Optimization of the Current Potential for Stellarator Coils

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Stellarator plasma confinement devices have no continuous symmetries, which makes the design of appropriate coils far more subtle than for axisymmetric devices such as tokamaks. The modern method for designing coils for stellarators was developed by Peter Merkel. Although his method has yielded a number of successful stellarator designs, Merkel's method has a systematic tendency to give coils with a larger current than that required to produce a stellarator plasma with certain properties. In addition, Merkel's method does not naturally lead to a coil set with the flexibility to produce a number of interesting plasma configurations. The issues of coil efficiency and flexibility are addressed in this paper by a new method of optimizing the current potential, the first step in Merkel's method. The new method also allows the coil design to based based on a freer choice for the plasma-coil separation and to be constrained so space is preserved for plasma access.

I. Introduction

A major advance in the design of stellarators was J. Nührenberg's concept of optimizing stellarator configurations by varying the shape of the outermost surface of the plasma. Early successes were stellarators with quasi-helical symmetry[1] and the design for the W7-X stellarator.[2] The shape of the outermost surface is determined by the external coils, and the magnitude of the magnetic field in the stellarator is set by the enclosed toroidal flux. The plasma equilibrium equation, $\nabla p = \vec{j} \times \vec{B}$, implies $\vec{B} \cdot \nabla p = 0$, so the normal field on the plasma surface due to the set of coils that is being designed must be equal and opposite to the normal field due to other coils, such as toroidal field coils, and plasma currents. In this paper, the toroidal flux will be assumed to be produced by a given set of toroidal field coils. The task is the design of an optimal set of supplemental coils that cancel the normal component of the magnetic field on the plasma surface. There are two optimizations: (1) the optimization of the stellarator configuration, which is carried out by varying the plasma shape, and (2) the optimization of the coils to produce that configuration.

The standard method to optimize the coils to produce a given stellarator configuration[3,4] is that of Peter Merkel. The first step is the determination of the current on a toroidal surface, the coil surface, that approximates the desired location of the coils. The current on a given toroidal surface is defined by a single function, the current potential $\kappa(\theta,\varphi)$, which is a function of the poloidal, θ, and the toroidal, φ, angles. The coil surface can be given in the form

$$\vec{X}_c(\theta,\varphi) = R(\theta,\varphi)\,\hat{R}(\varphi) + Z(\theta,\varphi)\,\hat{Z} \qquad (1)$$

with (R,φ,Z) cylindrical coordinates. An example of a simple toroidal surface is $R(\theta,\varphi) = R_0 + a$

$\cos\theta$ and $Z = a \sin\theta$ with R_o and a constants. The current density satisfies two constraints. It must be divergence-free and must lie in the coil surface, $\vec{j} \cdot \nabla r = 0$ with r any well-behaved radial coordinate such that $r = r_c$ gives the coil surface. These two constraints imply the current density has the form

$$\vec{j} = -\delta(r - r_c)\, \vec{\nabla} r \times \vec{\nabla} \kappa(\theta, \varphi), \tag{2}$$

which defines the current potential $\kappa(\theta,\varphi)$. The current flows along constant-κ contours because $\vec{j} \cdot \vec{\nabla}\kappa = 0$. The Dirac delta function $\delta(r\text{-}r_c)$ has the units of $1/r$, so κ has units of $\vec{j}$ times an area, which is Amperes. The current potential is found in Merkel's method by minimizing the mean-square of the normal field, $\int(\vec{B}\cdot\hat{n})^2 da$, on the desired plasma surface.[3] The turns of the coil system are then chosen to lie along constant-κ contours. The change in κ between contours, which has units of Amperes, gives the required current in each coil turn.

This paper proposes a new method for optimizing the current potential. The new method emphasizes the coil flexibility for producing many desirable plasma configurations, the coil efficiency (minimization of $\langle j^2 \rangle$ and Ohmic losses), and the preservation of space free of coils for plasma access (port space).

