Use of a Coil Complexity Measure in Configuration Development

N. Pomphrey, R. Hatcher, A. Boozer

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- **To date, we have focussed our design effort on baseline plasma configurations derived from the fixed-boundary VMEC optimizer.**

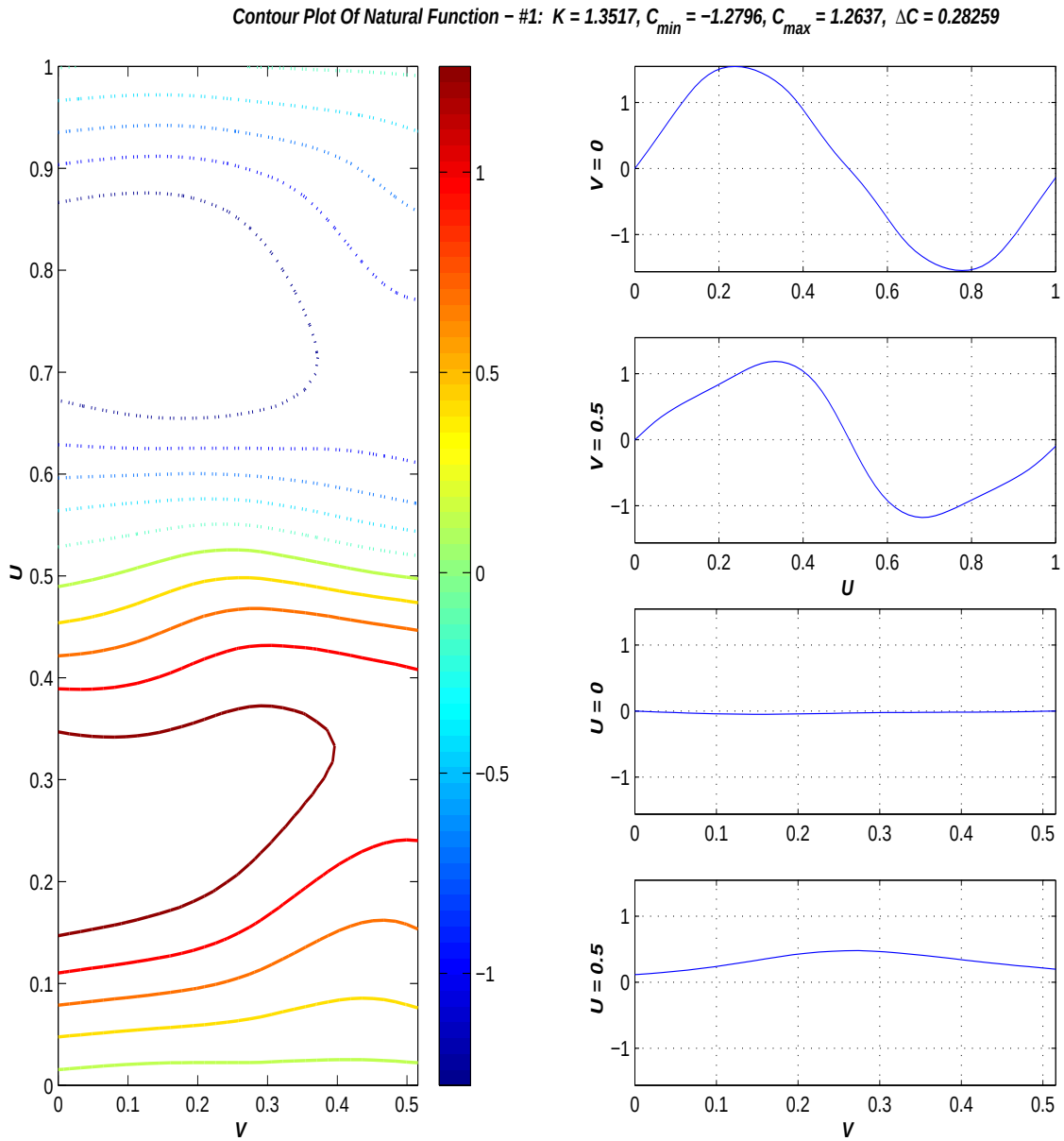
- **For the most part, the cost function depends on physics properties (Transport/Stability). By varying $N_\rho \sim 36$ independent combinations of the $\{R_{mn}, Z_{mn}\}$ describing the plasma boundary, L-P has obtained the c82 family of configurations. These have favorable physics properties, but are problematic for support by coils.**

