Strategies for Repairing Bad Magnetic Surfaces

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Heuristic Methods

- Set $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$ on a specified outer surface. Hope that this gives closed nested flux surfaces in the interior.

  Plasma Equilibrium, VMEC - <u>Assumes</u> good surfaces.

  Vacuum, NESCOIL – Has worked, until recently. (Aspect Ratio?).

  When it doesn't work, there is no logical next step.

- Residue Reduction. Residues of fixed points on good flux surfaces are zero. Hope that minimizing fixed point residues will improve flux surfaces.

  Hanson and Cary, Phys. Fluids **27**, 767 (1984).

  Cary and Hanson, Phys. Fluids **29**, 2464 (1986).

  Carreras, Dominguez, et. al., Nucl. Fusion **28**, 1195 (1988).

  Hanson, Doloc and Yuan, Plasma Phys. Reports **23**, 555 (1997).

  Not easily automated.

- NB. There are no guarantees.

Hamiltonian Perturbation Theory

- Stellarator Vacuum Fields

  Cary, Phys. Rev. Lett. **49**, 276 (1982).

  Cary, Phys. Fluids **27**, 119 (1984).

- Tokamak Error Correction

  Hanson, Nucl. Fusion **34**, 441 (1994).

  Hanson, IEEE Trans. On Plasma Science, to be published Dec. 1999.

- Corresponds to elimination of resonances at rational surfaces in first order perturbation theory.
- Can be written in many equivalent forms.

  Use correction coils to make a magnetic island of the same size and opposite phase as the island generated by the error.

Hamiltonian Perturbation Theory

- Linearization

$$\mathbf{B} = \mathbf{B}_0 + \varepsilon \mathbf{B}_1 = \mathbf{B}_0 + \varepsilon\left(\mathbf{B}_{error} + \mathbf{B}_{correction}\right)$$

- $\mathbf{B}_0$ has good flux surfaces. [u,θ,φ] are associated flux coordinates.
- Rational Surface, $\iota = \dfrac{n_0}{m_0} = \dfrac{\mathbf{B}_0 \cdot \nabla\theta}{\mathbf{B}_0 \cdot \nabla\varphi}$ at $u = u_r$.

$$\left.\begin{matrix} A_{pm_0, pn_0} \\ B_{pm_0, pn_0} \end{matrix}\right\} = \frac{1}{pm_0}\frac{1}{2\pi^2}\int_0^{2\pi} d\theta \int_0^{2\pi} d\varphi \frac{B_1^u(u_r,\theta,\varphi)}{B_0^\varphi(u_r,\theta,\varphi)} \left\{\begin{matrix} \sin(p(m_0\theta - n_0\varphi)) \\ -\cos(p(m_0\theta - n_0\varphi)) \end{matrix}\right\}$$

- A and B - Resonant Perturbation Amplitudes (RPAs).

Hamiltonian Perturbation Theory

- $\dfrac{B_1^u}{B_0^\varphi} = \dfrac{\mathbf{B}_1 \cdot \nabla u}{\mathbf{B_0} \cdot \nabla \varphi}$. ∇u and $\nabla \varphi$ factors are important.

- Vary $\mathbf{B}_{correction}$ so that A_{pm_0, pn_0} and B_{pm_0, pn_0} are zero.

  Use SVD (Singular Value Decomposition) to find $\mathbf{B}_{correction}$.

- Works well for tokamaks.
- Stellarator Code:

  Uses VMEC equilibrium for $\mathbf{B}_0$

  $\mathbf{B}_{error} + \mathbf{B}_{correction}$ fields are externally imposed, vacuum fields.

  Will be working RSN (Real Soon Now).

Comments

- Experience with tokamaks leads me to expect that Hamiltonian Perturbation approach will work very well for: small enough islands and well characterized error fields.
- Using VMEC currents to evaluate $\mathbf{B} \cdot \nabla u$ on rational flux surfaces is a good idea. The results could be included in an optimization loop.

Comments

- NCSX - It may not work.

  Even with $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$ on an outer surface, there may not exist good surfaces with specified pressure and current profiles.

  If you find pressure and current profiles that also give good surfaces, the pressure and current profiles may not be realizable.

  If you find good surfaces, they may be extremely sensitive to small values of $\mathbf{B} \cdot \hat{\mathbf{n}}$ on an outer surface. (Coil discretization may kill you.)

  When surfaces are completely destroyed, perturbation theory may not be able to recover them.

