MHD Stability of Compact Stellarators *

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Outline

1. Introduction
2. The 3D Stability Code Terpsichore
3. Stability of External Kink Modes
4. Stability of Vertical Modes
5. Stability of Ballooning Modes
6. Conclusions

I. Introduction

- We investigate the stability of MHD modes in compact stellarators.
- This work is motivated by recent development of compact stellarator concepts, such as

  Quasi-Axisymmetric Stellarators (QAS) and

  Quasi-Omnigeneous Stellarators (QOS).
- The MHD stability is a key issue in these two concepts.
- The proposed National Compact Stellarator Experiment (NCSX) is designed to be stable to external kink modes, vertical modes, and ballooning modes in absence of a conducting wall.

Highlights

- External Kink Stability:
  The external kink modes in QAS can be stabilized by 3D shaping at high beta ($\beta \sim 5\%$ at $R/a = 3.4$) without a conducting wall.

- Vertical Stability:
  The vertical mode in current-carrying stellarators is shown to be more stable than in tokamaks due to stabilizing effects of vacuum rotational transform.
  The NCSX candidate configuration is found to be robustly stable to vertical mode without a conducting wall.

II. The 3D Stability Code Terpsichore

- The TERPSICHORE[1] is a 3D ideal MHD linear stability code based on the Energy Principle.
- It uses a hybrid finite element method for radial discretization and Fourier decomposition in poloidal and toroidal angles (in Boozer coordinates).
- The code takes as input fully 3D numerical equilibria generated by the VMEC code[2].
- Equilibria are assumed to have closed flux surfaces and stellarator symmetry:

$$R = \sum_{m,n} R_{m,n}(s) \cos(m\theta - nN_p\phi)$$

$$Z = \sum_{m,n} Z_{m,n}(s) \sin(m\theta - nN_p\phi)$$

[1] D. V. Anderson, W. A. Cooper, R. Gruber, S. Merazzi, and U. Schwenn, Scient. Comp. Supercomputer II, 159 (1990).

[2] S. P. Hirshman and J. C. Whitson, Phys. Fluids **26** 3553 (1983).

The Energy Principle

$$\omega^2 \delta W_k = \delta W_p + \delta W_{vac}$$

$$\delta W_k = \frac{1}{2} \int_p d^3x [\rho \xi^2]$$

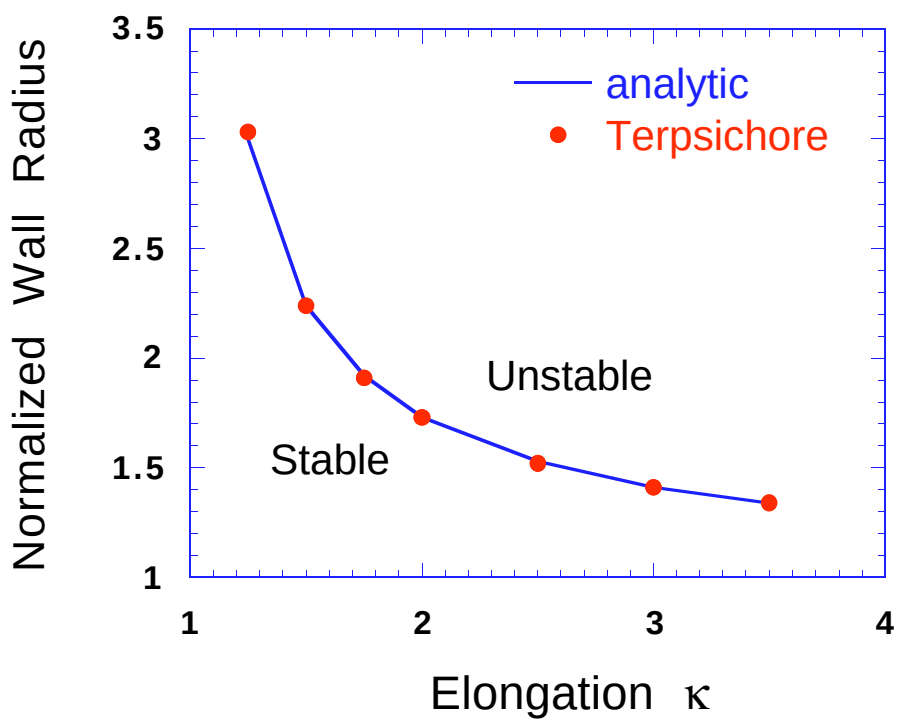
$$\delta W_{vac} = \frac{1}{2} \int_{vac} d^3x [\delta \mathbf{B}^2]$$

$$\delta W_p = \frac{1}{2} \int_p d^3x [\delta \mathbf{B}_\perp{}^2 + (\delta \mathbf{B}_\parallel - \mathbf{B} \frac{\xi \cdot \nabla p}{B^2})^2$$
$$+ \mathbf{j}_\parallel \cdot \xi \times \delta \mathbf{B} - 2\xi \cdot \nabla p \; \xi \cdot \kappa]$$

$\lambda = -\omega^2 > 0$ for instability.

