Recent Studies in Accelerating the PIES Code

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The PIES Algorithm

Given field in n'th iteration, $\mathbf{B}^{(n)}$,

1. Take p constant along field lines:

$$\mathbf{B}^{(n)} \cdot \nabla p^{(n+1)} = 0.$$

2. Calculate current, $\mathbf{j}^{(n+1)}$, satisfying

$$\mathbf{j}_{\perp}^{(n+1)} = \mathbf{B}^{(n)} \times \nabla p^{(n+1)} / (B^{(n)})^2,$$

$$\mathbf{B}^{(n)} \cdot \nabla \mu^{(n+1)} = -\nabla \cdot \mathbf{j}_{\perp}{}^{(n+1)}.$$

3. Update field.

$$\nabla \times \mathbf{B}^{(n+1)} = \mathbf{j}^{(n+1)}.$$

More generally, apply underrelaxation (blending). Let

$$\nabla \times \mathbf{C}^{(n+1)} = \mathbf{j}^{(n+1)}.$$

Then

$$\mathbf{B}^{(n+1)} = (1 - \alpha)\mathbf{C}^{(n+1)} + \alpha \mathbf{B}^{(n)},$$

where α is the underrelaxation parameter.

Resonant Currents Require Large Underrelaxation

Strong response of resonant current to small changes in $\mathbf{B}$ leads to requirement of large underrelaxation parameter.

Solve for pressure-driven current on good flux surfaces using magnetic coordinates,

$$\mathbf{B} = \nabla\psi \times \nabla\theta + \iota\nabla\phi \times \nabla\psi.$$

Current:

$$\mathbf{j} = \left(I'(\psi) - \frac{\partial\nu}{\partial\theta}\right) \nabla\psi \times \nabla\theta + \left(\frac{\partial\nu}{\partial\phi} - g'(\psi)\right) \nabla\phi \times \nabla\psi,$$

where

$$\nu = \frac{dp}{d\Psi} \sum_{n,m}{}' \frac{i\mathcal{J}_{n,m}}{(n - \iota m)} e^{-i(n\phi - m\theta)},$$

$$\mathcal{J} = (\mathbf{B} \cdot \nabla\phi)^{-1},$$

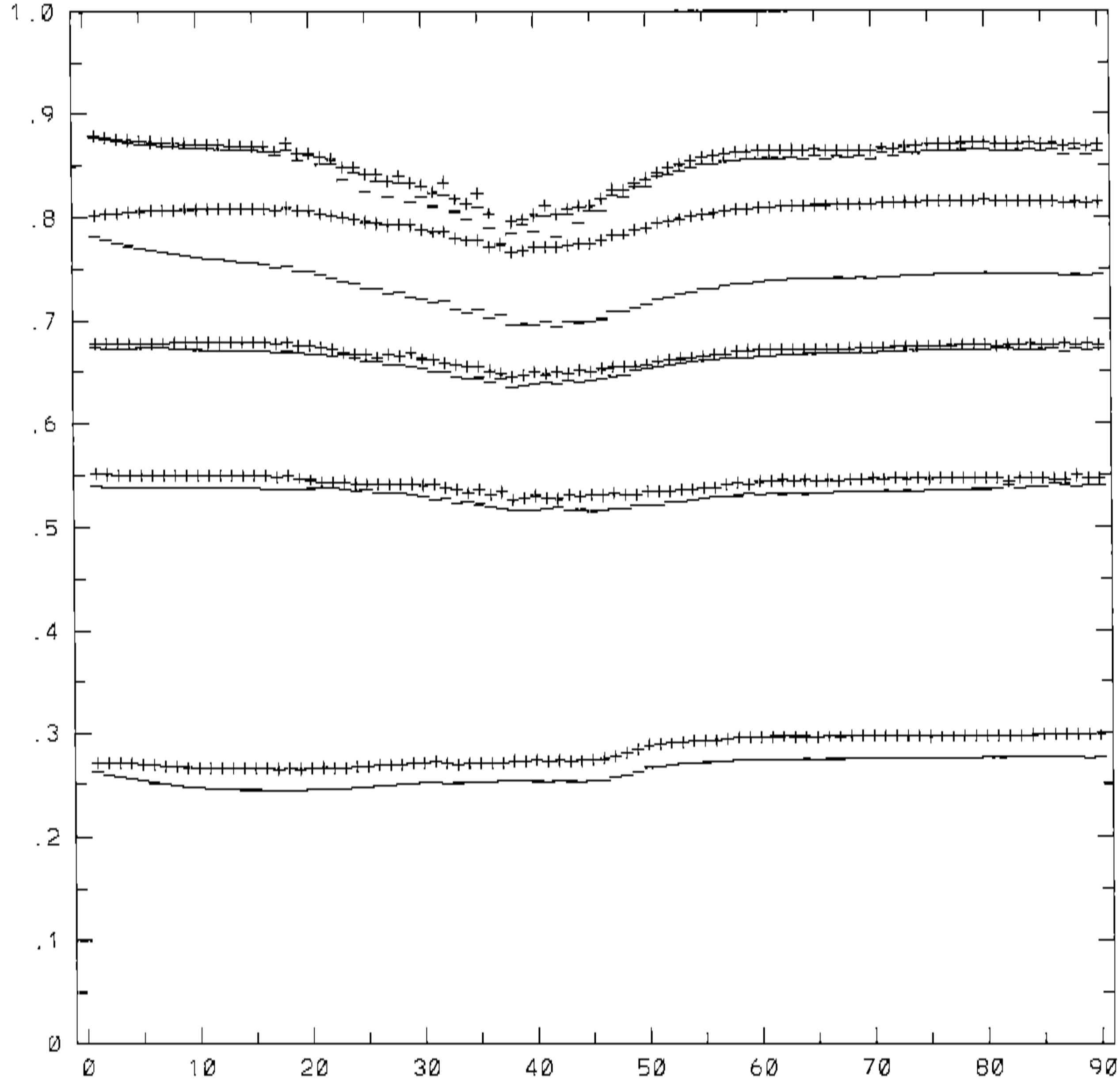
$$\mathcal{J}(\psi,\theta,\phi) = \sum_{n,m} \mathcal{J}_{n,m} e^{-i(n\phi - m\theta)}.$$

Code Displays Numerical Instability When Underrelaxation Not Large Enough

- When underrelaxation inadequate, get oscillatory, growing current.
- Problem most severe for low shear region near a low order rational surface.
- Underrelaxation parameters as large as .99 may be required.
- Can things be improved by applying underrelaxation to the current, using different blending parameters for different Fourier components? (A simple form of preconditioning.)

LI 383, $\alpha = .9$

Island Edges vs. iteration no.

