Control Matrix Approach for QAS Transport and Stability

Update of 9/2/99 presentation

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- For a successful NCSX design, we need to demonstrate independent control of the physics elements that define the mission. At a minimum, this means independent control of the transport and stability.
- In the experiment, (group) coil currents will be varied to effect the control.
  - A successful coil design will need to demonstrate this control

- A variation of coil currents produces a change in shape of the plasma boundary.
- A sensible first step in exploring plasma control is to understand what shape deformations are required to
  - Change the (kink) stability of a given configuration without affecting the transport, and
  - Change the transport without affecting the stability
- Assuming a tiered coil design (primary "EF" coils for producing the basic configuration, secondary "control" coils for physics scans, tertiary "trim" coils for error field correction), an understanding of these shape deformations should help in the placement and design of the control coils.

Control Matrix Approach

Consider a plasma configuration, $\mathbf{Z}$, which corresponds to a set of physics parameters, $\mathbf{P}$. For example, in the contest of the present VMEC optimization code, $\mathbf{Z}$ is the set of plasma boundary fourier coefficients R_{mn}, Z_{mn}. The physics parameters can be whatever you like that depends on the $\mathbf{Z}$, such as iota, $\chi^2_{\mathrm{transport}}$, λ_{kink}, $\lambda_{\mathrm{ballooning}}$, etc. The relationship between $\mathbf{P}$ and $\mathbf{Z}$ can be represented as

$$\hat{\mathbf{G}}\mathbf{Z} \Rightarrow \mathbf{P}. \tag{1}$$

Now change the configuration in some way so that $\mathbf{Z} \rightarrow \mathbf{Z} + \xi$. Then the physics parameters change to the new values $\mathbf{P} + \pi$ where π and ξ are related by

$$\pi_{\mathbf{i}} = \mathbf{G}^{(1)}_{\mathbf{ij}}\xi_{\mathbf{j}} + \mathbf{G}^{(2)}_{\mathbf{ijk}}\xi_{\mathbf{j}}\xi_{\mathbf{k}} + \cdots \tag{2}$$

Results (see later) show that the quadratic approximation is pretty good! By transforming coordinates this can be written as

$$\xi^{\mathbf{T}}\mathbf{G}\xi = \pi. \tag{3}$$

G is an "influence matrix" that relates the changes in shape to the consequent changes in physics.

- The **G** matrix determines the local topography of the cost function whose "minimum" has determined the plasma configuration (ie., the minimum found by Long-Poe by running the optimizer).

- The **G** matrix elements can be determined by a sequence of step response calculations (where individual ξ vector elements are excited and VMEC, +JMC +TERPSICHORE are run to determine the resulting π values.

- By examining the topography we can get a sense of the contollability of the plasma. A broad minimum is good, whereas a rough surface is bad.

- Once we have the surface shape, we can determine the directions orthogonal to $\nabla\chi^2$ and $\nabla\lambda$ (analysis of quadratic form, Eq. 3). These directions define the required shape deformations that the coils are to provide for independent control of the physics.

Some Results for C10

- For each of the 78 R,Z harmonics that specify the C10 reference configuration, we perturb the harmonic by up to +/- 0.02m and calculate the transport χ^2 and kink growth rate, λ. Typical results are plotted in Fig. 1. The dependence of χ^2 on the shape harmonics is typically much greater ($\sim$ factor of 100) than the dependence of λ. We can see this in Fig. 2 which scales the change in kink eigenvalue by a factor of 100.

- Fig. 3 shows a couple of harmonics that do have a significant influence on the kink stability. Decreasing the value of RBC(2,4) by 0.02m decreases the kink growth rate to about 25value. On the other hand, the $n = -3, m = 5$ harmonics are bad news for both λ and χ^2.

- Fig. 4 shows a comparison of plasma boundaries for C10 and C10 + δRBC(2,4) = -0.02 + δRBC(0,0) = +0.02. (Fig. 3 showed this was a good perturbation). Note the outboard indentation and compare with C82.

- Fig. 5 shows a comparison of plasma boundaries for C10 and C10 + δRBC(-3,5) = -0.02. (Fig. 3 showed this was a bad perturbation).

- Fig. 6 shows a comparison of plasma boundaries for C10 + δRBC(2,4) = -0.02 + δRBC(0,0) = +0.02 and C82. The C82 configuration has much more squareness.