Benchmark of the Terpsichore Code

- The Terpsichore results agree with PEST[3] and CAS3D[4] for the beta limit of an $n = 1$ external kink in an optimized ARIES-RS tokamak.
- The Terpsichore results agree with an analytic stability criterion for the vertical mode.



[3] R. C. Grimm, J. M. Greene, J. L. Johnson, Methods Comput. Phys. **16**, 273 (1975)

[4] C. Nuehrenberg, Phys. Plas. **3**, 2401 (1996).

A Few Subtleties in 3D Calculations

- Mode representation and phase dependence;
- Vacuum grid generation;
- Singularity in parallel current;
- Convergence study.

Mode Family and Phase Dependence

- Mode representation

$$\xi^s(s,\theta,\phi) = \sum_l \xi_l(s)\sin(m_l\theta - n_l\phi + \Delta)$$

$$\eta(s,\theta,\phi) = \sum_l \eta_l(s)\cos(m_l\theta - n_l\phi + \Delta)$$

where $\xi^s = \xi \cdot \nabla s$ and $\eta = \xi \cdot \mathbf{B} \times \nabla s / |\nabla s|^2$.

- Mode families

The mode number n is coupled to $n + kN_p$. There are $N_p/2+1$ families for even N_p and $(N_p-1)/2+1$ families for odd N_p. For example, there are $n = 0$ and $n = 1$ families for $N_p = 3$.

- Phase dependence

The stability of $n \neq 0$ families does not depend on the phase Δ except for the $n = N_p/2$ family with even N_p.

Resonant Detuning of Singularity in Parallel Current

- We find that parallel current is usually singular at the rational surfaces;

$$(\frac{j_{\|}}{B})_{m,n} = \frac{p'}{\sqrt{g}B^2}\frac{Im - Jn}{m\iota - n}(\sqrt{g})_{m,n}$$

- The singular parallel current drives unphysical Mercier modes and kink modes which make it impossible to determine real unstable modes;

- A resonant detuning is used to smooth the singular parallel current. One of possible mechanism for detuning is magnetic island.

$$\frac{1}{m\iota - n} \rightarrow \frac{m\iota - n}{(m\iota - n)^2 + (m\iota\delta)^2}$$

Quadratic Convergence in Radial Mesh and Exponential Convergence in Number of Equilibrium and Stability Harmonics