To make the distinctions clearer, Merkel's method will be described using the notation of the new method of optimizing the current potential. Merkel's method relates two matrix vectors, a flux vector $\vec{\Phi}$ and a current vector $\vec{I}$, by an inductance matrix $\vec{L}$. The normal magnetic field on the plasma surface due to sources other than the coils being designed is given by the magnetic flux vector, $\vec{\Phi}$. Let $f_i(\theta,\varphi)$ be any complete set of dimensionless functions on the plasma surface. One example of such a set of functions is the trigonometric functions; another is f_i constant in a $\delta\theta\delta\varphi$ cell on the plasma surface and $f_i = 0$ elsewhere. The i^{th} component of the flux vector is

$$\Phi_i \equiv \int_{\substack{\text{plasma}\\ \text{suface}}} f_i(\theta,\varphi)\, \vec{B}_o \cdot d\vec{a} \tag{3}$$

with $\vec{B}_o$ the field due to all other sources than the coils being designed. The current vector, $\vec{I}$, on the coil surface is defined using any general set of dimensionless functions, $g_i(\theta,\varphi)$, on that surface. The current potential is written as

$$\kappa(\theta,\varphi) = \sum_j I_j\, g_j(\theta,\varphi) \tag{4}$$

with the components of the current vector I_j having units of Amperes. The L_{ij} component of the inductance matrix $\vec{L}$ is defined by

$$L_{ij} I_j \equiv \int_{\substack{\text{plasma}\\ \text{surface}}} f_i(\theta,\varphi)\, \vec{B}_j \cdot d\vec{a} \tag{5}$$

with $\vec{B}_j$ the magnetic field produced by the current potential $\kappa = I_j g_j(\theta,\varphi)$ on the coil surface. (The convention of an implied sum over a repeated index is not being followed; all sums will be denoted explicitly.)

If the functions $f_i(\theta,\varphi)$ used to define the flux components, Eq. (3), are appropriately orthogonalized, Merkel's method for finding the current potential is equivalent to the minimization of the error $\mathcal{E}$ with

$$\mathcal{E}^2 \equiv (\vec{\Phi} - \overleftrightarrow{L}\cdot\vec{I})^T \cdot (\vec{\Phi} - \overleftrightarrow{L}\cdot\vec{I}). \tag{6}$$

The superscript T denotes the transpose of a matrix, $(L_{ij})^T = L_{ji}$ or the change of a column vector into a row vector. The minimum of $\mathcal{E}$ can be found by a number of techniques. These techniques are equivalent to solving, as well as is possible,

$$\overleftrightarrow{L}\cdot\vec{I} = \vec{\Phi}, \tag{7}$$

for the current $\vec{I}$.

Probably the best method for solving Equation (7), as well as is possible, is by the use of Singular Value Decomposition (SVD) techniques. These techniques were recently introduced into coil design by Neil Pomphrey. The SVD theorem[5] says any real matrix can be written as

$$\overleftrightarrow{L} = \overleftrightarrow{U}\cdot\overleftrightarrow{\ell}\cdot\overleftrightarrow{V}^T \tag{8}$$

with $\overleftrightarrow{\ell}$ a diagonal matrix with diagonal elements ℓ_i and $\overleftrightarrow{U}$ and $\overleftrightarrow{V}$ orthogonal matrices. (An orthogonal matrix multiplied by its transpose is the unit matrix.) Equation (7) can then be rewritten using the eigenvectors of the flux, $\vec{\Phi}^{(e)} \equiv \overleftrightarrow{U}^T\cdot\vec{\Phi}$, and the current, $\vec{I}^{(e)} \equiv \overleftrightarrow{V}^T\cdot\vec{I}$, as a large set of equations, one for each component of the current,

$$I_i^{(e)} = \frac{\Phi_i^{(e)}}{\ell_i}. \tag{9}$$

Multiplication by an orthogonal matrix does not change the magnitude of a vector, so the magnitude of the current squared is

$$\vec{I}^T\cdot\vec{I} = \sum_i \left(\frac{\Phi_i^{(e)}}{\ell_i}\right)^2. \tag{10}$$

The fundamental problem in solving Equation (7) is that the inductance matrix can have very small or zero diagonal elements ℓ_i. For example, in a cylinder of height h the inductance for a field that goes as $\cos(m\theta)$ is $\ell_m = m\pi\mu_o h\,(r_p/r_c)^m$ with r_p the plasma and r_c the coil radius. Since r_p/r_c is less than one, ℓ_m is very small when m >> 1. The components of the flux generally converge as $|\Phi_m| <$ A exp(-αm) with A and α independent of m, so Equation (10) gives a finite current, only when the plasma-coil separation is small, $r_c < r_p \exp(\alpha)$.