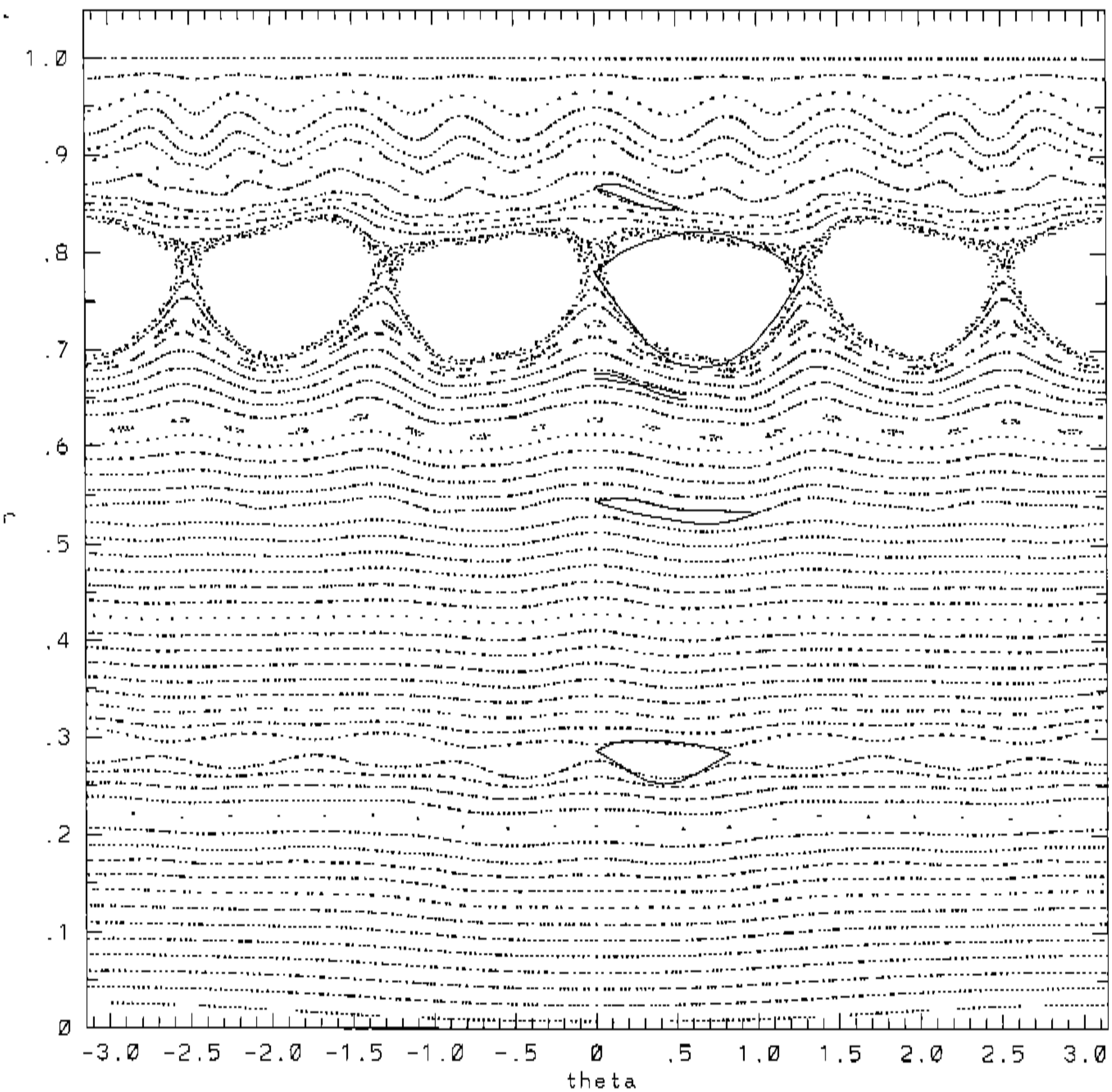


it= 90 itplt: edges of islands from Hudson

33

Plot 1481

widths have been multiplied by .5

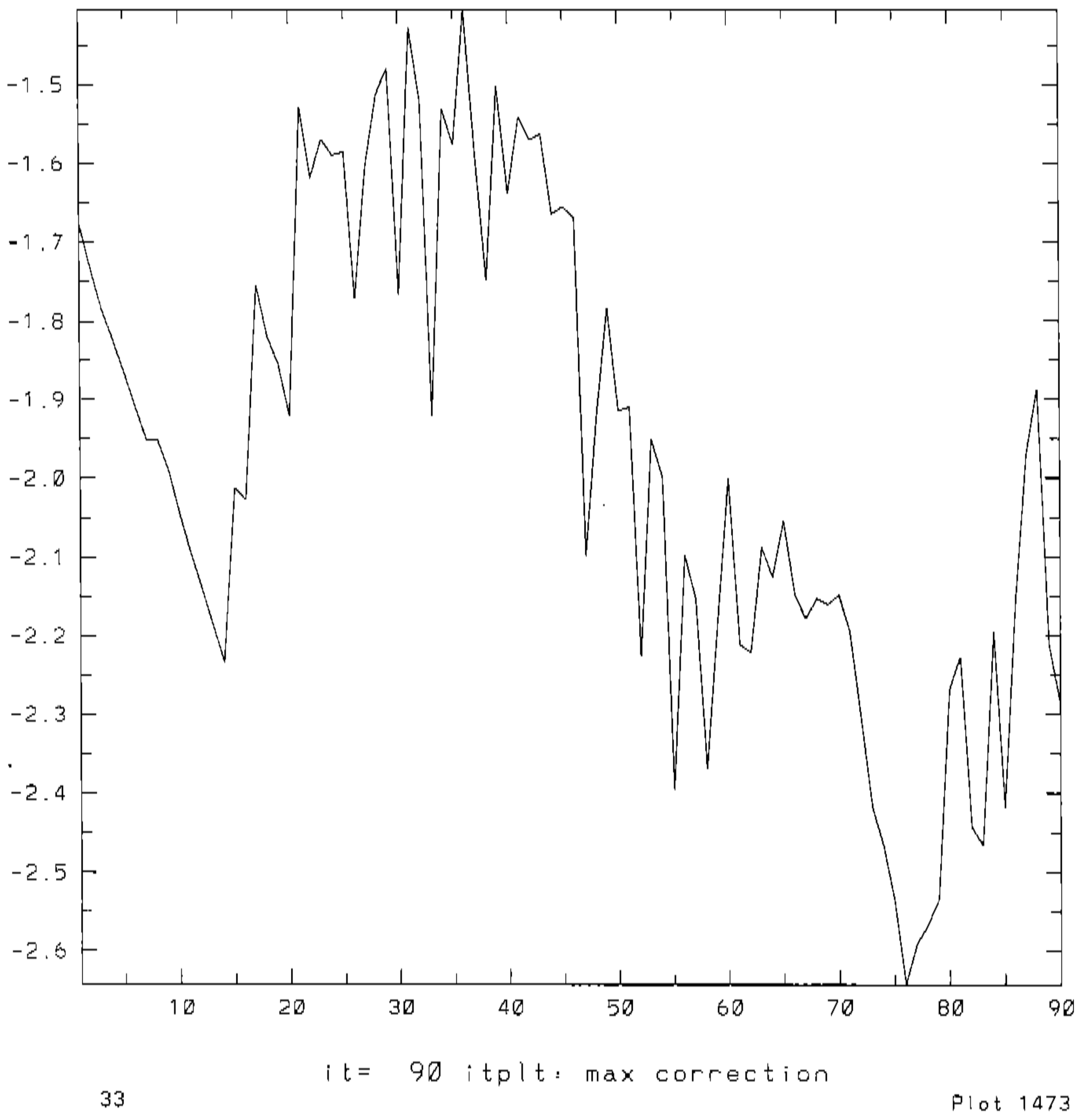


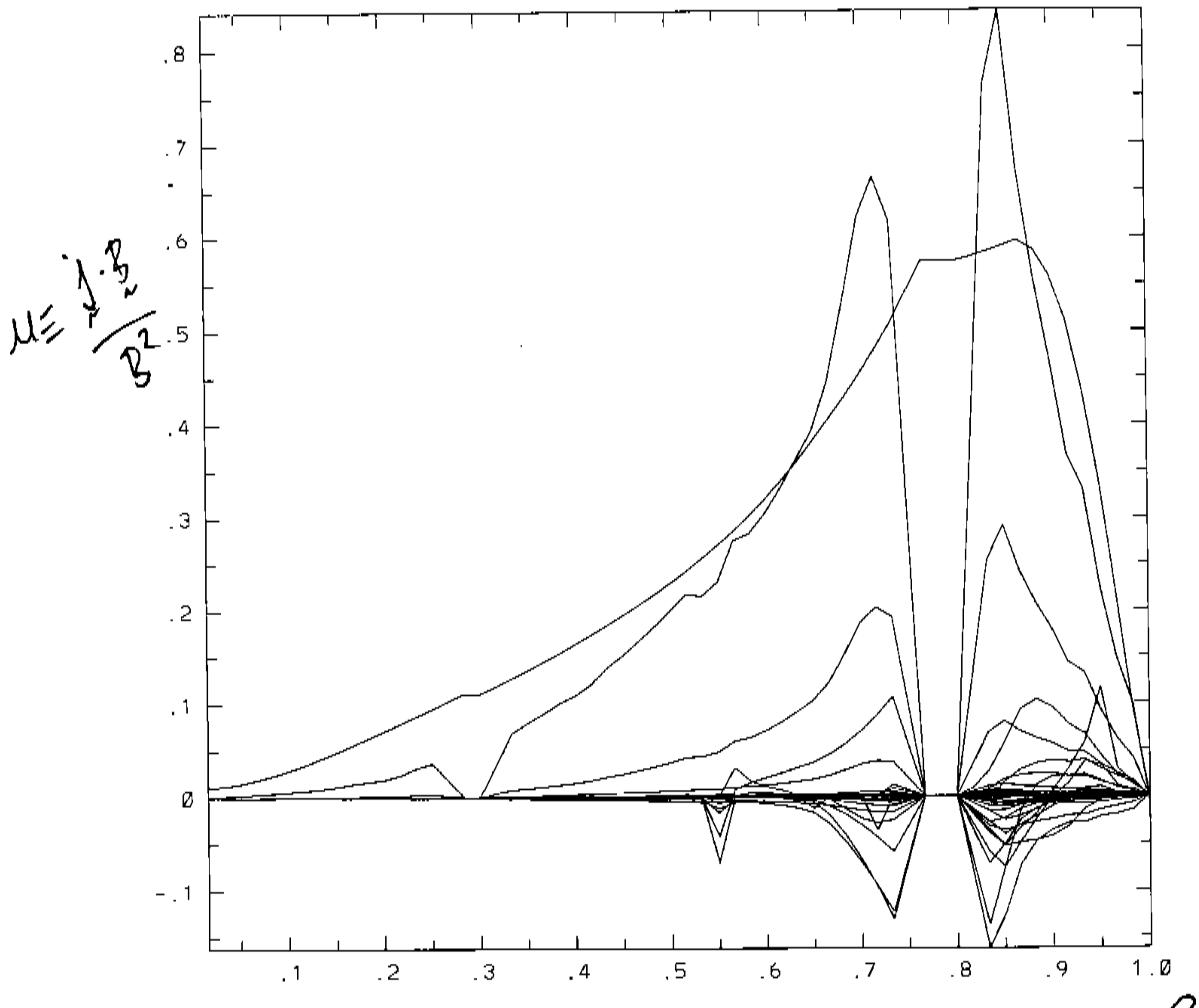
it= 90 rpoinc, background coordinates

33

Plot 1452

Max correction in B_{mn}
vs. iteration no.