- **The coil group has often used the "enstrophy" of the current sheet solution as a measure for coil "complexity", and for some time this measure has been incorporated in VMEC as an allowed penalty function.**
  - **However we have not always found a positive correlation between enstrophy and <u>actual</u> coil complexity.**
  - **A better measure of complexity is desirable.**
- **At the Jan 2000 NCSX Project Meeting (see Sec.II-I2 "Connecting Plasma Design to Coil Design") we proposed to build a penalty function into the VMEC optimizer based on so-called low-order "natural functions" which would automatically select configurations with low coil complexity.**

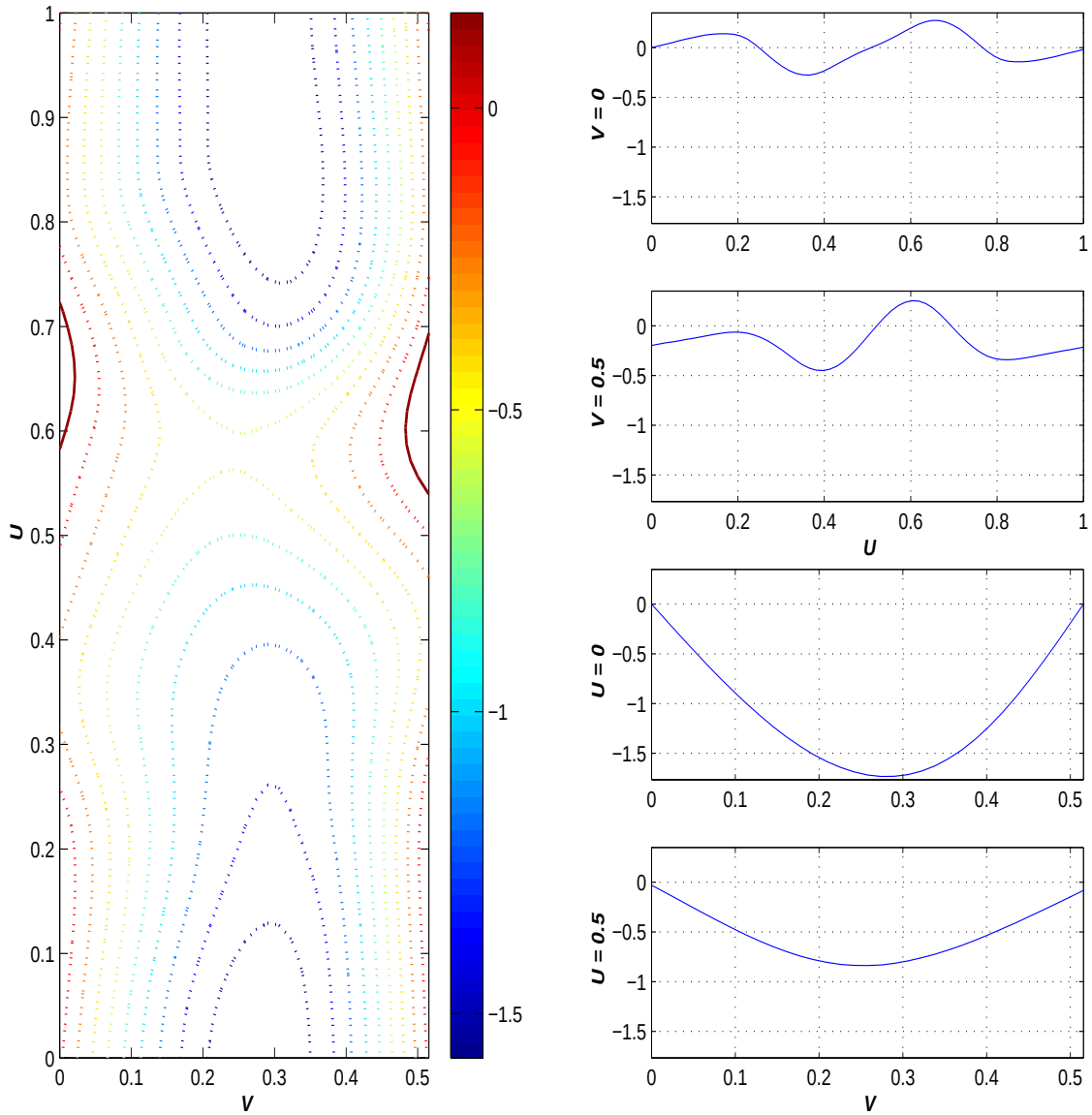
- **Natural functions are eigenfunctions of an "in-surface" operator whose elements depend only on the winding surface geometry. A surface current distribution can be expanded in this complete set of functions**

- **Each function is "labelled" by an associated eigenvalue which tells us how rapidly the magnetic field strength due to that distribution of current decreases as we move away from the surface.**

