Good Surface Targetting for the Optimizer

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Goal

Enable optimizers to find configurations with good surfaces

Method

Estimate island widths analytically using VMEC equilibria, even though VMEC's representation presumes good flux surface.

The estimate need not be absolutely accurate but needs to have the correct parametric dependencies.

Expressions

Hegna [6] has recently re-evaluated and summarized the analytic theory of island formation in stellarators [1-5] giving:

$$w = D/2 + \left(D^2/4 + \left|C + s_v w_v^2\right|\right)^{0.5}$$

in units of toroidal flux y ,

w_v is the 'vacuum width',

$s_v = \pm 1$ depending on whether the vacuum islands and the pressure driven islands are in (or out) of phase[4],

C represents the square of the island-width produced by the resonant component of the Pfirsch-Schluter current, and

D gives the effective island width due to non-linear flattening of pressure gradient modifying the Pfirsch-Schluter and bootstrap currents (the Glasser and neoclassical tearing effects).

Expressions, cont'd

$$D = k_1 \left[D_{nc} + D_R \right] / (-\Delta') \ ,$$

where

D_R is the resistive interchange stability parameter,

$$D_{nc} = -\mu_0 \frac{\left\langle \vec{J}_{boot} \cdot \vec{B} \right\rangle}{\left\langle B^2 \right\rangle} \frac{V'}{\iota'} \left\langle \frac{B^2}{|\nabla \psi|^2} \right\rangle ,$$

Δ' is the tearing mode asymptotic matching param.,

J_{boot} is the bootstrap current density, $' = \partial / \partial \psi$,

$$C = -k_0 \mu_0 \frac{p'}{B^2} \frac{\epsilon_{mn} R^2}{m \iota'^2 r^2 (-\Delta')} \ ,$$

$r \propto \sqrt{\psi}$ is the minor radius,

ϵ_{mn} is related to the Jacobian in Boozer coordinates by $\sqrt{g} = V' \left[1 + \sum \epsilon_{mn} \exp(im\theta - in\zeta) \right]$.

This expression for w is valid for arbitrary plasma shapes, except that the expression for C was derived at high aspect ratio.

Evaluation Strategy

- Evaluate D_R using the JMC code.

- Evaluate $\langle \vec{J}_{boot} \cdot \vec{B} \rangle$ using standard axisymmetric expressions, or use a full 3D code for less-symmetric cases.

- VMEC will numerically infer a resonant sheet current to close any island. If the parallel current density is expanded in a Fourier series as $\vec{J} \cdot \vec{B} / B^2 = \sum \lambda_{mn} \exp(im\theta - in\zeta)$ in Boozer coordinates, then the sheet current term will have the form $\hat{\lambda}_{mn} \delta(\psi - \psi_{mn})$. Following the analysis of [6], this produces an island width $w_d^2 = 4\mu_0 \hat{\lambda}_{mn} / (\Delta' \iota')$.

- VMEC's grid will not, typically, align with a given rational surface, but will have grid points spanning it. Calculate the resonant λ_{mn} in the zone spanning the rational surface and assume that it is all due to $\hat{\lambda}_{mn}$. This is equivalent to assuming that the entire jump in the resonant B_{mn} on the finite VMEC grid is due to $\hat{\lambda}_{mn}$.

- VMEC will calculate a sheet current for both the vacuum island and the island due to the resonant Pfirsch-Schluter current (the C term above). Thus,

  $$w = D/2 + \left(D^2/4 + s_\delta w_\delta^2\right)^{0.5}$$

- VMEC does not include the physics of pressure flattening, so will not be affected by the non-linear term D.

- Approximate Δ' by $-2m/(2\pi r^2 B_T)$. This standard approximation comes from a large aspect ratio expansion and the definition of the physical toroidal flux ψ .

Solution characteristics

Both C and D are proportional to p'.

1. If $D>0$, then the islands grow with p', without limit. This is the (observed) neoclassical tearing instability in tokamaks.

   This has already guided our focus to 'reversed shear configurations'.

2. For $D<0$, then
   - If $D^2/4 >> |C|$, which will occur at high p', get $w \approx |C/D|$
     which is independent of pressure!
     This is the maximum island width for any pressure. Thus, it is important to limit this ratio.

   - If $D/2 << \sqrt{|C|}$, then $w \approx \sqrt{C}$ and the island width grows as $\sqrt{p'}$.

Conclusions

Island widths should be calculable for VMEC equilibria and used for optimization.

The optimizer should search for equilibria with acceptable total island widths at all low-order rational surfaces.

This could be implemented in either of two ways:

- Limit the sum of calculated island widths over all the rational surfaces, at the highest β desired, or
- Optimize for $D < 0$, negligible $|C/D|$, and negligible w_d at each low-order rational.

This must be done over the range of expected ι values in the experiment, from vacuum to high-β.

References

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