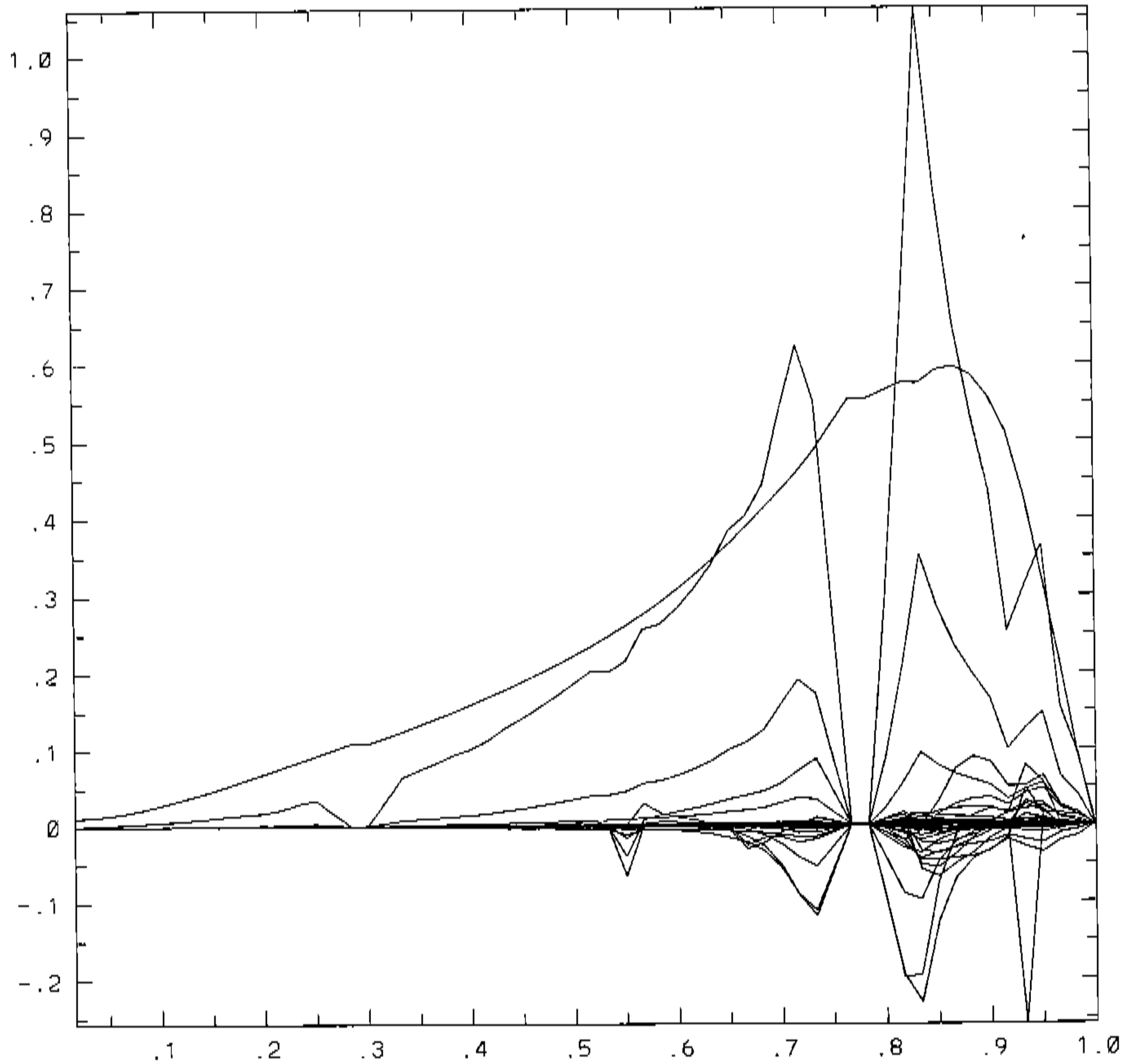




it= 84 mupspl: mu in mag. coord

33

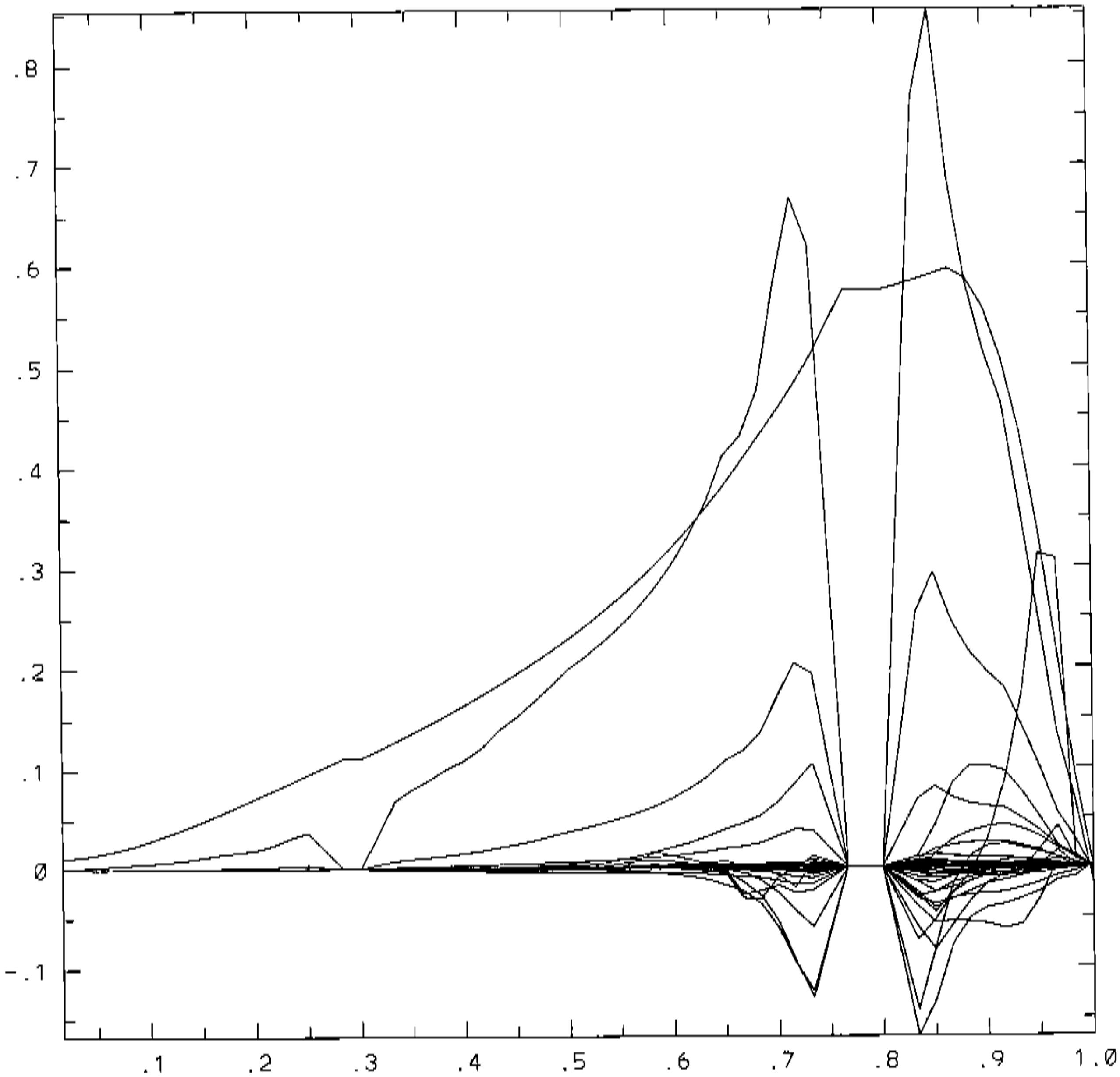
Plot 1358



it= 85 mupspl: mu in mag. coord

