Development of New Transport Optimization Targets: DKES, J-Invariant

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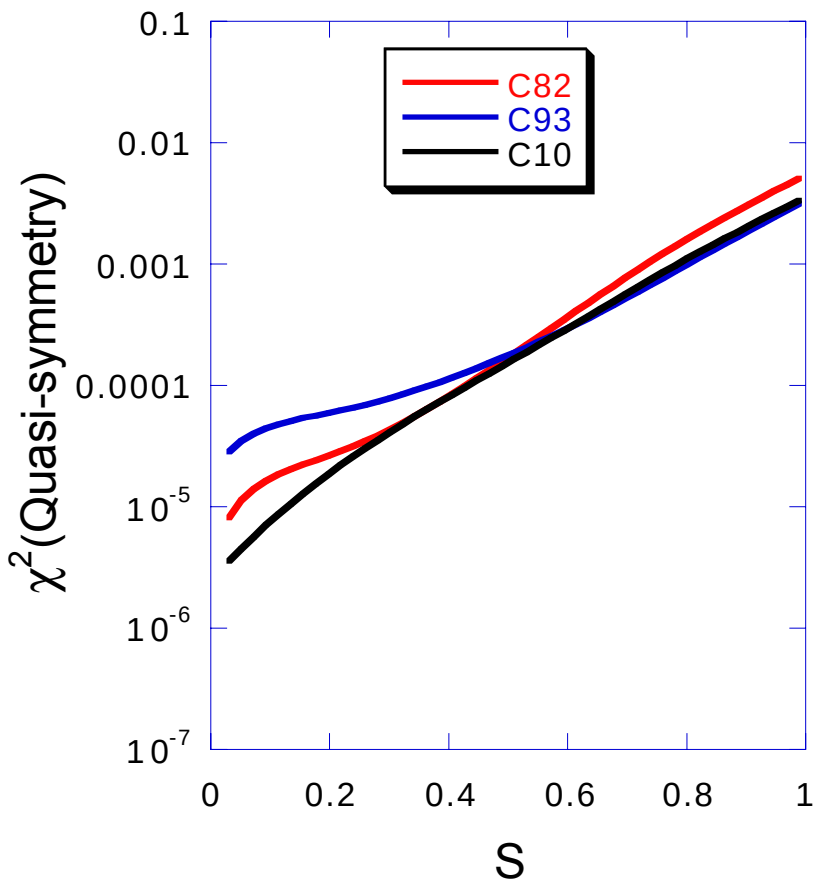
DKES and J-invariant targets offer a more direct approach to transport improvement:

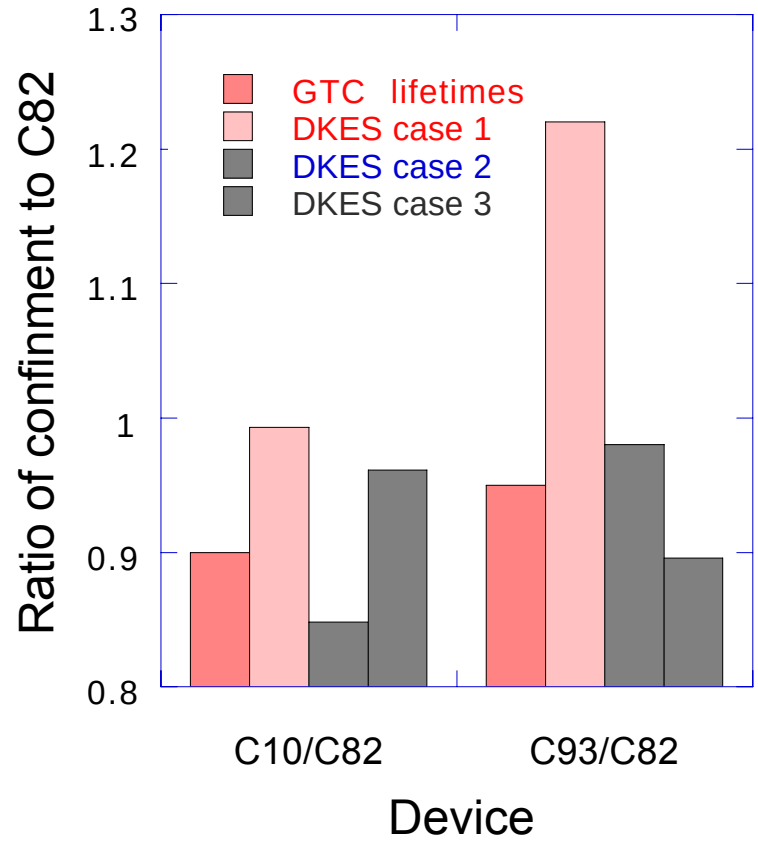
- Potential advantages of DKES/J-invariant targets
  - Minimization of deviations from quasi-symmetry is observed to have a practical lower limit for compact devices
  - Quasi-symmetry target does not always correlate well with GTC particle simulation transport evaluations
  - May allow more flexibility to achieve other physics requirements
- DKES and J offer complementary features:
  - DKES: moderate collisionality required, local diffusive model
  - J invariant: measures collisionless confinement, can treat non-local loss channels

DKES (Drift Kinetic Equation Solver) has been checked against GTC runs for a range of QA devices and has been recently incorporated into the optimizer by L-P Ku.

