Manipulating islands using boundary perturbations

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Abstract

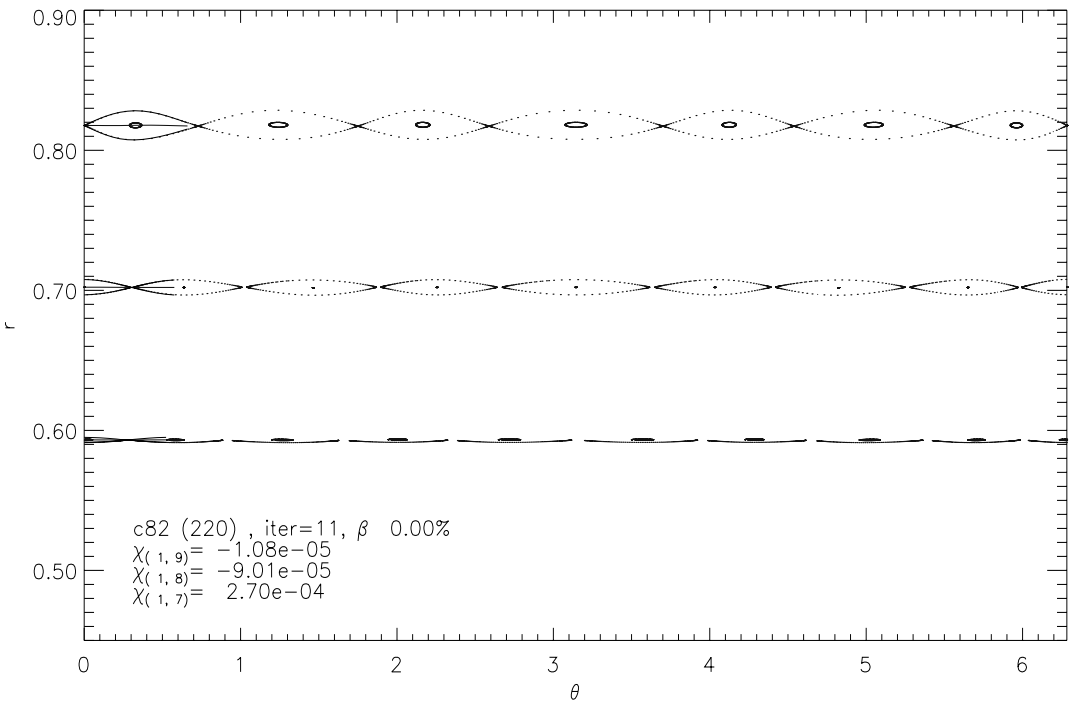
The extent to which islands may be manipulated using boundary perturbations is under examination. New computational diagnostics are being incorporated into PIES to provide accurate calculation of island widths. This calculation uses the normal component of the field to rational quadratic-flux-minimizing surfaces. Boundary perturbations are applied to PIES input files (prepared using VMEC). The perturbations are chosen to match the periodicity of selected island chains. The configuration C82 is used as a test problem.

At zero β, the islands may be manipulated; however, presently a poor numerical procedure is being used. This will be improved in future work. At full β, it is less clear that the islands can be controlled. Future work will clarify this and seek to identify the physical mechanism driving islands.