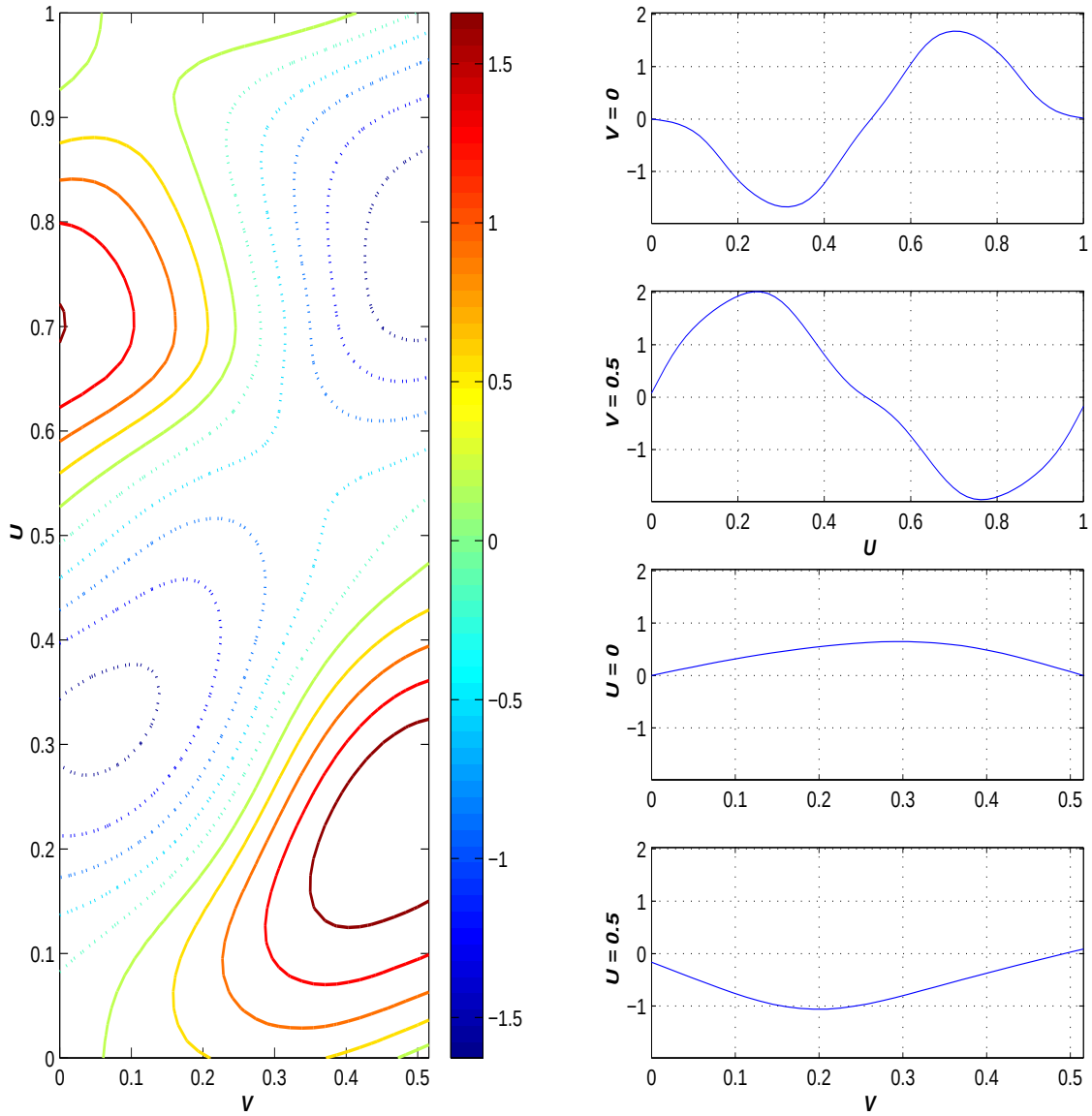
- **The lowest order natural functions have the smallest eigenvalues, and decay most slowly with distance from the surface. They are smooth (see Figs 1-6 for contour plots of selected functions). They couple efficiently with the plasma (see Fig 7 for plot of inductances calculated between the natural function distributions of current and the plasma).**


Contour Plot Of Natural Function − #1: K = 1.3517, C_min = −1.2796, C_max = 1.2637, ΔC = 0.28259
U
V
V = 0
V = 0.5
U = 0
U = 0.5