- Issues regarding DKES optimization target
  - Convergence of upper/lower bounds at collisionalities of interest
  - Computational time (for optimization, would like CPU time < 1 minute)
    - at low ν, representation of distribution function in narrow boundary layer regions requires progressively higher resolution
    - DKES spectrum optimizer
      - generates a more compact spectrum for f based on propagating the $\nabla_{||} \ln B^{-1}$ operator
    - Post-analysis: solve for distribution fcn. with large number of modes, then remove insignificant ones
    - explore other representations besides Fourier-Legendre
  - Comparisons with global GTC particle-based calculations
    - GTC shows confinement ratios of 1.0 : 0.95 : 0.90 between C82, C93 and C10

Our initial comparisons with GTC were based on the C10, C82 and C93 QA devices and had mixed success in matching the GTC trends:

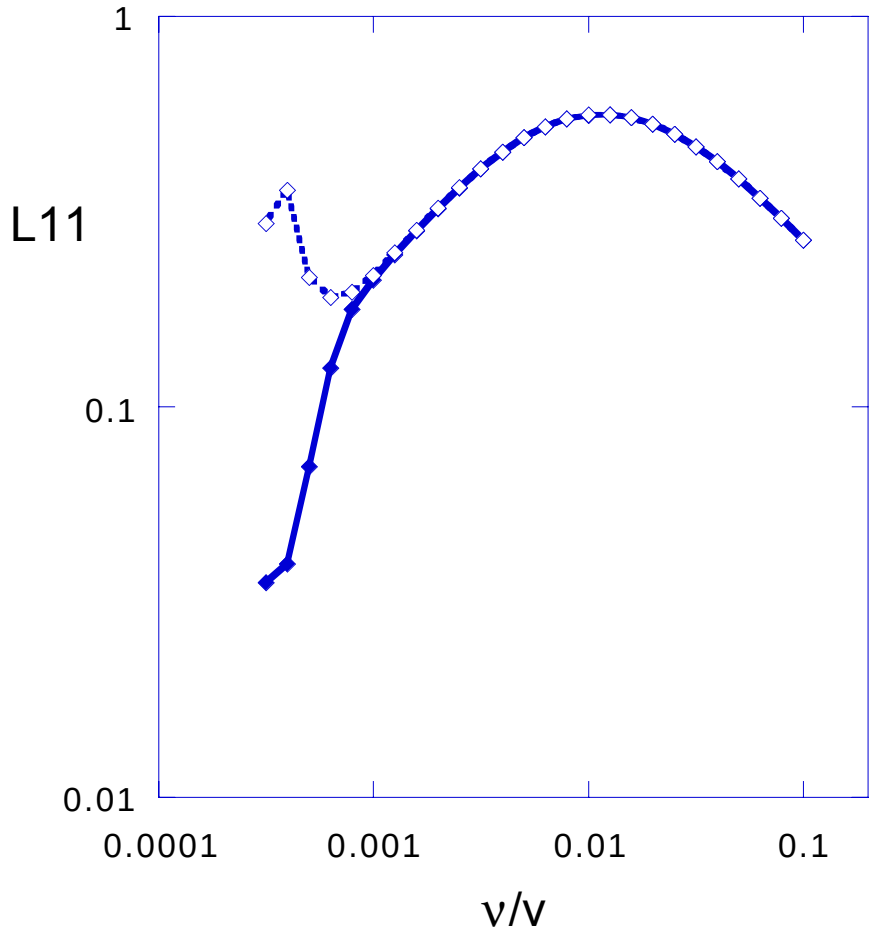




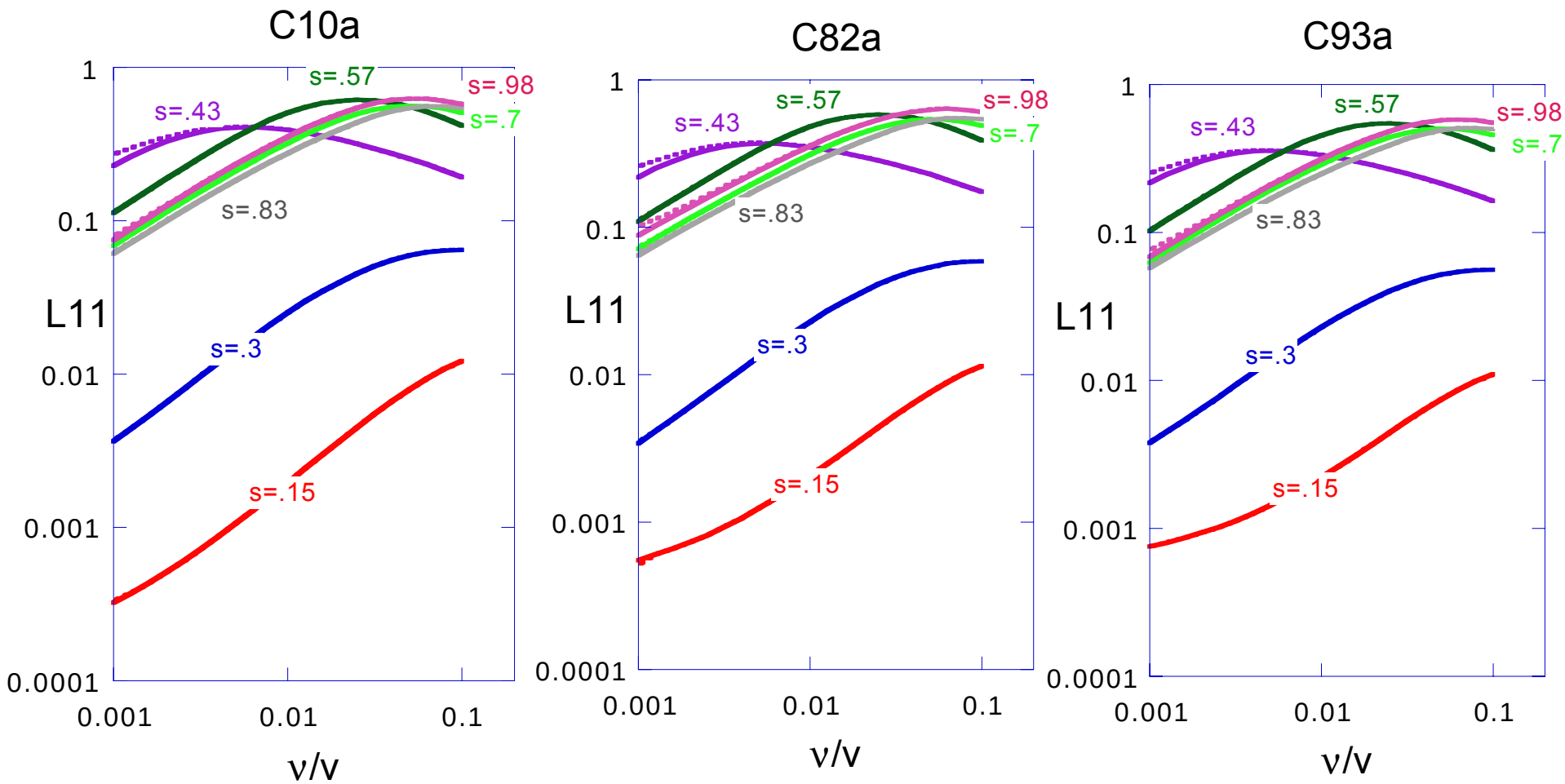
- DKES case 1: ν/v = 0.001, 200 Fourier modes, 50 Legendre modes, Cray time ~ 1 minute
- DKES case 2: ν/v = 0.0001, 200 Fourier modes, 50 Legendre modes, Cray time ~ 1 minute
- DKES case 3: ν/v = 0.0001, 500 Fourier modes, 100 Legendre modes, Cray time ~ 10-15 minutes

