VMEC2000 Developments II: NCSX Project Meeting

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Progress Since Last Meeting

- Tested temporal, spatial convergence of VMEC2000 with many modes (Isaev, et. al.-reported previously at theory meeting)
- Continued modifications to improve behavior of VMEC2000 in free-boundary optimizer
- Implemented radial current, radial force balance post-processing improvements

VMEC - Optimizer Interface Improvements

- Better method (mathematically “robust”) to initialize VMEC Jacobian for complicated boundaries (like c82…)

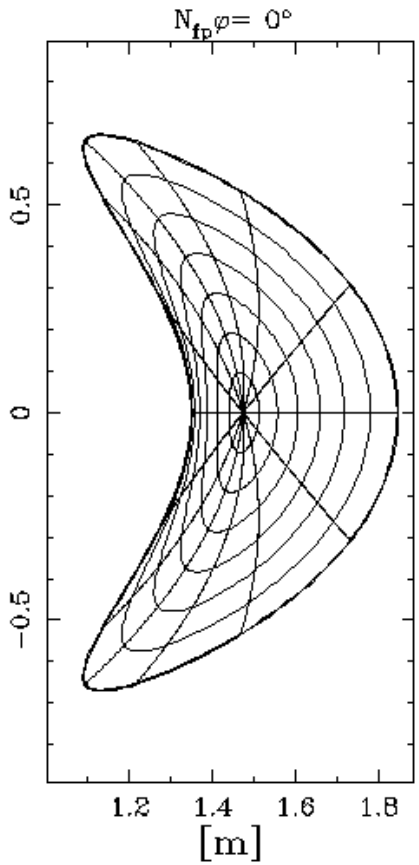
- Updated control logic (what to do if VMEC2000 *almost* converged, for example…) within VMEC code

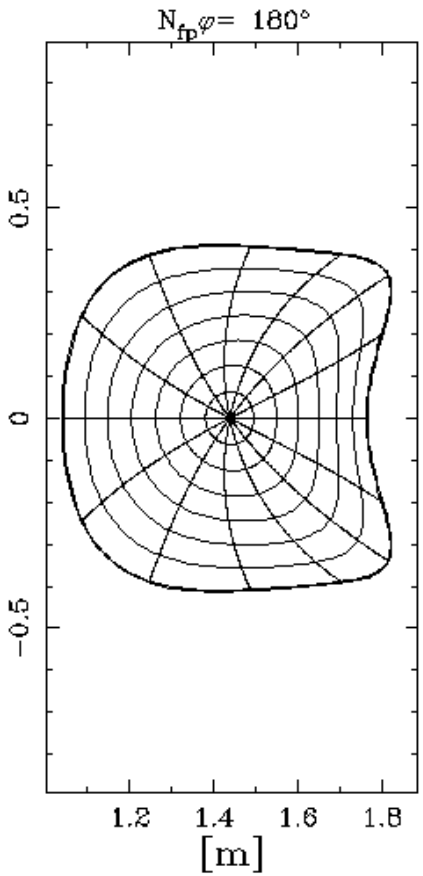
Handling the *dreaded* “Initial Jacobian changed sign” message

- For complicated boundaries, the initial guess for the VMEC jacobian can lead to non-nested flux surfaces (jacobian->0), even with a “good” guess for magnetic axis

- VMEC can not get started from here. It needs jac < (or >) 0 everywhere to work...

“Bad” axis guess (centroid) => Poor initial jacobian





Simple resolution to prevent VMEC2000 “crash” during optimization

- Not easy (in general geometry) to do better for axis guess than centroid…(want to maximize *minimum* axis - boundary sep.)
- “Solve” problem by choosing **3** surfaces (boundary, axis, 1 enclosed)
  - run VMEC on coarse mesh: improve axis guess
  - let VMEC interpolate to finer mesh

Force Balance

- Accurate force balance is important for
  - Stability analyses
    - Ballooning coefficients (bending) can change sign if there is poor force balance (Isaev)
  - Evaluation of island suppression algorithms
    - need accurate $j_{||}$

VMEC Inverse Equilibria

- Solve F = 0, where:

$$F = J \times B - \nabla p$$

- Decompose F in “flux” coordinate basis:

$$x^i = \{s, \theta, \varphi\}$$

Nested surfaces (VMEC)

- VMEC builds-in $B^s = 0$ (s = radial flux coordinate)
- Remaining 2 components of $F_i = 0$ are:

$$J^s = 0 \qquad \textit{radial current}$$

$$F_s \equiv \sqrt{g}\left(J^{\vartheta}B^{\varphi} - J^{\varphi}B^{\vartheta}\right) - \frac{dp}{ds} \qquad \textit{radial force balance}$$

VMEC Solves Cylindrical Components of Force Balance

- Because VMEC is inverse solver, it is natural to zero cylindrical components (R,Z) of forces
- These components are linear combinations (with angle-dependent coefficients) of F_i
  - leads to mode coupling of Fourier coefficients
  - aliasing (truncation in Fourier space) is an issue for recovering F_i