The exact solution of $\bar{\bar{L}} \cdot \bar{I} = \bar{\Phi}$, Equation (7), often gives a very large, and typically infinite, magnitude for the current. In the SVD method of solving Equation (7), only those components of the current in Equation (9) are solved that are associated with a sufficiently large inductance $\ell_i > \ell_{min}$. If the inductances elements are arranged so $\ell_i > \ell_{min}$ for $i < i_s$ and $\ell_i < \ell_{min}$ for $i \geq i_s$, the squared error in fitting the field is

$$\mathcal{E}^2 = \sum_{i=i_s}^{\infty} \left(\Phi_i^{(e)}\right)^2. \qquad (11)$$

In other words, no attempt is made to use currents in the coil surface to cancel the parts of the normal magnetic field on the desired plasma surface that are associated with flux components $\Phi_i^{(e)}$ with $i \geq i_s$. The more flux components that are canceled by coil currents the smaller ℓ_{min}, the smaller the error, but the larger the magnitude of the current. The SVD method finds the minimum current magnitude required to achieve a given level of error.

What are the limitations of Merkel's method for finding the current potential? Four important limitations are: (1) Components of the flux may be ignored that are essential to supporting the plasma. For example, flux components that resonate with the magnetic field lines and destroy magnetic surfaces in the plasma need to be retained even when their inductance coefficients are small. In the jargon of the field, missing components of the flux can lead to a poor reconstruction. (2) Components of the flux may be retained that are inessential to supporting the plasma. As the cylindrical problem illustrates, the highest mode number retained in the calculation will dominate the magnitude of the current if the coil surface is sufficiently displaced from the plasma. The retention of inessential components of the flux leads to inefficient coils and fictitious limits on the maximum tolerable separation between the coils and the plasma. (3) No concession is made to flexibility. The optimal current potential is found for one plasma configuration. Any flexibility is accidental that arises from varying the currents between the turns of the single coil set that is derived from such a current potential. (4) There is no constraint in the method, as normally applied, to reserve space for ports.

The four limitations of Merkel's method are addressed by the new method for finding the current potential. The new method optimizes the efficiency of the coil set by retaining all flux components that are essential to supporting a desirable plasma configuration but no more. The new method gives a current potential that depends independently on the components of the flux that must be controlled, which naturally gives a flexible coil set. Space can be reserved for ports, and the impact of port space on the efficiency and the flexibility of the coils can be studied.

II. New Method for Optimizing the Current Potential

The method that is being proposed for finding the optimal current potential differs strongly from the old. For example, only the important components of the flux vector, $\vec{\Phi}$, are retained--far fewer than the number of components of the current vector, $\vec{I}$, that is being considered. This means the desired flux vector, $\vec{\Phi}$, can be produced exactly by a non-unique current vector, $\vec{I}$. The freedom in the current vector is used to minimize the power dissipation in the coils that are being designed. In other words, the freedom in the current vector allows a minimization of the current density. At the end of the analysis, the number of independent current components is equal to the number of components of the flux vector that must be controlled. The analysis gives the current potential that most efficiently (minimal Ohmic dissipation in the coils) balances a given set of fluxes.

The new method for optimizing the current potential uses two matrices that play no role in Merkel's method. These are the quality matrix $\ddot{Q}$ and the resistance matrix $\ddot{R}$.