One issue is that DKES upper and lower bounds begin to significantly diverge at lower collisionalities ($\nu/v < 0.001$)

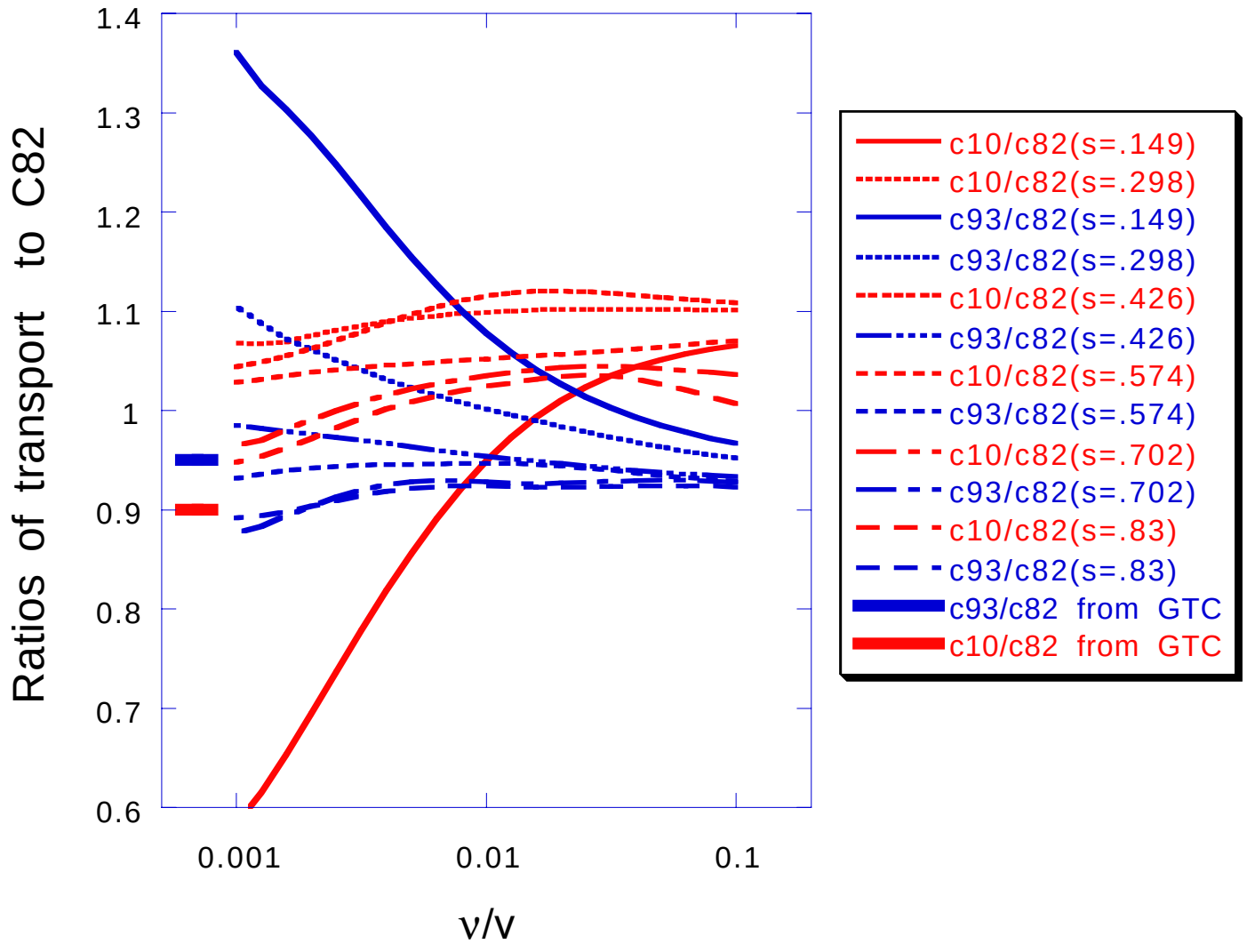
C82 at s = 0.49, E/v = 0.08, 40 B_{mn}'s, ~200 Fourier modes, 50 Legendre modes, J90 CPU time ~ 1 minute