The Future

Our picture book of harmonic perturbations shows that truncating Eq. 3 after the quadratic terms is a good model of the configuration space over the range of perturbations studied so far. We will now determine the orthogonal perturbations which control the plasma.

Note dependence of χ^2 on (shape) harmonic perturbation much more sensitive than dependence of λ_{KINK}.

$(\Pi - \Pi_0)/\Pi_0$

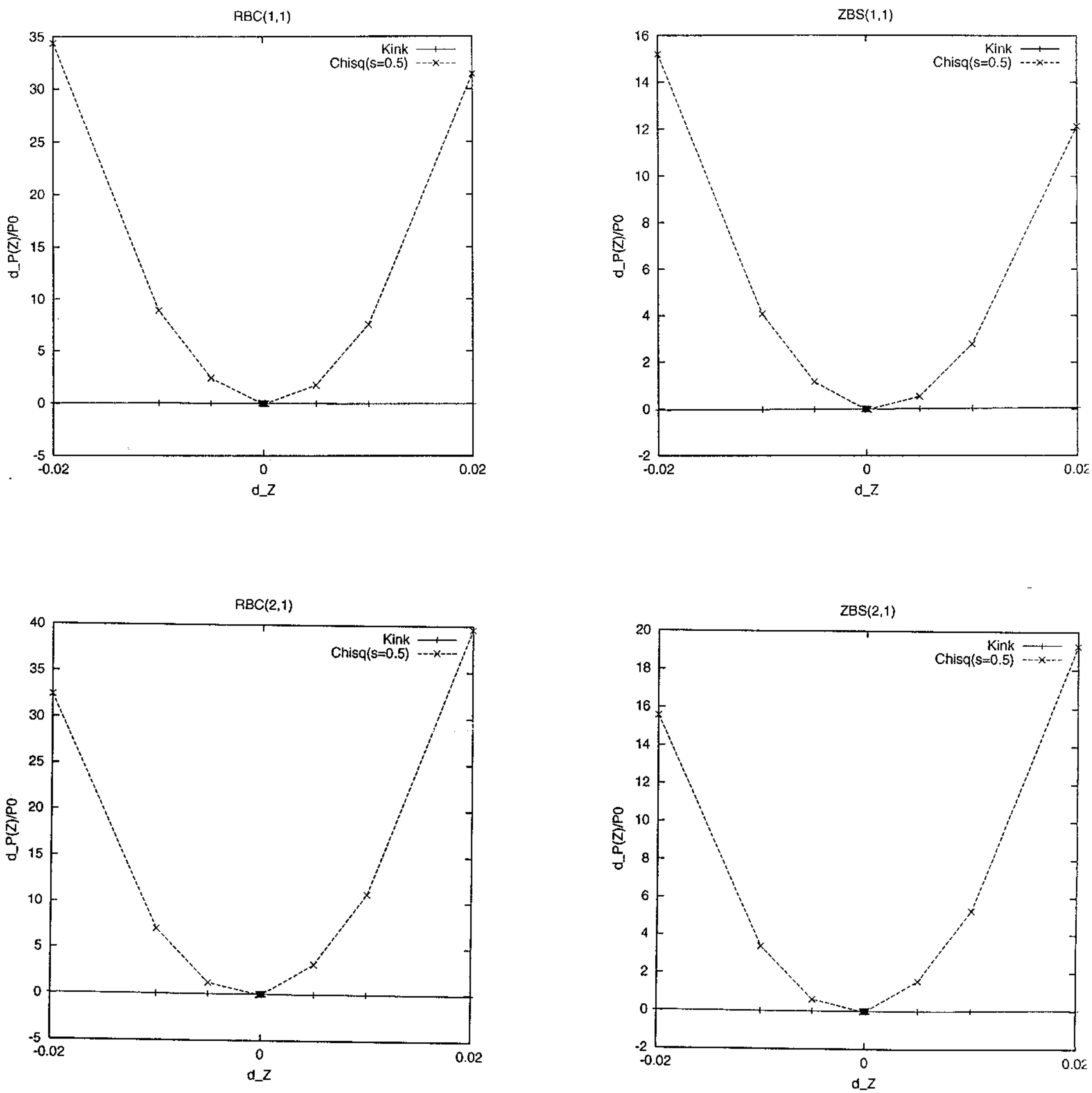


Fig. 1

Same ~~plot~~ as Fig. 1 except change in kink eigenvalue scaled by factor of 100.