The importance of the various components of the flux vector $\vec{\Phi}$ is measured by the quality matrix $\ddot{Q}$. Nührenberg's method[1] for finding stellarator configurations is an optimization of a target function which contains information on the magnetohydrodynamic and the neoclassical transport properties of the configuration. The target function is optimized by varying the Fourier coefficients in the shape of the outermost surface of the plasma. Once an optimum is obtained, one can find how the target function, or the quality of the configuration, is degraded by changes in the Fourier coefficients, or equivalently by displacements normal to the plasma surface, $\vec{\xi}\cdot\hat{n}$. (A tangential displacement gives a change in the parameterizaton of the plasma surface, not a new surface shape.) Except on a rational magnetic surface, the magnetic perturbation $\vec{b}$ associated with a displacement $\vec{\xi}$ is given[6] by $\vec{b}=\vec{\nabla}\times(\vec{\xi}\times\vec{B})$. This relation between the perturbed field and the displacement can be used to show that the perturbed flux is

$$\vec{\Phi} - \ddot{L}\cdot\vec{I} = \int_{\substack{\text{plasma}\\ \text{surface}}} f_i(\theta,\varphi)\,\vec{b}\cdot d\vec{a} = - \int_{\substack{\text{plasma}\\ \text{surface}}} (\vec{B}\cdot\vec{\nabla} f_i)\,\vec{\xi}\cdot d\vec{a}. \tag{12}$$

The target function for the stellarator configuration should deviate from its optimum value T_o with roughly a quadratic dependence on the normal displacement $\vec{\xi}\cdot\hat{n}$. Consequently, the target function depends on the perturbed flux as

$$T = T_o - (\vec{\Phi}-\ddot{L}\cdot\vec{I})^T\cdot\ddot{Q}\cdot(\vec{\Phi}-\ddot{L}\cdot\vec{I}), \tag{13}$$

which defines the quality matrix $\ddot{Q}$. A set of desired plasma shapes can only be reproduced by the coils to a certain tolerance. The quality matrix is like a metric tensor defining how far a set of coils miss reproducing a given target plasma.

The quality matrix, $\ddot{Q}$, is a symmetric, positive matrix and can be diagonalized. A diagonal element is important if it satisfies either one or both of the following criteria: (1) The diagonal matrix element q_i is large. (2) The component of flux Φ_i associated with the component is large.

The potential degradation in quality associated with the i^{th} element of the diagonalized quality matrix is $\delta Q = q_i \Phi_i^2$. The maintenance of a given quality tolerance on the target function of the stellarator configuration defines which components of the flux are important and which are not. Let N_Φ be the number of important components of flux. The irrelevant components of the flux are ignored in the remainder of this section.

The efficiency of the coil set is optimized by reducing the Ohmic power that is dissipated in the coils,

$$P = \vec{I}^T \cdot \vec{R} \cdot \vec{I}, \tag{14}$$

to a minimum. The method for calculating resistance matrix $\vec{R}$ is given in Section III. The resistance matrix is symmetric and positive definite. As shown in Section III, it can be used to impose the constraint that certain space on the coil winding surface must be free of coils in order to allow room for ports.

The number of components of the current vector $\vec{I}$, which is denoted by N_I, is assumed to be very large compared to the number of flux components that must be fit, N_Φ. To satisfy the constraint that the N_Φ components of the flux that must be fit are fit, the equation $\vec{\Phi} = \vec{L} \cdot \vec{I}$ must be solvable with $\vec{\Phi}$ the important part of the flux. The equation $\vec{\Phi} = \vec{L} \cdot \vec{I}$ can be solved by diagonalizing the matrix $\vec{L}^T \cdot \vec{L}$. If the $\vec{I}$ can indeed satisfy the required equation $\vec{\Phi} = \vec{L} \cdot \vec{I}$ then the matrix $\vec{L}^T \cdot \vec{L}$ will have precisely N_Φ non-zero eigenvalues, ℓ_i^2, the number of important fluxes. In other words, the matrix $\vec{L}^T \cdot \vec{L}$ has N_Φ eigenvectors $|s\rangle$ associated with non-zero eigenvalues and N_I - N_Φ eigenvectors $|n\rangle$ associated with zero eigenvalues. The eigenvectors $|n\rangle$ are said to span the null space of $\vec{L}^T \cdot \vec{L}$. The current can be written in the form

$$\vec{I} = \sum_{s=1}^{N_\Phi} I_s \, |s\rangle + \sum_{n=1}^{N_I - N_\Phi} I_n \, |n\rangle \tag{15}$$

with the I_s determined by the equation $\vec{\Phi} = \vec{L} \cdot \vec{I}$ and the I_n arbitrary. In other words, the N_Φ currents I_s can be chosen to reproduce the N_Φ fluxes exactly with the currents I_n totally unconstrained. The arbitrary components of the current I_n are then chosen to minimize the Ohmic power. That minimization yields

