Status of the Genetic Algorithm

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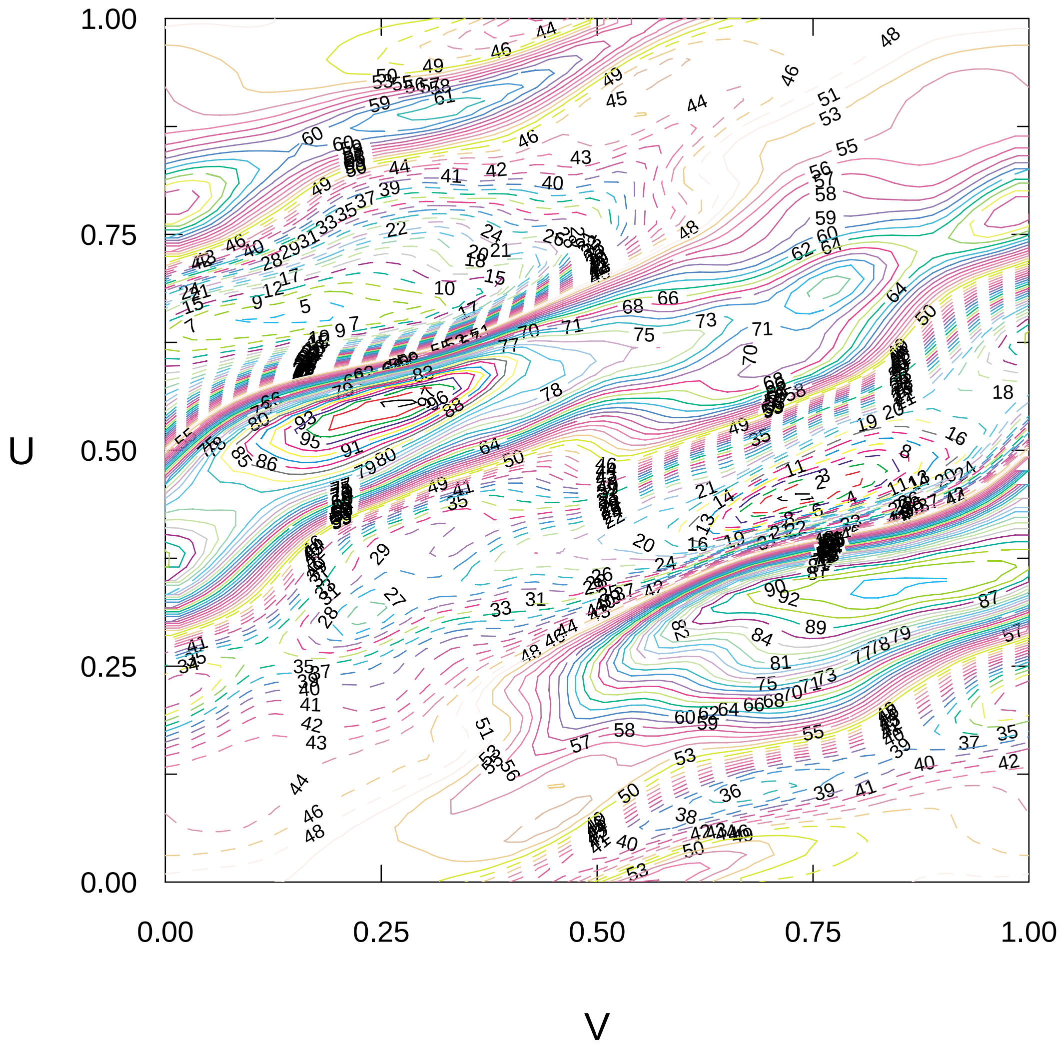
The Genetic Algorithm (GA) has previously been used to find saddle coils for several equilibria that were compatible with the PBX toroidal field coils and two of the existing vertical field coils. The GA has now been extended to work with modular coils and wavy poloidal field coils. The capability of adding fixed (in the geometrical sense) external coil sets will be included next.

