Control Matrix Studies

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In the Physics Meeting talks Neil and I gave on 12/2 and 12/16, we provided updates on what our Control Matrix (CM) studies have found so far, touching on where we plan to go from here. In our talks here, we will discuss the CM project with a somewhat different emphasis, summarizing the results of the earlier talks, and addressing how these will help provide improved design tools for NCSX.

∘Formulation:

∘Configuration space:
·'Full-space' $\mathbf{X}$ of amplitudes
$\mathbf{X} \equiv \{X_{j=1,..N_x}\} \equiv (R_{\mathbf{n}_1}, Z_{\mathbf{n}_1}, R_{\mathbf{n}_2}, ...Z_{\mathbf{n}_{N_x/2}})$
needed for $(e.g.)$ a VMEC equilibrium specification of the boundary. Here, $\mathbf{n} \equiv (\tilde{n} \equiv n/N_{fp}, m)$, and $N_x \sim 70$.
·'Reduced-space' $\mathbf{Z}$ of combinations of those X_j which capture the most important physics:
$\mathbf{Z} \equiv \{Z_{j=1,..N_z}\}$, where $N \equiv N_z \leq N_x$.

∘Over $\mathbf{X}$ or $\mathbf{Z}$, consider the behavior of $M \equiv M_p \sim 5$ physics figures of merit
$\mathbf{P} \equiv \{P_i(\mathbf{Z})\} = (\chi_1^2, \chi_2^2, W_1, W_2, \lambda)$.

·Expand $\mathbf{P}(\mathbf{Z} = \mathbf{Z}_0 + \mathbf{z}) = \mathbf{P}(\mathbf{Z}_0) + \mathbf{p}$ about $\mathbf{Z} = \mathbf{Z}_0$. (In component-form, summation over repeated indices assumed):

$$p_i(\mathbf{Z}_0 + \mathbf{z}) = C_{ij}(\mathbf{Z}_0) z_j + \frac{1}{2} H_{ijk}(\mathbf{Z}_0) z_j z_k + (h.o.). \quad (1)$$

For small enough $\mathbf{z}$, one has

$$\mathbf{p} = \mathbf{C}_0 \cdot \mathbf{z}, \quad (2)$$

with $\mathbf{C}_0 \equiv \mathbf{C}(\mathbf{Z}_0)$ the $M \times N$ 'control matrix' at design point $\mathbf{Z}_0$. It may be inverted, using SVD.

·Taking the particular basis set $\boldsymbol{\pi}^{i=1,M}$ which have 1 in the ith position and 0 elsewhere, have set $\boldsymbol{\xi}^i$ of displacements

$$\boldsymbol{\xi}^i \equiv \mathbf{C}_0^{-1} \cdot \boldsymbol{\pi}^i,$$

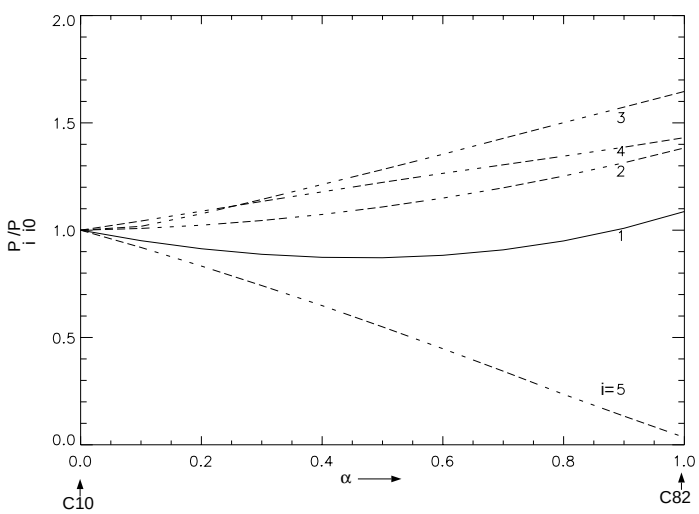
physically representing a set of displacements which vary a single physics parameter P_i, leaving the others unchanged. These span the 'range' of $\mathbf{C}$. The $(N-M)$ vectors $\mathbf{v}^{i=M+1,N}$ spanning the nullspace of $\mathbf{C}$ (change the configuration without modifying any of the P_i) also important for design flexibility.