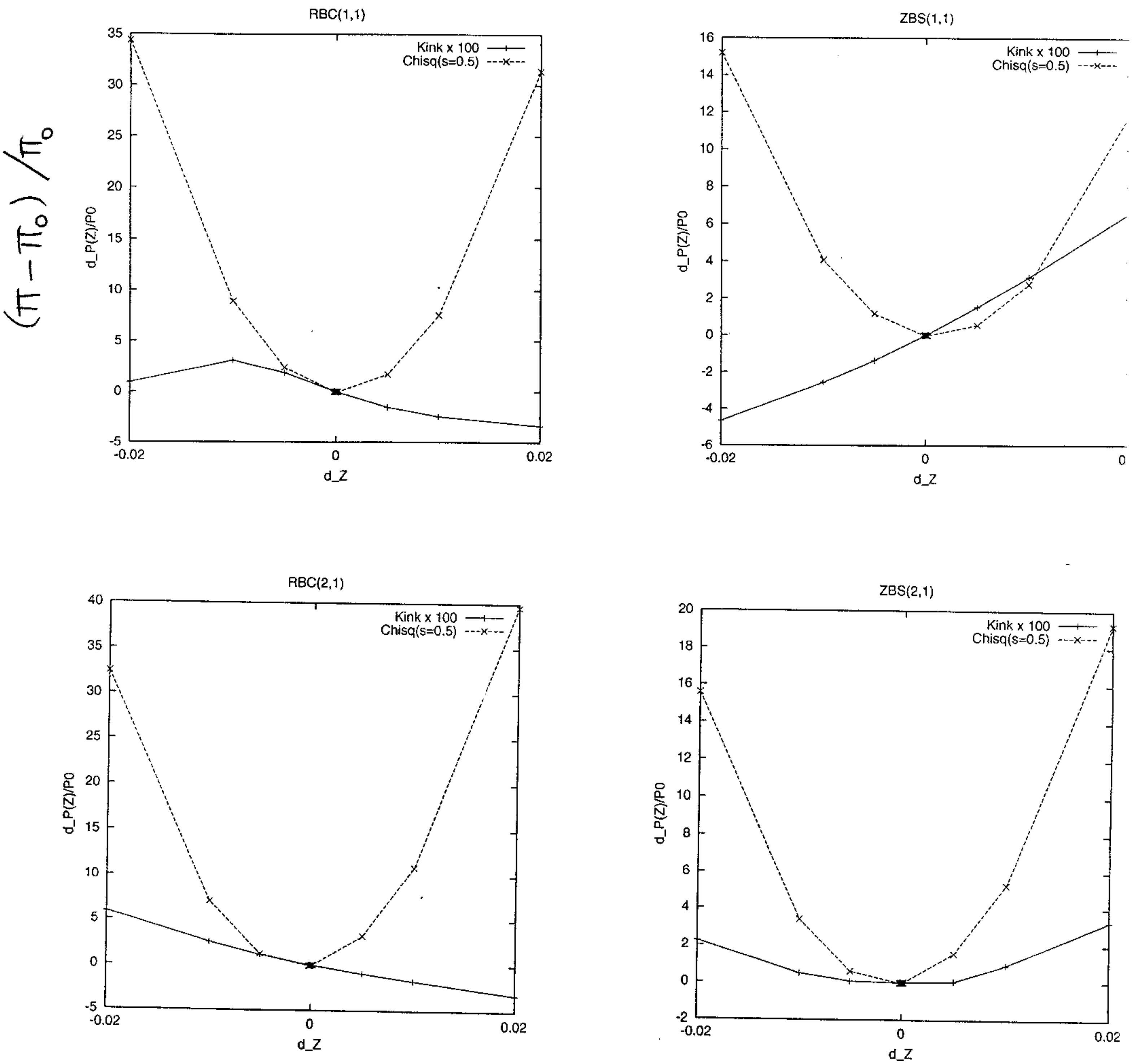


Fig. 2

THESE HARMONICS ARE AMONG THE FEW THAT HAVE A SIGNIFICANT INFLUENCE ON λ_{KINK}