The Problem



Outline

- The General Procedure
- Coil Cutting
- The Inductance Matrix
- The Reduced Matrix
- The Compressed Matrix
- Saddle Coils
- Modular Coils
- Wavy PF Coils
- Future Enhancements

The General Procedure

Once

- Build coils from the current potential produced by NESCOIL

Loop

- Let the GA decide which coils will be energized
- Compute the currents via Singular – Value Decomposition (SVD)

Cutting Coils

Saddle Coils ($I_{pol} = I_{tor} = 0$):

$nneg$	the number of coils obtained from negative contour values
$nzero$	the number of coils obtained from the zero contour value
$npos$	the number of coils obtained from positive contour values

Modular($I_{pol} = 1, I_{tor} = 0$) and
Wavy PF($I_{pol} = 0, I_{tor} = 1$) Coils:

$nneg$	the number of coils obtained from contour values $< .5$
$nzero$	the number of coils obtained from contour value $= .5$
$npos$	the number of coils obtained from contour values $> .5$

External Coils :

$nextbot$	the number of coils below the midplane
$nexttop$	the number of coils above the midplane

The Inductance Matrix

The inductance matrix relates the currents in the coils to the magnetic field at the plasma boundary, i. e.,

$$G_{ij}I_j = B_i$$

Since the geometry is fixed, this matrix needs to be computed only once for each equilibrium, coil type, and number of contours chosen.

Using the following notation

$$
\begin{aligned}
N_1 &= 1\\
N_2 &= nneg\\
N_3 &= nneg+1\\
N_4 &= nneg+nextbot\\
N_5 &= nneg+nextbot+1\\
N_6 &= nneg+nextbot+nzero\\
N_7 &= nneg+nextbot+nzero+1\\
N_8 &= nneg+nextbot+nzero+nexttop\\
N_9 &= nneg+nextbot+nzero+nexttop+1\\
N_{10} &= nneg+nextbot+nzero+nexttop+npos
\end{aligned}
$$

the inductance matrix can be also written as

$$
\begin{aligned}
\sum_{j=N_1}^{N_2} G_{ij}I_j &+ \sum_{j=N_3}^{N_4} G_{ij}I_j + \sum_{j=N_5}^{N_6} G_{ij}I_j\\
&+ \sum_{j=N_7}^{N_8} G_{ij}I_j + \sum_{j=N_9}^{N_{10}} G_{ij}I_j = B_i
\end{aligned}
$$

The Reduced Matrix

Using stellarator symmetry the problem can be reduced by noting that the coils (except for the zero level contour for saddle coils and the .5 level contour for for modular and wavy pf coils) come in symmetric pairs such that

$$I_j = \begin{cases} -I_{sp} & \text{saddle coils} \\ I_{sp} & \text{modular or wavy pf} \end{cases}$$

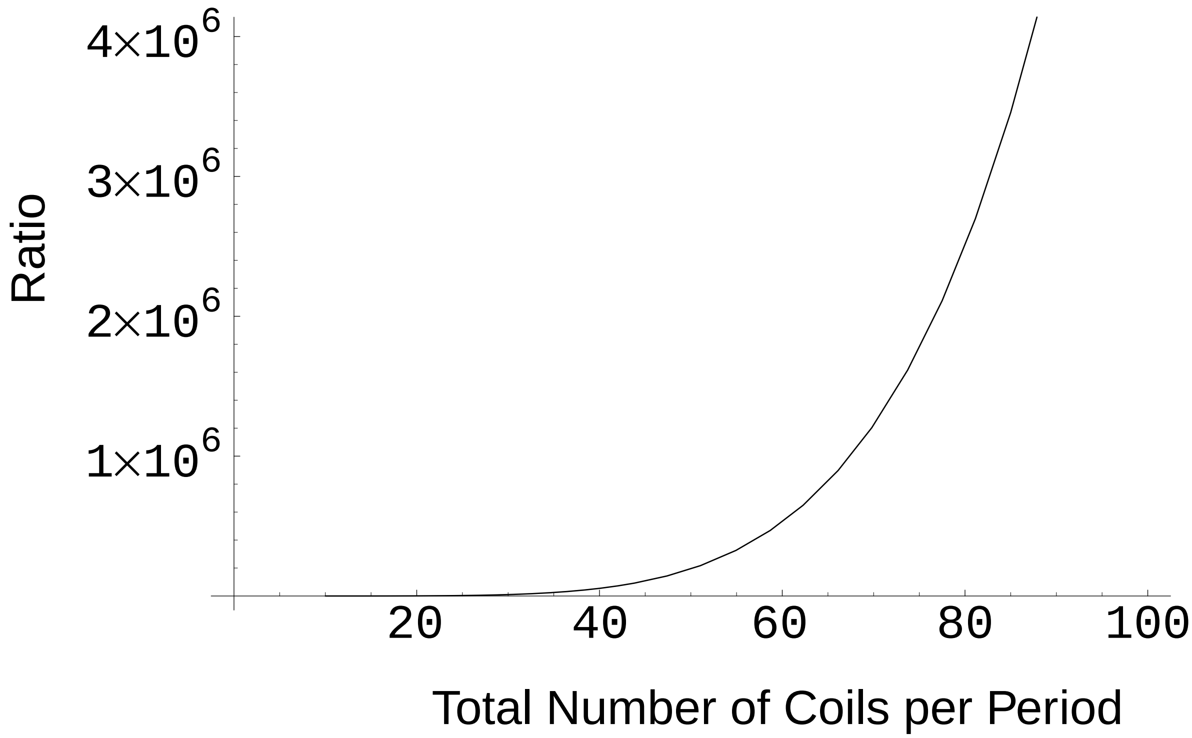
Why is this important? We are looking for m coils to be energized out of n total coils. Therefore the number of possible combinations is

