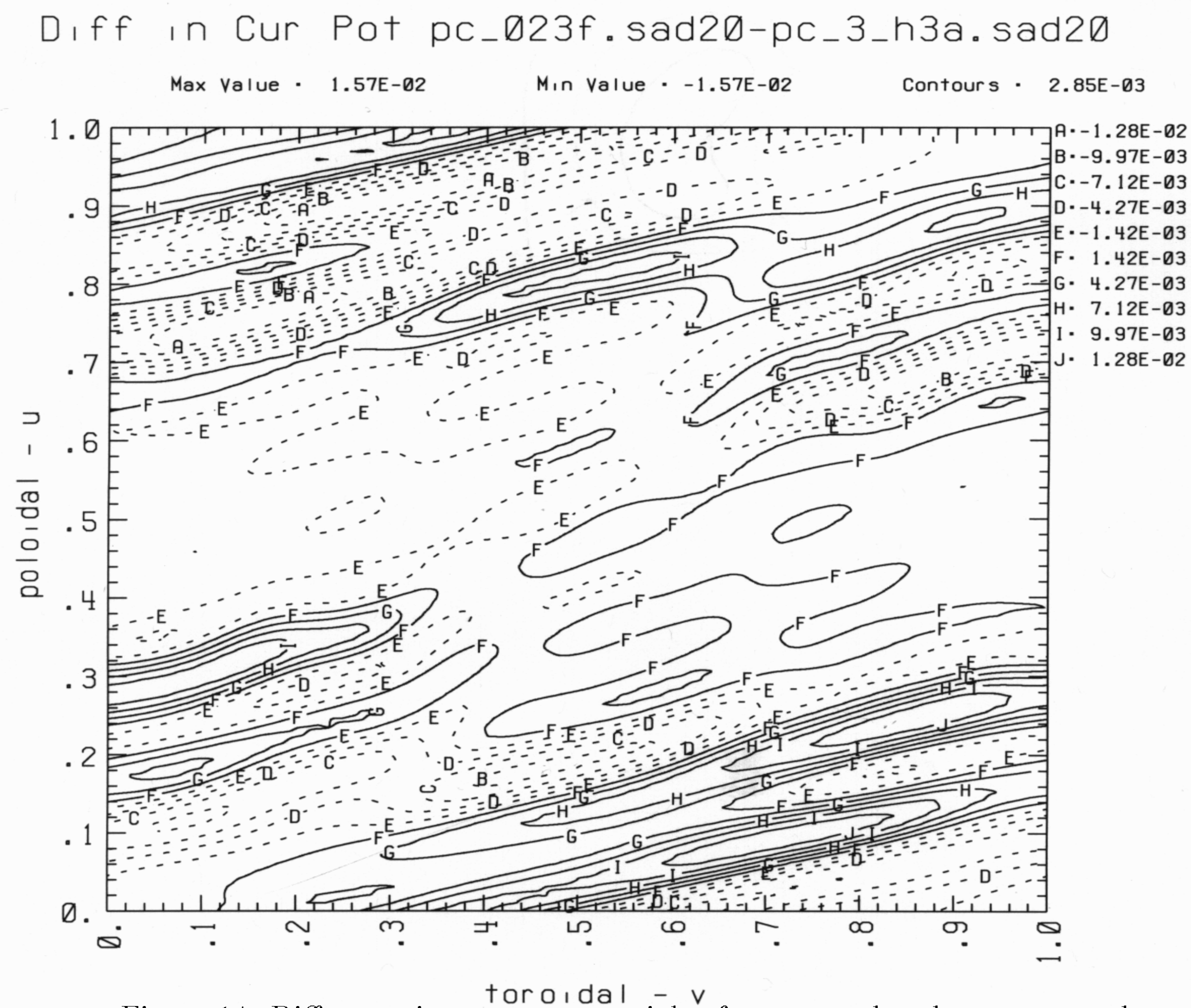


Figure 14: Difference in current potentials of corrugated and uncorrugated congigurations.

