Free-Boundary Natural Function Approach

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Introduction

"Conventional" approach to configuration development and coil design:

1. Vary plasma shape harmonics with fixed-Boundary optimizer to minimize

$$\chi^2 = \sum_{i=1}^{\text{physics targets}} \chi_i^2.$$

2. Run NESCOIL to obtain a current sheet solution $\kappa(u, v)$ on a chosen winding surface which can support the fixed-boundary shape.

   - an ill-conditioned problem that makes it difficult to obtain accurate smooth solutions on distant winding surfaces (even for smooth plasma boundary shapes).
   - situation is worse if the plasma boundary has regions of small curvature - now the NESCOIL winding surface must be chosen carefully if a solution is to exist at all! (eg., PBX tokamak).

3. Apply a coil-cutting algorithm (eg GA) to cut discrete coils from the current sheet.

"Alternative" approach to configuration development:

- let optimizer vary the current distribution on the current sheet, solving for a sequence of free-boundary equilibrium shapes.
  - optimizer seeks to find a current sheet solution which minimizes χ^2, as before.
  - the problematic Step 2 of the "conventional" approach is completely eliminated.
- Start by choosing a toroidal winding surface on which to evaluate the current sheet potential $\kappa(u,v)$. The precise shape of this surface is not important - it should allow for a range of high performance plasma shapes.
- Choose an appropriate set and number of basis functions, $f_i(u,v)$, with which to expand $\kappa(u,v)$:

$$\kappa(u,v) = \sum_{i=1}^{\mathrm{NF}} c_i f_i(u,v)$$

  - "natural functions" are a good choice for efficient coils.
- The c_i are the independent variables of the optimization.

Expectations for new approach

- C82 was obtained using the conventional approach using MPOL=5, NTOR=3 (~ 35 independent plasma boundary harmonics).
- Since each c_i is an independent control knob, NF$\sim$ 35 should gives us enough degrees of freedom within the optimizer to obtain free-boundary configurations with the same quality of physics as C82 (although the resulting configuration will have a different shape).
- Note: The resulting configuration would be represented exactly by a current sheet with NF$\sim$ 35 functions, whereas for C82 Standard NESCOIL requires 221 functions (for $m_f = n_f = 10$), and SVD Nescoil typically requires 121 eigenfunctions of GG^T ($G =$ inductance matrix).

A few calculation details

- The winding surface chosen for the current sheet is the C82 18cm surface.
- A set of orthonormal "natural functions" current distributions, $f_i(u,v)$, (ordered w.r.t. increasing eigenvalue) are calculated.
- Magnetic field grids (mgrids) are calculated corresponding to each of the f_i.
- At each iteration the optimizer chooses a set of c_i values, in effect selecting a current sheet solution. VMEC superimposes the pre-calculated mgrids, each weighted by a c_i, to determine the B-field corresponding to the optimizer's chosen current sheet $\kappa(u,v) = \sum_i c_i f_i(u,v)$. A free-boundary equilibrium solution is then calculated, from which the various physics targets can be evaluated. The optimizer then updates the c_i. obtained.
- To start the optimizer iterations, NF initial values c_i values are required. To obtain these, we select an initial target plasma shape (which for the calculations reported here is C82), and calculate a NESCOIL current sheet solution $\kappa(u,v)$. The c_i are obtained by a Least Squares fit.

To "shake down" the new method, we have made a sequence of runs with different values of NF (=10, 20, 30 so far), and various physics targets in the χ^2. Our initial goal is to SIMULTANEOUSLY minimize χ^2_{Bmn} and χ^2_{Kink} while holding the edge iota at the C82 target value ($\iota(1) = 0.47$), with the iota profile monotonic.

1. With NF=10, we find there is not enough freedom to effectively constrain iota and minimize χ^2_{Bmn}. However, the isolated iota constraint is easily met: Fig. 1 shows a plot of the initial and final plasma configurations (dashed lines) for a run which only targeted iota. The C82 boundary shape is included in the figure (solid line) for comparison purposes. The initial plasma state had $\iota(1) = 0.31$; the final state has $\iota(1) = 0.47$ (same as target!). With regard to Quasi-symmetry, $\chi^2_{Bmn} = 350.9$ for the initial state, and 801.6 for the final configuration. This can be compared with $\chi^2_{Bmn} = 23.6$ for C82.....The iota constraint has been met at some expense to the quasisymmetry. By relaxing the iota constraints somewhat, we can improve the quasisymmetry to obtain $\chi^2_{Bmn} = 315.3$ (the profile was quite flat, with $\iota_{min} = \iota(0) = 0.40, \iota(1) = 0.50$, with $\iota_{max} = 0.52$ at $s = 0.81$).