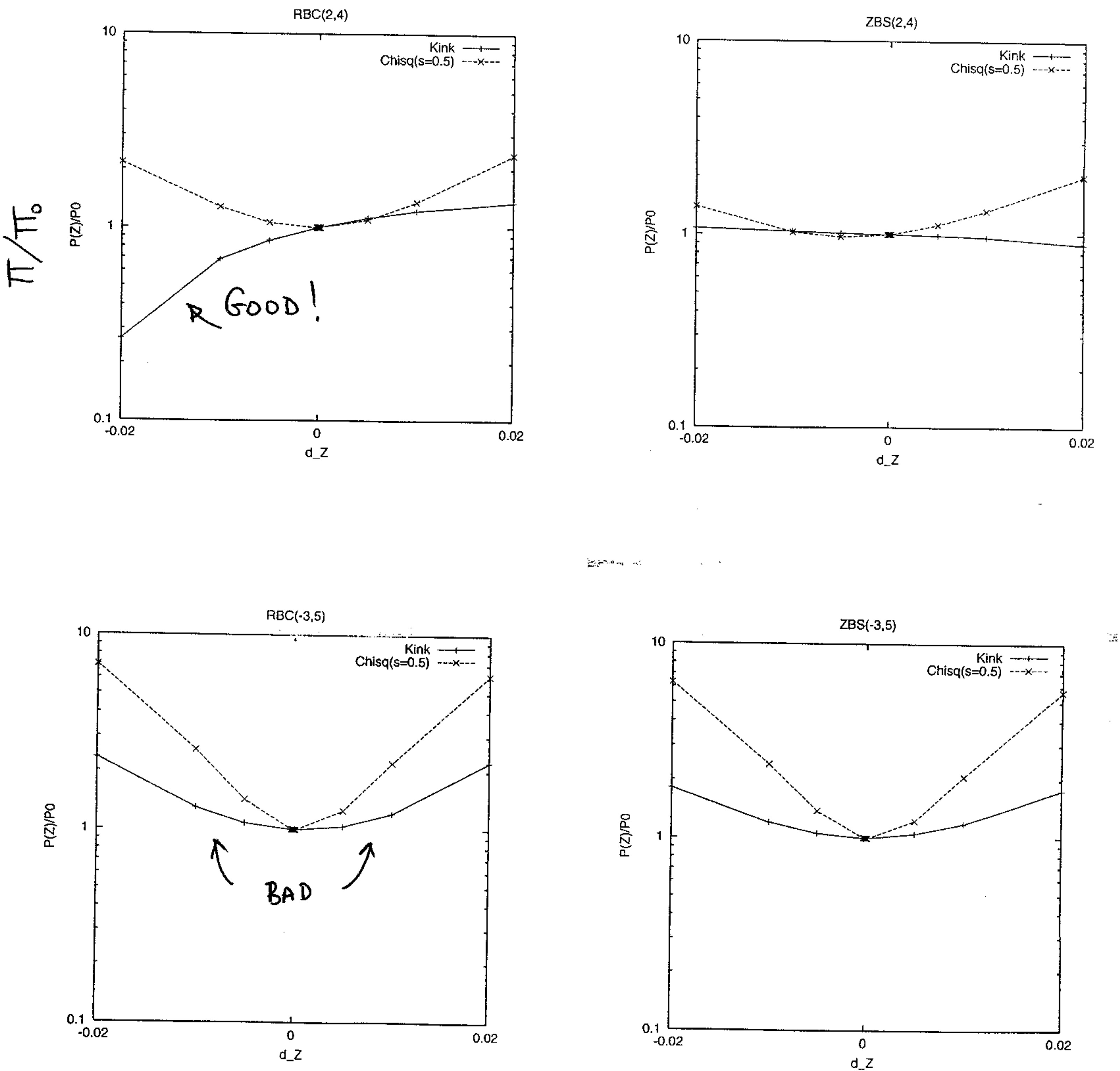
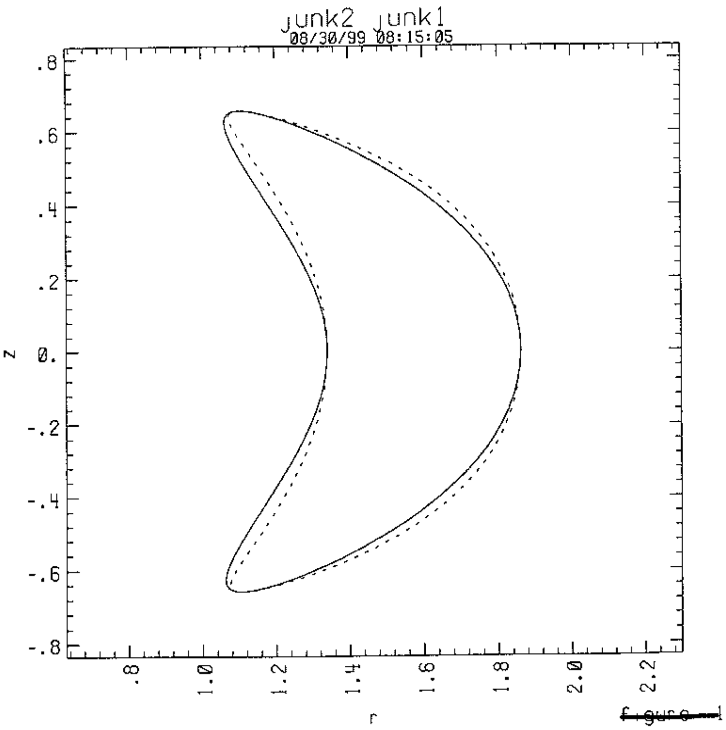
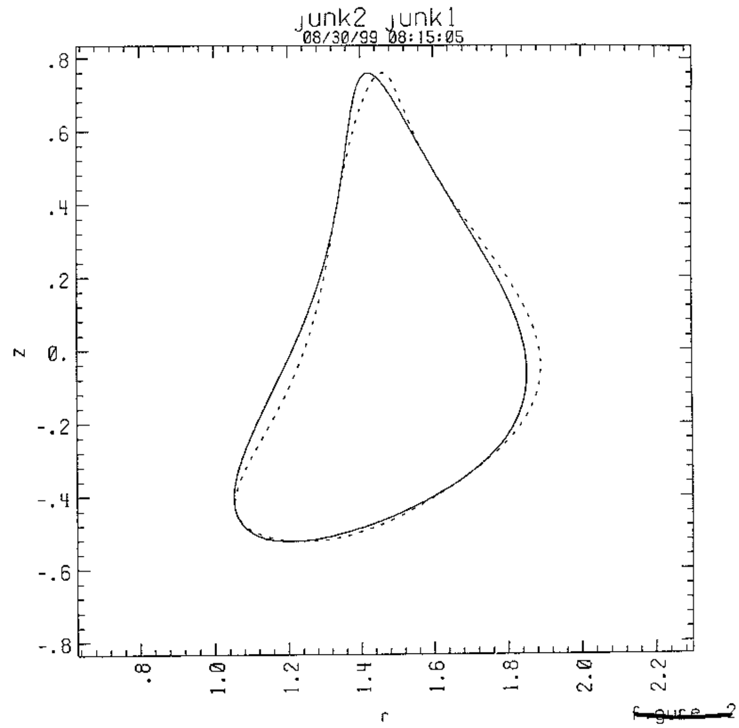