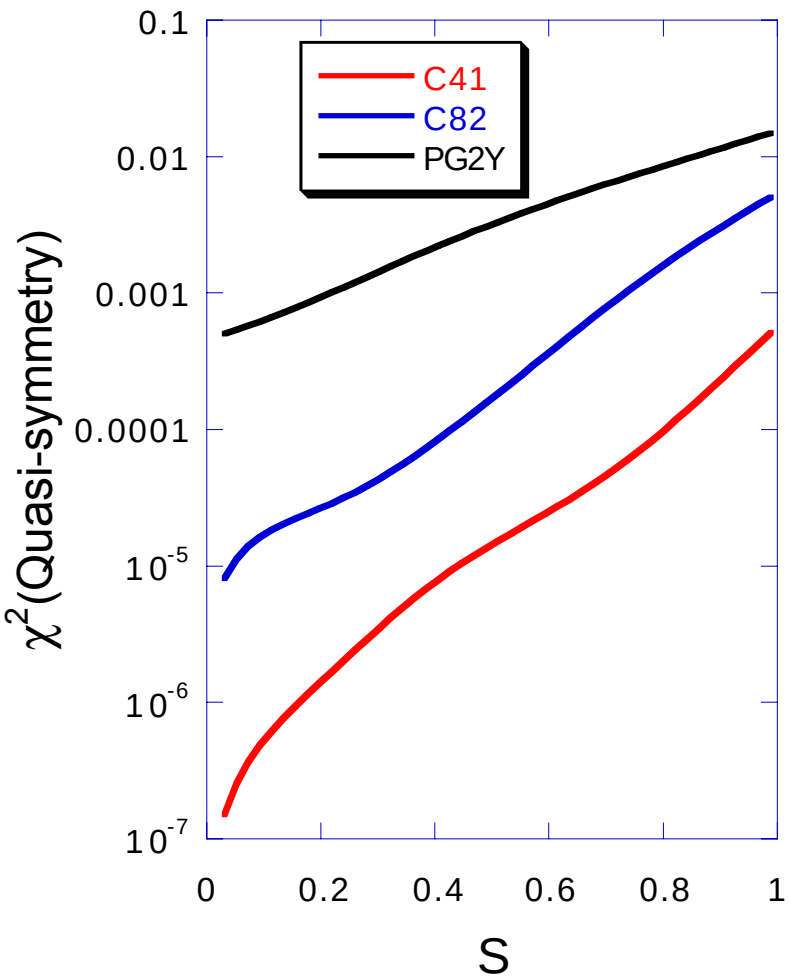
Also, GTC samples global confinement characteristics while DKES provides local diffusion coefficients. More recent DKES calculations show that these vary substantially with radial position (s = flux surface label)



Ratios of the DKES L_{11} coefficient for c10, c93 to c82 are plotted as a function of flux surface location and collisionality. At lower collisionalities the DKES results show similar trends as GTC (C93 > C10) while at high collisionalities the trend of the DKES coefficients reverses (C10 > C93):



More recently DKES/GTC have also been compared for more disparate devices (i.e., differences between C10, C82 and C93 may be too subtle to be a good test)



Comparison of Transport Target Calculated by GTC, Simple Sum of Bmn (n≠0), and DKES

	C41	C82	PG2Y	
τ_i^{NC} (ms)	33.2±3.4	17.8±1.7	14.9±0.9	GTC
χ^2 (s=0.5)	$1.5 \cdot 10^{-5}$	$1.8 \cdot 10^{-4}$	$3.3 \cdot 10^{-3}$	B_{mn}'s
$\Gamma_{11}(\upsilon/v=10^{-3})$	$3.3 \cdot 10^{-2}$	$5.7 \cdot 10^{-2}$	$1.1 \cdot 10^{-1}$	DKES
$\Gamma_{11}(\upsilon/v=10^{-4})$	$5.3 \cdot 10^{-3}$	$1.8 \cdot 10^{-2}$	$2.1 \cdot 10^{-2}$	DKES

- These results show a better agreement in trends between GTC and DKES
  (based on on s=0.51 surface, E_r/v=0.008, 80 modes for B, ~400 modes for the distribution function, and 100 Legendre modes)
- Similar results are obtained with half the Legendre modes and result in a Cray run time of about 1 minute