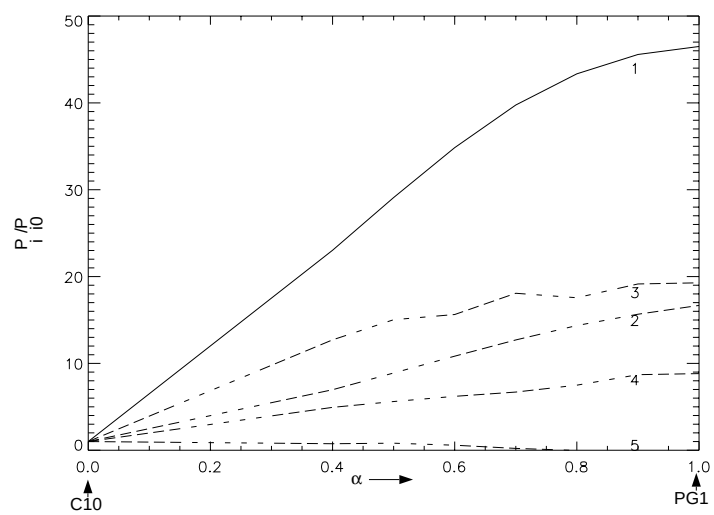
∘Summary of results of CM studies so far:

∘Established and tested machinery for application of CM analysis, mostly independent of particular $\mathbf{Z}$-space specialization.

∘Examined topography of $\mathbf{Z}$-space, both locally in the vicinity of design points $\mathbf{Z}_0$, and globally, on trajectories between different $\mathbf{Z}_0$. Most attention focussed on C10, but C82 and PG1 also examined.

∘Found that the variation of the P_i is rather smooth and unstructured, even over distances in $\mathbf{Z}$ generally considered large. In an appreciable neighborhood of C10 ($\Delta Z_j \sim 1$cm) the P_i may be modeled by a quadratic function of $\mathbf{z} = \mathbf{Z} - \mathbf{Z}_0$.





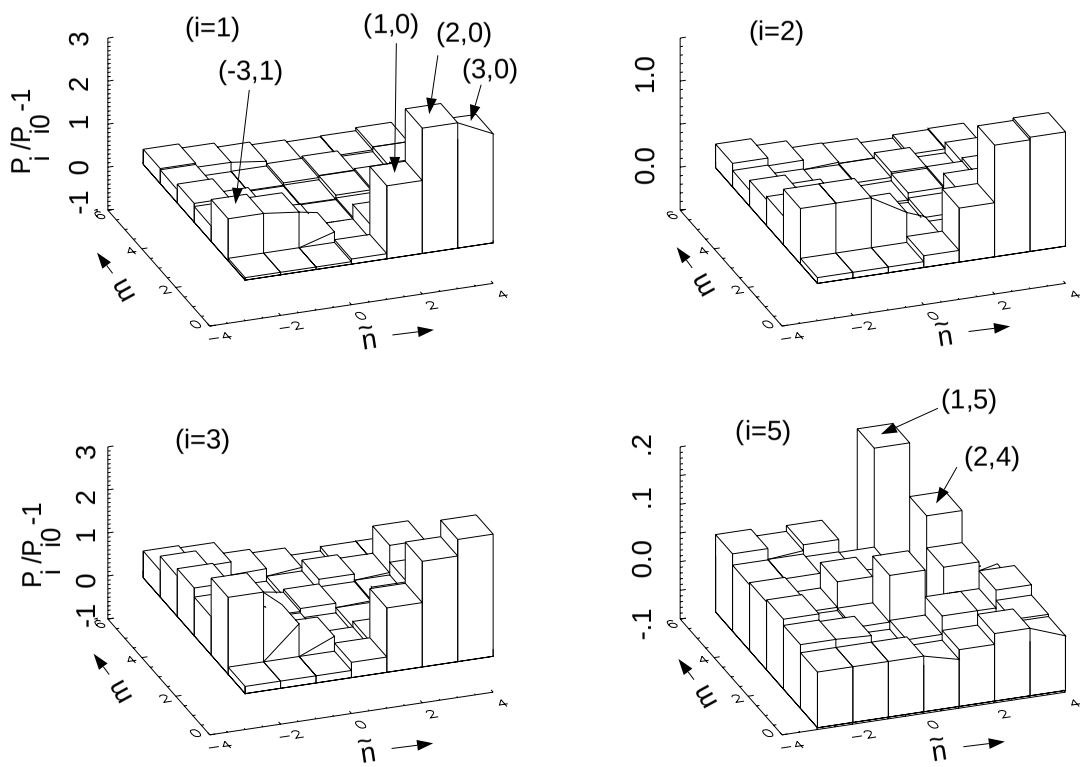
∘Explicitly constructed this quadratic representation about C10 for a reduced set ($N_z = 8$) of perpendicular displacements of the C10 boundary, computing both the 1st and 2nd order coefs C_{ij} and Hessian H_{ijk} for

this set.