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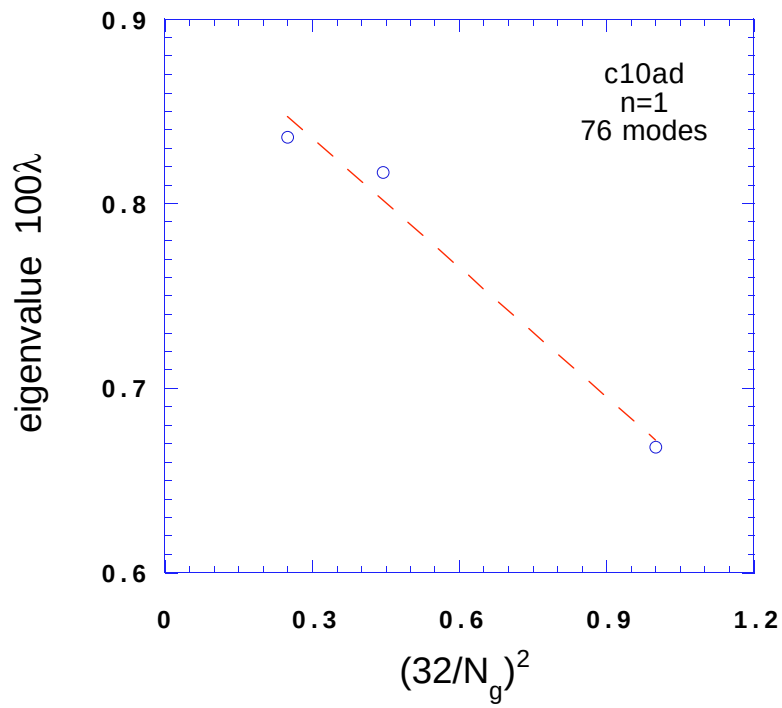
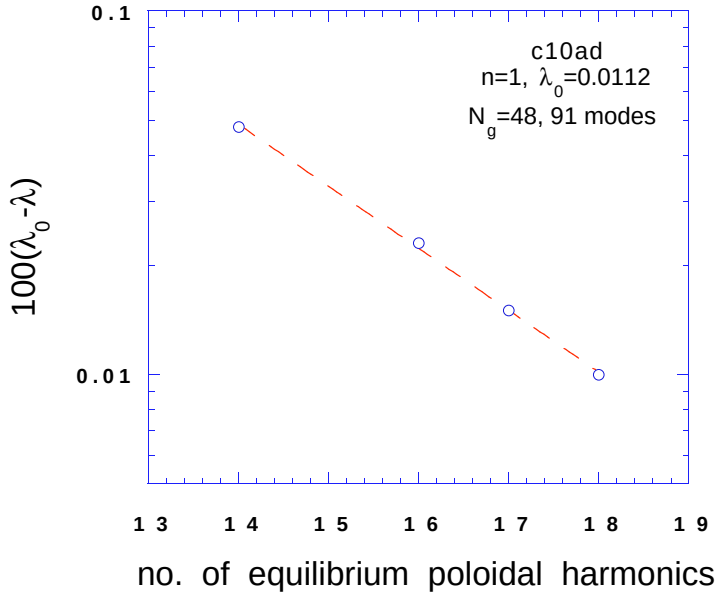
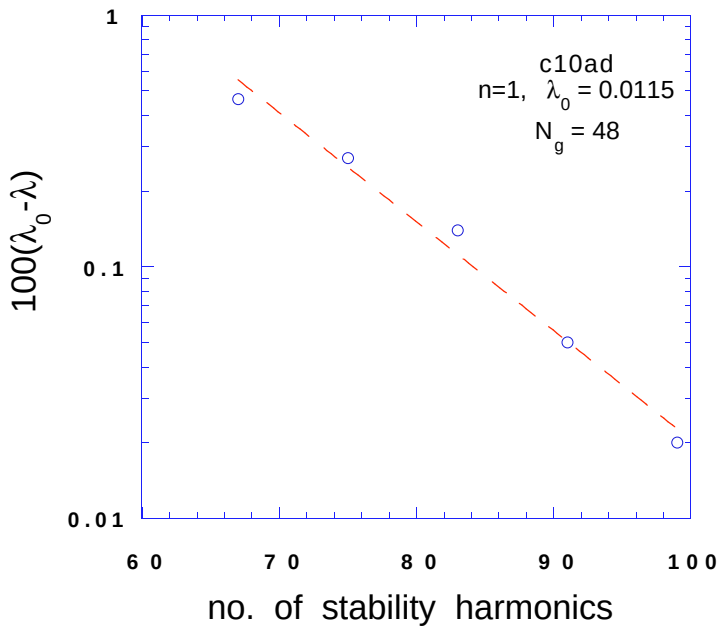



c10ad
n=1, λ0=0.0112
Ng=48, 91 modes
0.1
0.01
100(λ0-λ)
13
14
15
16
17
18
19
no. of equilibrium poloidal harmonics


c10ad
n=1, λ0 = 0.0115
Ng = 48
1
0.1
0.01
100(λ0-λ)
60
70
80
90
100
no. of stability harmonics

III. Stability of External Kink Modes

- Bootstrap current in QAS is comparable to tokamak's and can drive external kinks.
- Advanced tokamaks with high bootstrap fraction rely on wall stabilization for kink stability at high beta.
- In QAS, 3D non-axisymmetric shaping can be used as an extra knob to control the kink stability.
- In QAS, the external kinks can be stabilized at high beta by edge magnetic shear and 3D shaping.