33

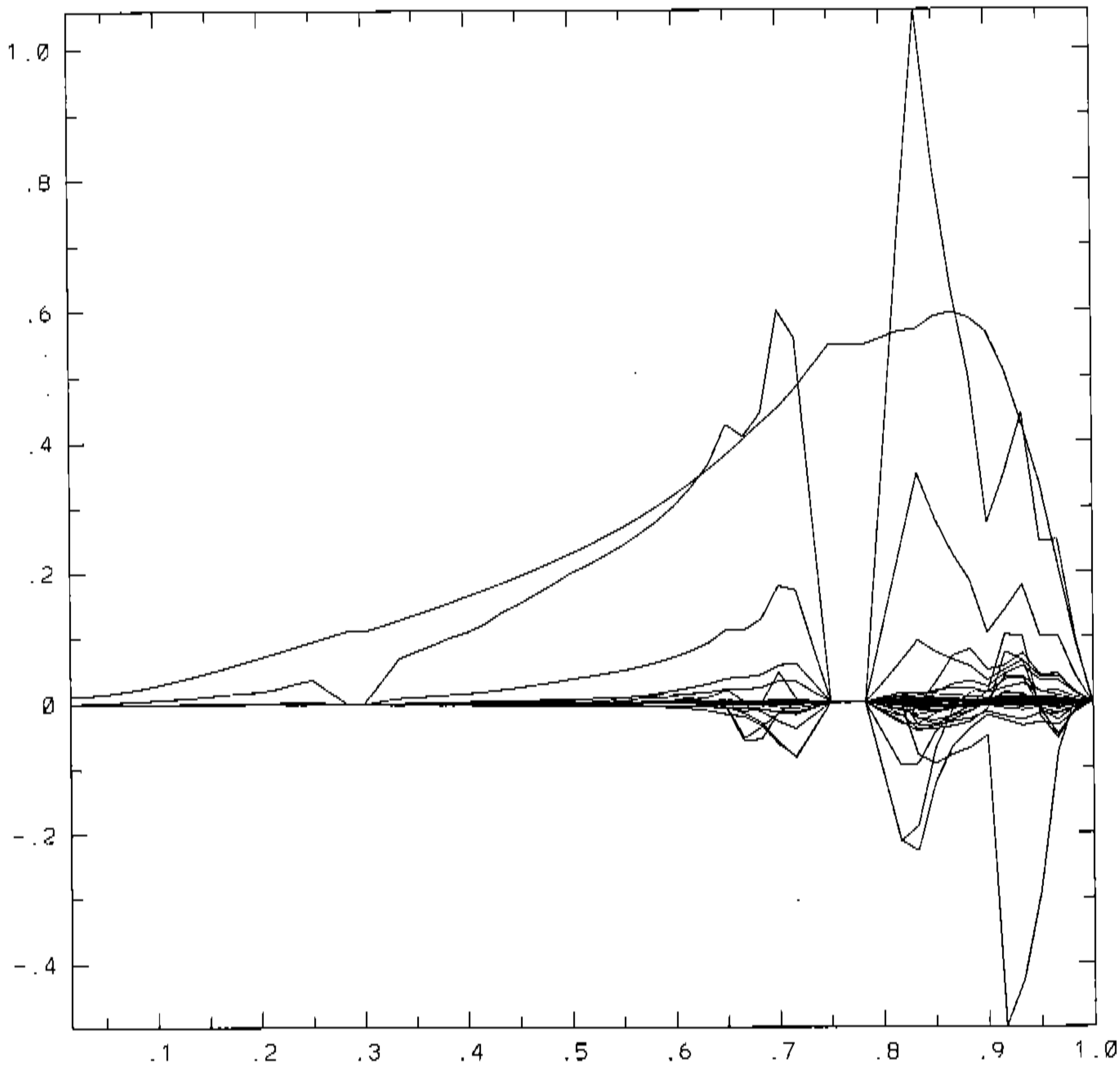
Plot 1374



it= 86 mupspl: mu in mag. coord

33

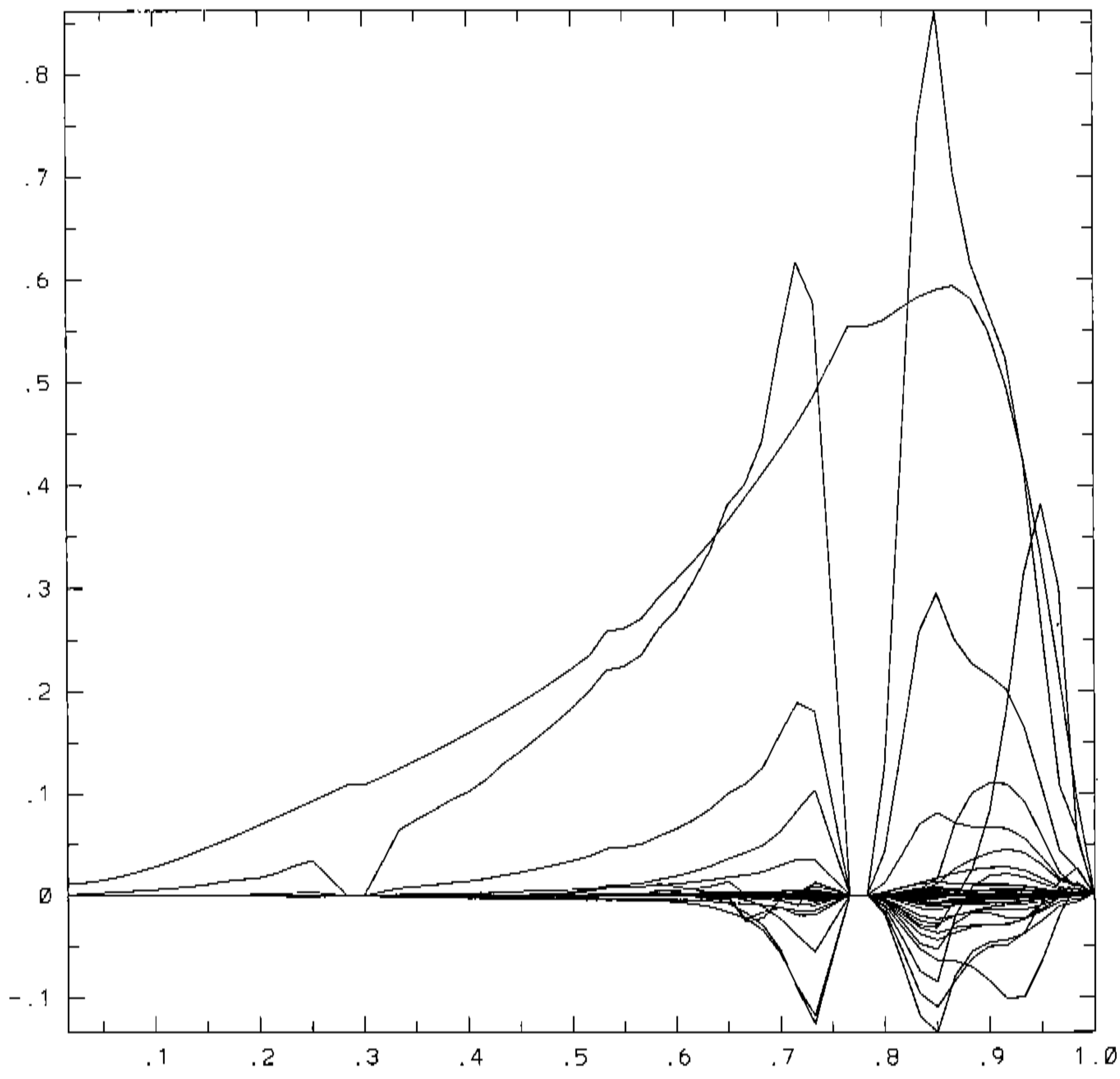
Plot 139Ø



