MHD Stability Calculations of Current-carrying Quasi-axisymmetric Stellarators

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Abstract

We present recent stability results of external kink modes and vertical modes in Quasi-axisymmetric Stellarators. The vertical mode is found to be stabilized by externally generated poloidal flux. A simple stability criteria is derived in the limit of large aspect ratio and constant current density. For wall at infinite distance from the plasma, the amount of external flux needed for stabilization is given by $f = (\kappa^2 - \kappa)/(\kappa^2 + 1)$ where κ is the axisymmetric elongation and f is the fraction of the external rotational transform. A systematic parameter study shows that the external kinks in QAS can be stabilized at high beta ($\sim 5\%$) without a conducting wall by combination of edge magnetic shear and 3D shaping. It is found the external kinks are mainly driven by the parallel current. The pressure contributes significantly to the overall drive through the curvature term and the Pfirsch-Schluter current.

I. INTRODUCTION

The design of the proposed National Compact Stellarator Experiment(NCSX)[1,2] aims at combining the best features of both tokamaks and stellarators to achieve reactor-relevant plasma performance: high beta, good particle confinement, disruption-free steady state operation with little need for current drive, and compact size. The NCSX is based on the concept of quasi-axisymmetric stellarator[3,4] (QAS) which is a hybrid of advanced tokamak and stellarator. The quasi-axisymmetry means that the magnetic field strength is nearly axisymmetric in Boozer coordinate so that the particle confinement is as good as that in an axisymmetric tokamak. QAS are obtained from tokamaks by adding rotational transform generated by 3D shaping (or nonaxisymmetric coils) while maintaining quasi-axisymmetry.

In a high beta QAS, quasi-axisymmetry necessarily leads to substantial bootstrap currents. The bootstrap current generates an internal rotational transform which always adds to the externally generated transform. A large current helps to generate poloidal flux needed for good confinement, but it may also lead to external kink instability as in tokamaks. In this work, we investigate the stability of ideal MHD modes, such as external kink modes and vertical mode, in QAS.

Early work had considered current-driven external kink modes where the stabilizing role of magnetic shear was recognized[5–7]. More recently, Mikhajlov and Shafranov[8] have shown analytically that a sufficient magnetic shear generated by helical coils can stabilize the external kink modes. Most previous work had assumed large aspect ratio and zero beta. In this work, we use fully 3D calculations for high beta compact stellarators where the external kinks are driven by both current and pressure gradient.

We find that the external kink modes in high beta QAS can be stabilized by edge magnetic shear as well as by appropriate 3D shaping without a conducting wall. In contrast, advanced tokamaks with high bootstrap fraction tend to have lower beta limits without wall stabilization. Initial results of external kink stability in QAS have been reported elsewhere[9]. Here the stability results are made more accurate by using more Fourier harmonics for the

perturbation.

The vertical mode is studied for the first time in QAS. It is found the QAS are much more stable to the vertical mode than in tokamaks. Physically, this is because the externally generated poloidal flux enhances the field line bending energy relative to the current-driven term.

II. THE 3D STABILITY CODE TERPSICHORE

In this work, the three dimensional ideal MHD stability code Terpsichore[10] is used to calculate the stability of free boundary external kink modes. The code determines the eigenvalues of the ideal MHD equations by minimizing the plasma potential energy as defined in the Energy Principle. Full 3D numerical equilibria obtained by the VMEC[11] code are used as input to the Terpsichore code. It uses a finite hybrid element method for radial discretization and Fourier decomposition in poloidal and toroidal angles. The radial and surface component of the plasma displacement vector ξ is represented by

$$\xi(s,\theta,\phi) = \sum_l \xi_l(s)\sin(m_l\theta - n_l\phi + \delta) \tag{1}$$

$$\eta(s,\theta,\phi) = \sum_l \eta_l(s)\cos(m_l\theta - n_l\phi + \delta) \tag{2}$$

where $\xi = \xi \cdot \nabla s$ and $\eta = \xi \cdot \mathbf{B} \times \nabla s/|\nabla s|^2$ with s being the flux variable.