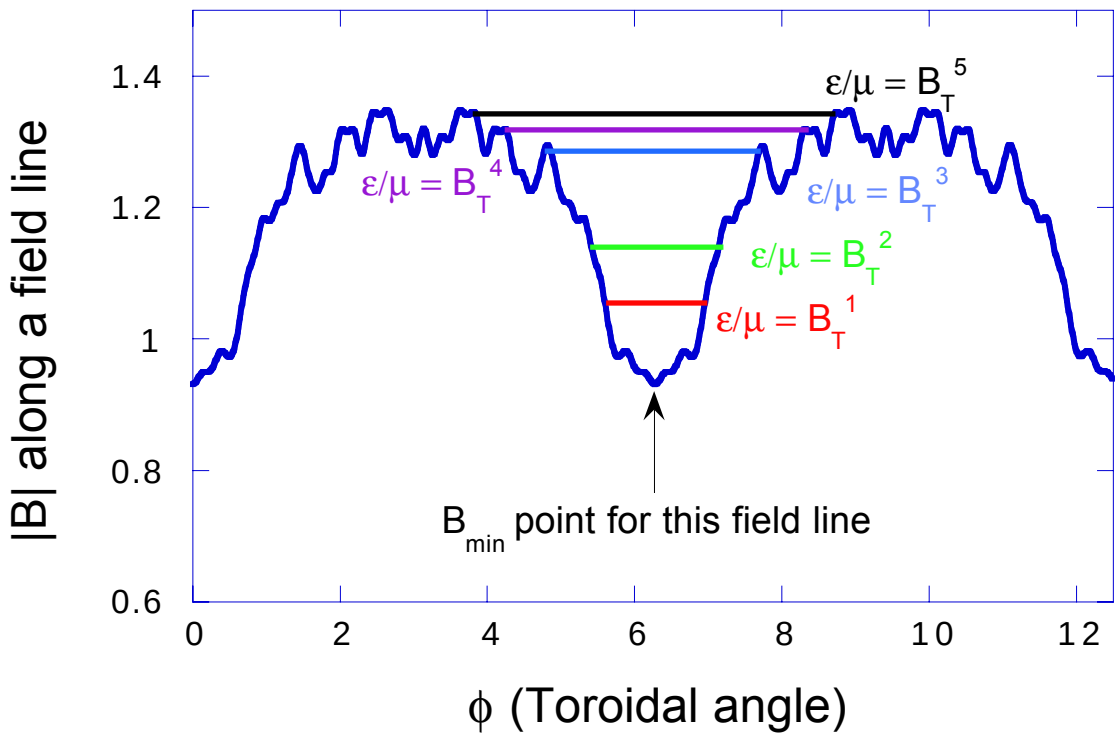
Longitudinal Invariant (J) optimization target

- $J = \oint dl\, v_{\parallel}$ $\qquad$ $\langle v_d \rangle = \dfrac{c}{eB\tau_B} \vec{\nabla} J \times \dfrac{\vec{B}}{B}$

where τ_B = bounce time = $\oint \dfrac{dl}{v_{\parallel}} = \dfrac{\partial J}{\partial \varepsilon}$

- References: T. G. Northrup, E. Teller, "Stability of the Adiabatic Motion of Charged Particles in the Earth's Field," Phys. Rev. Vol. 117, Jan., 1960, pg. 215; Northrup's book: *The Adiabatic Motion of Charged Particles* (Interscience, 1963), ...etc.
- J contours show trajectories of banana centers for trapped orbits
  - Any variation of J within a magnetic surface implies drifts away from the surface
  - Perfectly toroidal symmetry: banana centers precess only in toroidal direction
  - Slight asymmetries away from toroidal lead to drifts across flux surfaces
  - with finite banana width effects orbits move inside and outside of J surfaces (i.e., closure of J surfaces is necessary, but not sufficient for confined orbits)
- Application of J to passing orbits
  - longitudinal motion must be periodic for J to exist
  - implies J can only be applied to passing orbits on rational surfaces

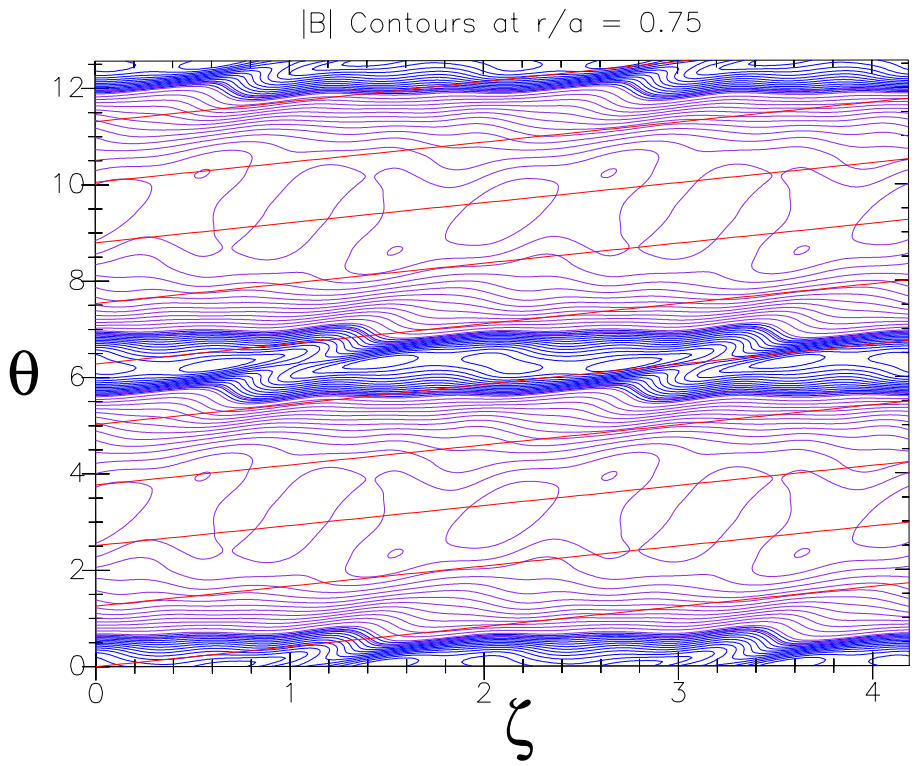
Algorithm used to calculate J:

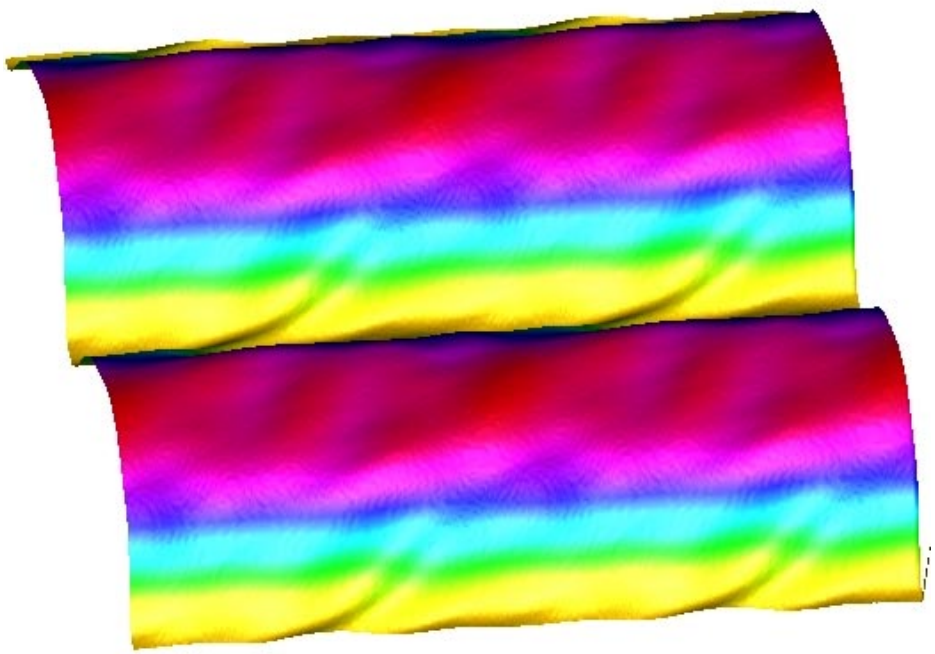


- On each surface, follow field lines starting at a succession of toroidal angles within a field period
- On first field line identify location of global minimum in |B| (generally near $\theta = 0$ for QA devices)
- Follow location of this global minimum in |B| as you move to successive neighboring field lines
- Determine the global maximum and minimum in |B| over each surface
- Calculate J on each field line for a range of ε/μ's running from this global B_{min} to B_{max}
  - Start at B_{min} location and integrate forward and backward along field line until $B = \varepsilon/\mu$ (turning point) is encountered (this avoids trapping in local wells)
  - Step to next field line, do the same, etc.
- Three loops: flux surfaces, field lines on each surface, ε/μ values
- Note that there will be discontinuous jumps in J (but not its ψ,ϕ derivatives) with turning point jumps (i.e., $\varepsilon/\mu = B_T^3$ to $\varepsilon/\mu = B_T^4$ to $\varepsilon/\mu = B_T^5$ in the above figure)