$$\sum_{n=1}^{N_I - N_\Phi} R_{n'n} \, I_n = - \sum_{s=1}^{N_\Phi} R_{n's} I_s \tag{16}$$

with $R_{n'n} = \langle n' | \vec{R} | n \rangle$ and $\langle n|$ the transpose of the eigenvector $|n\rangle$. Since the resistance matrix is positive definite (has no null space for non-zero $\vec{j}$), one can solve Equation (16) to find the I_n in the form

$$I_n = -\sum_{s=1}^{N_\Phi} I_s c_{sn} \tag{17}$$

with c_{sn} a matrix of constants. The current that exactly reproduces N_Φ fluxes with minimal Ohmic power is

$$\vec{I} = \sum_{s=1}^{N_\Phi} I_s \left(|s\rangle - \sum_{n=1}^{N_I - N_\Phi} c_{sn} |n\rangle \right). \tag{18}$$

The new method of finding the current potential defines a set of N_Φ independent currents that exactly control N_Φ flux components. If one assumes the important N_Φ components of the flux are similar for a set of stellarator configurations, then the set of N_Φ currents is sufficiently flexible to produce all the configurations in the set. Why might one expect this to be the case? The important flux components associated with different eigenvalues of the quality matrix affect the target function differently, so one would assume that at least that many flux components must be controlled. The most important flux components are presumably associated with either low or resonant Fourier harmonics of the normal magnetic field. By low Fourier harmonics is meant poloidal harmonics m=0,1,2,3 and low toroidal harmonics of the number of periods of the stellarator. This is analogous to saying the properties of a tokamak plasma are largely determined by the aspect ratio, the ellipticity, and the triangularity. In any case, a study of a number of interesting plasma equilibria will establish which set of flux components must be controlled by the coils being designed and which flux components, if any, are consistent with a fixed current potential over an interesting range of plasma configurations.

III. The Resistance Matrix

The resistance matrix $\vec{R}$ is defined so the Ohmic power dissipated by the coils

$$P = \int \eta j^2 d^3x. \tag{19}$$

can be written as $P = \vec{I}^T \cdot \vec{R} \cdot \vec{I}$. The current density is given in terms of the current potential by Equation (2) and the current potential is expressed in terms of the current components I_i by Equation (4). The theory of general coordinates implies

$$\vec{\nabla} r \times \vec{\nabla} \kappa = \frac{1}{\mathcal{J}} \left(\frac{\partial \vec{X}_c}{\partial \varphi} \frac{\partial \kappa}{\partial \theta} - \frac{\partial \vec{X}_c}{\partial \theta} \frac{\partial \kappa}{\partial \varphi} \right). \tag{20}$$

with $\vec{X}_c(\theta,\varphi)$ the equation for the coil surface, Eq. (1), and $\mathcal{J}$ the Jacobian of (r,θ,φ) coordinates.

Although the current carrying region can be arbitrarily thin--so for most purposes it can be viewed as a surface, a finite resistance matrix implies the thickness cannot be zero. The volume of the current carrying region associated with a small change in θ and φ will be denoted by

$v(\theta,\varphi)\delta\theta\delta\varphi$. That is

$$\frac{\int d^3x}{v(\theta,\varphi)} = \delta\theta\ \delta\varphi = \int \delta(r-r_c)dr\delta\theta\delta\varphi. \tag{21}$$

Using $d^3x = \mathcal{J}\ dr\ \delta\theta\ \delta\varphi$, one finds that

$$\frac{1}{v} = \frac{\delta(r-r_c)}{\mathcal{J}}. \tag{22}$$

The Ohmic power can then be written as

$$P = \int \frac{\eta}{v} \left\{ \left(\frac{\partial \vec{X}_c}{\partial\varphi}\right)^2 \left(\frac{\partial\kappa}{\partial\theta}\right)^2 - 2\left(\frac{\partial \vec{X}_c}{\partial\theta}\cdot\frac{\partial \vec{X}_c}{\partial\varphi}\right)\frac{\partial\kappa}{\partial\theta}\frac{\partial\kappa}{\partial\varphi} + \left(\frac{\partial \vec{X}_c}{\partial\theta}\right)^2 \left(\frac{\partial\kappa}{\partial\varphi}\right)^2 \right\} d\theta d\varphi. \tag{23}$$