OUTLINE

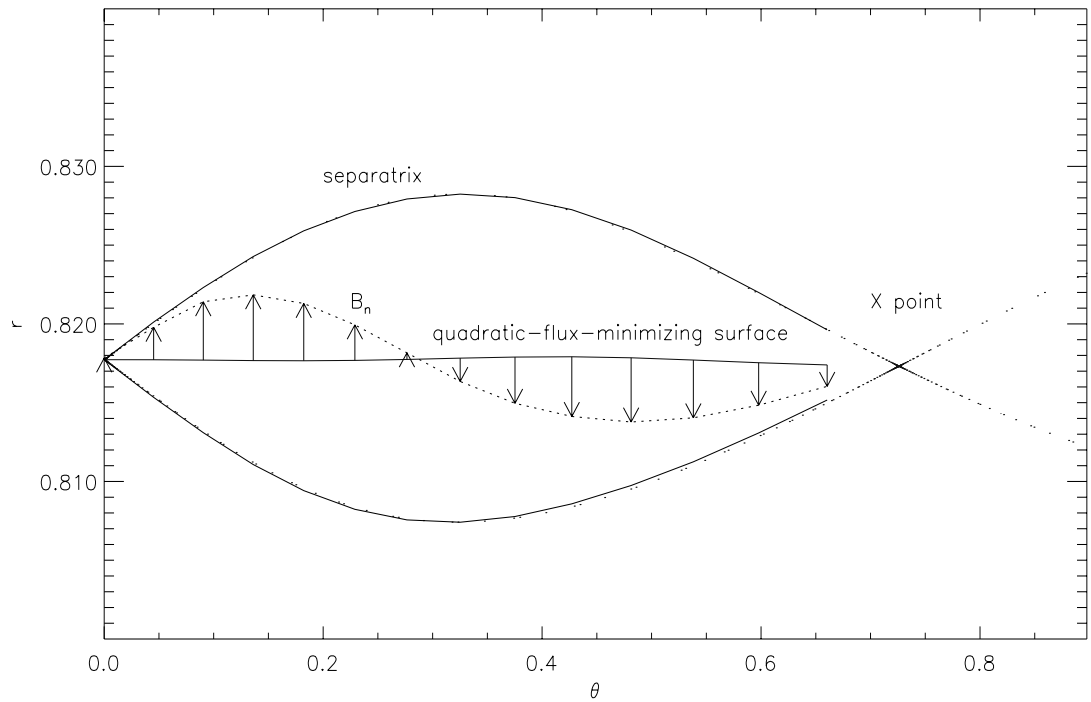
- new diagnostic routines based on quadratic-flux-minimizing surfaces determine island width
- resonant boundary perturbations applied to PIES input files
- island response examined
- **NB. This is work in progess. The method and results to be shown are a test run.**

Islands chains in the configuration are selected.



- C82 at $\beta = 0\%$ has been used as a test case.
- Note that only 10 PIES iterations has been performed, so this is not a converged equilibrium.
- The islands selected are the $\iota = 3/7, 3/8, 3/9$. The 3/9 is already very small. The 3/10 is present, but ignored.

Quadratic-flux-minimizing surfaces are used to locate islands and determine widths and resonant fields.



- Rational surfaces, island widths and resonant fields are located using algorithms developed by S.R.Hudson and R.L.Dewar and is based on the construction of quadratic-flux-minimizing surfaces [1].

- The quadratic-flux functional is:

$$\varphi_2 = \frac{1}{2}\int_0^{2\pi}\int_0^{2\pi} d\theta d\zeta \left(\frac{\partial S}{\partial \theta}\right)^2$$

$$S = \oint \mathbf{A}\cdot\mathbf{dl}$$

- The minimizing surface is found as a family of periodic orbits of the pseudo-field :

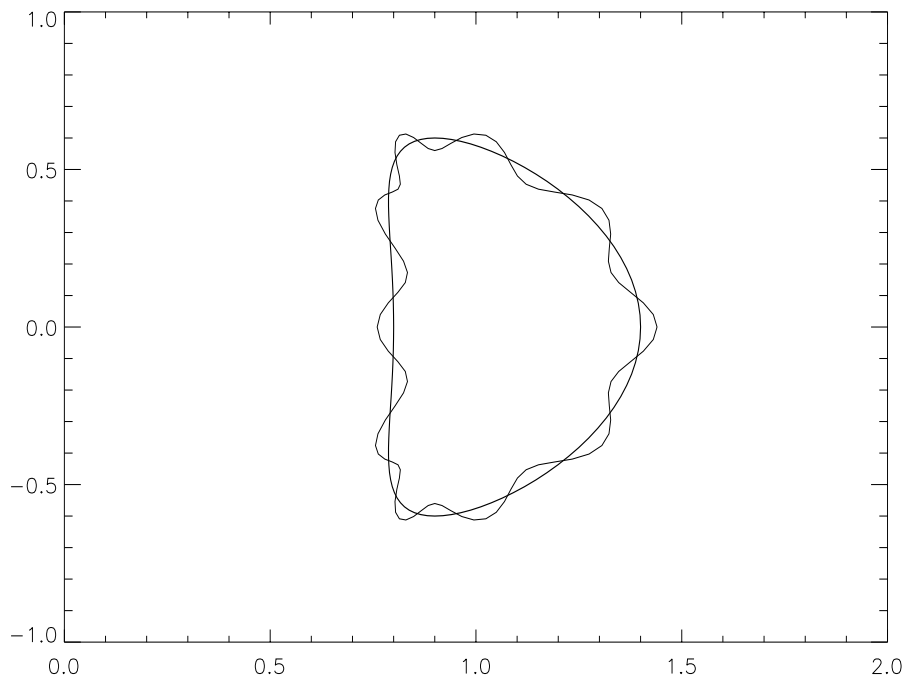
$$B_\nu = (B^\rho - \nu\mathcal{J}^{-1})\mathbf{e}_\rho + B^\theta\mathbf{e}_\theta + B^\zeta\mathbf{e}_\zeta$$