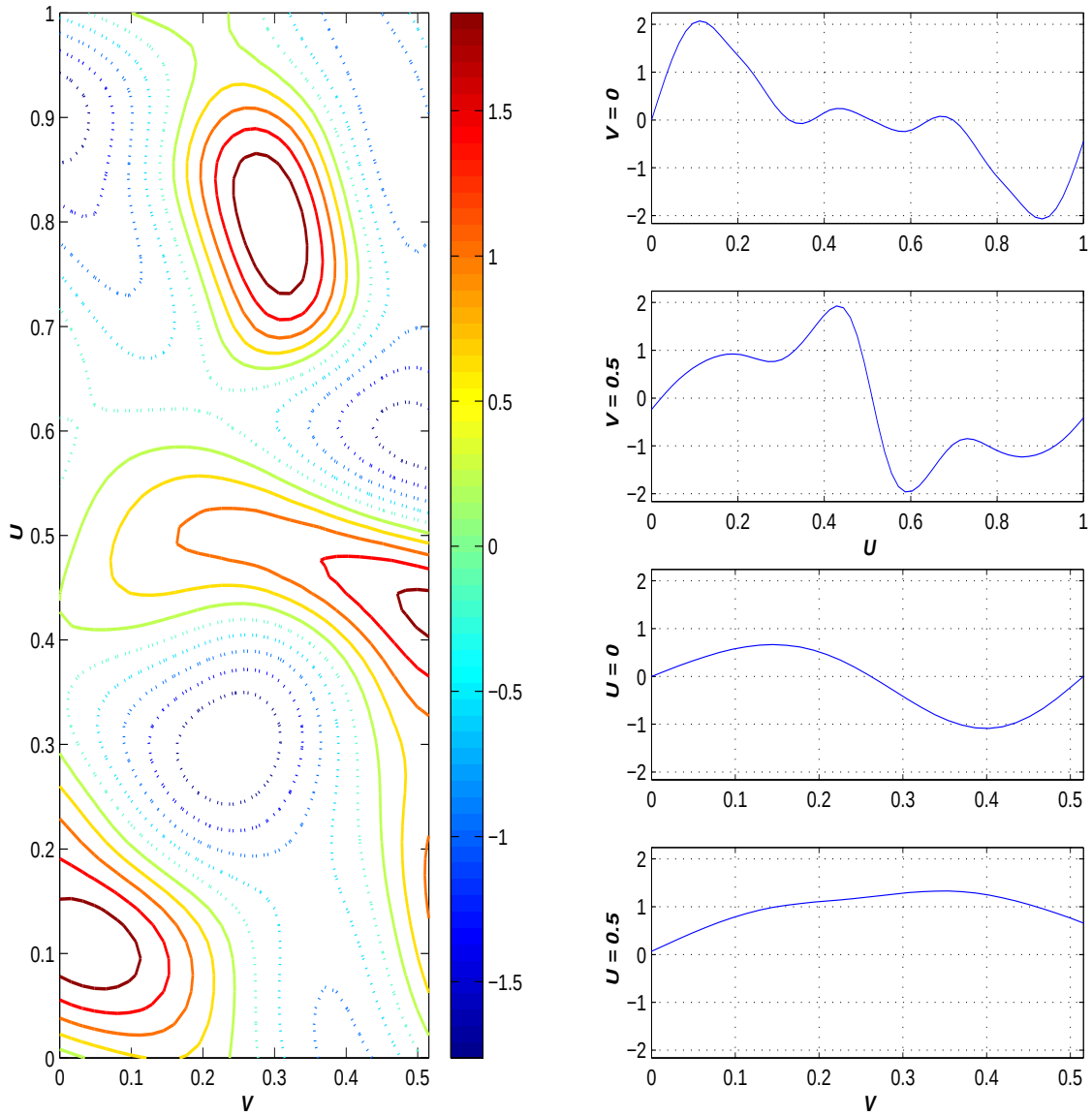
Contour Plot Of Natural Function − #2: K = 2.0133, C_{min} = −1.5736, C_{max} = 0.15728, ΔC = 0.19231