∘Two methods applied to reduce the dimensionality of the **Z**-space need to work with from the initial $N_x = 78$.

(a)Removing the redundancy in the **X**-specification. This reduces N_z from $N_x = 78$ to $N_x/2 = 39$. (VMEC internally also does this.)

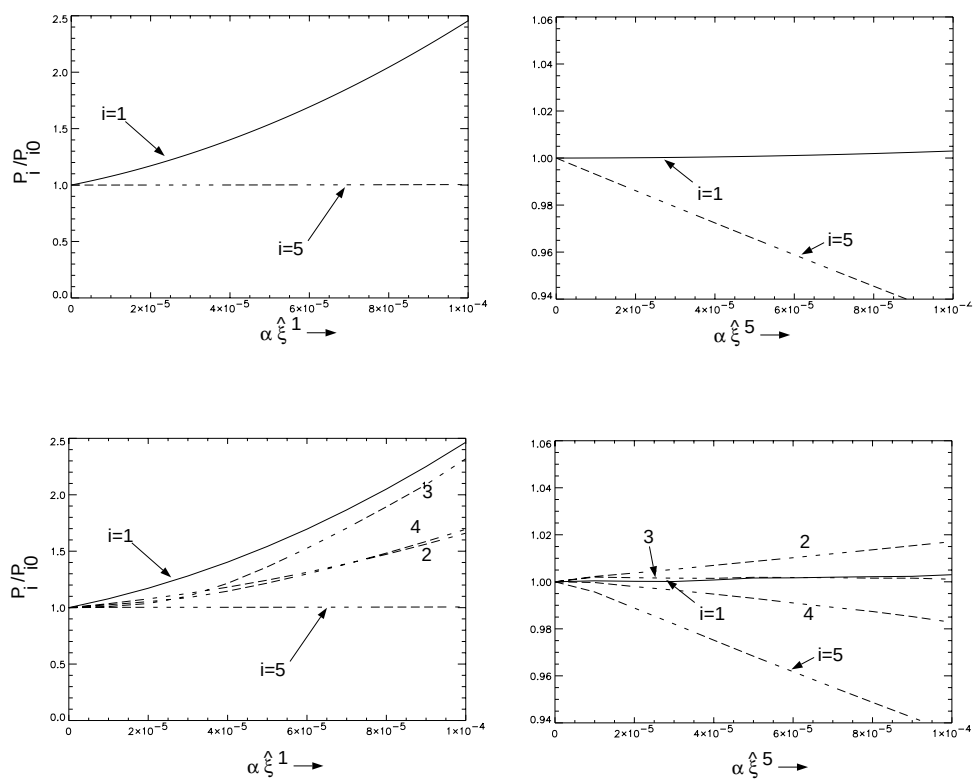
(b)Taking only the perturbations most effective in varying some P_i, as seen from 'sensitivity histograms' of $P_i/P_{i0} - 1$ over the $(\tilde{n} \equiv n/N_{fp}, m)$ plane. We chose the 4 most effective for P_1, and for P_5, thus further reducing N_z from 39 to 8.