Radial Current: $J^s = 0$

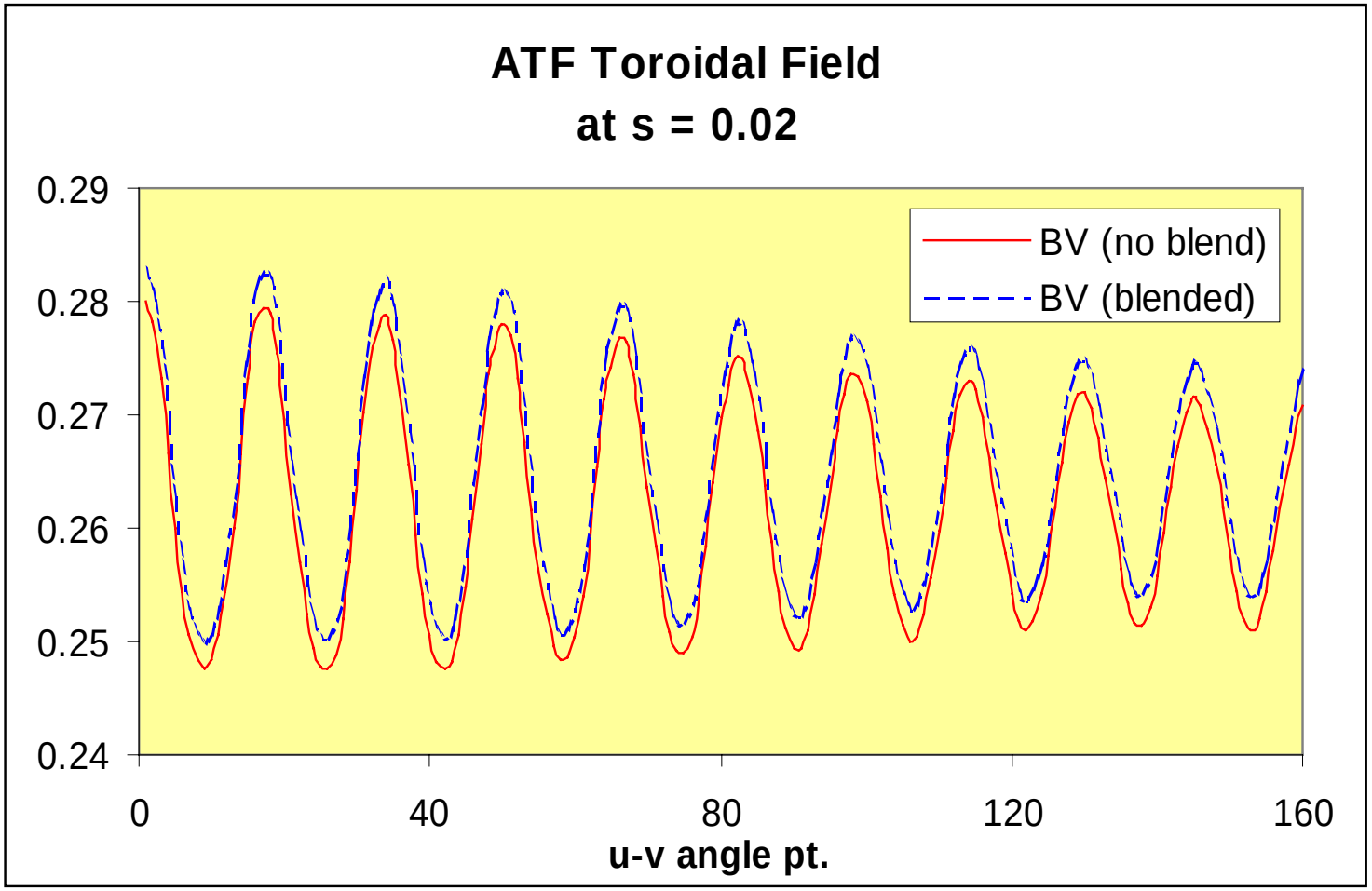
- This component is related to the λ-force:

$$J^s = \frac{1}{\sqrt{g}}\left(\frac{\partial B_\varphi}{\partial \vartheta} - \frac{\partial B_\vartheta}{\partial \varphi}\right)$$

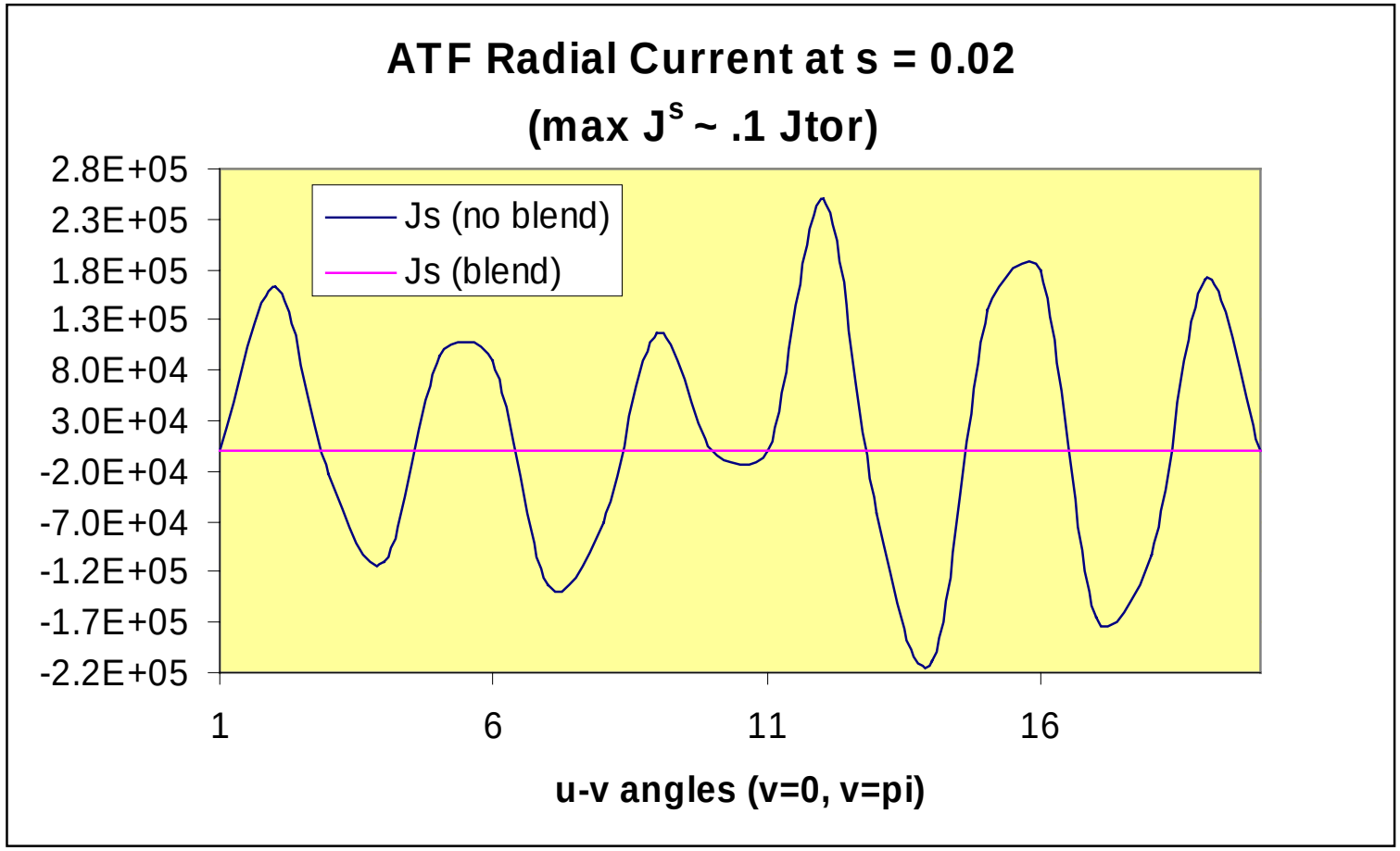
$$B_\alpha = B^\vartheta(\lambda_\varphi) g_{\alpha\vartheta} + B^\varphi(\lambda_\vartheta) g_{\alpha\varphi}$$