$$\frac{n!}{m!(n-m)!}$$

If the symmetry is used the problem reduces to

$$\frac{\frac{n}{2}!}{\frac{m}{2}!(\frac{n}{2}-\frac{m}{2})!}$$

The ratio of these two quantities gives the reduction in the number of combinations that need to be tried. For the case where $m = 10$



The reduced matrix problem can be expressed as

$$
\begin{pmatrix}
R_{11} R_{12} \cdots R_{1N} \\
R_{21} R_{22} \cdots R_{2N} \\
\vdots \\
\vdots \\
\vdots \\
R_{M1} R_{M2} \cdots R_{MN}
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{pmatrix}
=
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
\vdots \\
\vdots \\
B_M
\end{pmatrix}
$$

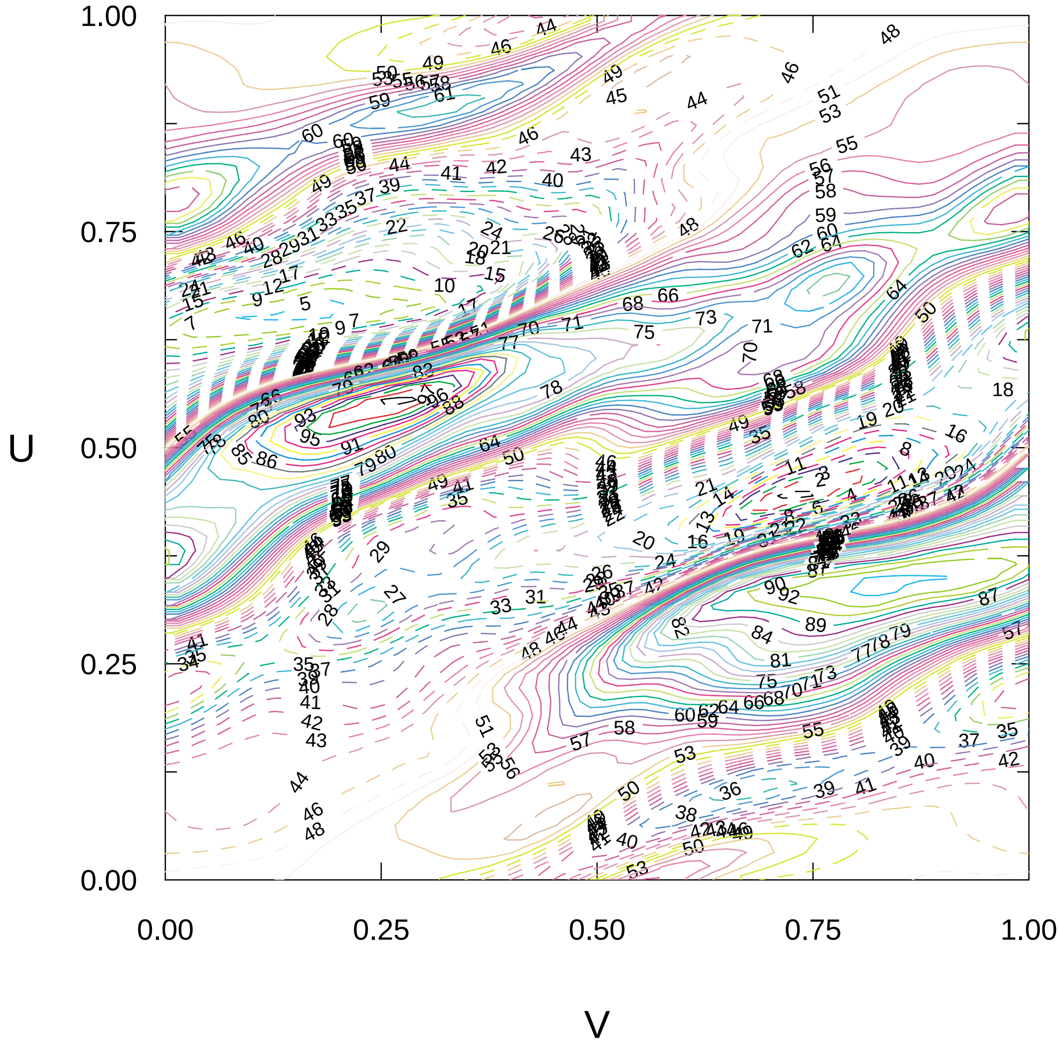
Since only a few of the coils will be energized in the GA, we do not need to solve the entire problem.