it= 87 mupspl: mu in mag. coord

33

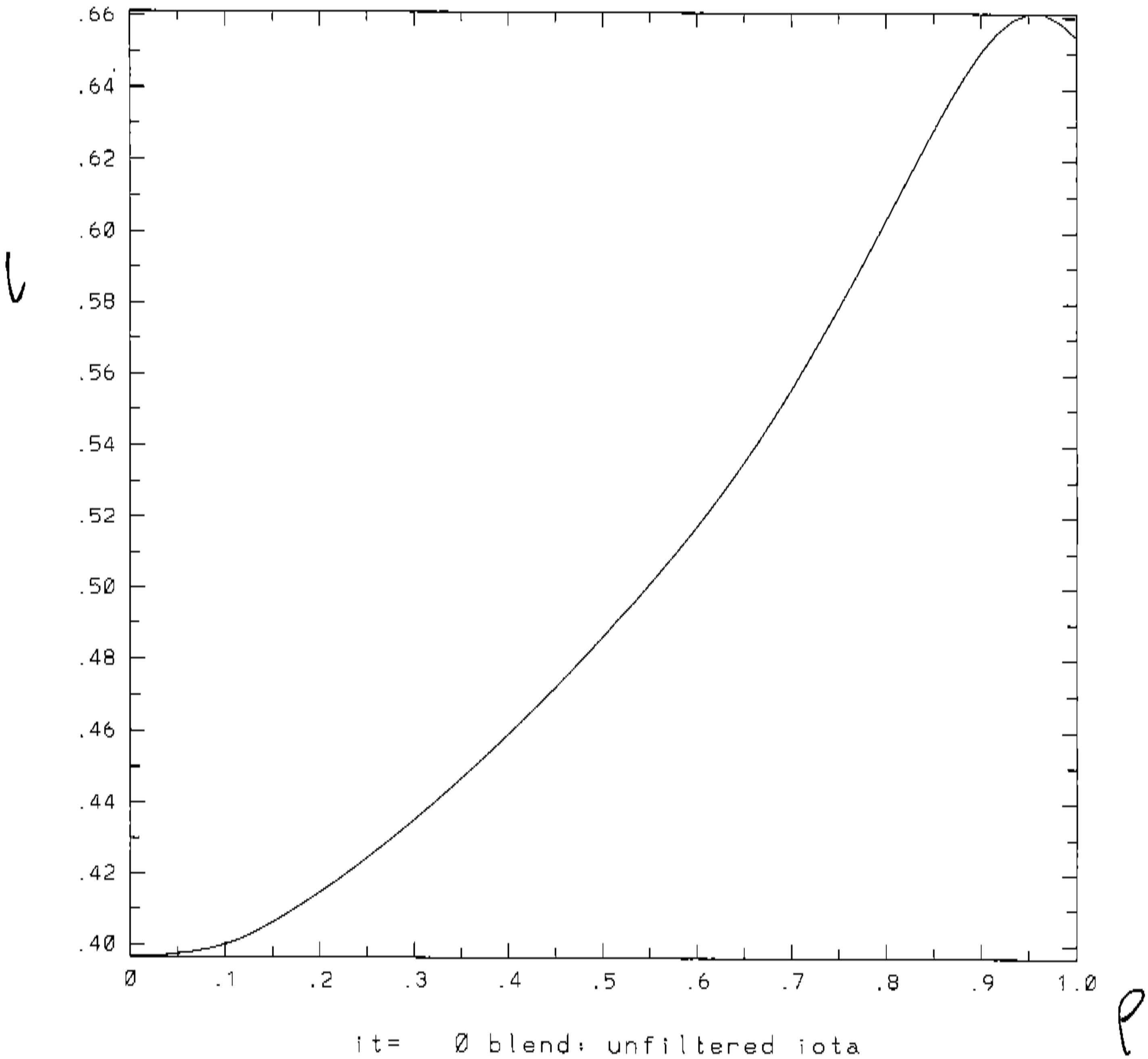
Plot 14Ø6



it= 88 mupspl: mu in mag. coord

33

Plot 1422