- Leads to 2nd-order elliptic equation for λ

Use “Blended” Currents



Correct blend has **HUGE** impact on MHD force (J^s in this case)



Local Radial Force Balance: $F_s = 0$

- Implemented solution of magnetic differential equation for B_s :

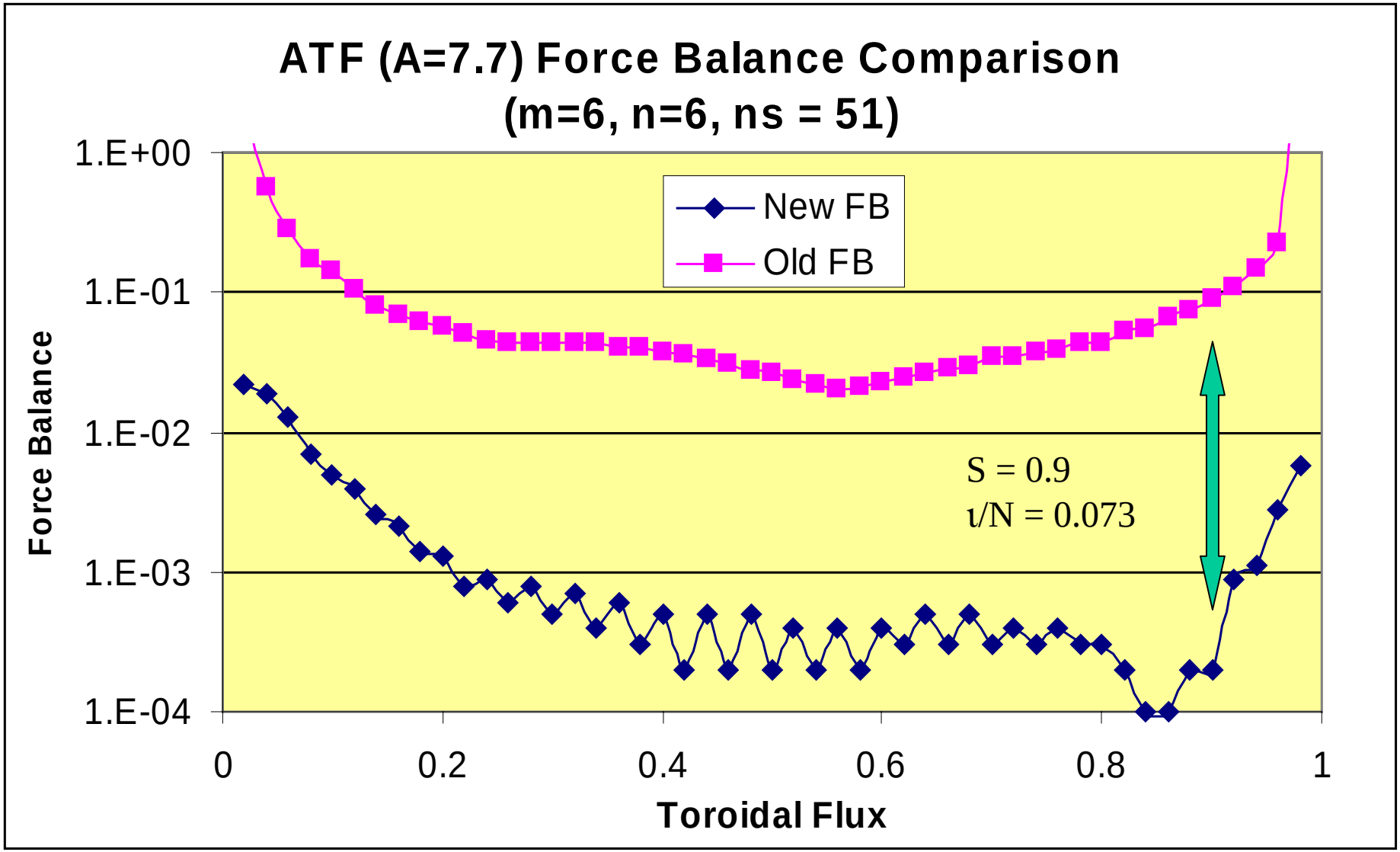
$$B \bullet \nabla B_s = dp/ds + B^{\vartheta} dB_{\vartheta}/ds + B^{\varphi} dB_{\varphi}/ds$$