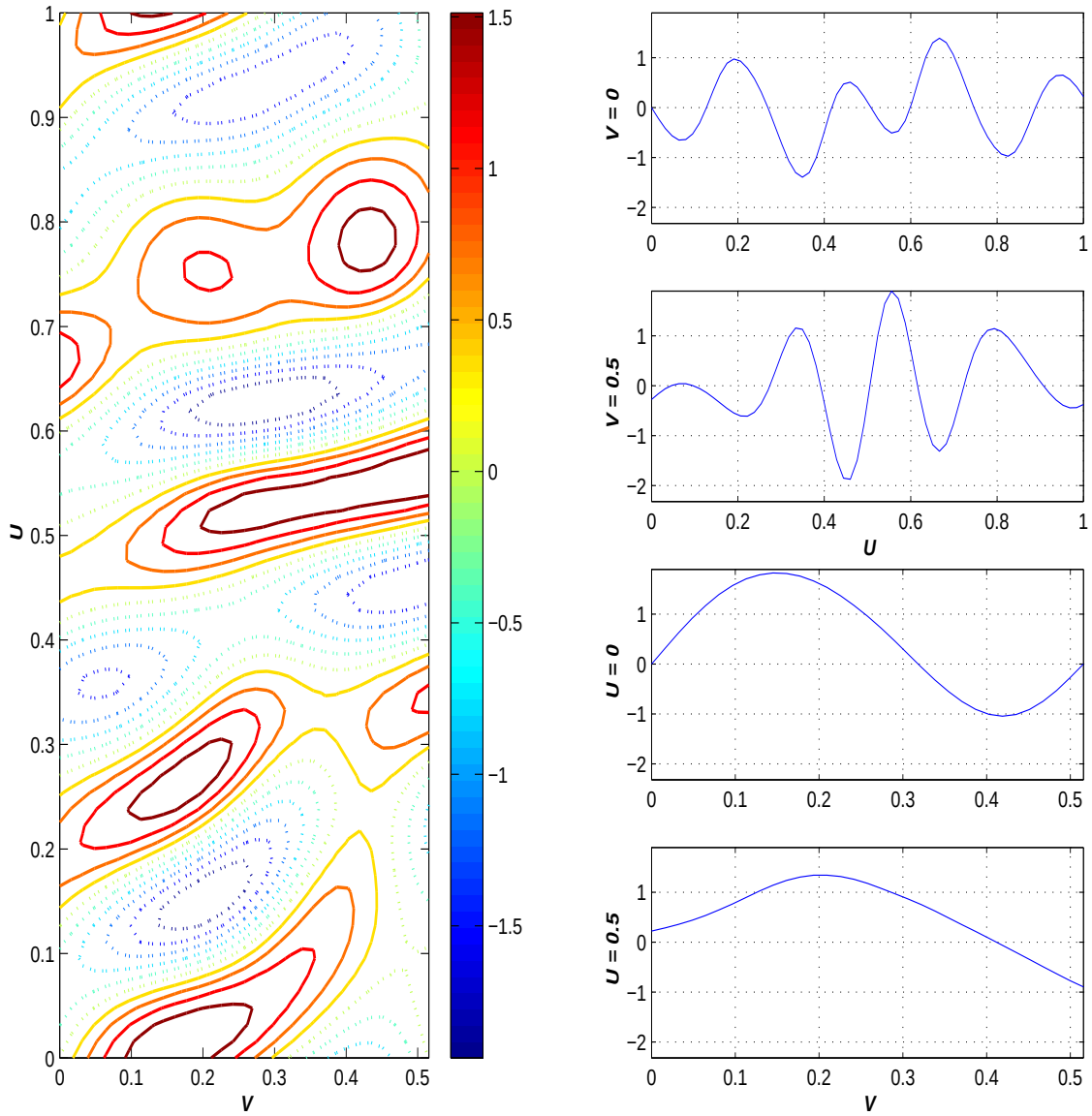
Contour Plot Of Natural Function − #3: K = 2.3866, C_{min} = −1.6272, C_{max} = 1.66, ΔC = 0.36523