NF = 10

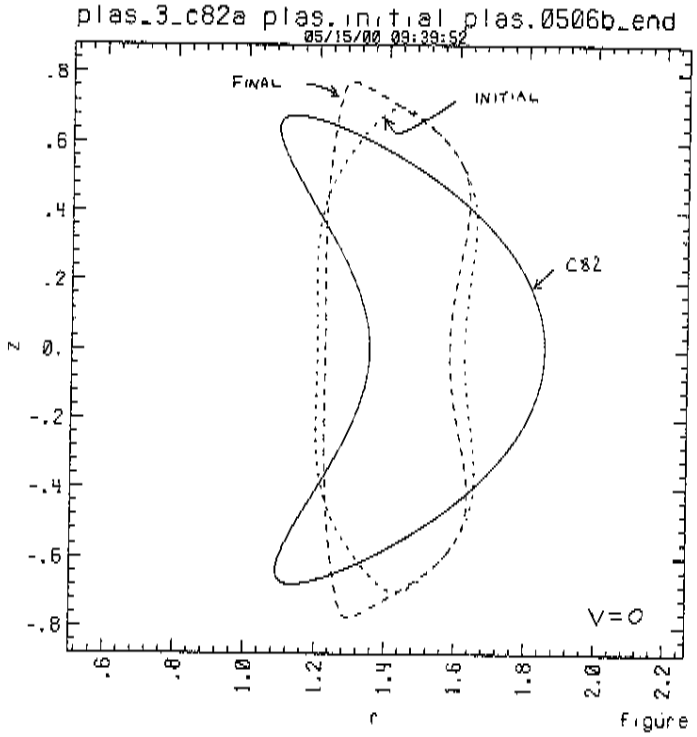

figure 1

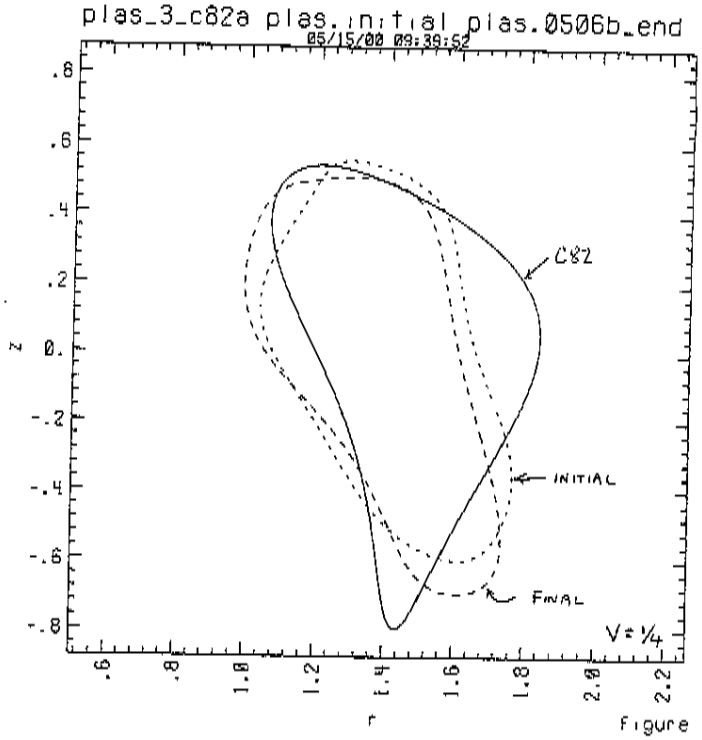

figure 2

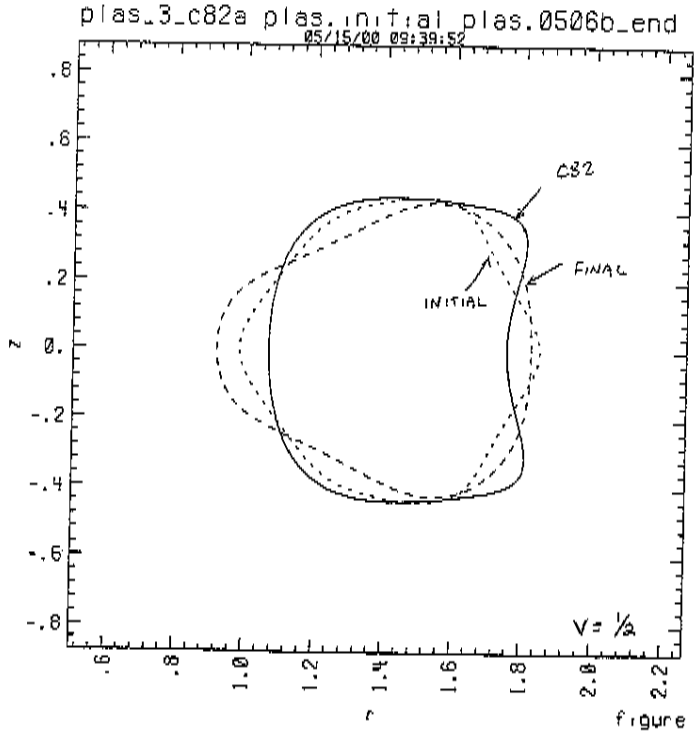

figure 3

Figure 1

No geometric constraints were imposed on the configurations, so we can see that plasma-to-wall minimum separations would be violated if we cared. We do not care in the present proof-of-principle calculations!

NOTE: Having a version of VMEC that constrains the plasma boundary to be in contact with a physical limiter is an essential modification for the future, and would be desirable to have available now.

For the VMEC calculations, we used MPOL=6, NTOR=3 (see later discussion).

2. Using NF=20, we do a little better at simultaneously satisfying the iota constraint and minimizing χ^2_{Bmn}, but nothing very exciting! A breakthrough occurs when we increase NF to 30:

3. Case NF=30: Here, we are able to satisfy the iota constraint AND get good quasisymmetry. In particular, we have obtained $\chi^2_{Bmn} = 8.2$ (<u>lower</u> by a factor of 3 than the value for C82!) for a monotonic iota profile with $\iota(0) = 0.22$ and $\iota(1) = 0.46$. For the VMEC calculations, we used MPOL=9, NTOR=5. (Assuming a straight periodic cylinder approximation for the winding surface, the natural functions forming a basis for the current sheet with NF=30 correspond to poloidal and toroidal modenumbers of 4 or less).

- Fig. 2 shows a plot of this configuration (dashed line) at various toroidal angles. Once again, C82 is shown for comparison.
- The configuration shape seems somewhat bizarre - what is causing the m=5 rippling of the boundary at each cross section .... is it real? (i.e., a true consequence of the currents in the winding surface - push-me pull-you arguments should help clarify) .... do we have enough VMEC modes in the calculation? .... is VMEC calculating a sufficiently accurate free-boundary equilibrium for these difficult low-aspect-ratio configurations? .... is the boundary rippling due to islands? ....