∘The sensitivity histograms for the 4 QA-associated

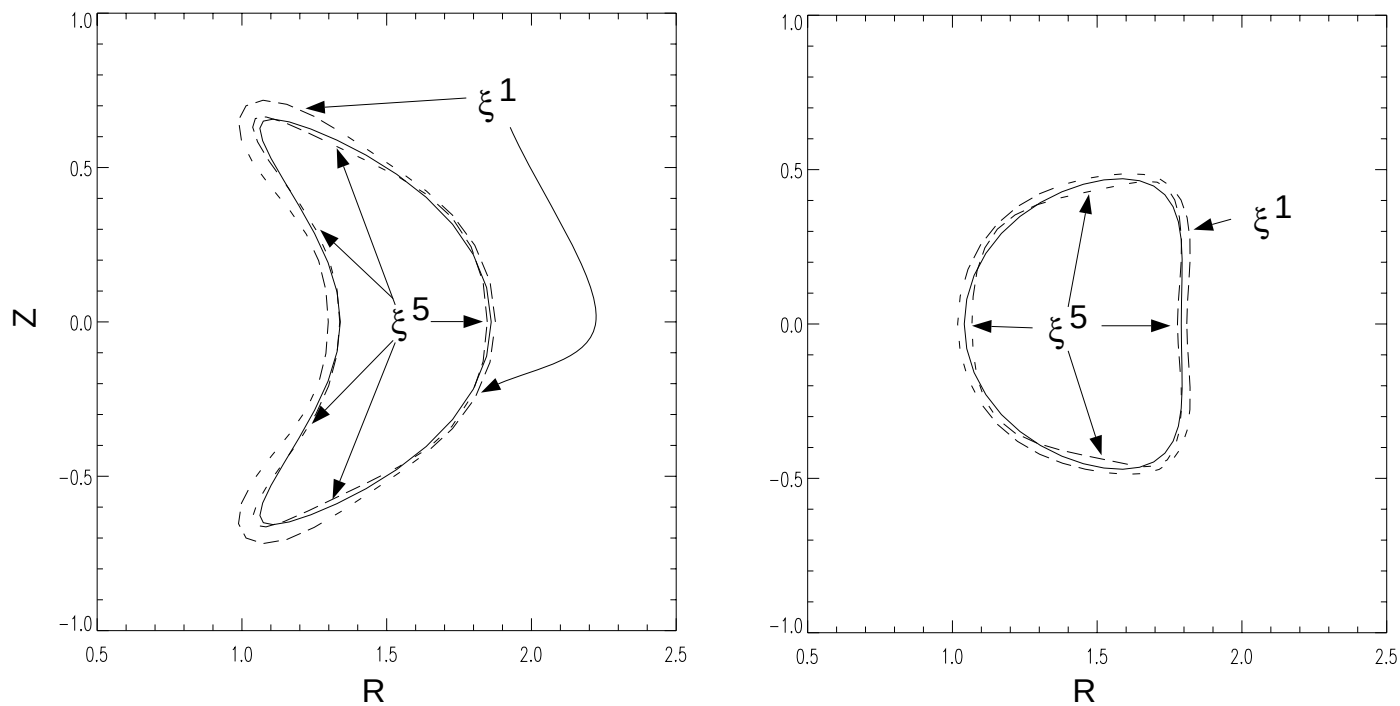
P_i look similar, and these differ from the kink–associated one P_5. The former have small m and a range of $\tilde{n}$, while the P_5 set have $\tilde{n} \sim 1$ or 2 and a range of m.

∘Applying the CM formalism locally (near C10), numerically demonstrated that the mathematically–computed deformations $\boldsymbol{\xi}^i$ which vary a single P_i and the null-space basis $\mathbf{v}^i$ which vary none of the P_i (yet do produce a different configuration) do in fact have the desired effect.



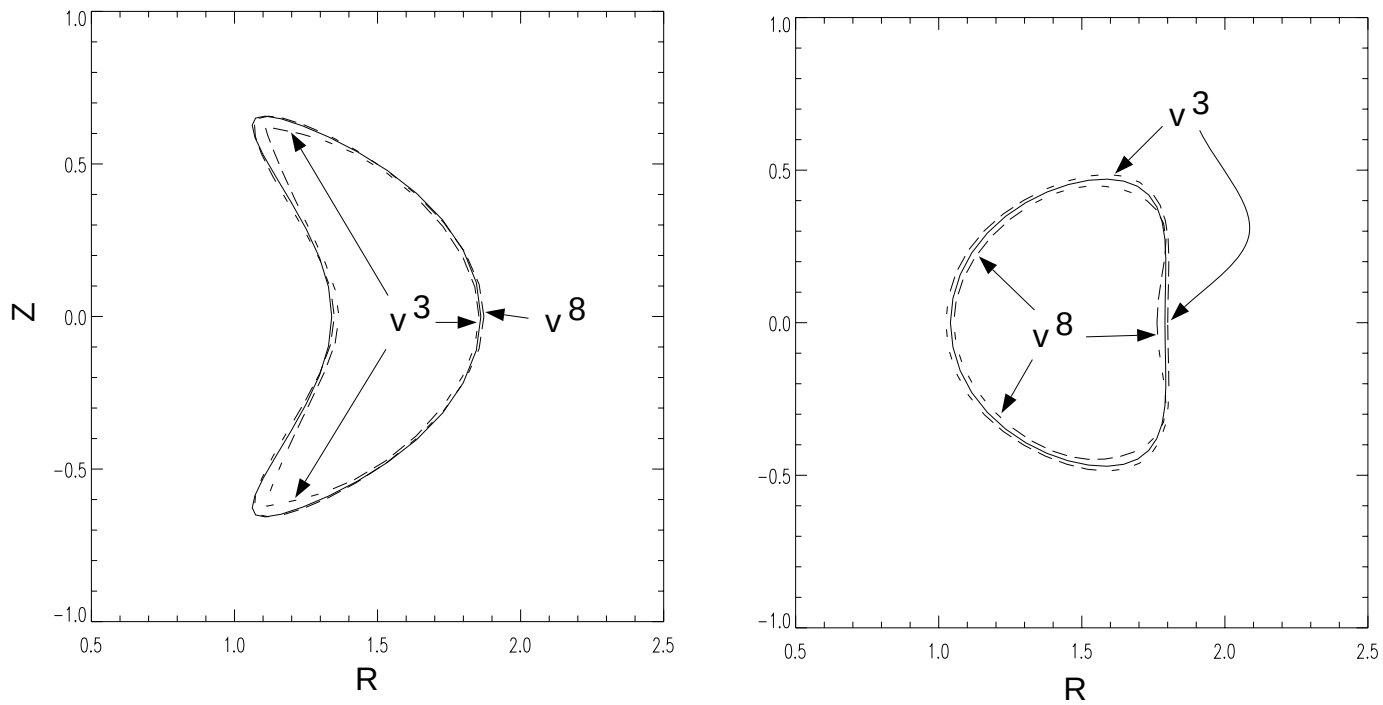
∘The 4 QA-associated P_i produce boundary displacements $\boldsymbol{\xi}^i(\theta, \zeta)$ similar in appearance, and different from the kink–associated one P_5.