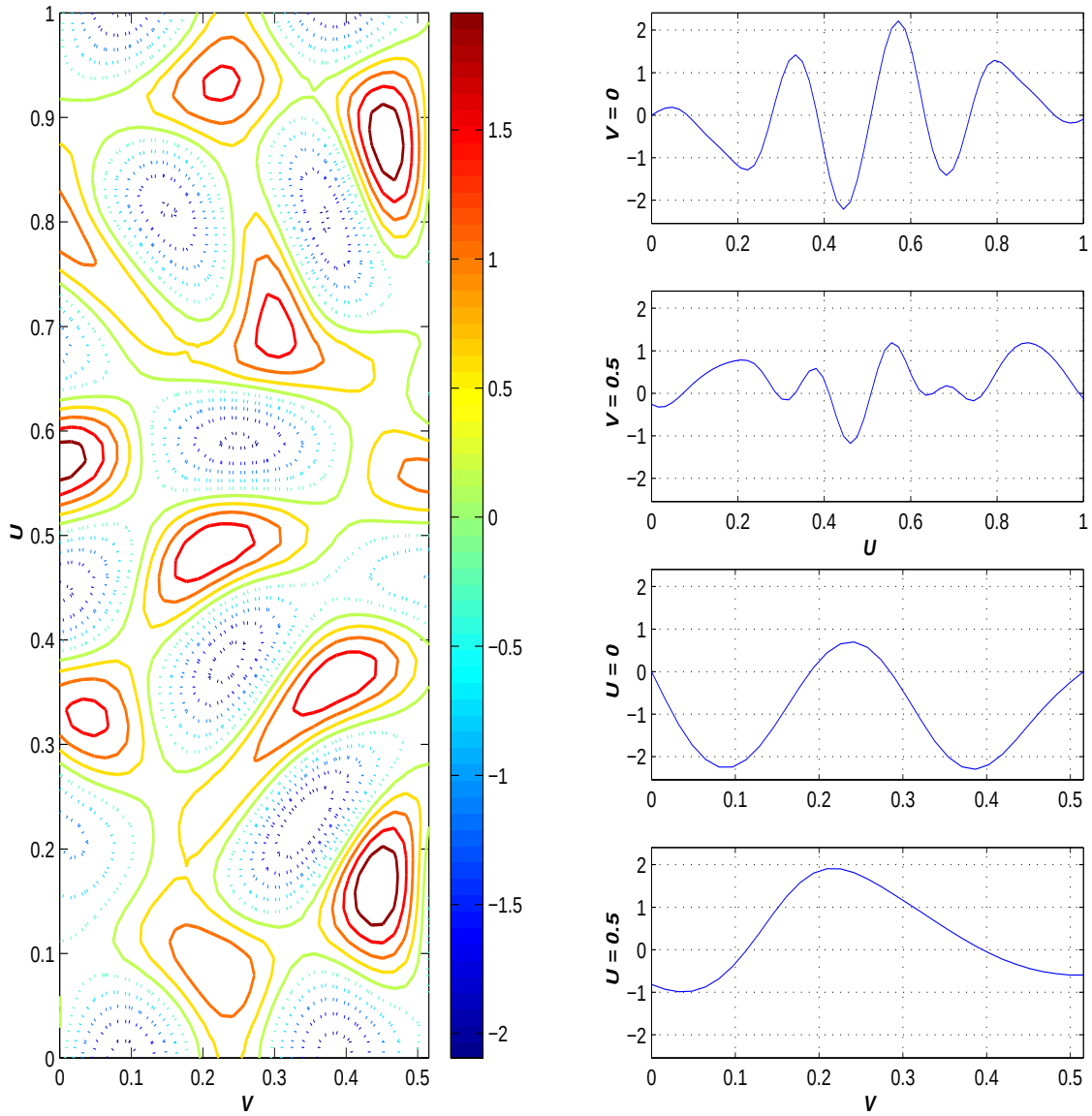
Contour Plot Of Natural Function − #10: K = 4.2329, C_{min} = −1.7626, C_{max} = 1.837, ΔC = 0.39996



Contour Plot Of Natural Function − #20: K = 6.1512, C_{min} = −1.9362, C_{max} = 1.513, ΔC = 0.38324



Contour Plot Of Natural Function − #30: K = 7.5938, C_{min} = −2.0945, C_{max} = 1.9528, ΔC = 0.4497