The current potential κ can be written as $\kappa(\theta,\varphi) = \sum I_j\, g_j(\theta,\varphi)$ with $g_j(\theta,\varphi)$ any general set of dimensionless functions, Equation (4). Equation (23) implies the components of the resistance matrix are

$$R_{ij} = \int \frac{\eta}{v} \left\{ \left(\frac{\partial \vec{X}_c}{\partial\varphi}\right)^2 \frac{\partial g_i}{\partial\theta}\frac{\partial g_j}{\partial\theta} - \left(\frac{\partial \vec{X}_c}{\partial\theta}\cdot\frac{\partial \vec{X}_c}{\partial\varphi}\right)\left(\frac{\partial g_i}{\partial\theta}\frac{\partial g_j}{\partial\varphi} + \frac{\partial g_i}{\partial\varphi}\frac{\partial g_j}{\partial\theta}\right) + \left(\frac{\partial \vec{X}_c}{\partial\theta}\right)^2 \frac{\partial g_i}{\partial\varphi}\frac{\partial g_j}{\partial\varphi} \right\} d\theta d\varphi, \tag{24}$$

and the Ohmic dissipation is $P = \sum I_i R_{ij} I_j$.

The constraint that there be no coils in a region occupied by a port is imposed by making the $g_j(\theta,\varphi)$ constant in any region that is to be occupied by ports or by making η/v very large.

IV. Summary

The scientific usefulness of a stellarator experiment is largely determined by the flexibility of its coils to produce a number of important plasma configurations and the access if offers for heating and diagnostics. The cost and technical limitations of an experiment are largely determined by the efficiency with which the required magnetic fields can be produced. In this paper a new method of finding the current potential is given which can serve as the basis for designing coils that optimize flexibility, efficiency, and access.

Stellarators are designed by maximizing a target function through variations in the shape of the plasma. The target function defines how far a particular stellarator configuration is from the optimum. The matrix that measures this distance is the quality matrix $\ddot{Q}$, which is a positive symmetric matrix like a metric tensor in ordinary space. The coils that are being designed cancel the normal magnetic field on the plasma surface due to all other sources. The eigenvectors and eigenvalues of $\tilde{Q}$ determine which parts of the normal magnetic field must be carefully canceled and which can be ignored. The current distribution on the coil surface is chosen to cancel the important parts of the normal magnetic field. This does not uniquely determine the current distribution since there are only a finite number of such parts of the field. The current distribution is made unique by maximizing the efficiency (minimizing the required Ohmic power) and constraining the current to be zero in regions to be occupied by ports. Since each non-degenerate eigenvector of the quality matrix engenders a different response by the target function, it is expected that the important eigenfunctions must be independently controllable by the coils that are being designed to have a flexible coil set. The most critical parts of the normal magnetic field to control are presumably related to simple features of the plasma cross section (like aspect ratio, ellipticity, or triangularity) or to resonances that can destroy the magnetic surfaces in the plasma.

The method that has been used until now to optimize the current distribution in the coil surface forces the coils to cancel unimportant parts of the normal magnetic field on the plasma surface which (1) needlessly increases the current that is required to support the plasma and (2) reduces the maximum acceptable separation between the coils and the plasma. It is preferable to have the coils far from the plasma for two reasons (1) to provide space for freedom in the plasma shape and (2) to simplify the coils. The tolerable space between turns of a coil set is less than the distance from the coils to the plasma; the further back the coils the fewer the turns that are required to obtain a good representation of a continuous distribution of surface current.

The concepts that have been introduced in this paper give a definite procedure for designing coils that are flexible, efficient, and have good plasma access. These are the most critical features of the coils of an attractive experiment.

Acknowledgements:

The author would like to thank members of the Nation Compact Stellarator Experiment (NCSX) design group for useful discussions. He would like to particularly thank Neil Pomphrey and Wayne Reiersen. This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-95ER54333 and by agreement S-04054-G with the Princeton Plasma Physics Laboratory.

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