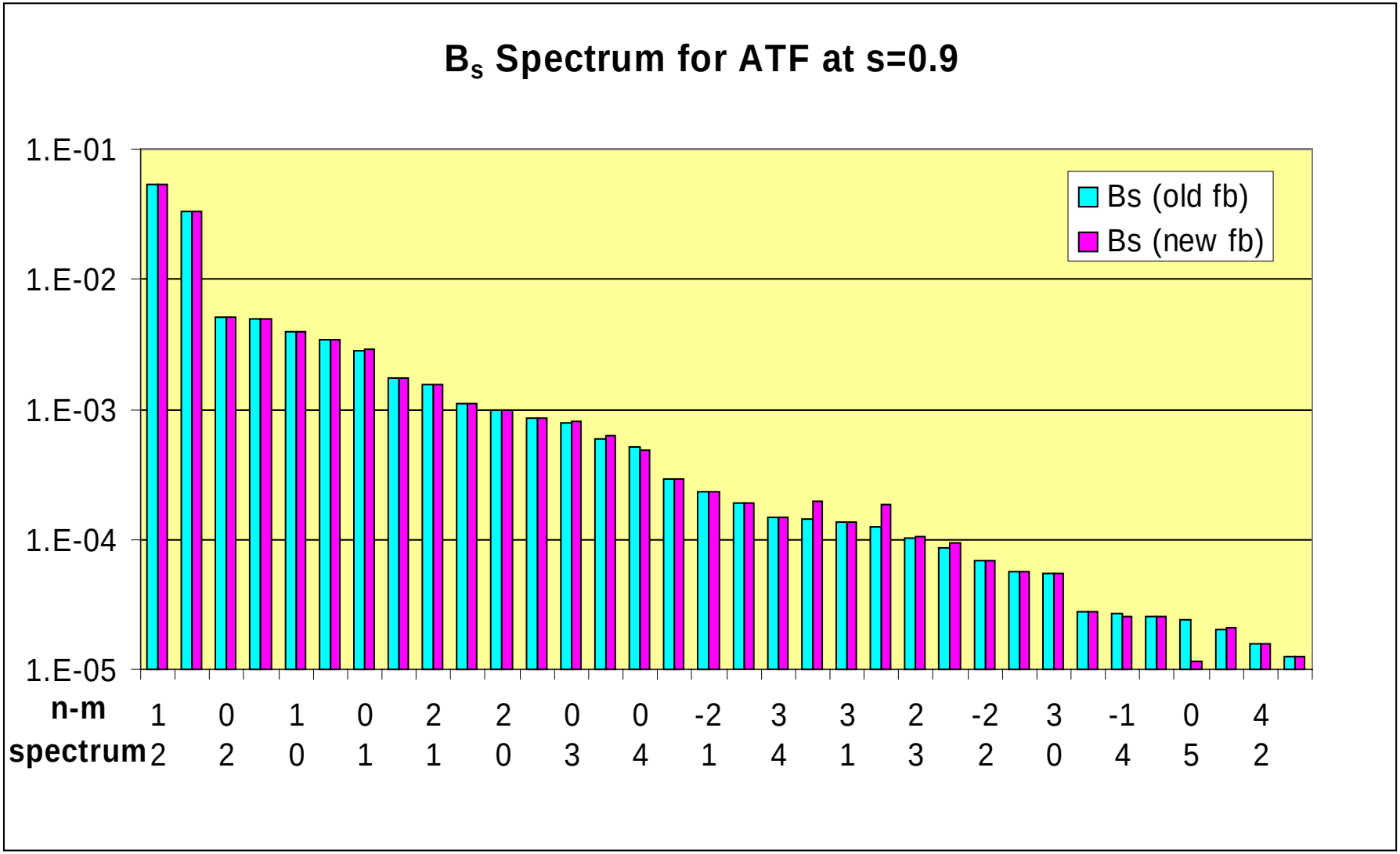
- B_s derivatives arise from currents J^{ϑ} *and* J^{φ}
  - has major impact on parallel current