- These surfaces pass directly through the island chains. The normal component of the field to these surfaces $\chi_{m,n}$ produces islands.

- Diagnostic routines that locate rational surfaces and calculate separatrices are presently being incorporated into PIES.

Boundary perturbations are applied to manipulate islands.

- The following is an artistic impression of boundary perturbations



$$X = r\cos\theta \qquad Y = r\sin\theta$$

$$r \to r + 2\delta_{mn}\cos(m\theta - n\phi)$$

$$X \to X + \delta_{mn}\cos((m+1)\theta - n\phi) + \delta_{mn}\cos((m-1)\theta - n\phi)$$

$$Y \to Y + \delta_{mn}\sin((m+1)\theta - n\phi) - \delta_{mn}\sin((m-1)\theta - n\phi)$$

- A set of perturbation harmonics $\{\delta_{m_i,n_i}\}$ is chosen to match the periodicity of islands.

A function measuring island widths is minimized.

- A function is defined

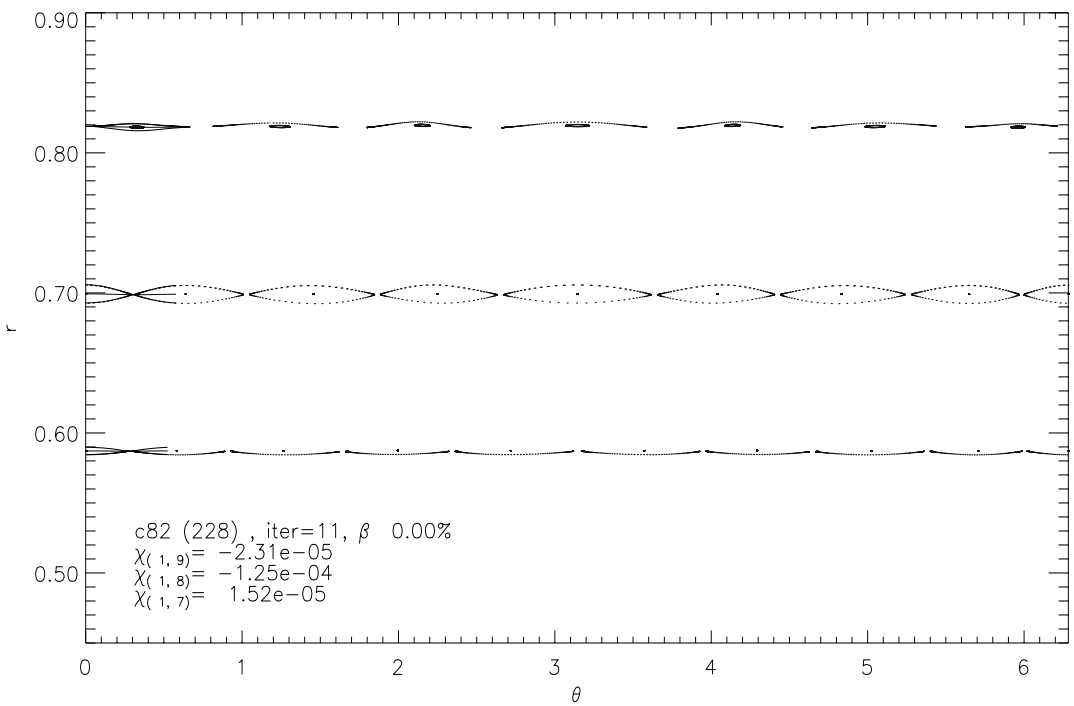
$$\Delta = \Delta(\{\delta_{m_i,n_i}\}) = \sum_i \chi^2_{m_i,n_i}$$

- A numerical routine (NAG E04JYe) is used to vary the $\delta_{m,n}$'s to find the minimum of Δ, which corresponds to zero island width. The routine E04JYe uses function values only to evaluate the minimum.