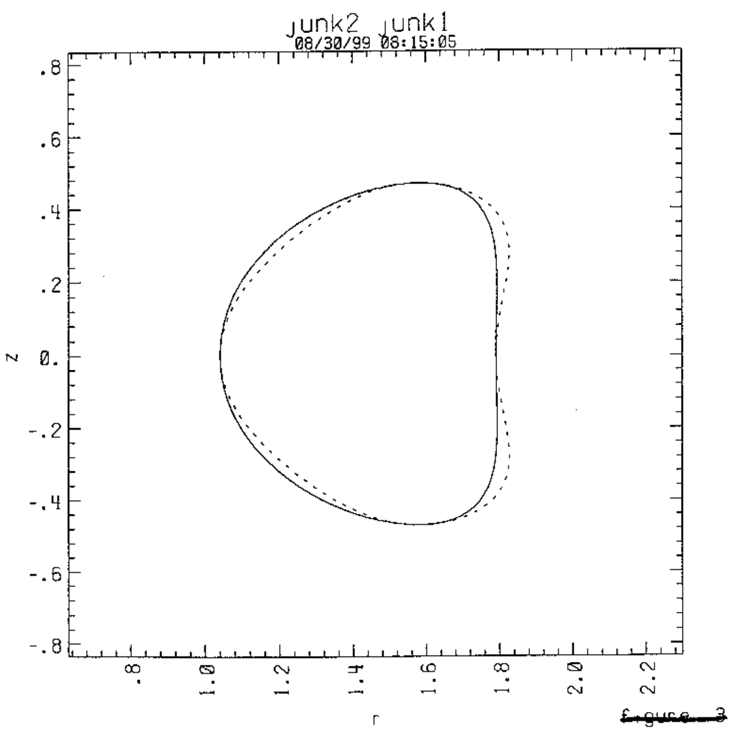
Fig. 3

COMPARISON OF C10 (SOLID) with C10 + δRBC(2,4) = −0.02 + δRBC(0,0) = +0.02 (DASH)