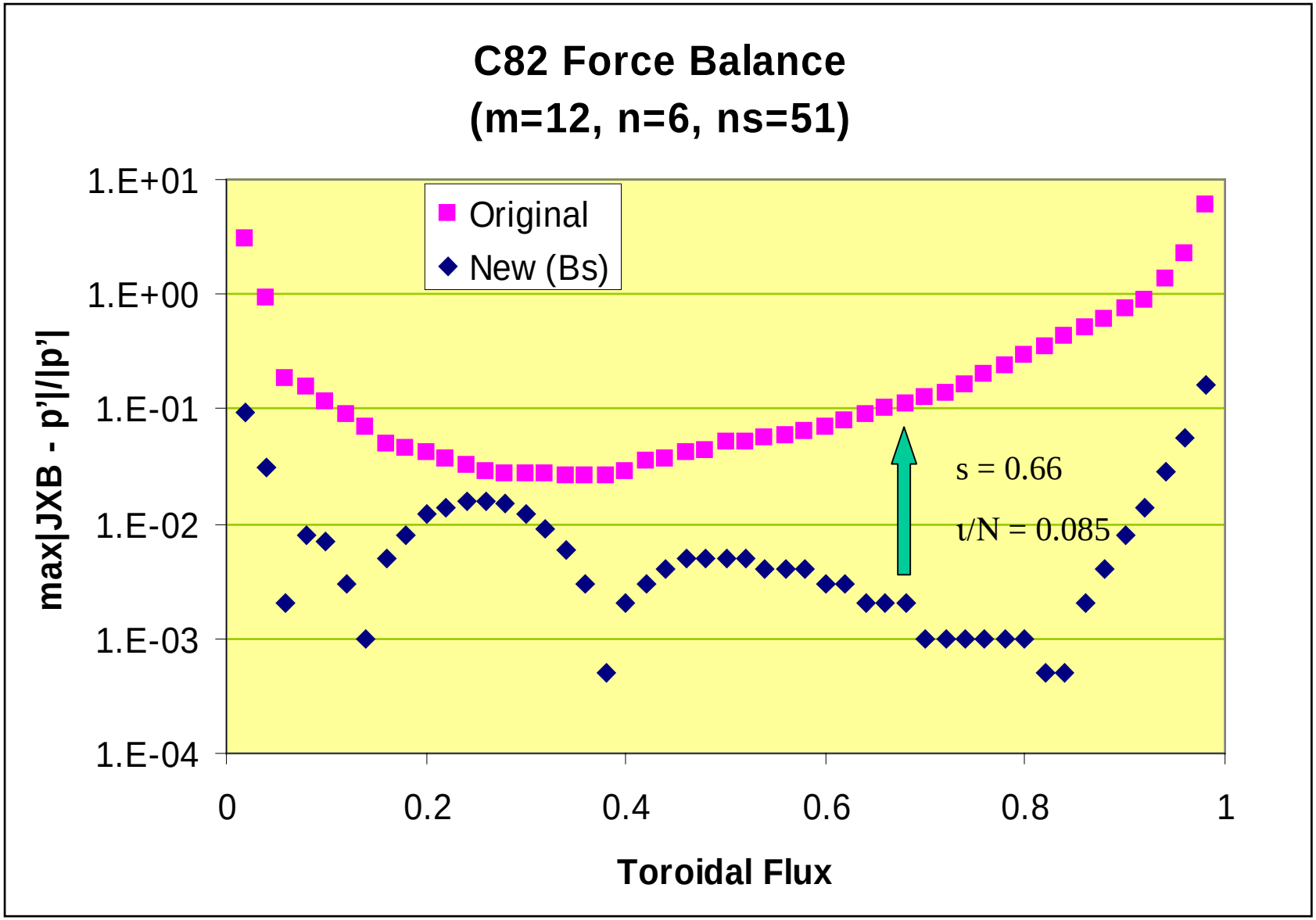
Calculation of B_s Spectrum

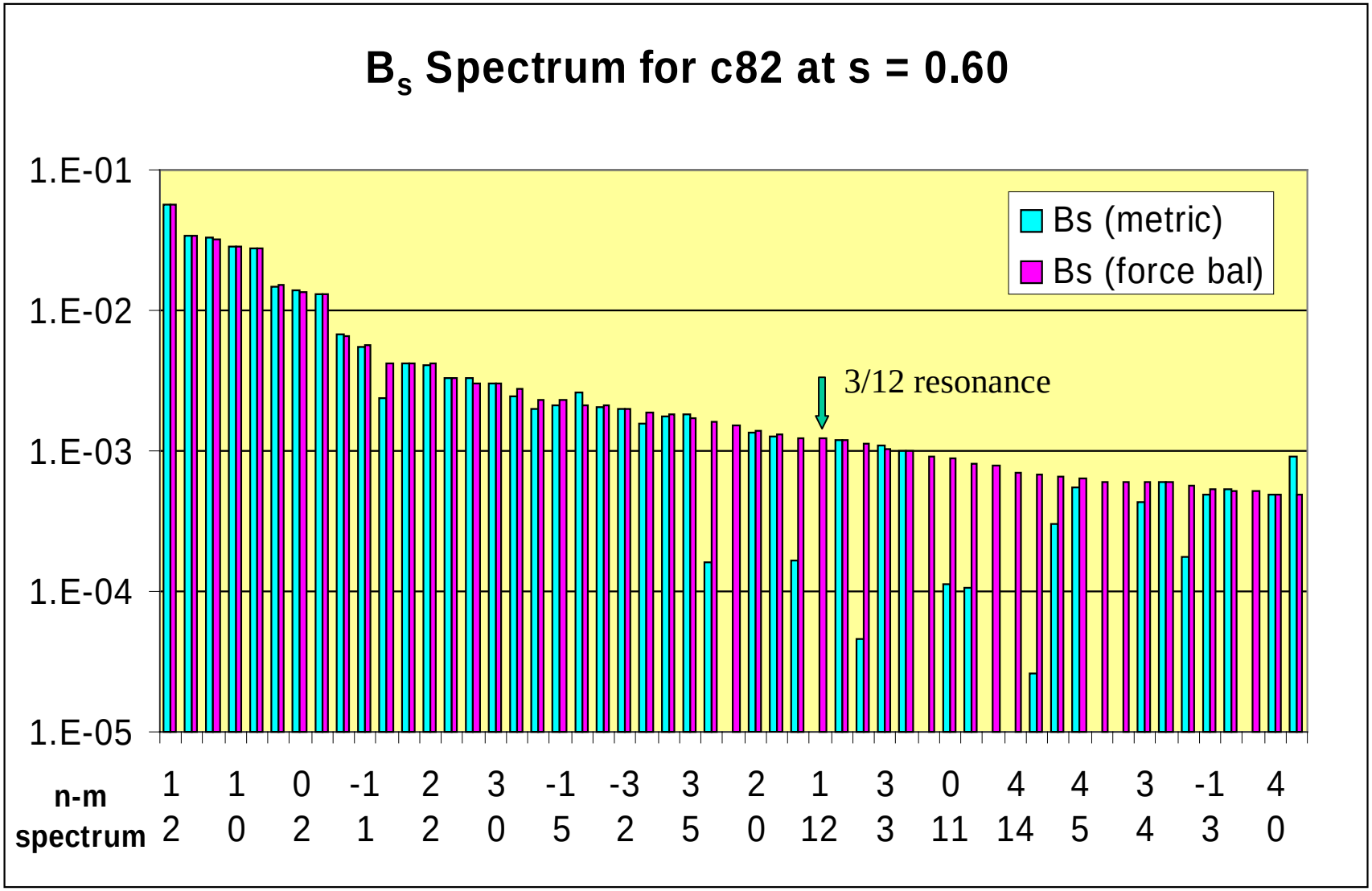
- Used dual-space (force equations in real space, B_s in Fourier space) solution method