Assuming stellarator symmetry and field period N_p, modes with mode number n are coupled to $n + kN_p$, where k is an arbitary integer. There are $N_p/2 + 1$ families for even N_p and $(N_p - 1)/2 + 1$ families for odd N_p. For example, there are $n = 0$ and $n = 1$ families for $N_p = 3$ and there are $n = 0$, $n = 1$ and $n = 2$ families for $N_p = 4$. Usually, the $n \neq 0$ families are called kink modes and $n = 0$ family is called vertical mode. However, when the dominating component is of $n \neq 0$, the $n = 0$ family of modes are physically kink modes and should be called as such.

It can be shown that the stability of $n \neq 0$ families does not depend on the phase δ except for the $n = N_p/2$ family when N_p is even. The phase must be zero for the $n = 0$

family because this breaks the stellarator symmetry.

III. STABILITY OF EXTERNAL KINK MODE

We consider stability of low-n external kink modes driven by current and pressure using the Terpsichore code. A systematic convergence study has been done in terms of number of equilibrium modes, radial grid points, and number of Fourier modes for perturbations. Figure 1 show convergence of n=1 external kink mode in terms of (a) number of radial grid points, (b) number of equilibrium Fourier modes and (c) number of stability Fourier modes. The results are obtained for a $N_p = 3$ QAS with $R/a = 3.5$ and $\beta \sim 4\%$. These results indicate a resolution of about 50 grid points, 100 equilibrium modes and 100 stability modes is required for typical QAS under considerations.

We now consider the effects of magnetic shear on external kink stability. Figure 2 shows the calculated n=1 external kink mode eigenvalue (i.e., square of growth rate) as a function of edge shear defined by $\iota(1) - \iota(0.75)$ at fixed value of edge $\iota(1) = 0.46$. It is shown that the external kink modes is stabilized by edge magnetic shear. The results are obtained for a $N_p = 4$ QAS with $R/a = 2.1$ and $\beta \sim 6.3\%$. The variation of shear is controlled entirely by the helical coils or the 3D plasma boundary shape. The current and pressure profiles are fixed.

However, a complete stabilization by shear alone tends to reduce central rotational transform to low values (~ 0.1) and result in poor particle confinement. This problem can be solved by appropriate 3D shaping. It is found that external kinks can be stabilized by appropriate 3D shaping at moderate edge shear. In order to find the optimal shaping for kink stability, we have incorporated the terpsichore code into a configuration optimizer which includes kink stability as well as quasisymmetry in its objective function. The optimizer is used to determine the necessary 3D shaping for kink stabilization. Figure 3 shows plasma cross-sections before (left) and after (right) optimization. The initial configuration (called c3) is unstable to n=1 kink with eigenvalue of $\lambda = 0.002$. The final configuration (called

c82) is marginal stable to external kinks at $\beta \sim 4\%$. We note that the major change in shaping from c3 to c82 is an indentation of plasma boundary on the outboard side at the half-period cross section which is found to be most effective for stabilization. It should also be noted that the change in shaping results in minor change in the iota profile. The optimized configuration c82 has $N_p = 3$ at $R/a = 3.5$ It has good quasi-axisymmetry. It will be shown to be stable to vertical mode as well as external kinks. Currently, it is the reference configuration for NCSX.

IV. STABILITY OF VERTICAL MODES

It is known that tokamaks with high elongation suffer from vertical instability which results in disruptions. Thus, feedback stabilization is needed for stable operation.

We find that the vertical mode (i.e., n=0 family) is much more stable in QAS than in tokamaks. The configuration c82 is calculated to be robustly stable to the vertical mode. This result has been confirmed by the CAS3D calculations[12]. In order to understand the physics, we have evaluated stability for a series of equilibria varying the degree of nonaxisymmetric shape of c82. Figure 4 shows the eigenvalue of the vertical mode as function of fraction of nonaxisymmetric shape f at fixed current profile and zero beta, where $f = 1$ corresponds to the full c82 shape and $f = 0$ corresponds a tokamak with just the axisymmetric shape of c82. The equilibria in between are obtained by linear interpolation. We observe that there is a big stability margin for vertical mode in c82 and only 60% of the full 3D shape is needed for stabilization. The results of Fig. 4 are obtained at zero beta because of equilibrium convergence problem at small f due to low transform. at finite f, the effects of beta are found to be stabilizing. Thus, even larger margin is expected at finite beta.