NOTE BENEFICIAL OUTBOARD INDENTATION



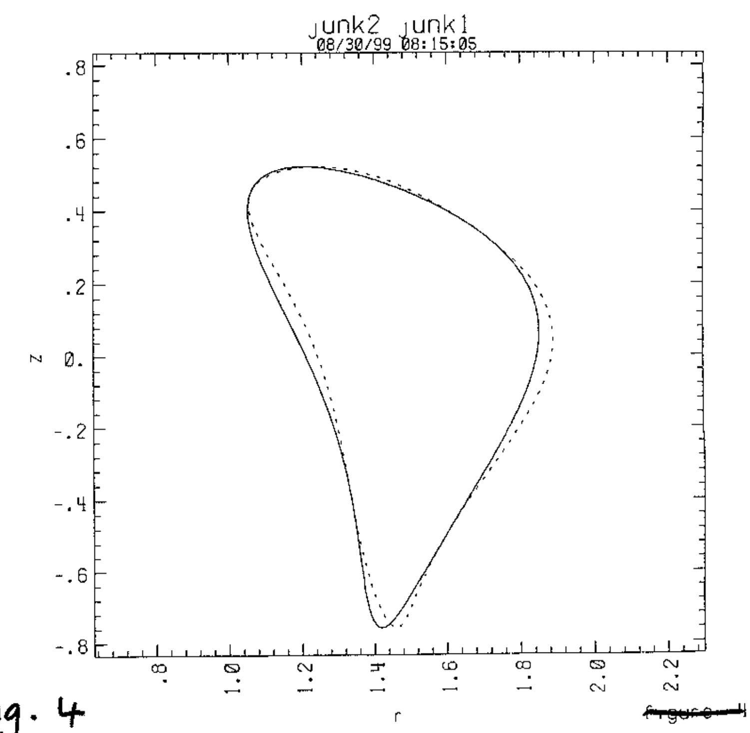
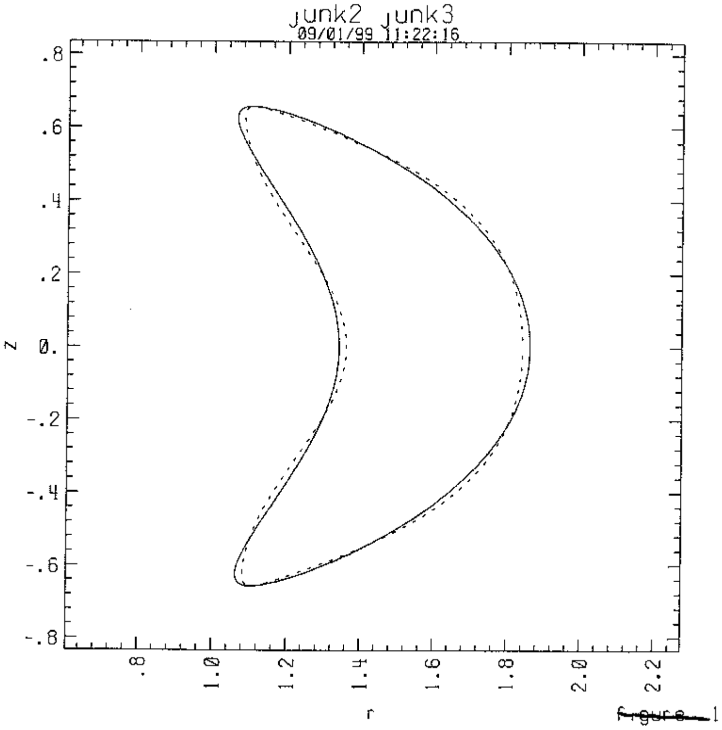
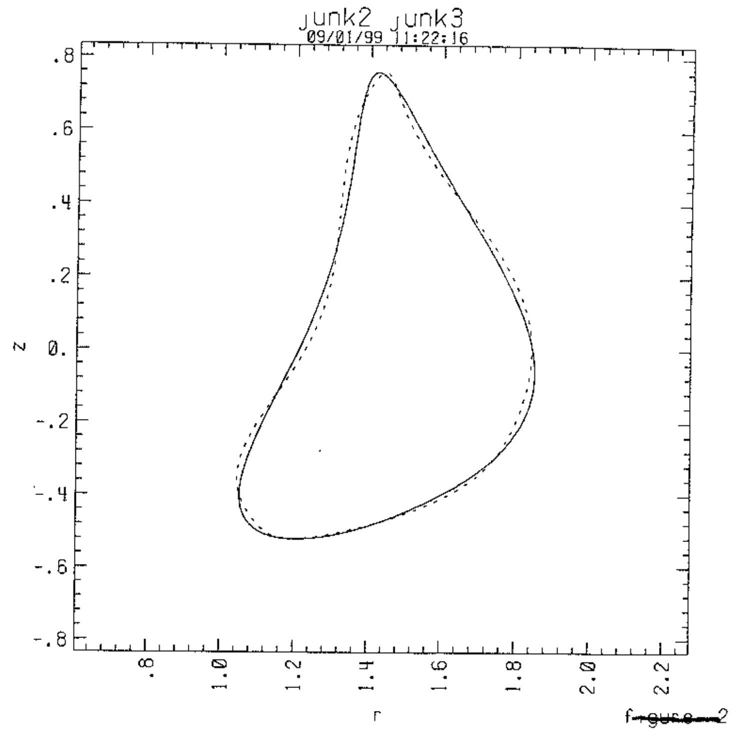