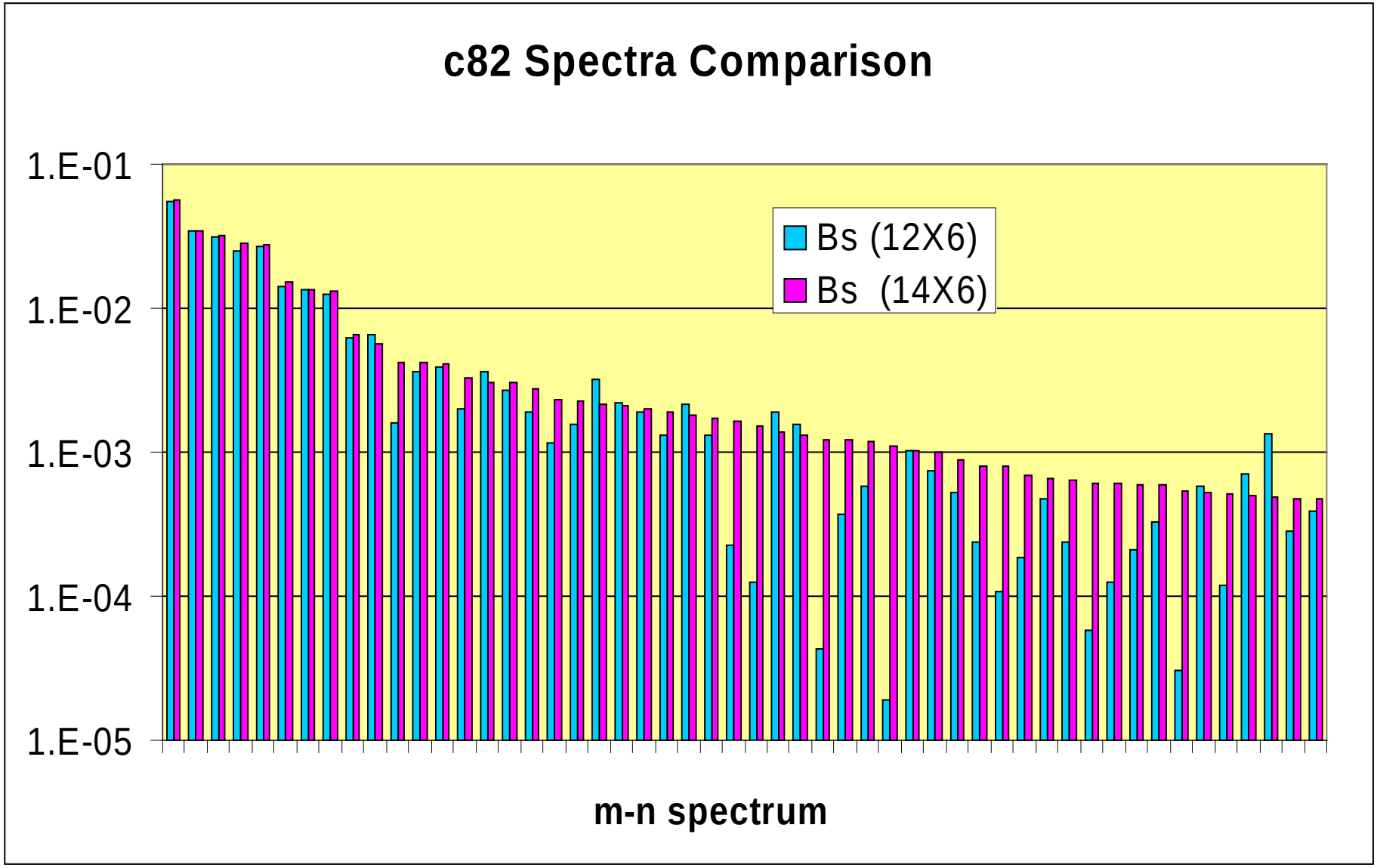
- Compare B_s spectrum with VMEC
  - if deviations are small, equilibrium is “good”.


ATF (A=7.7) Force Balance Comparison
(m=6, n=6, ns = 51)
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
Force Balance
0
0.2
0.4
0.6
0.8
1
Toroidal Flux
New FB
Old FB
S = 0.9
ι/N = 0.073


B_s Spectrum for ATF at s=0.9
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Bs (old fb)
Bs (new fb)
n-m
spectrum
1 2
0 2
1 0
0 1
2 1
2 0
0 3
0 4
-2 1
3 4
3 1
2 3
-2 2
3 0
-1 4
0 5
4 2


C82 Force Balance
(m=12, n=6, ns=51)
Original
New (Bs)
s = 0.66
ι/N = 0.085
1.E+01
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
0
0.2
0.4
0.6
0.8
1
max|JXB - p'|/|p'|
Toroidal Flux


B_s Spectrum for c82 at s = 0.60
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Bs (metric)
Bs (force bal)
3/12 resonance
n-m spectrum
1 2
1 0
0 2
-1 1
2 2
3 0
-1 5
-3 2
3 5
2 0
1 12
3 3
0 11
4 14
4 5
3 4
-1 3
4 0