NF=30

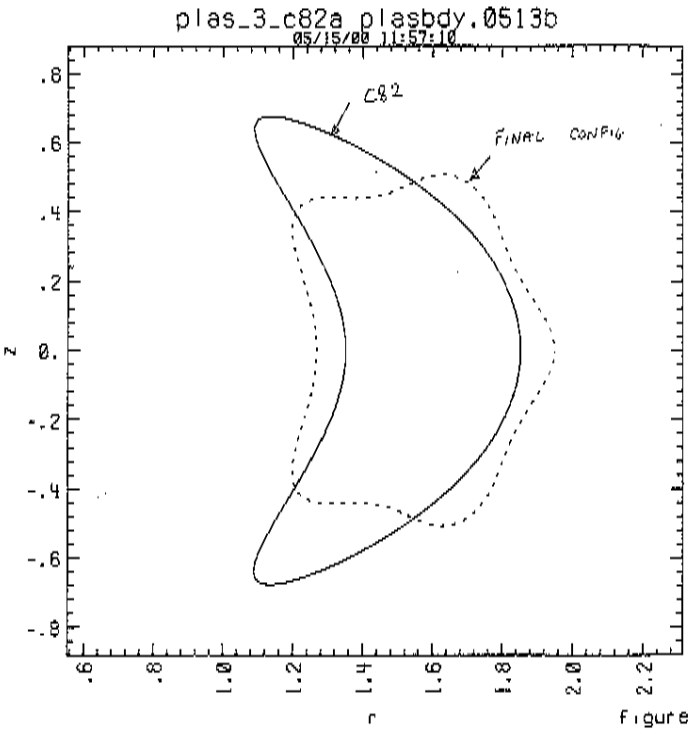

figure 1

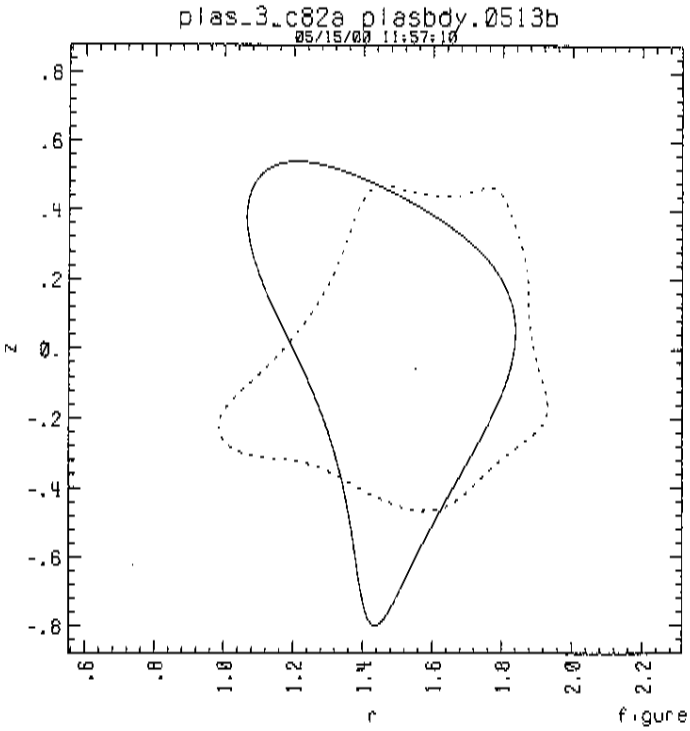

figure 2

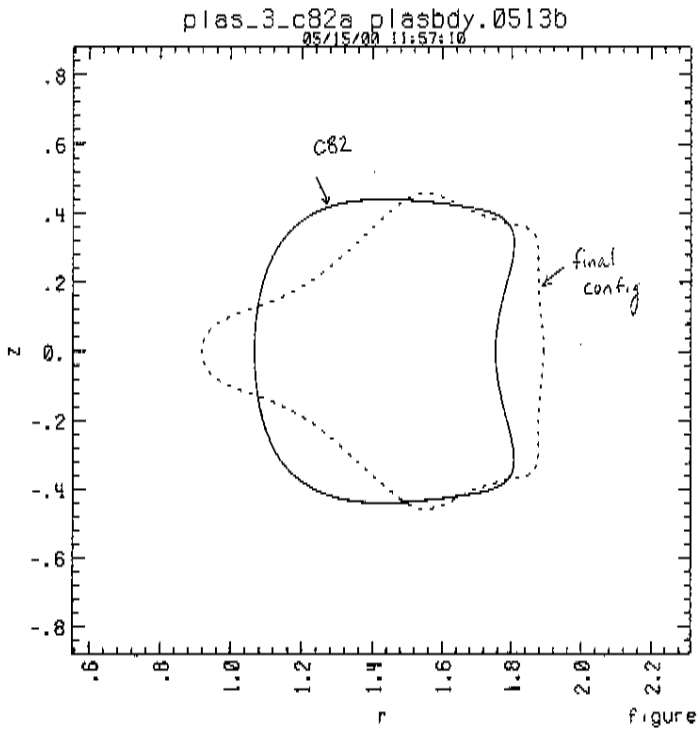

figure 3

Figure 2

Future

- Turn on the Kink optimization with NF=40.
  - Preliminary results (today's runs) indicate that NF=30 is unable to improve the kink very much .... The quasi-symmetric configuration discussed above for NF=30 has a kink growth rate = 5.5 x C10, and 150 x C82. When we turn on the Kink optimizer, we are only able to decrease λ by a factor of 2.
- Turn on ballooning calculation.
- Use a less prejudicial winding surface (probably axisymmetric), and use a less prejudicial initial condition.
- Examine the final current sheet solutions .... Are contours of the current potential sufficiently simple to warrant cutting coils?
- Examine whether we need to penalize large coefficients of the highest of our low-order natural functions.
- Run with secular additions to current potential so that modulars are included.