The Compressed Matrix

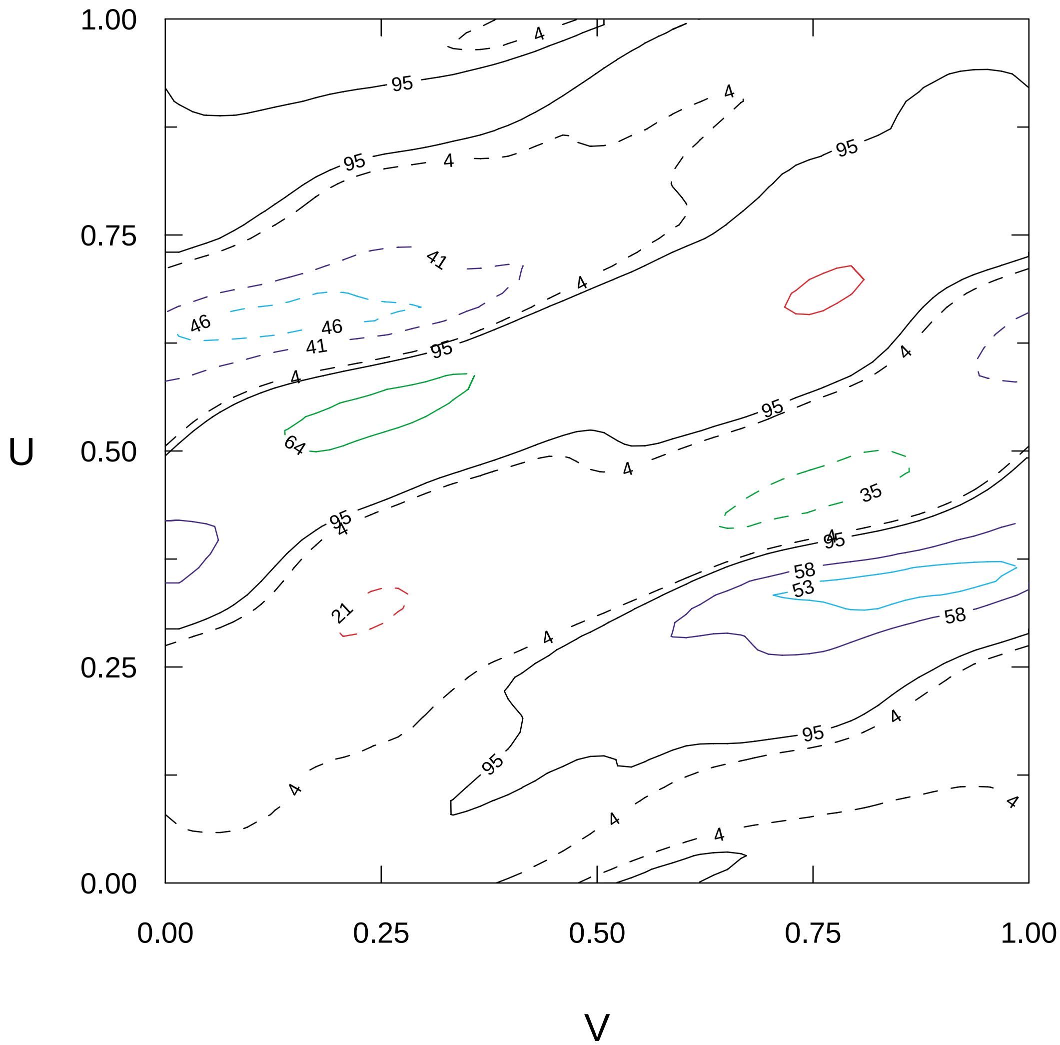
The inductance matrix is compacted by eliminating the columns that correspond to coils that are not energized. For example, if only coils i, j, k, and l were energized, the matrix equation to be solved would be,