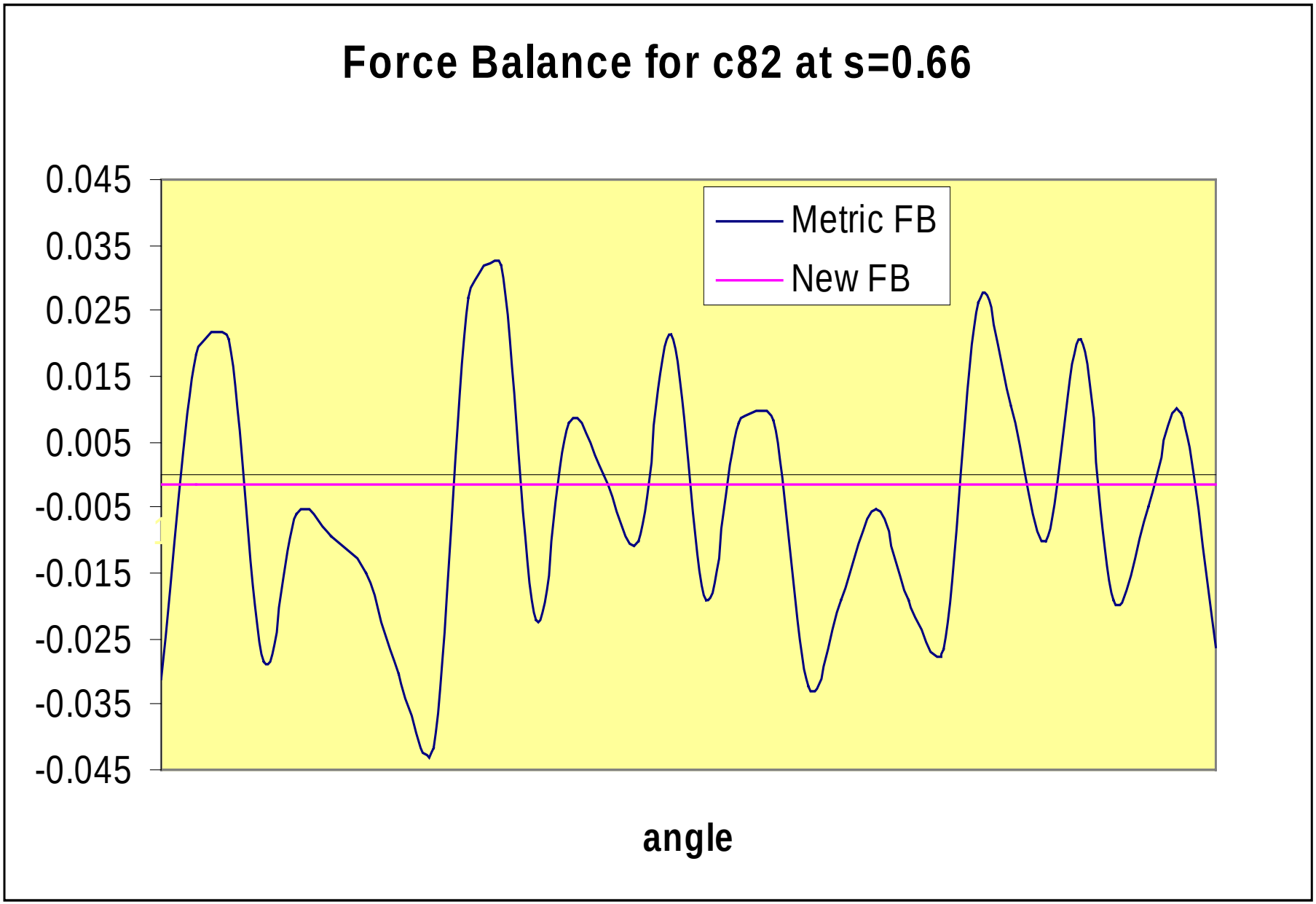

c82 Spectra Comparison
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Bs (12X6)
Bs (14X6)
m-n spectrum