- Varying currents in VF, TF, and saddle coils will help. However, I expect that additional correction coils will be needed

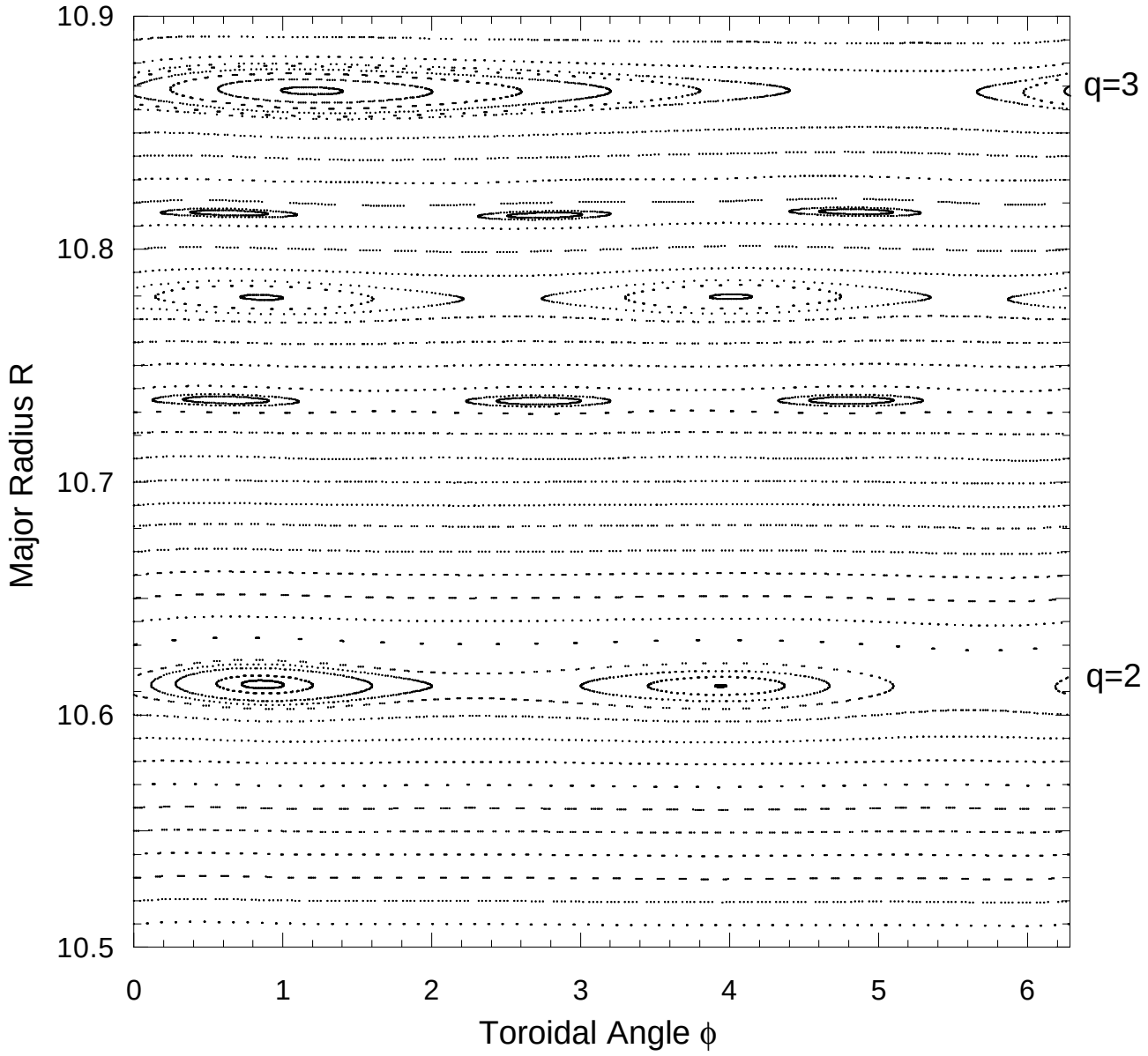

Figure 1. Surface of section plot of the outer 40 cm of the plasma region.
The perturbation field is only due to a vertically shifted toroidal field coil.