◦For C10, $\boldsymbol{\xi}^5$ manifests the outboard indentation at $\tilde{\zeta} \equiv N_{fp}\zeta = \pi$ previously noted by Long-Poe and Neil to stabilize the kink, enhancing C10's negative triangularity at $\tilde{\zeta} = \pi$.



However, this behavior is not generic: for PG1, $\boldsymbol{\xi}^5$ enhances its *positive* triangularity, consistent with tokamak intuition on kink stabilization.

∘For C10, outboard indentation at $\tilde{\zeta} = \pi$ is not enough to help kink stability. E.g., null vector $\mathbf{v}^8$ also causes such indentation, but has a different ζ-dependence.



∘Implications for plasma configuration:

∘Our topographic findings suggest that, as for the P_i, there also might not be many hills and valleys in the objective function $F(\mathbf{P})$. This needs to be squared with what the optimizer is doing. Long-Poe Ku and Chuck Kessel are working with us on the optimizer side, conducting experiments with the optimizer to help us better understand the relationship between the CM topography findings and the optimizer results.

∘The smooth topography also raises the possibility of using an MC code to provide a *real* transport P_i.

∘The CM machinery is giving us boundary deformations which affect given P_i. Further physical interpretation of these should improve our intuition about how to make deformations that do what we want.

∘Implications for operational flexibility, startup, & coil design:

∘Use of H_{ijk} in weighting matrix $\mathbf{W}$ in QM approach (see Neil's talk).

∘As in my 12/2 talk, the same CM analysis can be applied with a new specialization of the $\mathbf{Z}$ from boundary amplitudes $\mathbf{X}$ to coil current amplitudes $\mathbf{I}$, and using free– instead of fixed–boundary VMEC:

∘With specialization $\mathbf{Z} \to \mathbf{I} \equiv \{I_{j=1,..N_z}\}$ describing the external currents: One may choose
(a)$I_i \to K_{\mathbf{n}_i} \equiv$ the Fourier amplitudes of the current potential $K(\theta, \zeta)$ (for coil design), or
(b)$I_i \to J_i \equiv$ the amount of current in the ith coil of a given coil set (for operational flexibility).
∘Then the $P_i(\mathbf{Z})$ can be computed almost as now, but using free-boundary VMEC.

∘Sometimes useful to extend the Z_j, P_i:
·Extend the Z_j to include $Z_\beta \equiv \langle\beta\rangle$, (and maybe a parameter characterizing the peakedness of the pressure profile), and
·Extend the P_i to include P_6, = current–sheet complexity measure like that which Steve H developed last year.

∘Then applying the CM method just as earlier:
·$\boldsymbol{\xi}^6$ gives current–sheet changes which reduce the coil complexity, while keeping the physics performance unchanged.
·Including $\langle\beta\rangle$, the relative size of components of $\langle\beta\rangle$ to the other Z_j tell what changes need to be made as $\langle\beta\rangle$ is raised to maintain physics performance.