3-D Flux Evolution in NCSX with Inductive and Non-Inductive Current Sources

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Outline

- Flux evolution in 3-D
  - Currents and fluxes
  - The susceptance matrix
  - Evolution equation for ι
- Components of the THRIFT code — THRee dimensional Inductive Flux Evolution in Toroidal plasmas
  - Geometry from VMEC (optional dynamic coupling)
  - NCLASS
  - BEAM 3-D beam deposition and CD
  - Global power balance
- Verification and examples

Basic definitions and relations

Poloidal and Toroidal fluxes

$$\Psi(\rho) = \int_0^{2\pi} d\zeta \int_0^{\rho} \sqrt{g} B^{\theta} d\rho$$

$$\Phi(\rho) = \int_0^{2\pi} d\theta \int_0^{\rho} \sqrt{g} B^{\zeta} d\rho$$

Toroidal and <u>external</u> poloidal currents from Ampere's Law ,

$$I(\rho) = \int_0^{2\pi} d\theta \int_0^{\rho} \sqrt{g} J^{\zeta} d\rho = \frac{1}{\mu_0} \int_0^{2\pi} B_{\theta} d\theta$$

$$F(\rho) = \int_0^{2\pi} d\zeta \int_{\rho}^{\infty} \sqrt{g} J^{\theta} d\rho = \frac{1}{\mu_0} \int_0^{2\pi} B_{\zeta} d\zeta$$

Currents and fluxes are related through a susceptance matrix

$$\mu_0 I = \mathsf{S}_{11}\Psi' + \mathsf{S}_{12}\Phi' = (\mathsf{S}_{11}\iota + \mathsf{S}_{12})\Phi'$$

$$\mu_0 F = \mathsf{S}_{21}\Psi' + \mathsf{S}_{22}\Phi' = (\mathsf{S}_{21}\iota + \mathsf{S}_{22})\Phi'$$

The Susceptance Matrix

VMEC description of the magnetic field,

$$\mathbf{B} = \frac{1}{2\pi\sqrt{g}}\left[(\Psi' - \lambda_\zeta \Phi')\mathbf{e}_\theta + (1+\lambda_\theta)\Phi'\mathbf{e}_\zeta\right]$$

where λ is the stream function and $\lambda_\theta = \partial\lambda/\partial\theta$ and $\lambda_\zeta = \partial\lambda/\partial\zeta$

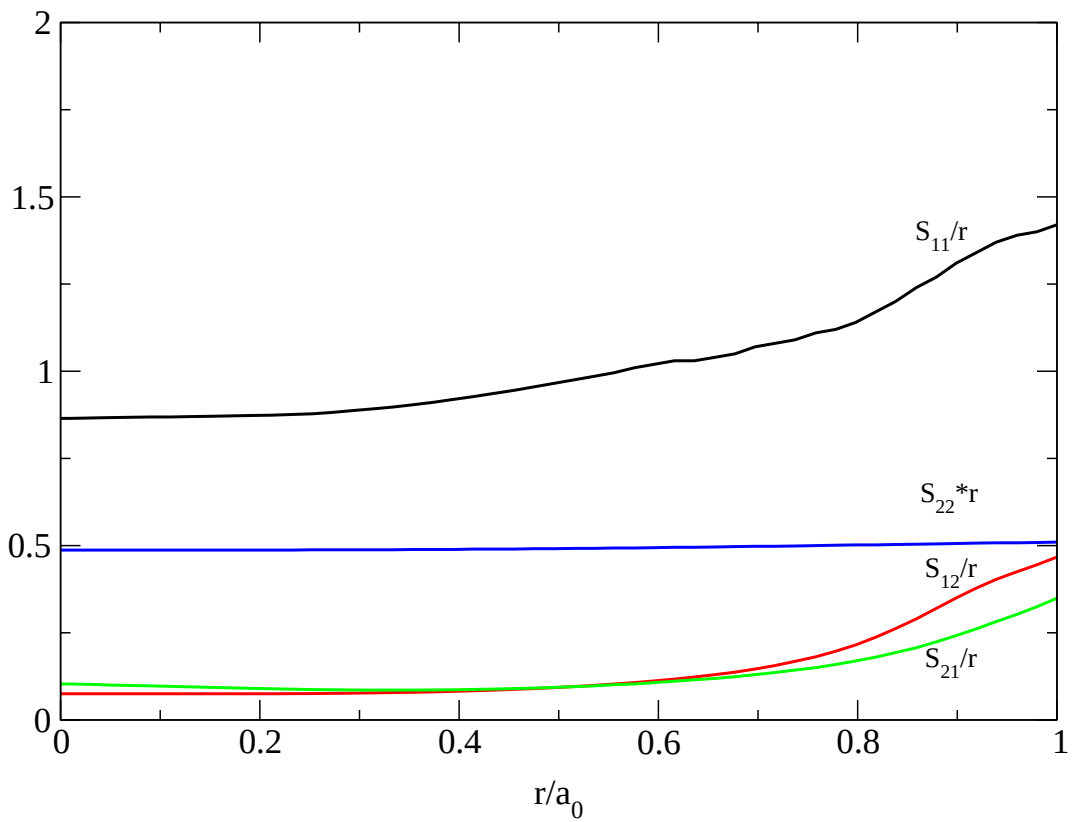
$$\begin{aligned}
\mathsf{S}_{11} &= \frac{V'(\rho)}{4\pi^2}\langle\frac{g_{\theta\theta}}{g}\rangle \\
\mathsf{S}_{12} &= \frac{V'(\rho)}{4\pi^2}\langle\frac{g_{\theta\zeta}(1+\lambda_\theta) - g_{\theta\theta}\lambda_\zeta}{g}\rangle \\
\mathsf{S}_{21} &= \frac{V'(\rho)}{4\pi^2}\langle\frac{g_{\zeta\theta}}{g}\rangle \\
\mathsf{S}_{22} &= \frac{V'(\rho)}{4\pi^2}\langle\frac{g_{\zeta\zeta}(1+\lambda_\theta) - g_{\zeta\theta}\lambda_\zeta}{g}\rangle
\end{aligned}$$