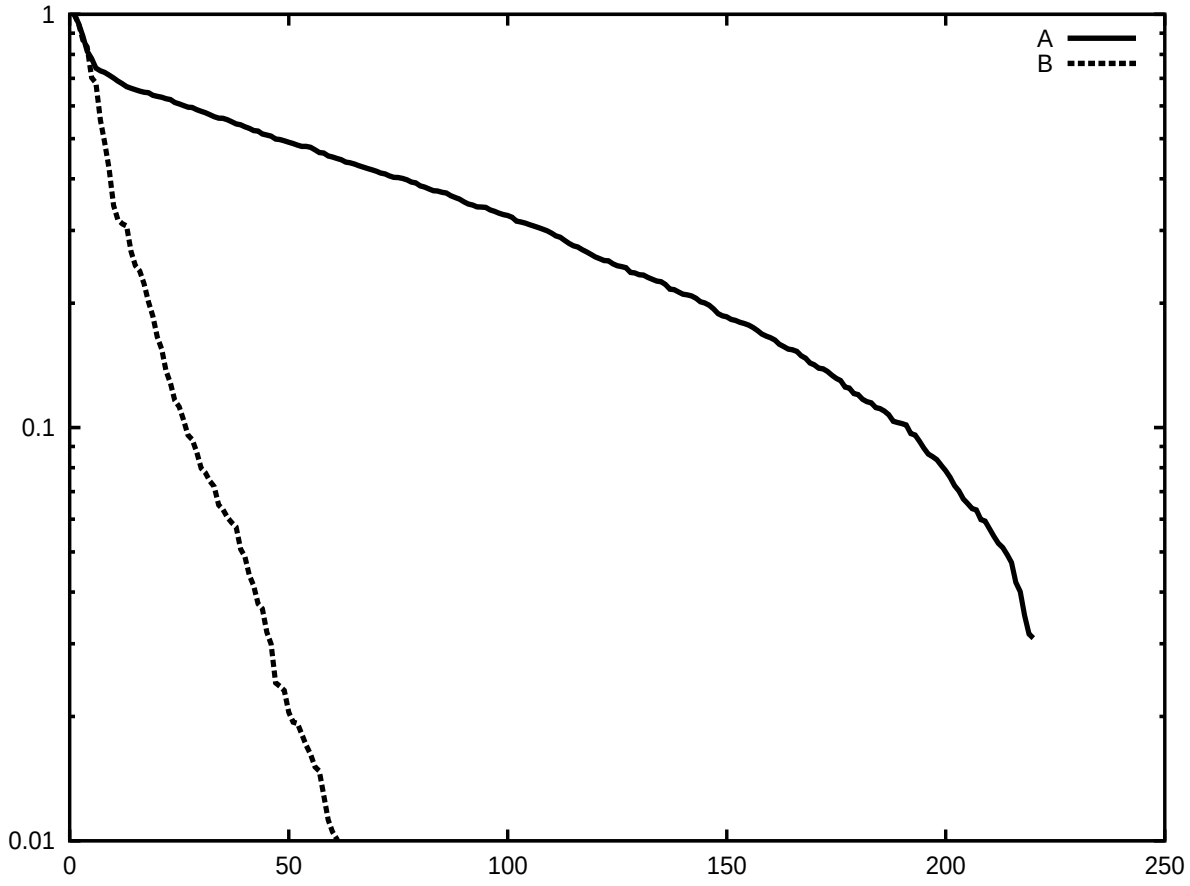


Figure 1: Inductance vs Natural Function Index for

(A) c82 plasma (25cm minimum belly radius) with 18 cm conformal winding surface.

(B) 10 cm circular cross section toroidal plasma with c82 winding surface.