$$\begin{pmatrix} R_{1i} R_{1j} R_{1k} R_{1l} \\ R_{2i} R_{2j} R_{2k} R_{2l} \\ \vdots \\ \vdots \\ \vdots \\ R_{Mi} R_{Mj} R_{Mk} R_{Ml} \end{pmatrix} \begin{pmatrix} I_i \\ I_j \\ I_k \\ I_l \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ \vdots \\ B_M \end{pmatrix}$$

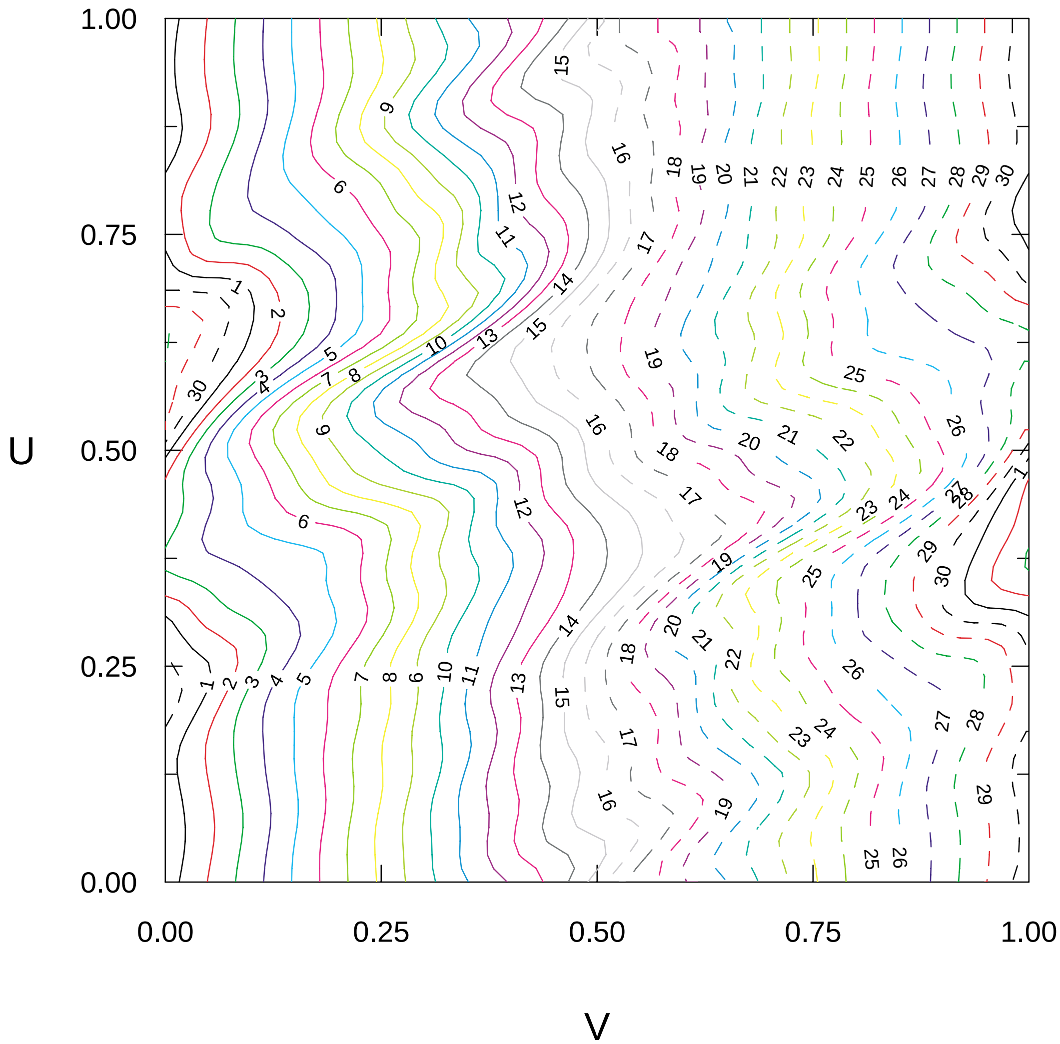
Saddle Coils



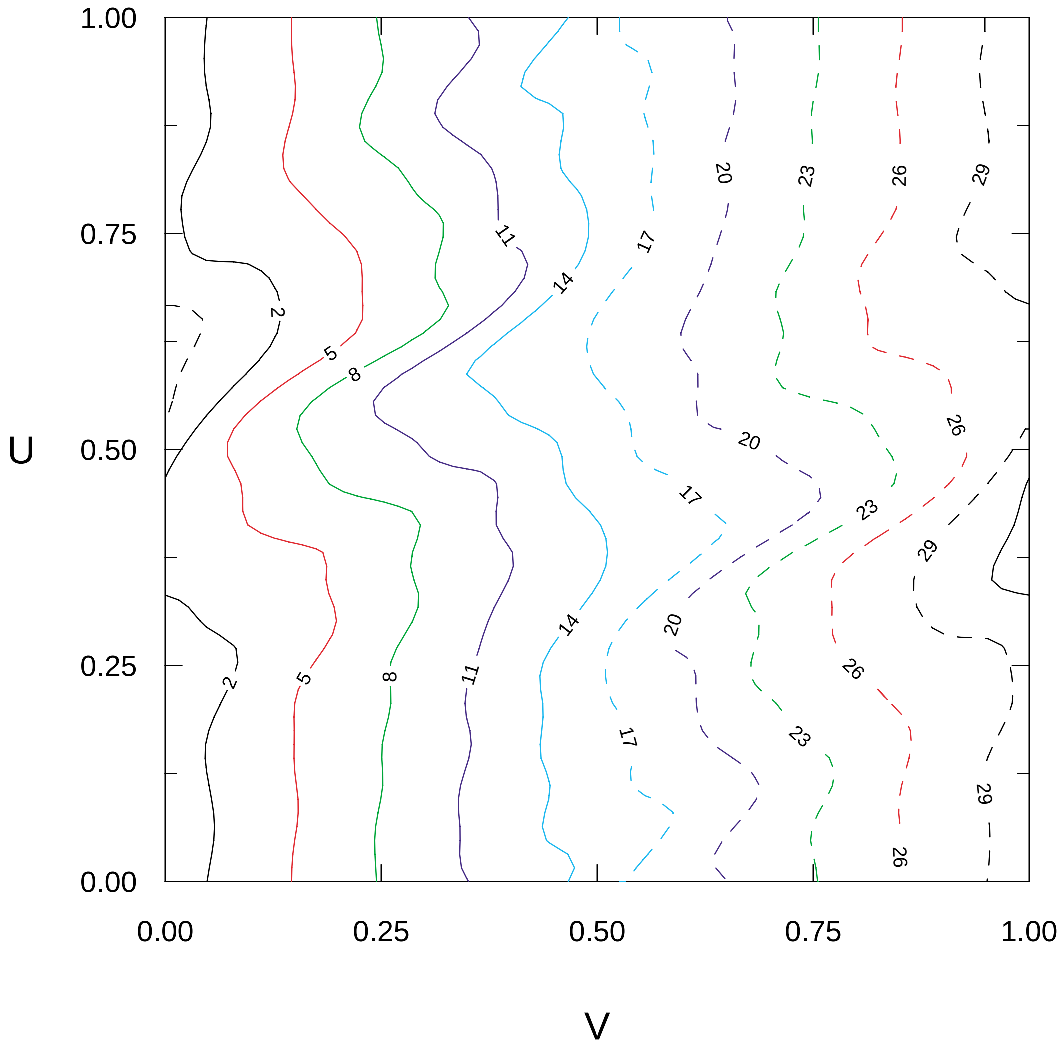
	no. coils per period	Berror (%) avg.	max.	Jmax (kA/cm_2)
EC	26	0.95	7.0	14.7
GA	10	1.04	4.9	15.4
GA	10	1.60	7.6	11.0