$$\iota = \frac{\Psi'}{\Phi'} = \frac{\mathsf{S}_{22}I - \mathsf{S}_{12}F}{\mathsf{S}_{11}F - \mathsf{S}_{21}I} = \frac{\mu_0 I}{\mathsf{S}_{11}\Phi'} - \frac{\mathsf{S}_{12}}{\mathsf{S}_{11}}$$

where the vacuum rotational transform is

$$\iota_{vac} = -\frac{\mathsf{S}_{12}}{\mathsf{S}_{11}}$$

Components of the Susceptance Matrix



The dominant radial dependence close to the axis in S is:

$$\mathsf{S}_{11} \sim \mathsf{S}_{12} \sim \mathsf{S}_{21} \sim r$$

$$\mathsf{S}_{22} \sim 1/r$$

Choosing an r(m) makes S_{ij} dimensionless. We use

$$r = a_0 \rho = a_0 \sqrt{\frac{\Phi}{\Phi_a}}$$

Evolution equation for ι

Averaging over the poloidal component of Faraday's law yields

$$\frac{\partial \Psi'}{\partial t} = \frac{\partial}{\partial \rho}\left\{\langle \mathbf{E}\cdot\mathbf{B}\rangle \frac{\partial V}{\partial \Phi}\right\}$$

Introducing $\langle \mathbf{E}\cdot\mathbf{B}\rangle = \eta_{\|}\langle(\mathbf{J}-\mathbf{J}_s)\cdot\mathbf{B}\rangle$, and assuming $\dot{\Phi}=0$

$$\frac{\partial \iota}{\partial t} = \frac{\partial}{\partial \Phi}\left\{\eta_{\|}\langle(\mathbf{J}-\mathbf{J}_s)\cdot\mathbf{B}\rangle \frac{\partial V}{\partial \Phi}\right\} = \frac{\partial}{\partial \Phi}\left\{\eta_{\|}\left[\mu_0 F^2 \frac{\partial}{\partial \Phi}\left(\frac{I}{F}\right) - \langle \mathbf{J}_s\cdot\mathbf{B}\rangle \frac{\partial V}{\partial \Phi}\right]\right\}$$

where $\langle \mathbf{J}\cdot\mathbf{B}\rangle V' = \mu_0(FI' - IF')$. Finally,

$$\frac{\partial \iota}{\partial t} = \frac{\partial}{\partial \Phi}\left\{\frac{\eta_{\|}}{\mu_0}\Phi'\left(\mathsf{S}_{21}\iota + \mathsf{S}_{22}\right)^2 \frac{\partial}{\partial \rho}\left[\frac{\mathsf{S}_{11}\iota + \mathsf{S}_{12}}{\mathsf{S}_{21}\iota + \mathsf{S}_{22}}\right] - \eta_{\|}\langle \mathbf{J_s}\cdot\mathbf{B}\rangle \frac{\partial V}{\partial \Phi}\right\}$$

The THRIFT code

THRee-dimensional Inductive Flux evolution in Toroidal Plasmas.