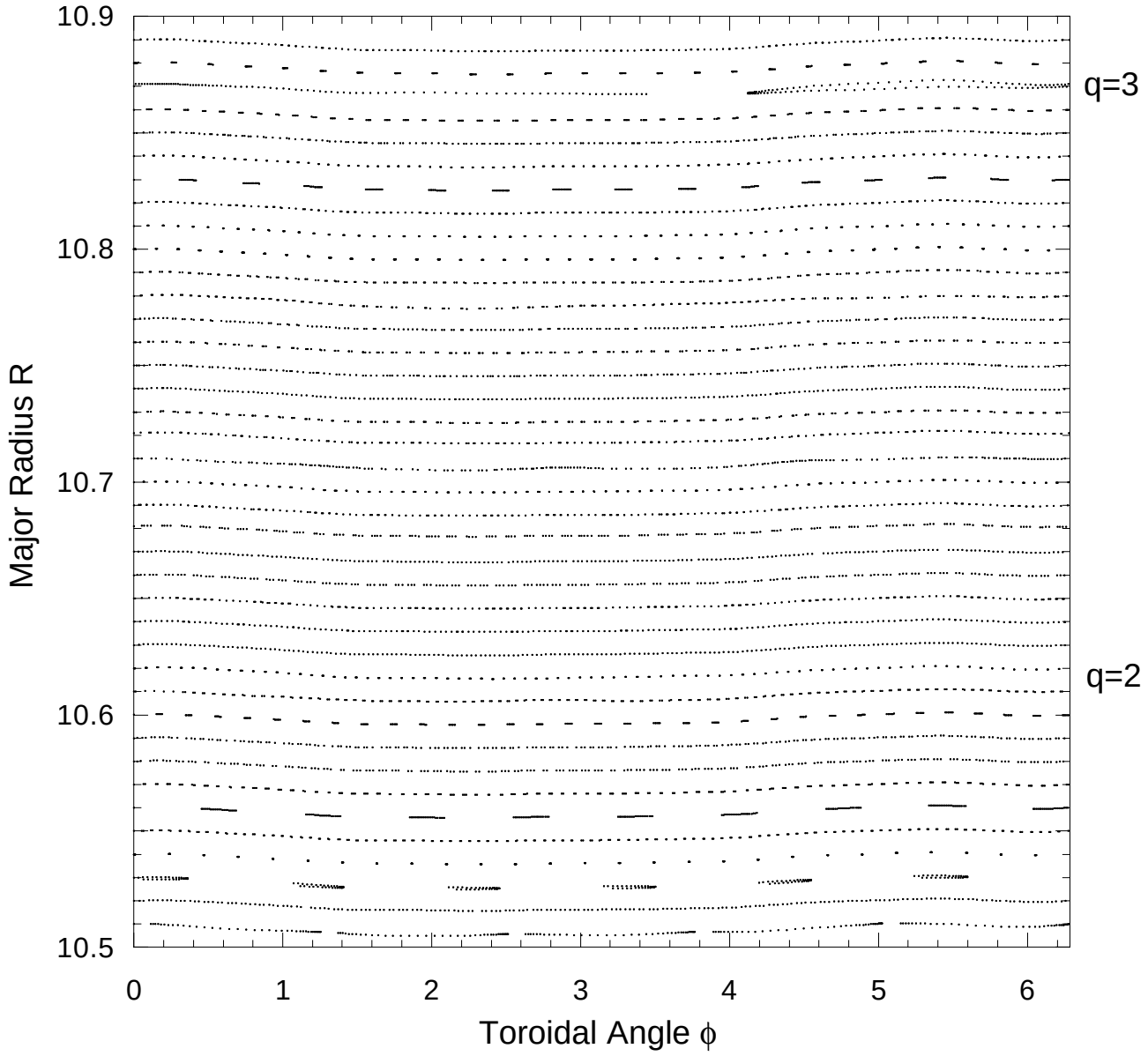

Figure 2. Surface of section plot of the outer 40 cm of the plasma region. The perturbation field is due to a vertically shifted toroidal field coil and to to correction coils with currents chosen to minimize all RPAs with n=1, 2, and 3, and with q values between q=1 and q=3, inclusive.

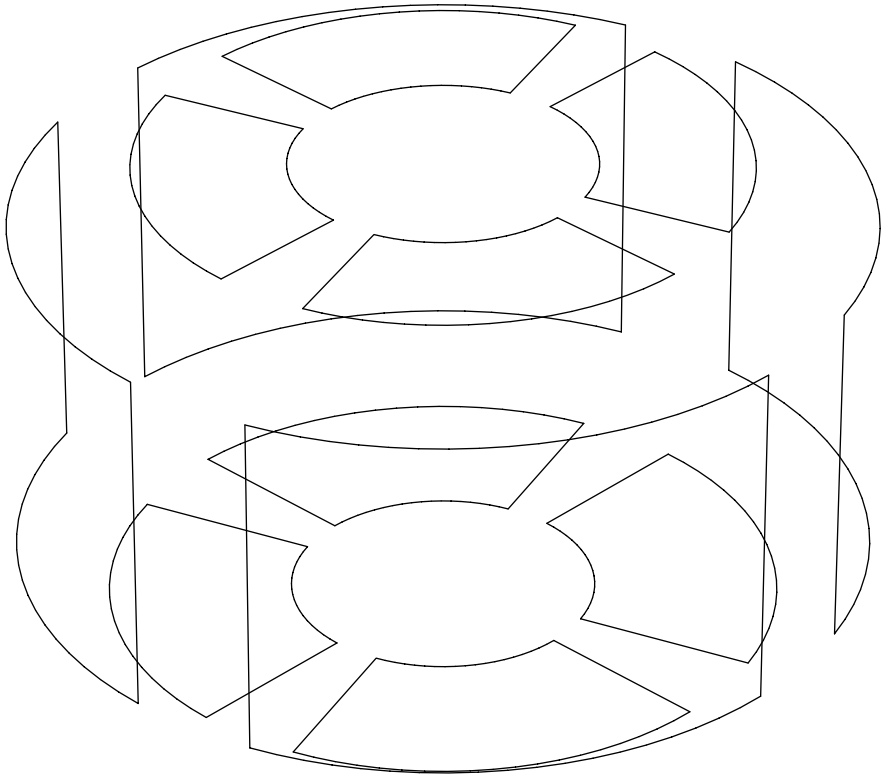

Figure 3. Correction Coils for a tokamak.