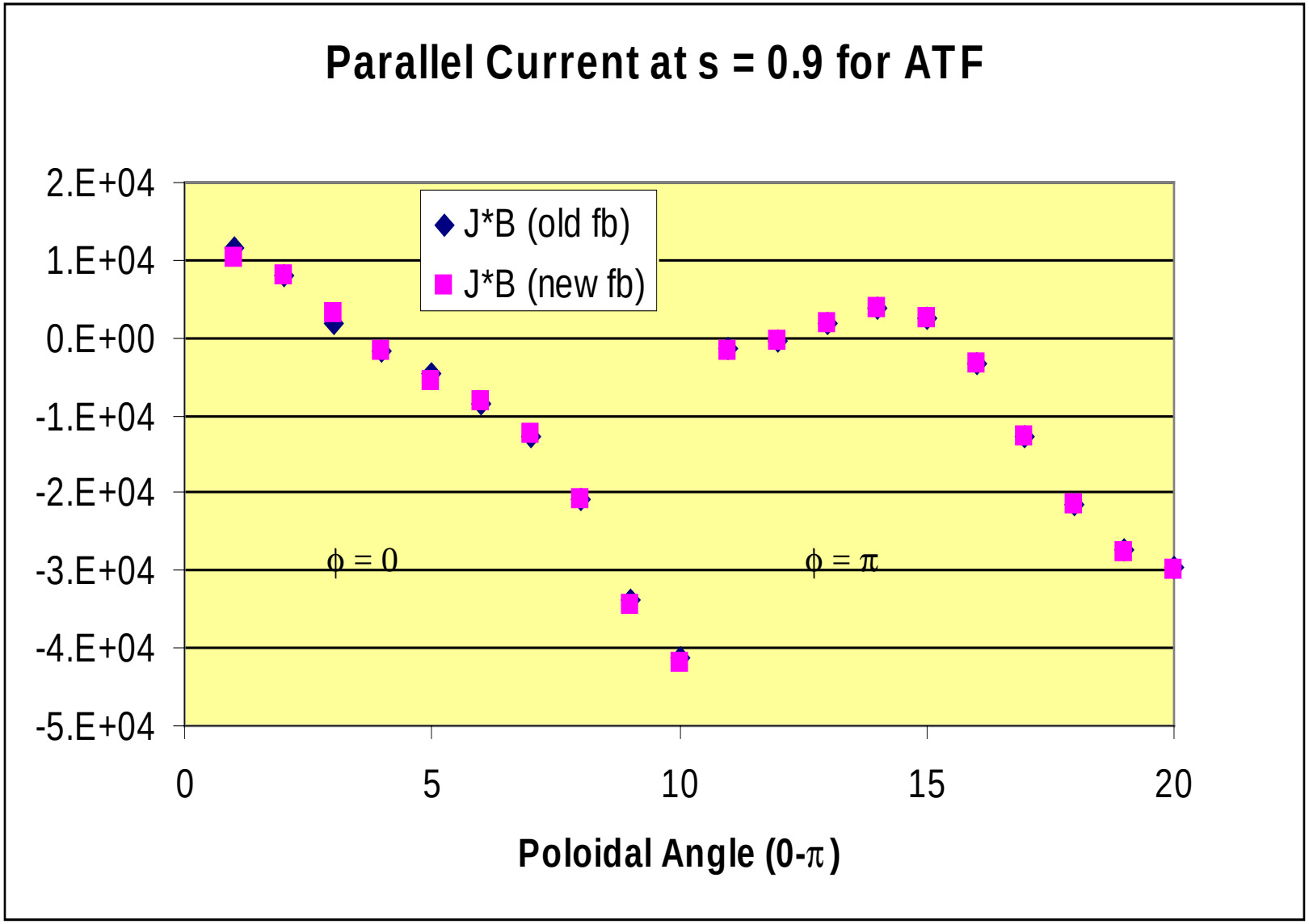
Observations

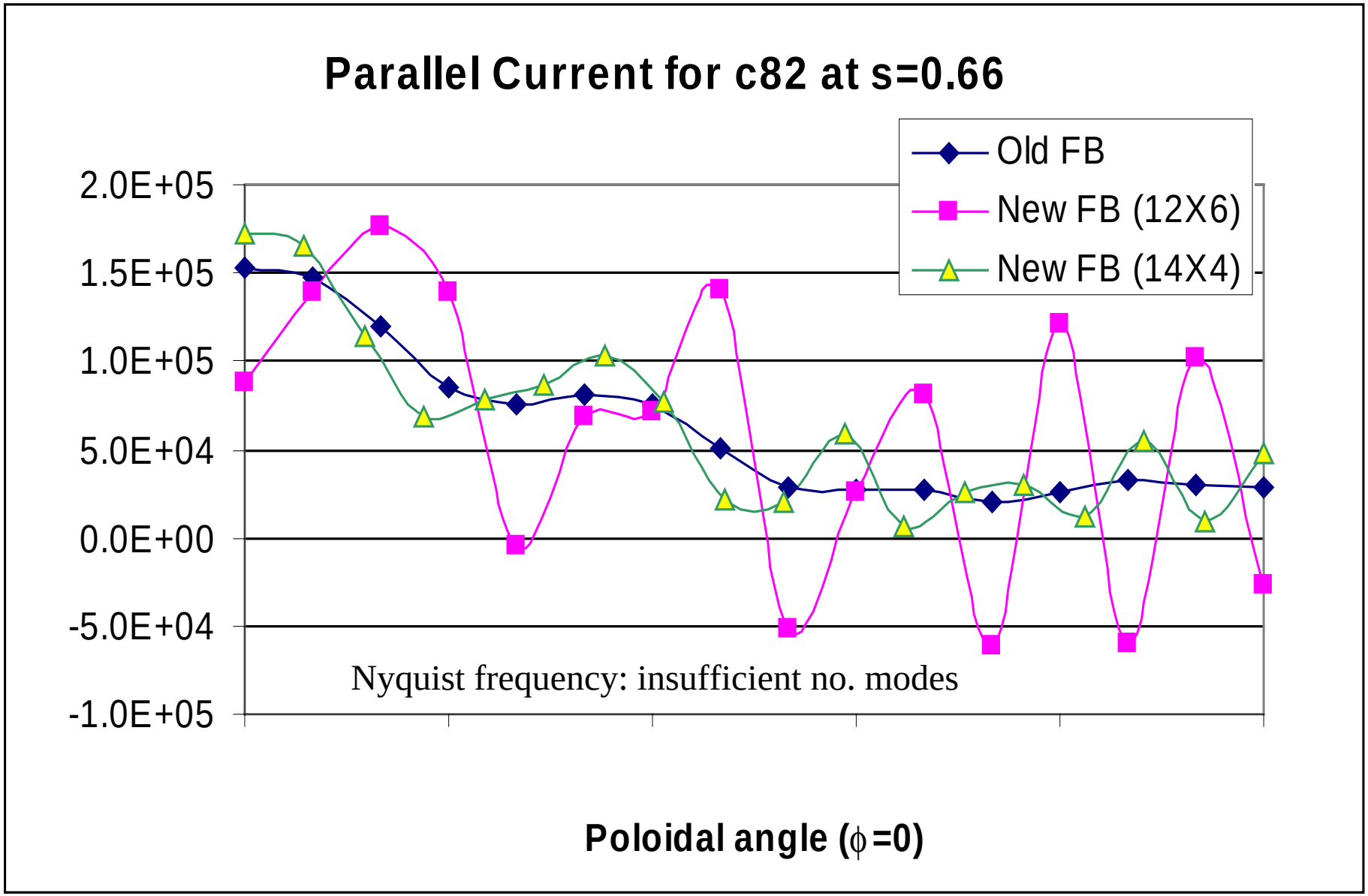
- B_s spectra agree well only for ~4 decades at higher A, ~2 decades for lower A
- The metric - B_s spectrum
  - missing (or inaccurate) higher-*m* terms after 2nd decade (for c82)
  - missing near-resonant component (N=3,m=12)
- Will spectrum ever converge?
  - expect large changes in parallel current m-n spectrum (but <J*B> largely unaffected)

Radial Force Balance is now **GREATLY** Improved

- Now, for c82, max LOCAL force deviation is < 1% over **MOST** of the discharge radius
- This is ***much*** better than previously achieved, where force balance deviations were ~10% or larger
  - B_s spectrum comparison not *terrific*
  - m-convergence very *sluggish*
- Limited by accuracy of *average* force balance


Force Balance for c82 at s=0.66
0.045
0.035
0.025
0.015
0.005
-0.005
-0.015
-0.025
-0.035
-0.045
Metric FB
New FB
angle


Parallel Current at s = 0.9 for ATF
2.E+04
1.E+04
0.E+00
-1.E+04
-2.E+04
-3.E+04
-4.E+04
-5.E+04
J*B (old fb)
J*B (new fb)
ϕ = 0
ϕ = π
0
5
10
15
20
Poloidal Angle (0-π)


Parallel Current for c82 at s=0.66
Old FB
New FB (12X6)
New FB (14X4)
2.0E+05
1.5E+05
1.0E+05
5.0E+04
0.0E+00
-5.0E+04
-1.0E+05
Nyquist frequency: insufficient no. modes
Poloidal angle (ϕ=0)

Summary

- Made VMEC2000 more robust
  - Jacobian initialization improved
  - gain access to larger phase space of configurations (previously *verboten*)
- Improved radial current, radial force balance
  - provides criterion for convergence in m/n space
  - assess impact on parallel current, stability