2. SVD for Improving the Current Potential Calculation in NESCOIL

- NESCOIL seeks to calculate a sheet current distribution (on a coil surface ∂_C that encloses the plasma) which satisfies continuity of B-normal at the plasma edge

$$\mathbf{B}_{tot} \cdot \hat{\mathbf{n}} = 0. \tag{1}$$

  $\mathbf{B}_{tot}$ is the total magnetic field, part of which is known (internal plasma current contribution and external axisymmetric fields), and part of which is unknown (the contribution of saddle coils).

- Let b_i be the known normal field at a point "i" on the plasma boundary Then Eq. (1) can be re-written as

$$(\mathbf{B}_{\mathbf{saddle}} \cdot \hat{\mathbf{n}})_i = b_i.$$

- The current potential for the sheet current is identical to a dipole moment distribution $m(\theta, \phi)$ corresponding to dipoles normal to ∂_C.

- If we wish to take into account forbidden regions for the placement of coils, Allen Boozer has pointed out that approximating $m(\theta, \phi)$ by a discrete set of dipoles is appropriate.
- If we ignore ports, etc, a more efficient procedure is to expand m in Fourier modes (Merkel, Varenna Workshop Aug 1987, p25):

$$m(\theta, \phi) = \sum_{m=0,n=-N}^{M,N} \hat{\Psi}_{mn} sin(m\theta - n\phi)$$

in which case Eq. (1) becomes

$$\sum \hat{\Psi}_{mn} k_{mn}(\theta_i, \phi_i) - b_i = 0$$

where

$$k_{mn}(\theta, \phi) = \int_0^{2\pi} \int_0^{2\pi} d\theta_j d\phi_j k(\theta_j, \phi_j) sin(m\theta_j - n\phi_j),$$

$$k(\theta_j, \phi_j, \theta, \phi) = \frac{\mu_0}{4\pi} \left[\frac{3(\hat{\mathbf{n}}_j \cdot \mathbf{r}_{ij})(\hat{\mathbf{n}}_i \cdot \mathbf{r}_{ij})}{r_{ij}^5} - \frac{\hat{\mathbf{n}}_j \cdot \hat{\mathbf{n}}_i}{r_{ij}^3} \right]$$

- Whether or not one is dealing with dipoles directly (see my presentation of 9/9/98), or the above Fourier method of NESCOIL, a matrix equation of the type

$$\sum_\sigma D_{i\sigma} k_\sigma = b_i.$$

In NESCOIL, σ is an index labelling a particular Fourier mode. If there are N_p points on the plasma boundary and N_σ Fourier coefficients of $m(\theta, \phi)$, the matrix D has dimension $N_p \times N_\sigma$. Assuming $N_\sigma < N_p$ a Least Squares solution can be sought.

- The original form of NESCOIL does not deal with the non-square matrix, D, directly, and forms the "normal L-S" equations (from Eq. (2)), by premultiplying both sides of the matrix equation by D^T. This is equivalent to minimizing $\Sigma_i(\mathbf{B} \cdot \hat{\mathbf{n}})_i^2$. The normal equations are ill-conditioned. Without regularization, a "rat's nest" is obtained if a large number of Fourier modes are retained. Regularization (smoothing) is achieved by throwing out high m, n Fourier modes. However this is at the expense of accuracy (in satisfying the true boundary conditions for B-normal at the plasma edge).

- We have confronted the regularization issue by solving the original (non-square) matrix equation by a Singular Value Decomposition (SVD) of D, removing ill-conditioning of the pseudoinverse by explicitly zeroing the small singular values.

- In the 9/9/98 presentation, we showed promising results using SVD applied to the pure dipole representation: For a 4-period QAS, current potential solutions were obtained which were (a) smoother than solutions obtained by NESCOIL, and (b) had smaller Max and Mean errors. We also showed that for a 3-period QAS, smooth solutions were obtained for coil-to-plasma separation distances that were greater than those achievable with the old version of NESCOIL.
- Prashant Valanju has now incorporated the SVD method into NESCOIL and repeated the improved behavior of the 4-field period QAS test configuration.