It uses

- VMEC for the geometry description:
  - Reads VMEC output files for static equilibria or
  - couples dynamically to VMEC as profiles evolves (still under development/testing)
- Global power balance to evolve fixed shape pressure profiles.

$$\dot{W} = P - \frac{W}{\tau_E}$$

  is solved and the pressure (temperatures) are adjusted accordingly.
- NCLASS and BEAM for current sources and related quantities

NCLASS

Features

- Multiple species radial and parallel force balances for axisymmetric plasmas
  - Advanced viscosity models
- Extensive validation with tokamak experiments and other codes
  - Bootstrap current, electrical resistivity, particle transport, ...

Application to NCSX design

- Standard tokamak model for Pfirsch-Schluter viscosity
  - Shaing low A corrections turned off
- 3-D calculation of trapped fraction using Lin Liu-Miller model
  - Good agreement with full integrals for both tokamaks and stellarators
- NCLASS $\eta_{\|}$ and $\langle \mathbf{J}_{BS} \cdot \mathbf{B} \rangle$ should yield reasonable results

BEAM

Features

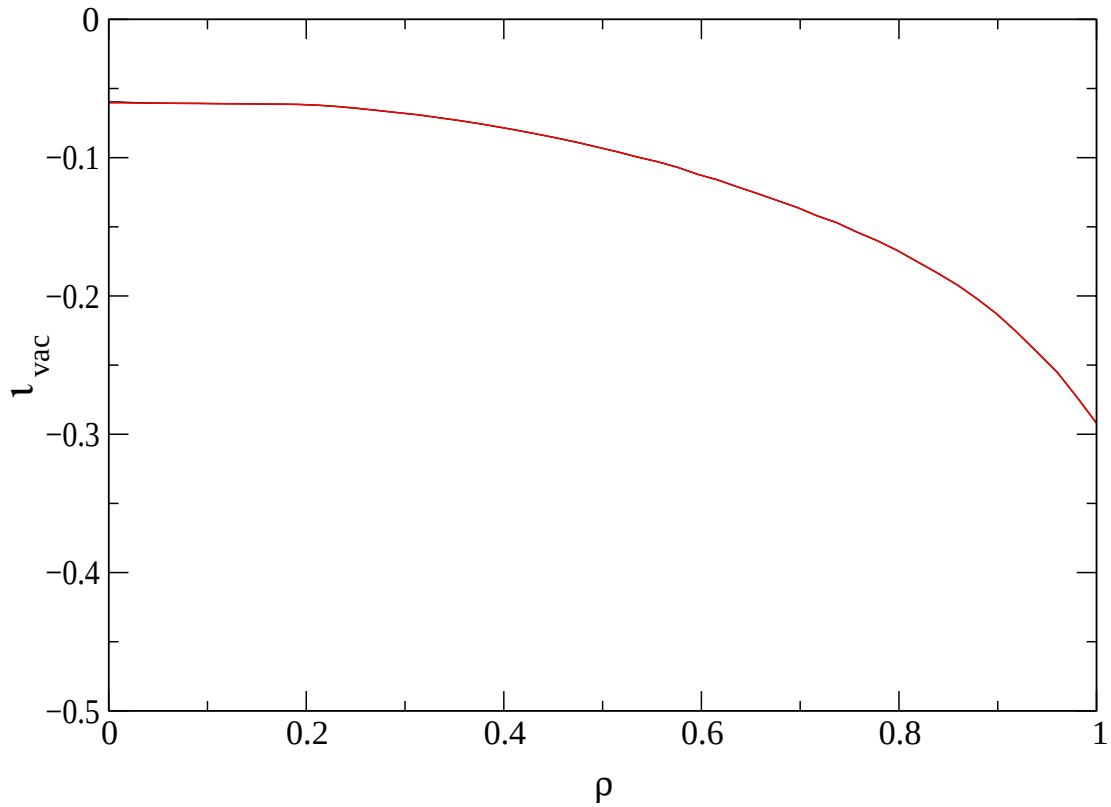
- Multiple parallel beamlet calculation in full 3-D geometry
  - No beam divergence or focus
- Rectangular or circular cross-section with Gaussian widths
  - Single beam source could be modified for multiple beams
- Parallel momentum and current source from $\langle \mathbf{v}_{BM} \cdot \mathbf{B} \rangle$ summed over beamlets
  - Input to Fokker-Planck solution
- $\langle \mathbf{J}_{BM} \cdot \mathbf{B} \rangle$ and shielding factor
  - Hirshman tokamak models for steady-state beam distribution