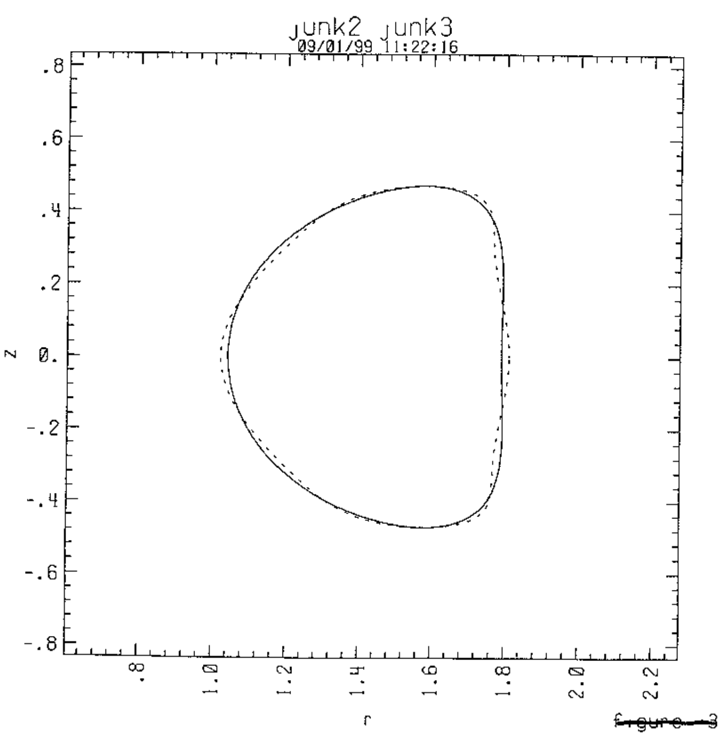
Fig. 4

COMPARISON OF C10 (SOLID) WITH C10 + $\delta RBC(-3,5) = -0.02$ (DASH)





(THIS IS BAD PERTURBATION)