- This is not the optimal numerical procedure for this task, but it was simple to implement as a quick check of the routine. I will implement a procedure that maintains information about the sign of the $\chi_{m,n}$

Results of minimization procedure.

- The minimization procedure had some success. The (3/7) island has been reduced, but the (3/8) appears unchanged.

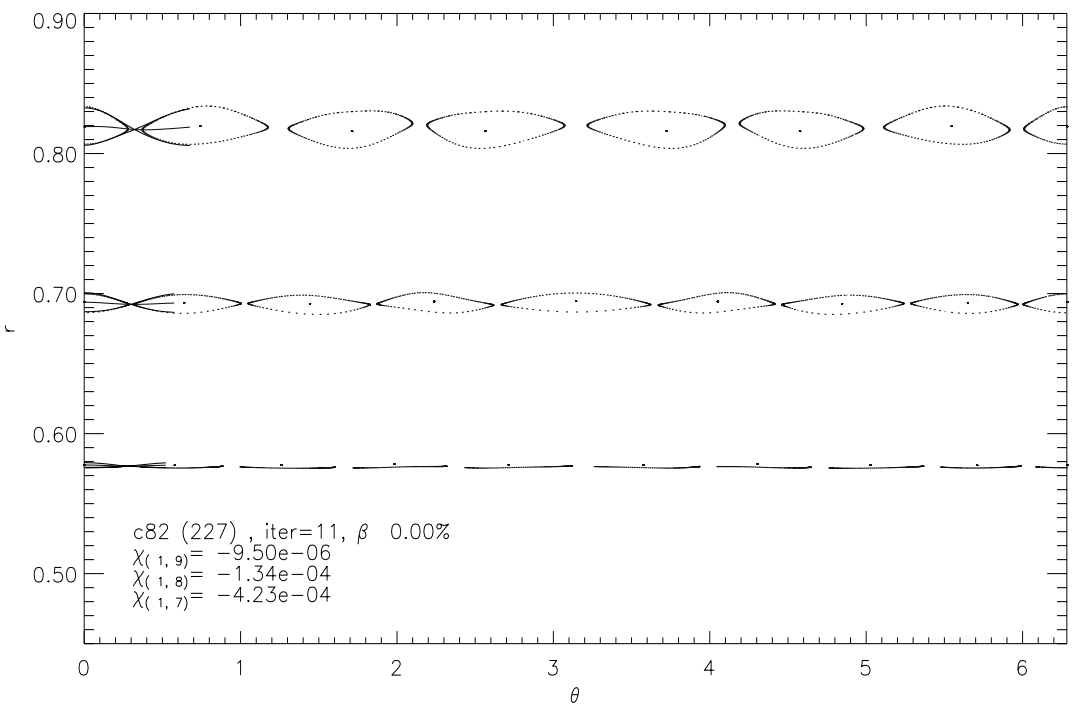


- The boundary perturbations are :

RBC(1 , 6)	=	5.0094585E-04	ZBS(1 , 6)	=	-5.0094585E-04
RBC(1 , 7)	=	1.8664403E-03	ZBS(1 , 7)	=	-1.8664403E-03
RBC(1 , 8)	=	-4.6504055E-04	ZBS(1 , 8)	=	1.4669322E-03
RBC(1 , 9)	=	1.8664403E-03	ZBS(1 , 9)	=	1.8664403E-03
RBC(1 , 10)	=	-9.6598640E-04	ZBS(1 , 10)	=	-9.6598640E-04

The phase of islands may be swapped – possible self healing ?

- It is possible to completely eliminate the (3/7) island. During the minimization, a set of δ_{mn}'s was tried that swapped the phase of the (3/7) island.



Comments

- The results shown are at zero β.
- The manipulation of the phase of the island chains raises the possibility of self-healing phenomena. This will be investigated.
- I have tried to cancel the same islands at full β ($\approx 4\%$). but it seems that the islands are more difficult to control. Perhaps pressure effects overwhelm boundary changes ?
- Using the accurate calculations of island widths, analytic expressions derived by Hegna etal will be investigated to determine the physical mechanisms driving the islands.
- Configurations relevant to NCSX will be used.

References

[1] S.R. Hudson and R.L. Dewar. Analysis of perturbed magnetic fields via construction of nearby integrable field. *Phys.Plasmas*, 1999.