We have derived an analytic stability criterion for vertical mode in a large aspect ratio QAS with constant current density and constant external rotational transform. The external rotational transform needed for stability is given by:

$$f = \frac{\kappa^2 - \kappa}{\kappa^2 + 1} \tag{3}$$

where $f = \iota_{ext}/\iota_{total}$ is the fraction of external rotational transform and κ is the axisymmetric elongation. This criterion has been confirmed by the Terpsichore code. Physically, the external transform is stabilizing because the external poloidal flux enhances the field line bending energy relative to the current-driven term for the vertical instability.

V. CONCLUSIONS

The MHD stability of current-carrying quasi-axisymmetric stellarators are investigated using fully 3D calculations. The vertical mode in quasi-axisymmetric stellarators is studied for the first time. It is found that the vertical mode is much more stable in QAS than in tokamaks due to stabilizing effects of externally generated poloidal flux. The external kink modes in QAS can be stabilized by combination of edge magnetic shear and 3D shaping at high beta without conducting wall. The results found here demonstrate that there exists a new class of stellarators with quasi-axisymmetry, large bootstrap current, high MHD beta limit without conducting wall, and compact size.

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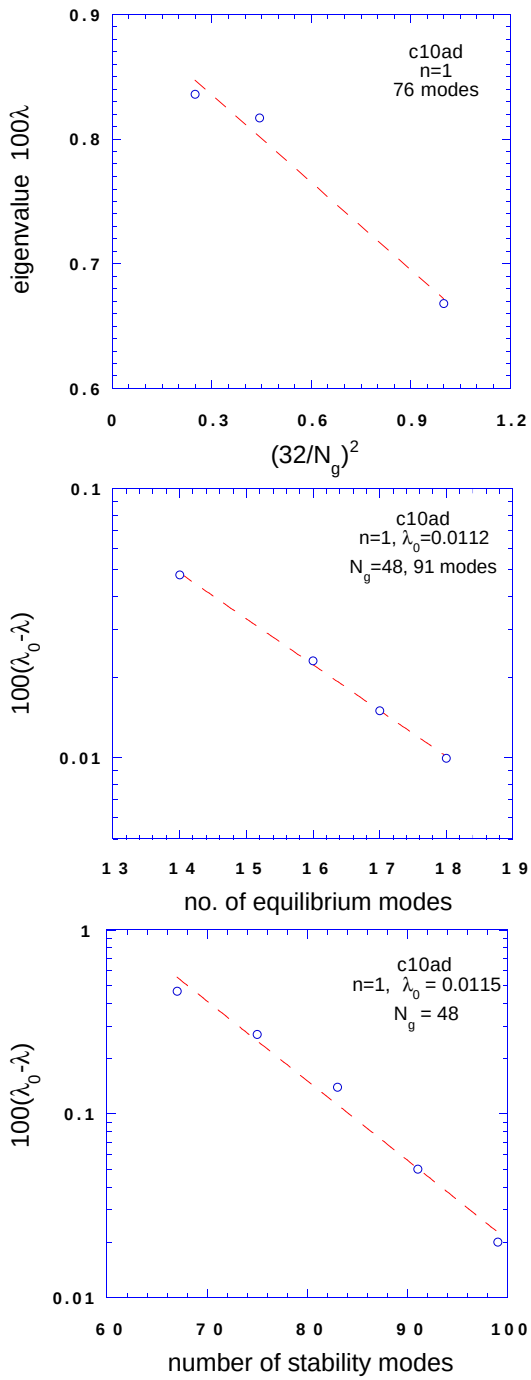