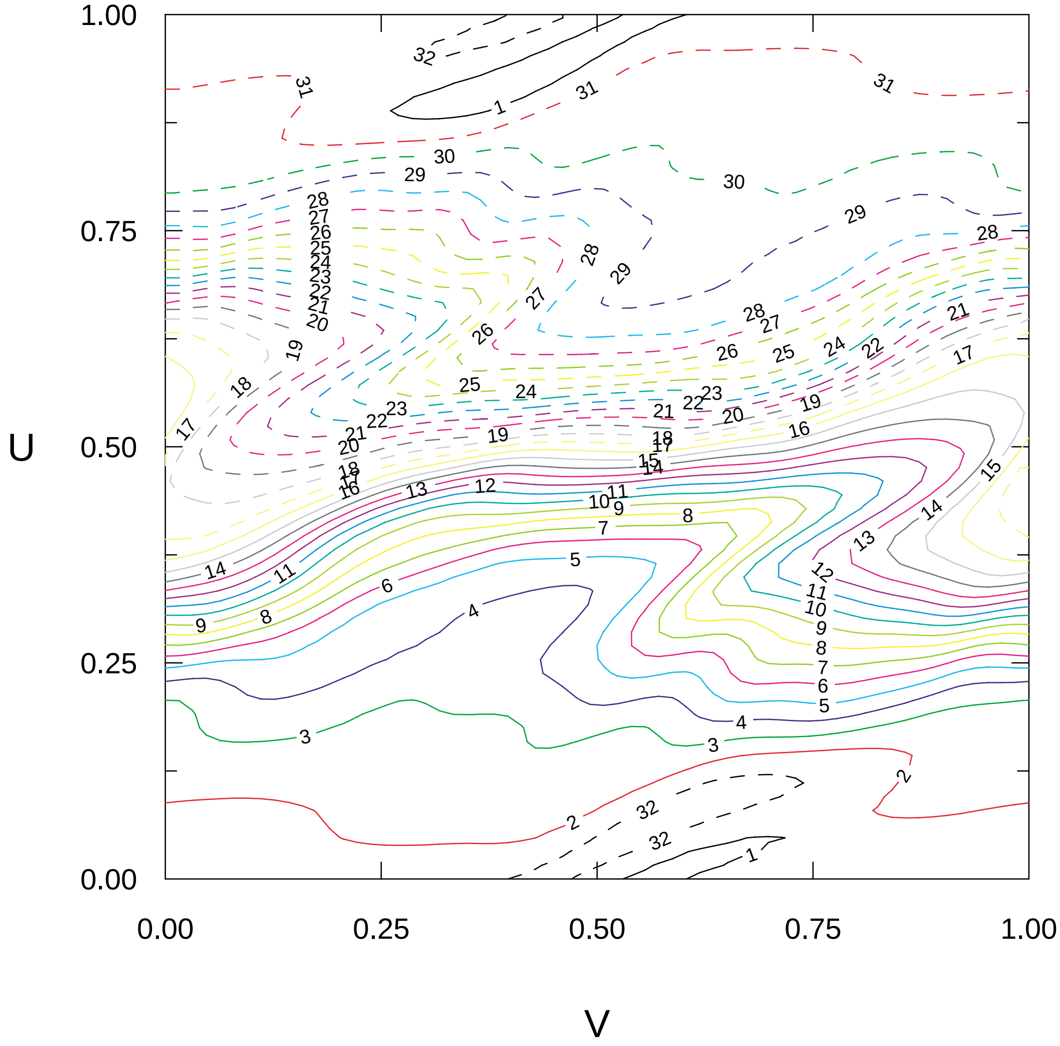
Modular Coils



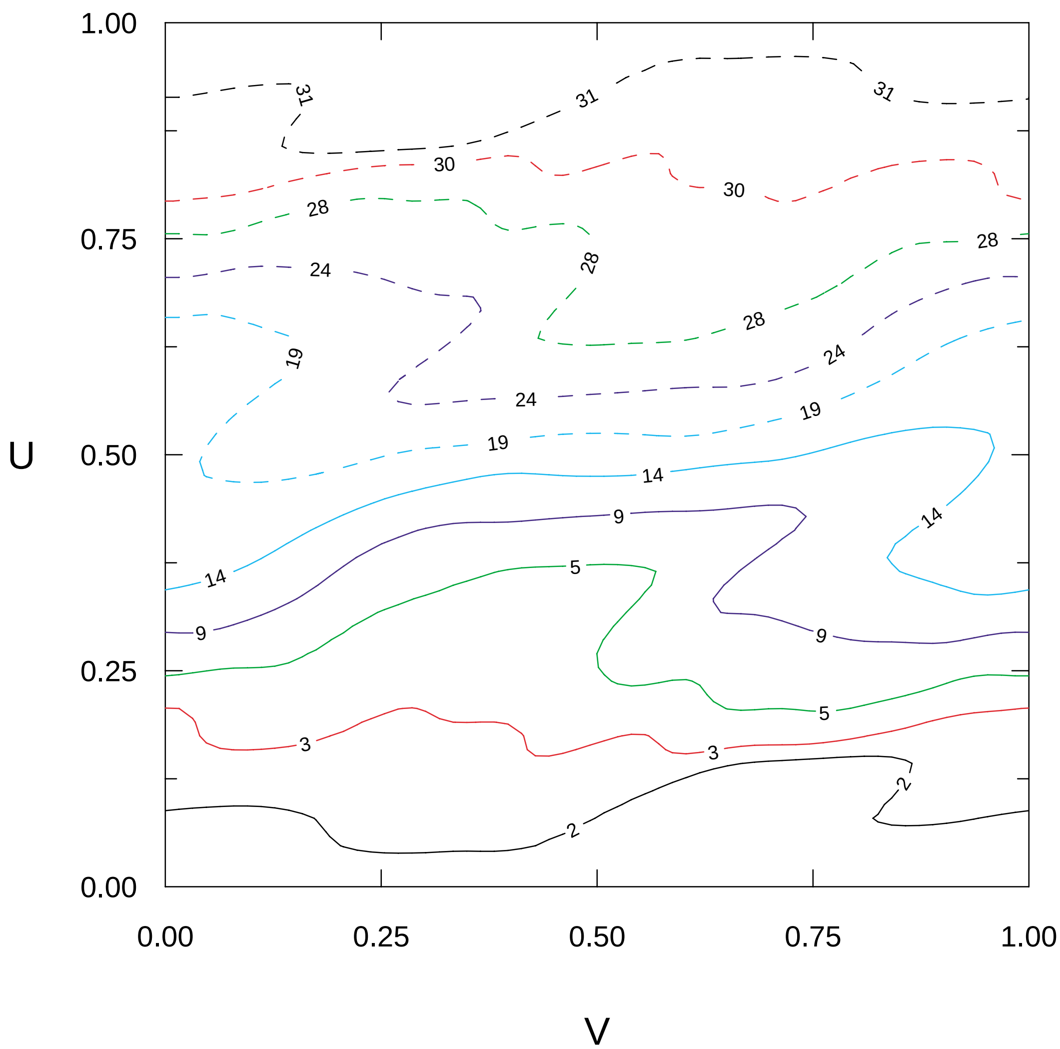
	no. coils per period	Berror (%) avg.	max.
EC	10	2.17	11.28
GA	10	2.17	11.11



Wavy PF Coils



	no. coils per period	Berror (%) avg.	max.
EC	16	2.92	11.93
GA	10	3.21	17.76



Future Enhancements

Additional external coil sets whose currents are determined by SVD

Search coils cut from multiple winding surfaces

Target multiple equilibria and look for common features