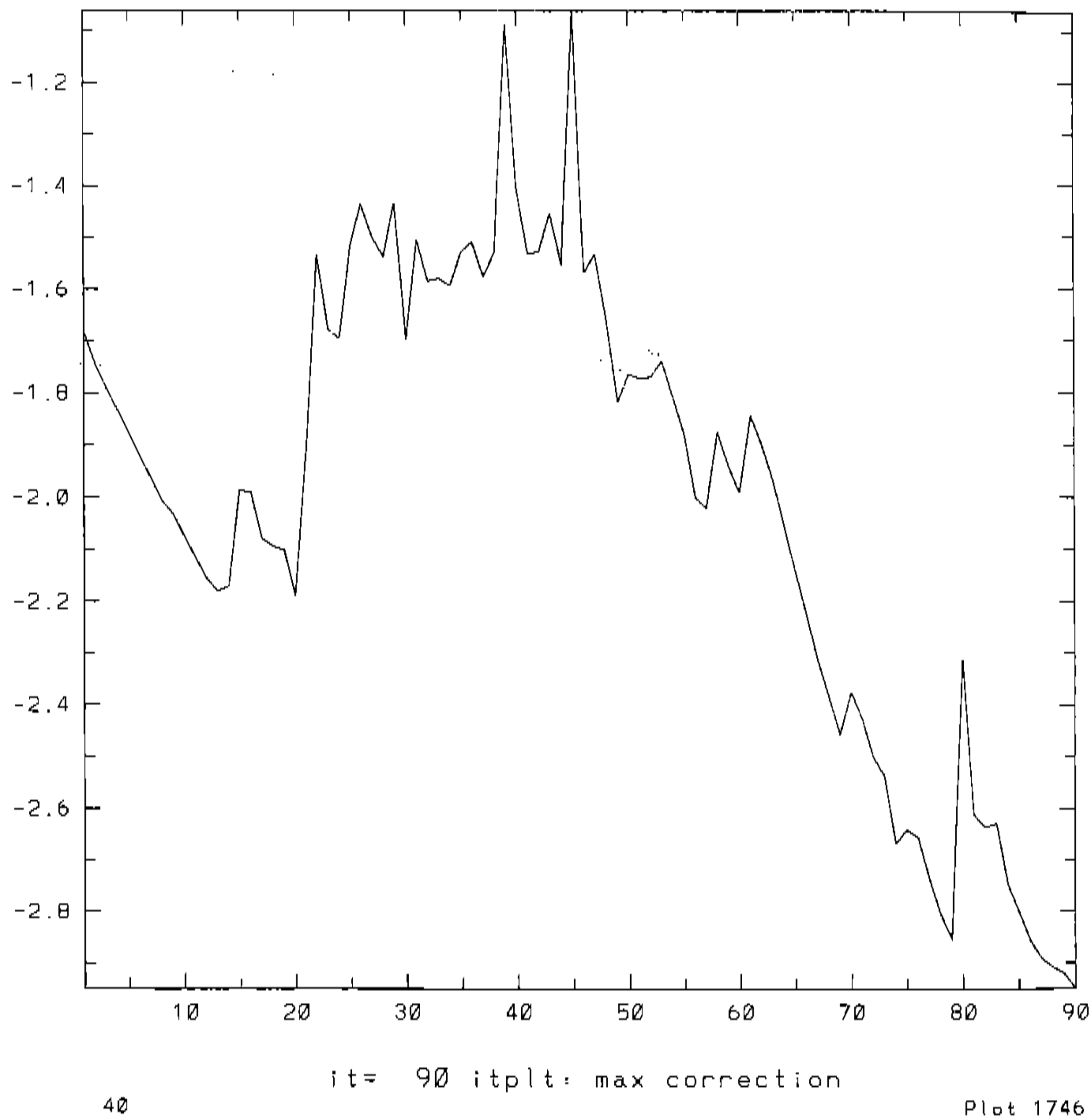


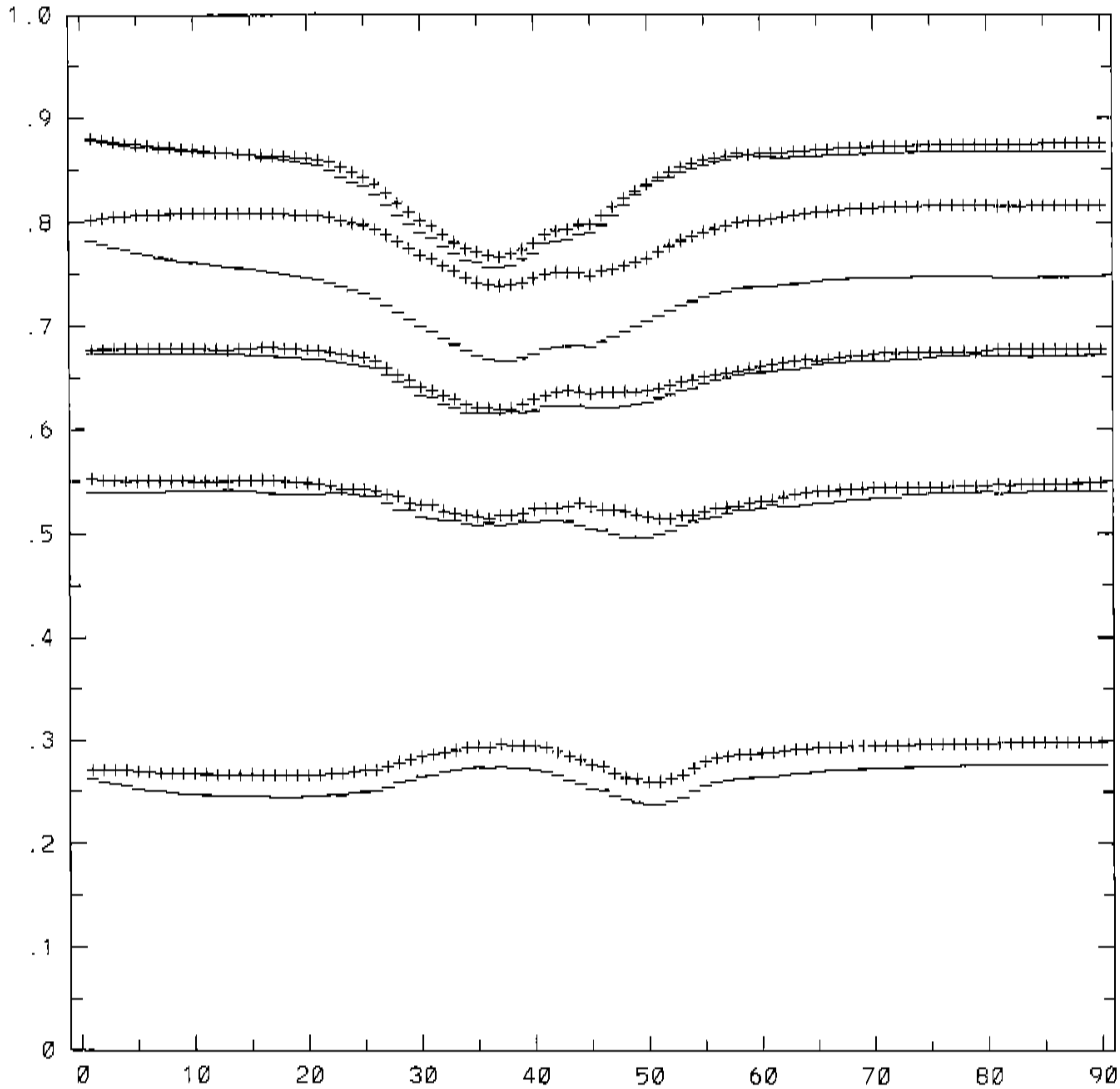
it= Ø blend: unfiltered iota

33

Plot 18

Blending also on m=9, n=2 component of current.