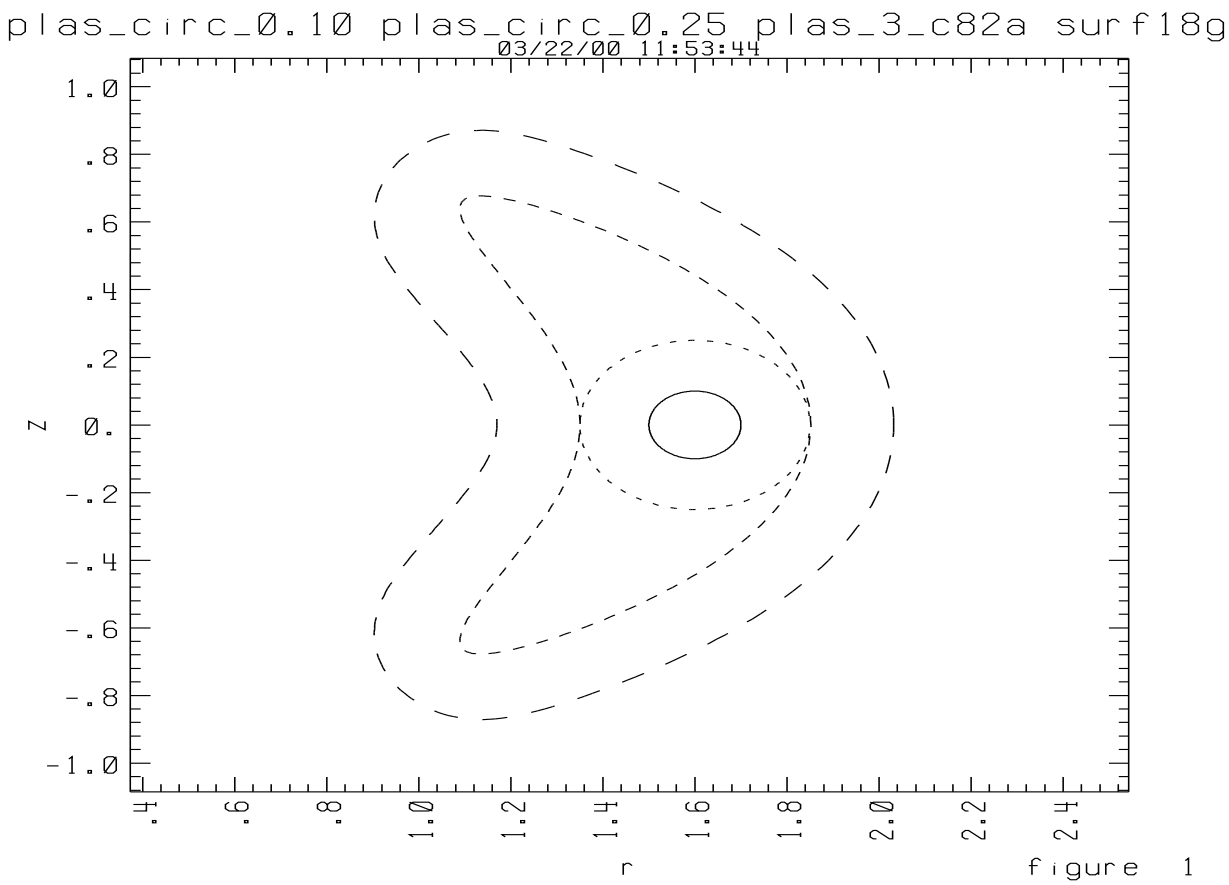


Figure 2: Nested surfaces for inductance calculation shown at $v = 0$ plane are:

(a) c82 winding surface

(b) c82 plasma boundary

(c) 25 cm radius circular cross section torus inscribed by c82 plasma boundary

(d) 10cm radius circular torus

THE GOAL

- **Obtain a plasma configuration with desirable physics properties (as good as c82) whose sheet current distribution can be described by a linear combination of ONLY low-order natural functions.**

- **With the free-boundary VMEC optimizer the procedure for obtaining configurations consistent with such sheet current distributions is clear:**

  - **Choose a coil winding surface.**
  - **Choose N_I natural functions, $f_j(u,v)$, to be retained in the current sheet soln. Presumably $N_I \sim N_\rho = 36$ will provide sufficient variation to support configurations with good physics.**

$$\kappa(\mathbf{u},\mathbf{v}) = \sum_{\mathbf{j=1}}^{\mathbf{N_I}} \mathbf{I_j f_j(u,v)} \tag{1}$$

  - **Form N_I "MGRID" files of the B-field from each surface current distribution, $f_j(u,v)$, assuming unit amplitude.**
  - **Let the optimizer vary the coefficients, I_j, to minimize the physics penalty functions.**

Implementation of Coil Complexity Penalty Function

- **In Jan/Feb, the free-boundary VMEC optimizer was not yet working. For back-up, we have implemented a penalty function in the fixed-boundary VMEC optimizer as follows: For each step of the optimizer that the configuration shape changes,**
  - **Generate a winding surface 18 cm separated and conformal to the plasma boundary.**
  - **Calculate N_I natural functions on the winding surface.**
  - **Penalize the optimizer cost function with a term representing the failure to fit the calculated B-normal at the plasma boundary with the N_I functions:**

$$\chi^2_{nat} = w_{nat}\Big[b(u,v) - \sum_{j=1}^{N_I} b_j f_j(u,v)\Big]^2$$

RESULTS:

- **Starting from c82 using $N_I = 30$ natural functions, and TURNING OFF ALL PHYSICS PENALTY FUNCTIONS, χ^2_{nat} was driven down by a factor of 10. The final plasma boundary at the "usual" viewing v planes appeared to have larger minimum radii of curvature than the original c82, suggesting a smoother current potential solution, HOWEVER contour plots of a 1/R TF field on the plasma boundary (surrogate NESCOIL) showed a MORE complicated solution than for c82! Closer analysis showed that the final plasma surface was "lumpy" due to large amplitudes of $R_{mn=3}, Z_{mn=3}$. The problem is that the VMEC description of the plasma used by the optimizer ($m_{max} = 5, n_{max} = 3$) was not sufficient for the given N_I. (See Allen Boozer's Sherwood Poster for a "solution" to the problem of conistency between N_I and N_ρ.)**

- **The same run was repeated using a smaller N_I(=18). Now, a similar decrease in χ^2_{nat} produces a smooth**

plasma boundary (rather similar to c10).

- **Turning ON the transport minimization, in conjunction with the χ^2_{nat} (complexity) penalty finds a similar configuration with good quasisymmetry.**

- **However turning on the kink eigenvalue minimizer seems to be problematic FOR THE GIVEN SMALL NUMBER OF NATURAL FUNCTIONS. ie., we are unable to drive down the kink eigenvalue to an unacceptable level (in the c82 range).**

Conclusions

- **Particularly with the availability of the free-boundary VMEC optimizer, we have a much clearer idea of a promising strategy which couples the plasma and coil designs. The strategy will hopefully lead to plasma configurations with satisfactory physics and simpler coil solutions.**

- **A straightforward modification of the mgrid generator will allow the free-boundary VMEC optimizer to be run in such a way that current sheet solutions are naturally smooth and consistent with the plasma shape.**

- **The fundamental ill-conditioning inherent in using NESCOIL to reverse engineer the coils is avoided in the proposed approach.**