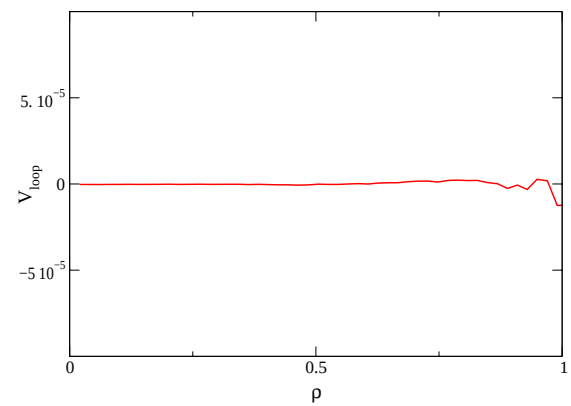
Application to NCSX design

- Large shielding factor from low iota-bar and high effective collisionality
  - Needs validation
- Possible upgrade to calculate electron response using NCLASS

Numerical Test — Maintaining ι_{vac}

Current free C82 configuration





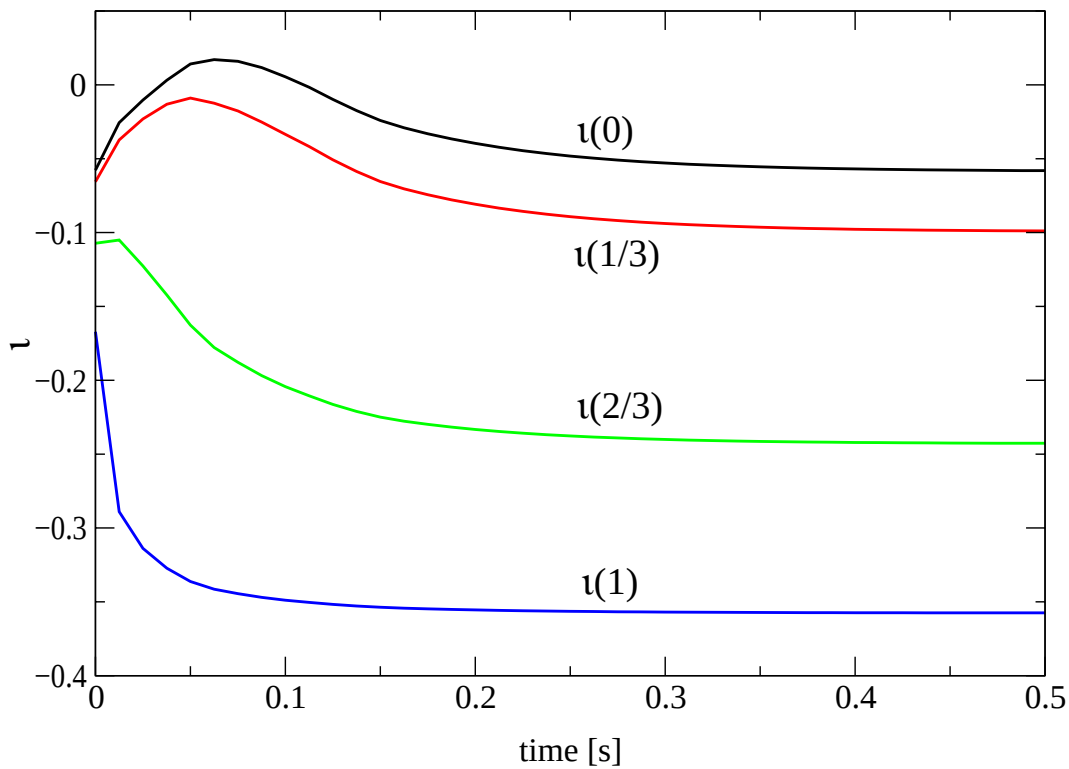
V_{loop} goes to numerical fluctuation levels over the full profile and ι_{vac} remains invariant.