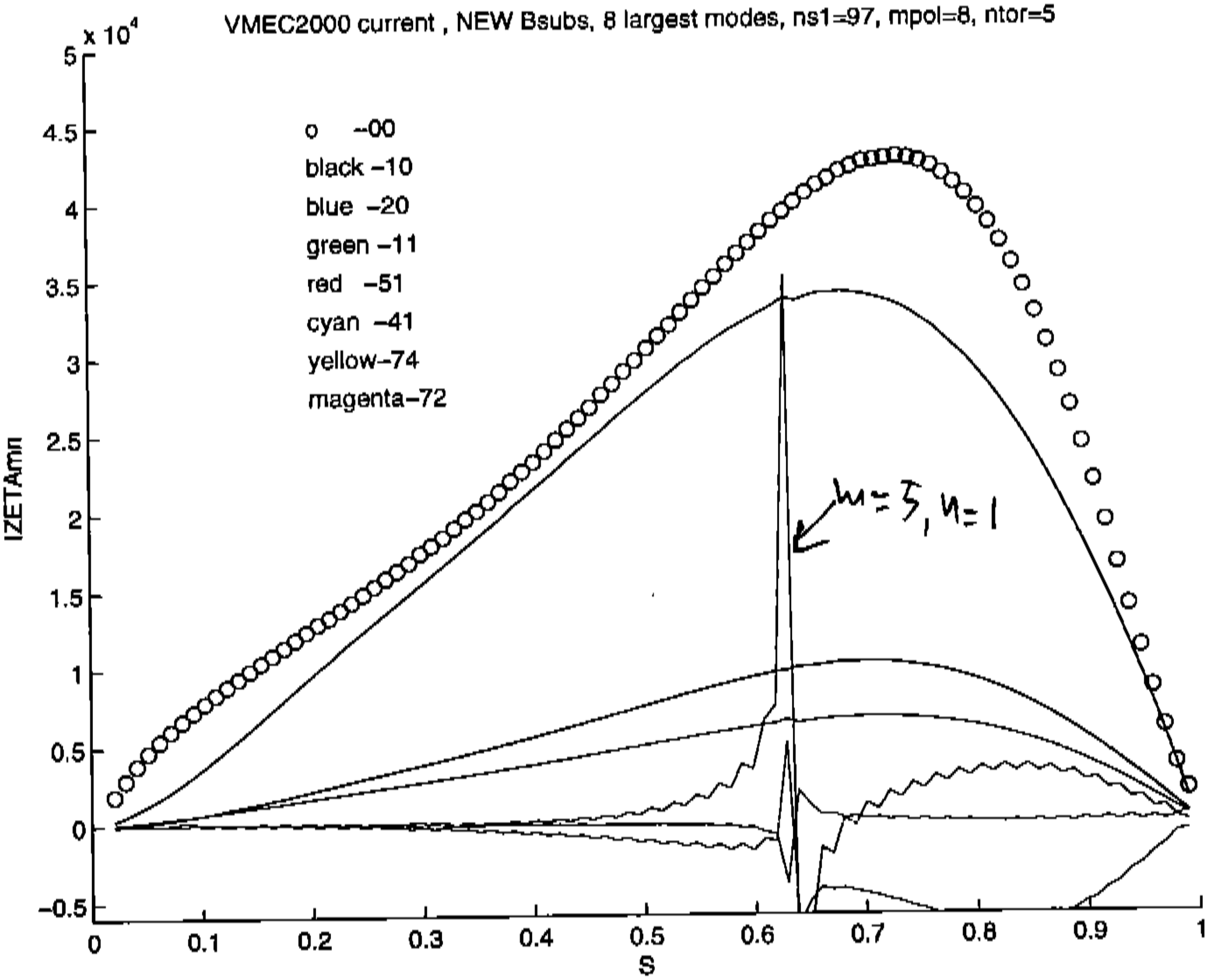


it= 90 itplt: edges of islands from Hudson (from iter1)
40
Plot 1755

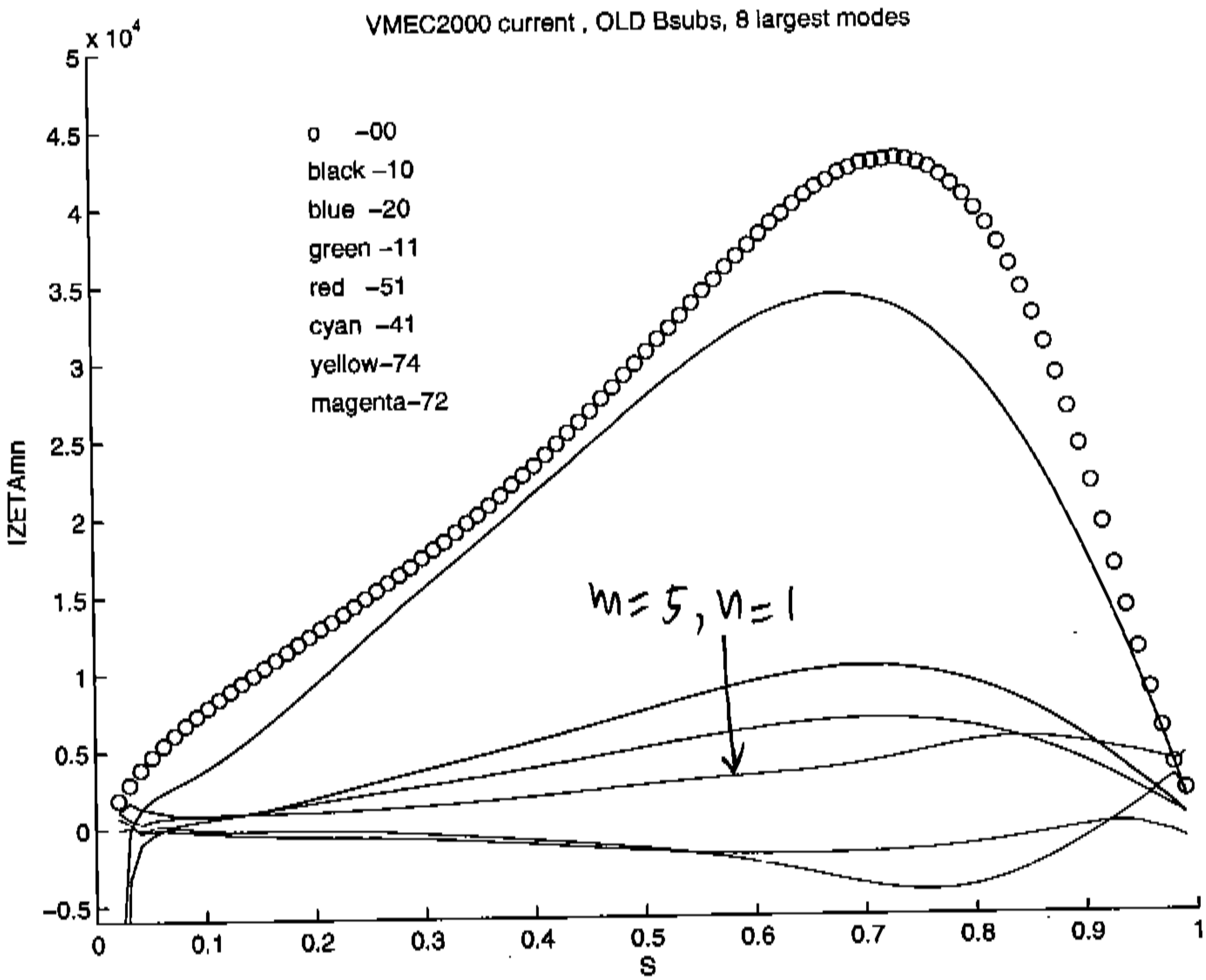
Blending on Currents

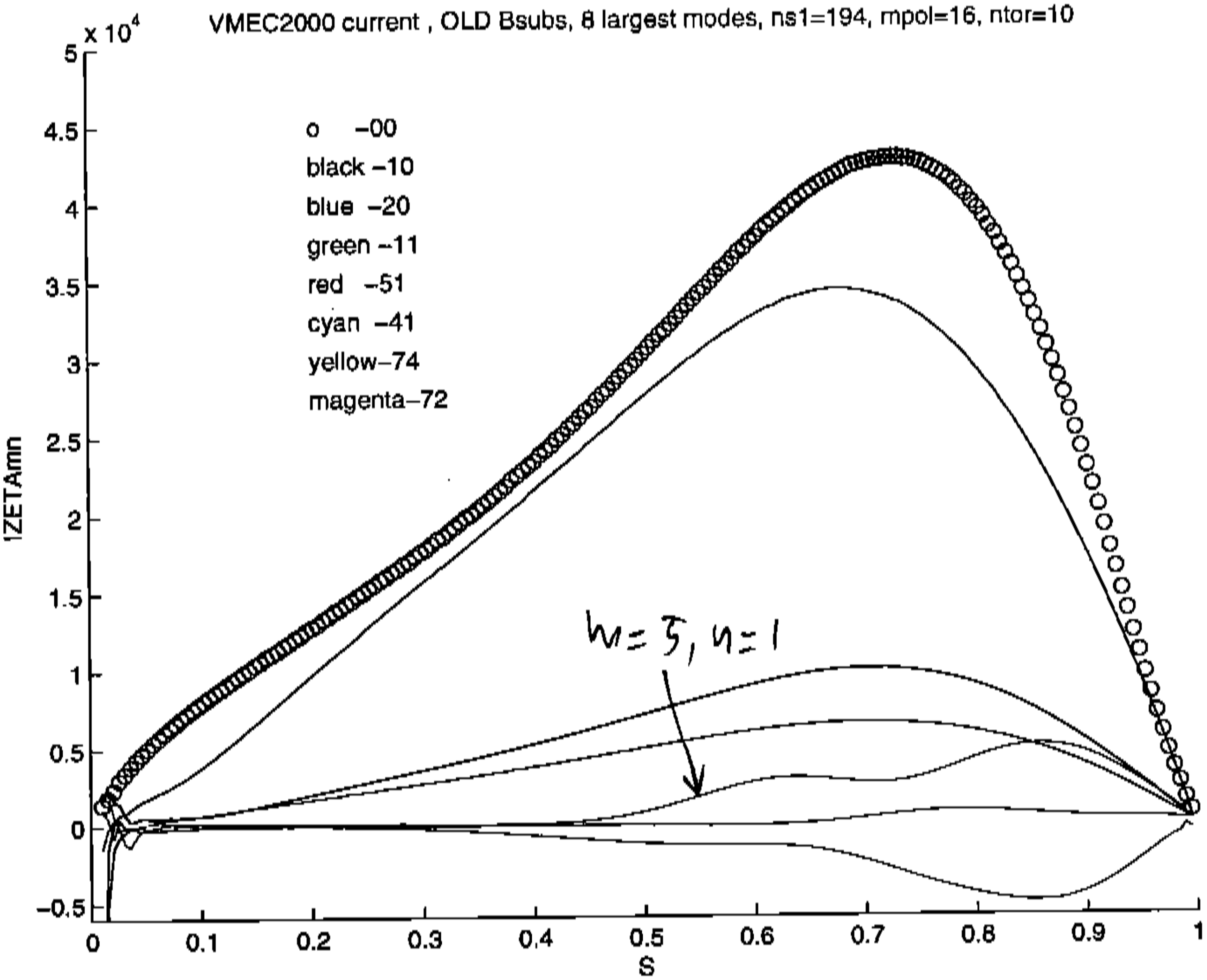
- Blending on a single Fourier component of current shows improved convergence properties. Suggests desirability of more general scheme for blending currents.
- An issue: are currents provided by VMEC sufficiently accurate to use for initializing blending.

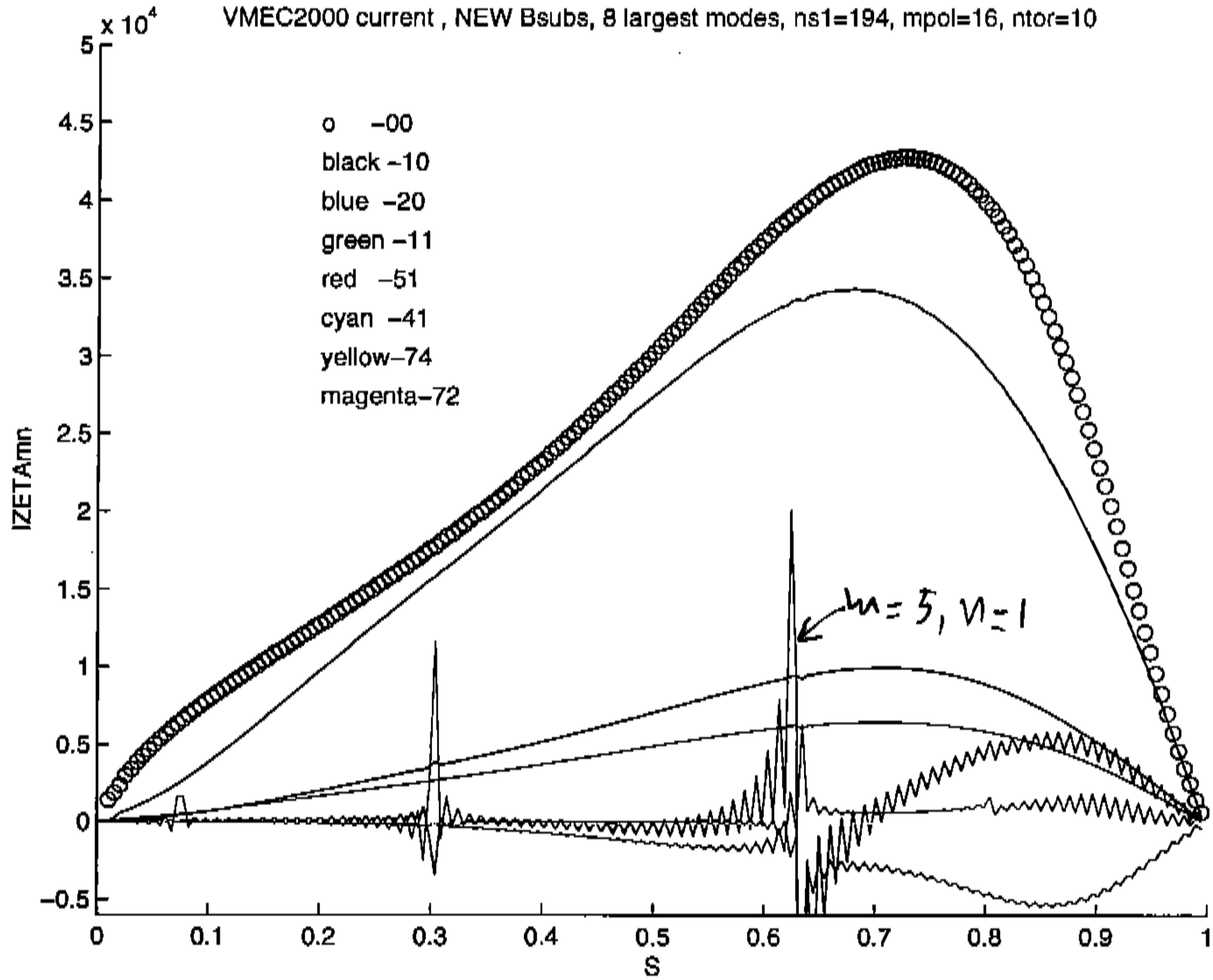
Current recalculated from
VMEC field using force balance.
(VMEC postprocessor)



VMEC Current




VMEC2000 current , OLD Bsubs, 8 largest modes, ns1=194, mpol=16, ntor=10
x 10^4
IZETAmn
S
o -00
black -10
blue -20
green -11
red -51
cyan -41
yellow-74
magenta-72
m=5, n=1


VMEC2000 current , NEW Bsubs, 8 largest modes, ns1=194, mpol=16, ntor=10
x 10^4
o -00
black -10
blue -20
green -11
red -51
cyan -41
yellow-74
magenta-72
m=5, n=1
IZETAmn
S

Conclusions

- A simple form of preconditioning shows promise for accelerating the convergence of the PIES code.
- Higher resolution VMEC equilibria may be helpful in providing more accurate currents, allowing blending on the currents rather than the fields.