FIG. 1. *The $n = 1$ external kink eigenvalue versus (a) number of radial grid points N_g, (b) number of equilibrium Fourier modes and (c) number of stability Fourier modes.*

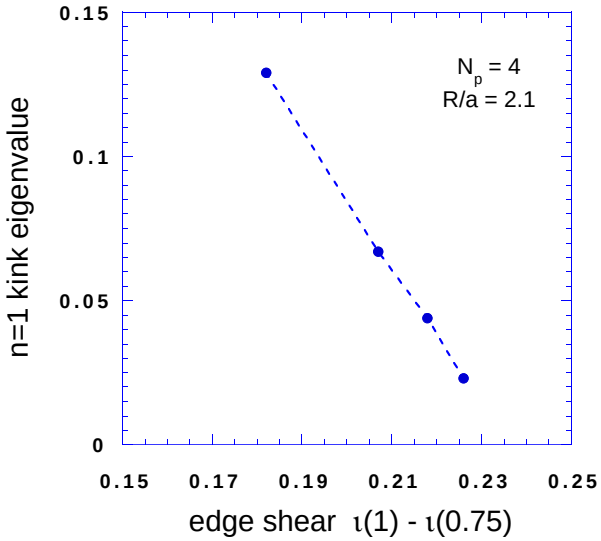

FIG. 2. *The $n = 1$ external kink eigenvalue versus edge magnetic shear for a four field period QAS with $R/a = 2.1$ and $\beta \sim 6.3\%$.*

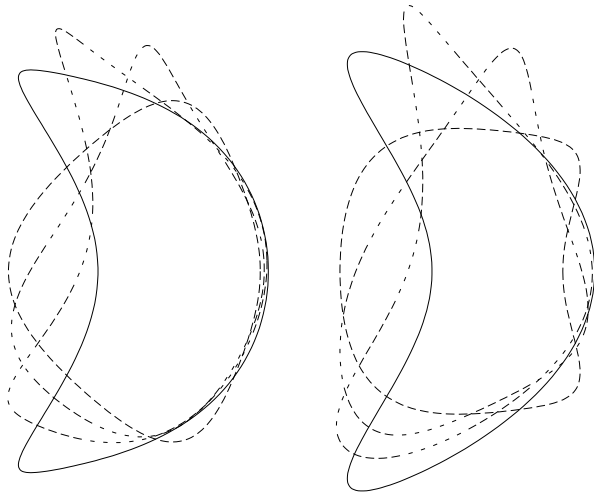

FIG. 3. *Plasma cross-sections of a three field period QAS before optimization (left) and after optimization (right).*

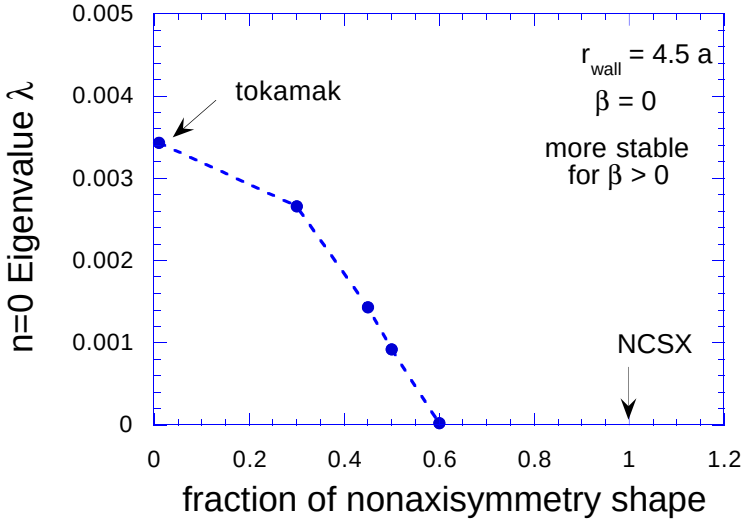

FIG. 4. *The eigenvalue of the vertical mode versus fraction of c82's nonaxisymmetric shape*