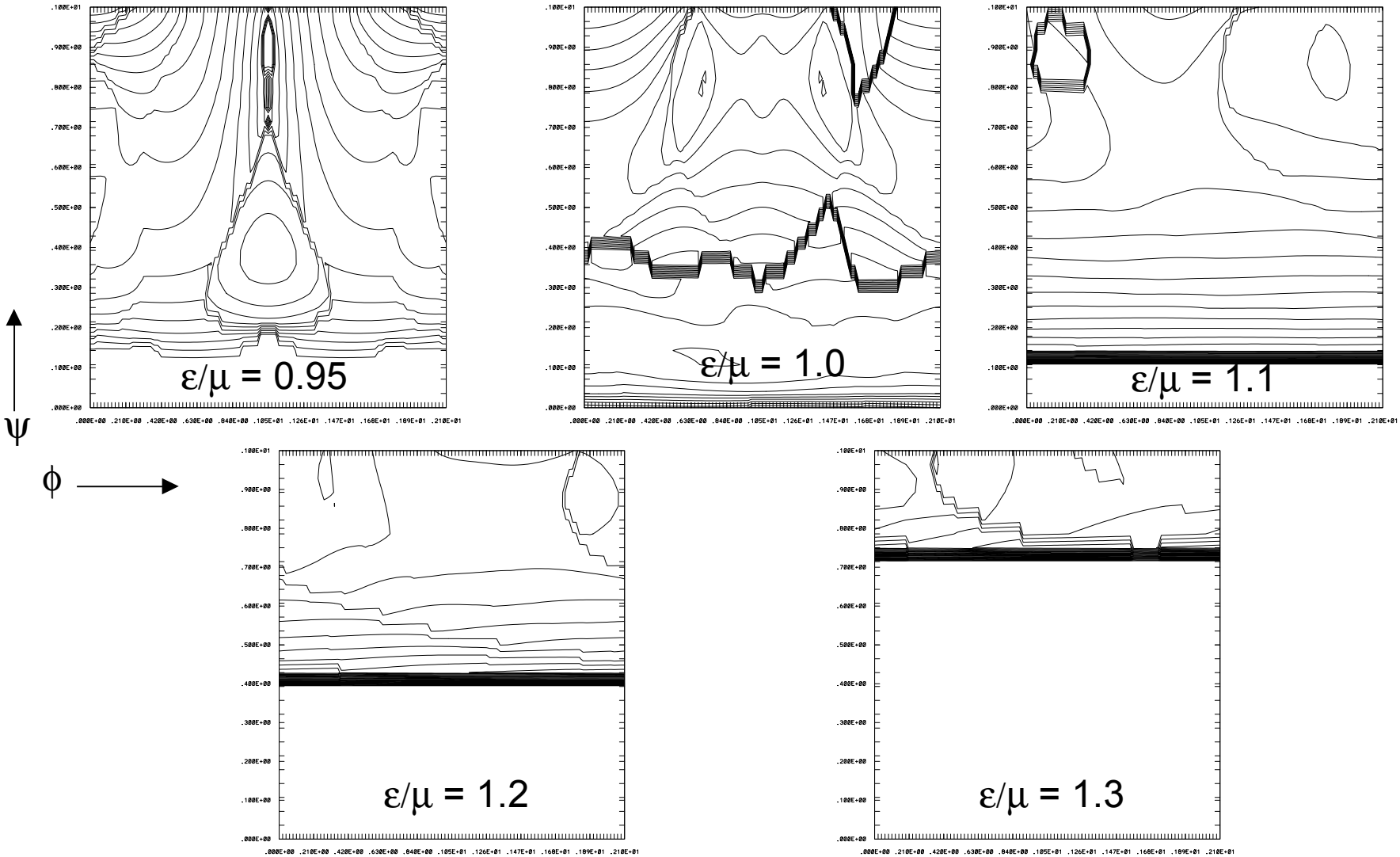
|B| contours for QAS devices

- For QAS we calculate J about the $\theta = 0$ equatorial plane
  (surface of section for the main group of trapped particles)
  - J contours are then plotted on this plane
- For QOS, the surface of section is a helical surface
  - J contours are then plotted on a constant ϕ plane



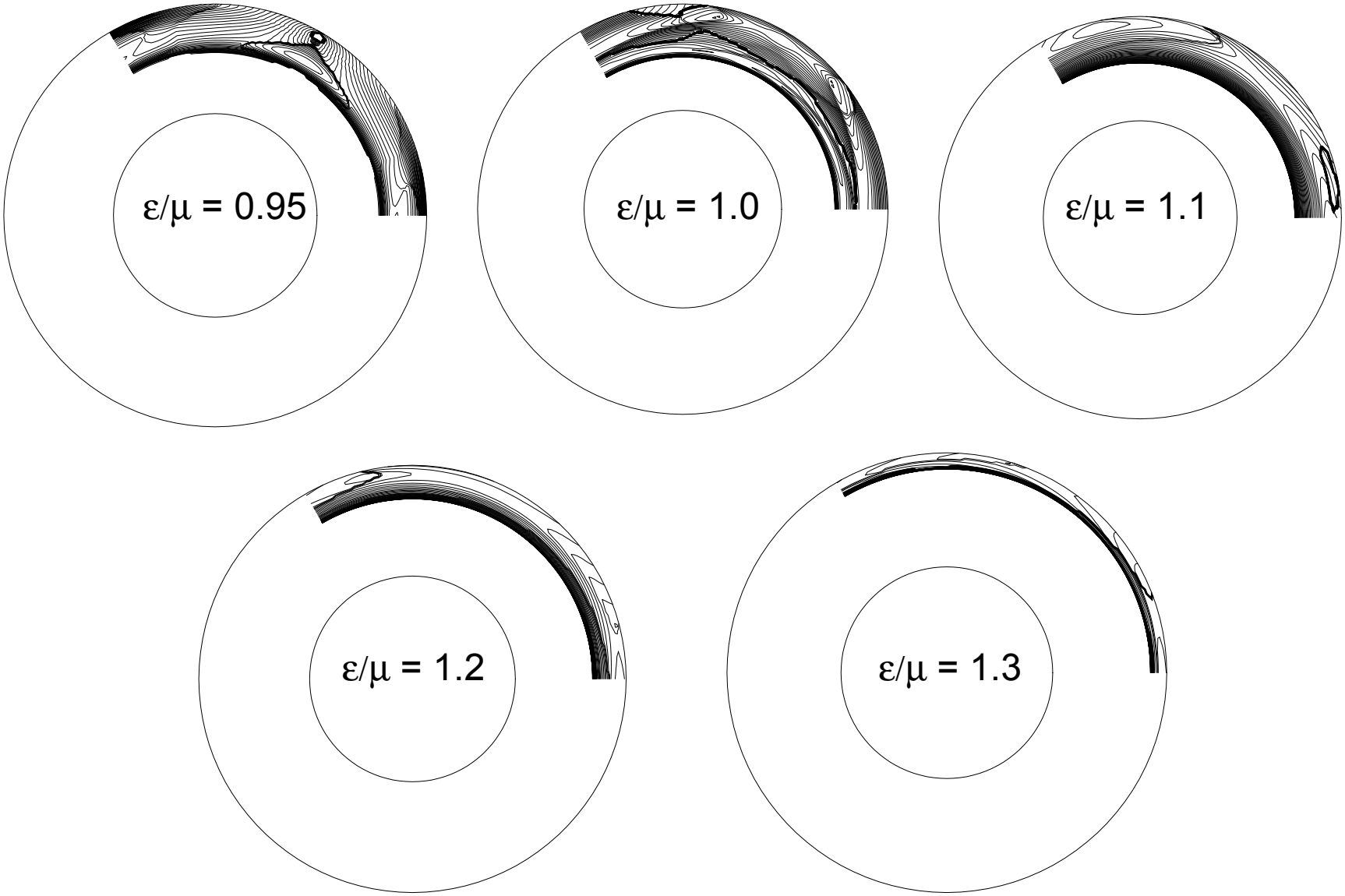


J contours for C82 vs. toroidal flux and toroidal angle (in Boozer coordinates) over 1/3 field period:



Regions of whitespace at the bottom of the ε/μ = 1.1, 1.2, 1.3 plots indicate that only Passing particles are present for these positions and ε/μ values

J contours for C82 looking down on equatorial plane (in Boozer coordinates) over 1/3 field period:



J invariant optimization targets:

- Minimize deviations of J within a flux surface [i.e., from J = J(ψ)]

$$\chi_J^2 = \sum_{flux\ surfaces} \sum_{\varepsilon/\mu} \sum_{\varphi} \left[J(\psi, \varepsilon/\mu, \varphi) - J_{average}(\psi, \varepsilon/\mu) \right]^2$$

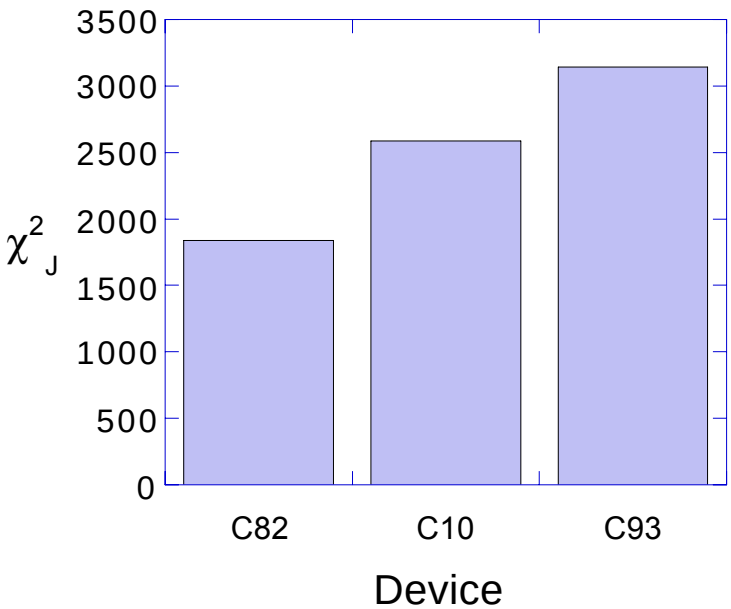
  - fast, easy to implement, can be done on only a few surfaces
  - previously used with J* on QOS devices
  - local in nature, never achieved exactly, may be overly constraining

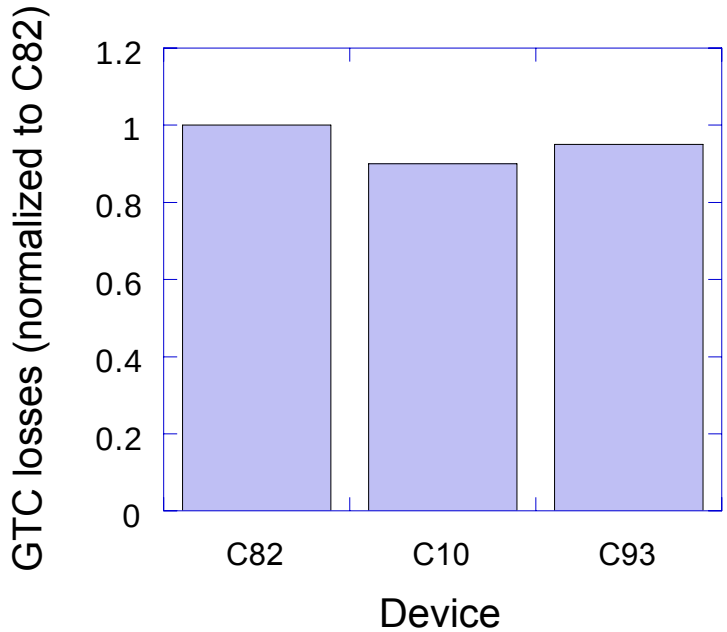
- Minimize ψ component of $\langle v_{drift} \rangle = \dfrac{c}{eB\tau_B} \vec{\nabla} J \times \dfrac{\vec{B}}{B}$

  - similar to above, except for $1/B\tau_B$ weighting
  - doesn't credit the fact that $\langle v_{drift} \rangle$ may change sign over the orbit

- Avoid J contours which connect over significant radial regions
  - should provide a better target of direct losses
  - Requires evaluation of J at fixed ε/μ and possibly over more surfaces than currently used
  - to be developed

Initial calculations of the J = J(ψ) optimization target for C82, C10 and C93 show a different trend than the GTC results:





However,

- J is collisionless and only applied to trapped particles
- No electric field is currently included for J, but was present for GTC
- J is averaged over a range of flux surfaces and pitch angles with equal weights

Summary

- Optimization targets based on DKES transport coefficients and the J longitudinal invariant have been developed
  - are of interest for both QA and QO devices
  - These have been incorporated into optimization codes and are currently being tested.
- DKES
  - Collisional, local diffusive approximation
  - Between devices with large differences in transport (C41, C82, PG2Y) trends correlate well with GTC evaluations
  - For devices with more subtle differences (C82, C10, C93), the radial structure and collisionality dependence of the diffusion coefficients can lead to correlation/anti-correlation with GTC trends
- J invariant
  - Collisionless, includes local as well as direct losses
  - Provides global phase space view of the location of unconfined trapped particle trajectories (analogous to P_ϕ in tokamaks)
  - A variety of optimization targets are being developed/tested