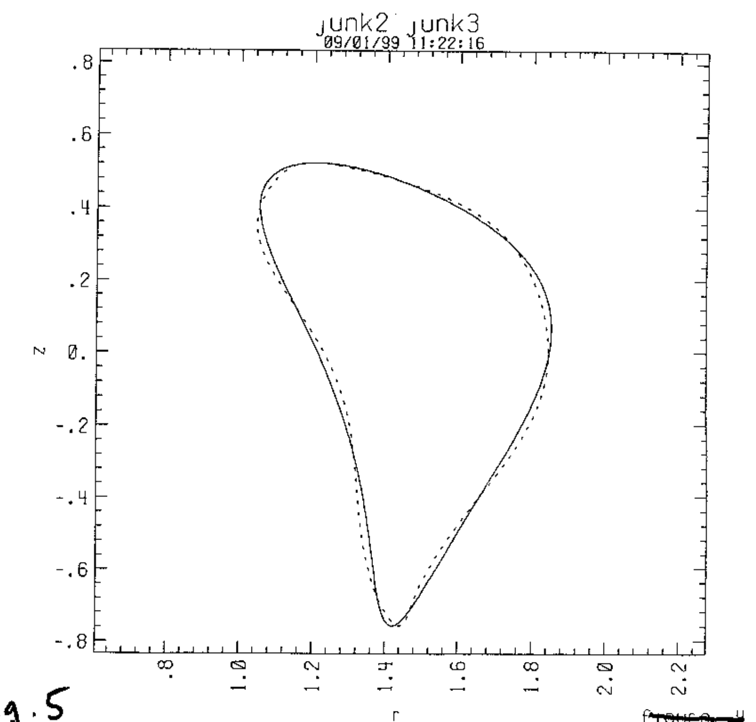


Fig. 5

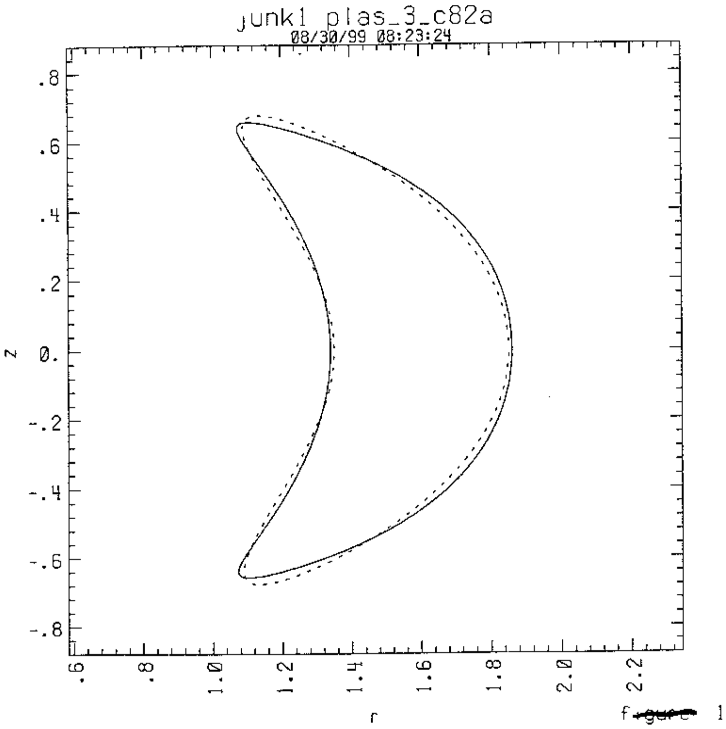
COMPARISON OF C10 $+ \delta RBC(2,4) = -.02$

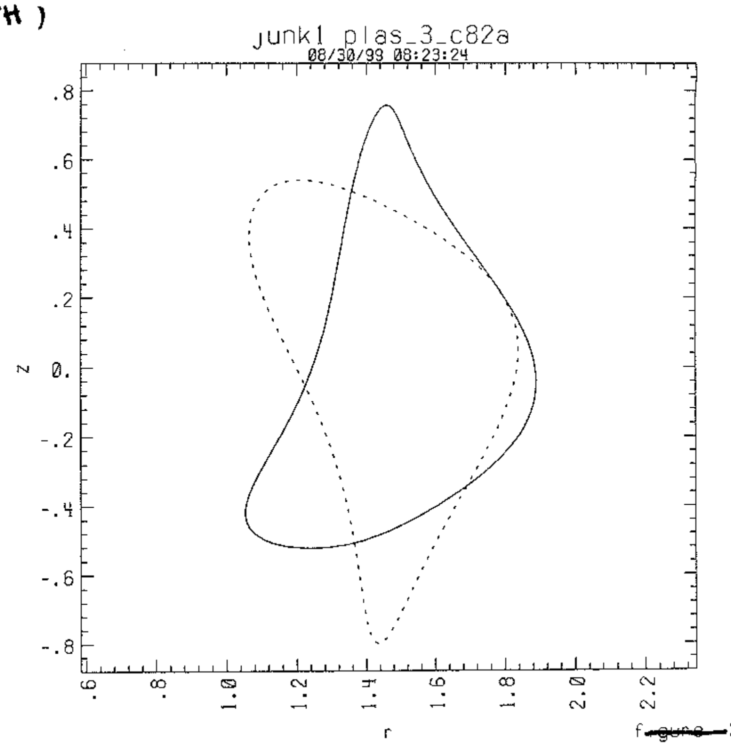
$+ \delta RBC(0,0) = +.02$

(SOLID)

WITH C82

(DASH)





C82 HAS MORE SQUARENESS.