$\Rightarrow$

Discretization OK and numerics converges.

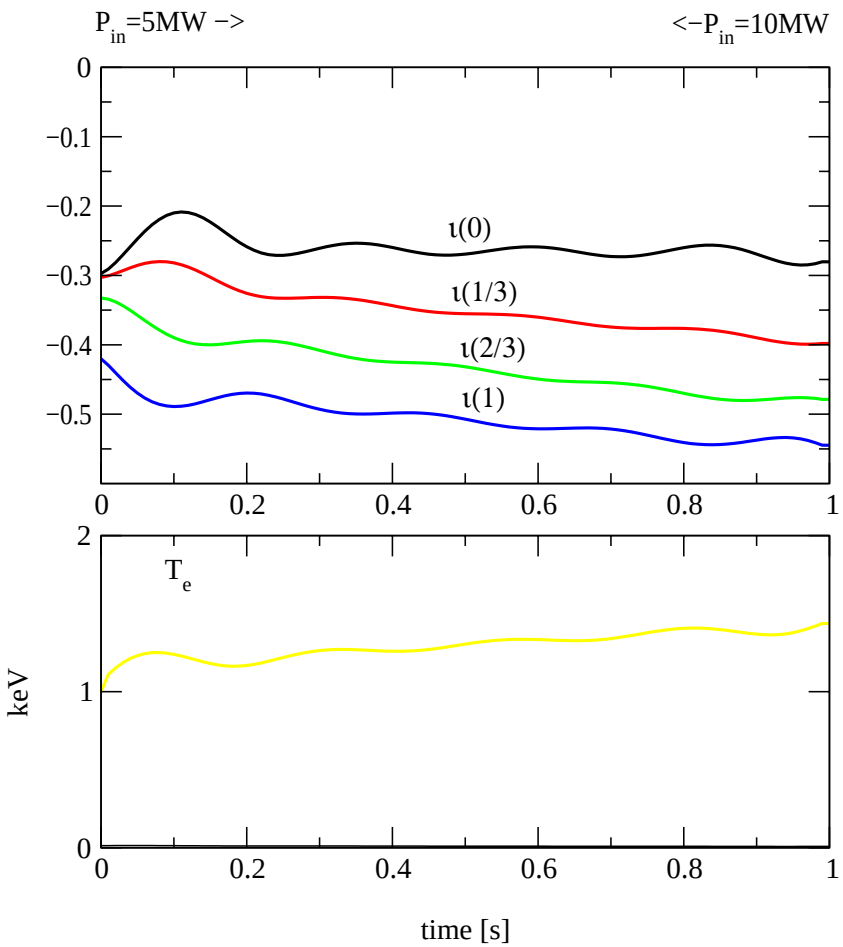
Numerical test – Bootstrap Current

The same C82 configuration but now add the bootstrap current and set $I = I_{bs}$



- ι BV drops in response to I_{bs}
- BC change propagates inwards (resistive timescale)
- $\iota(axis)$ responds transiently to local gradients in $\mathbf{J}_{BS}$ and
- original $\iota(0)$ is regained in stationary state.

Global Power Balance – Timedependent Pressure



Use a C82 configuration prescribe it's current to our generic source term. Allow the bootstrap current to vary in accordance to the pressure varying according to ISS95

Summary

- A flux evolution equation for updating the poloidal flux through the rotational transform have been derived for 3-D geometries.
- A standalone test code THRIFT are being developed/tested, evolves ι using
  - VMEC equilibria, with (optional) dynamic coupling being tested.
  - Neoclassical (NCLASS) components included, bootstrap current, resistivity, etc
  - Beam driven currents from BEAM, implemeted but not really performing well yet. Needs more detailed beam configuration/setup info?!
  - Global power balance, shifts pressure (temperature) profiles according to a predefined global scaling law.
  - Facilitates a generic source term and has been designed with the intent of having more source modules included when available.

Outstanding issues

- Boundary conditions: What are relevant, useful BC to use in actual applications.
- What about toroidal flux? Can we implement Grad's alternating dimensions approach. I.e., use the fluxaveraged radial force balance (Kruskal-Kulsrud)

$$-p'V' = F'\Phi' + I'\Psi',$$

"in between" calls to VMEC to obtain a new Φ_a to feed VMEC with. Problems: Non-linear. The grid is tied to toroidal flux. Not obvious how to implement in a numerically stable fashion. Any other options?