Stabilization of Kinks by Edge Shear

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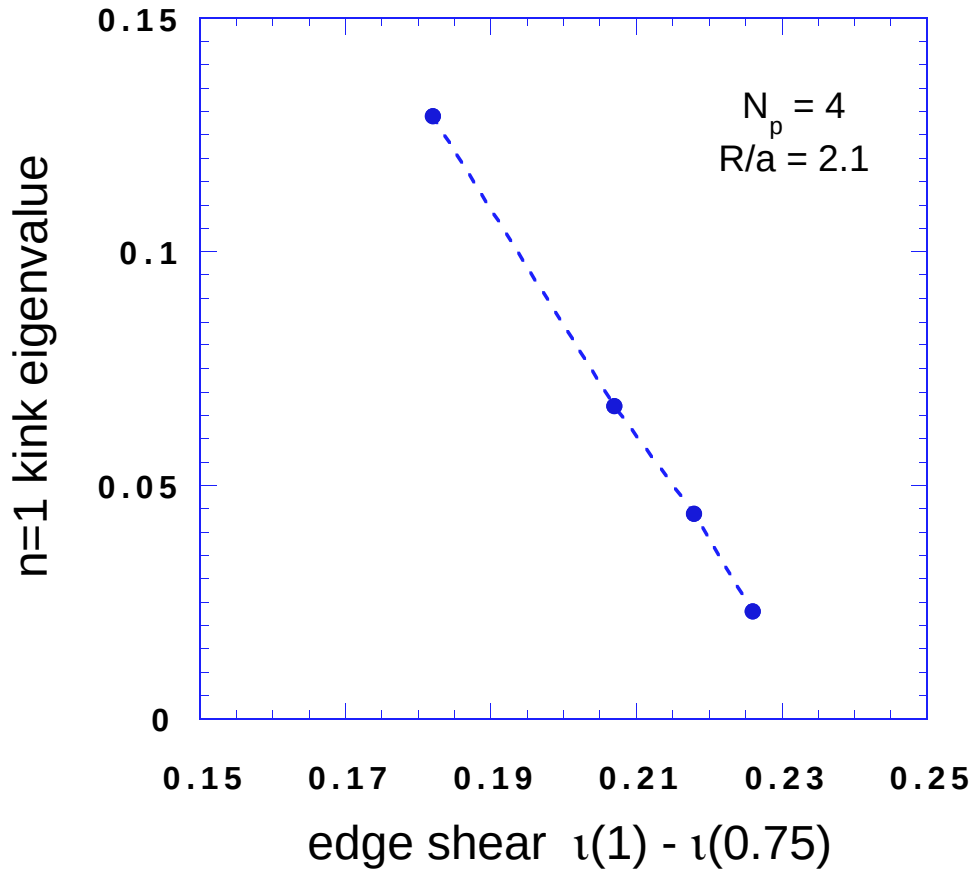

However, a complete stabilization by shear alone tends to reduce the rotation transform to a low value.

Stabilization of Kinks by 3D shaping

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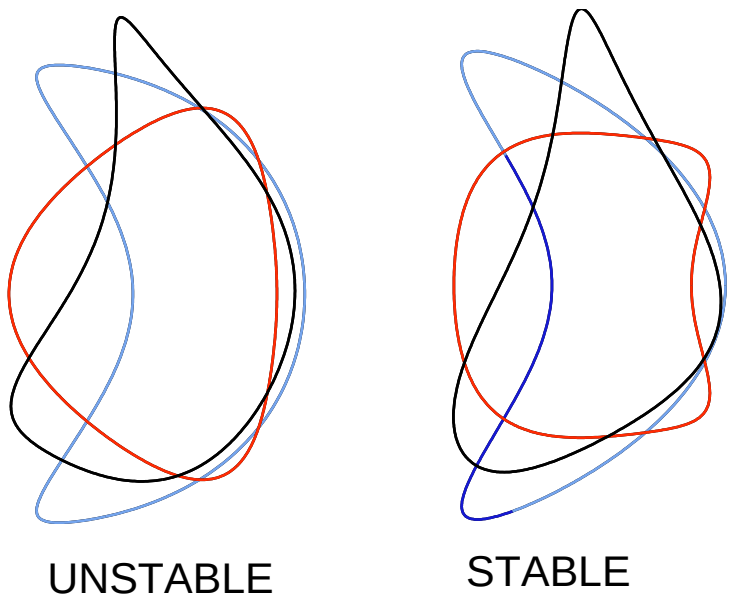
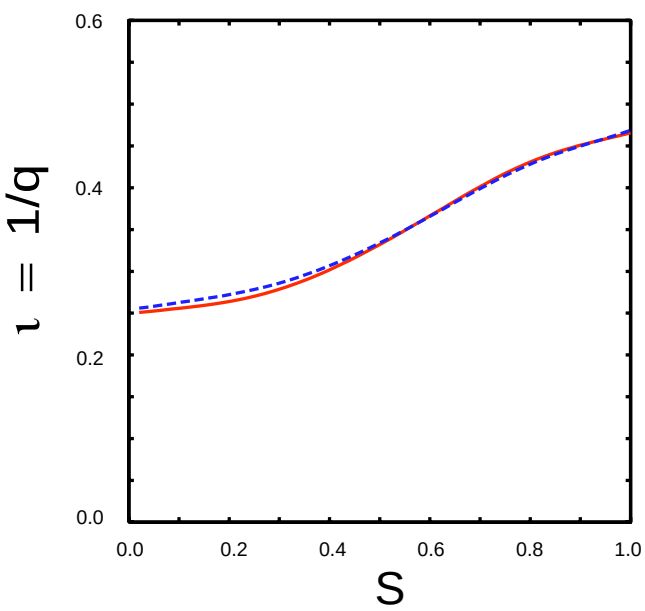
Cross-sections of a QAS before and after 3D shaping.





The Mode Structure of External Kinks Exhibit Strong Ballooning Feature

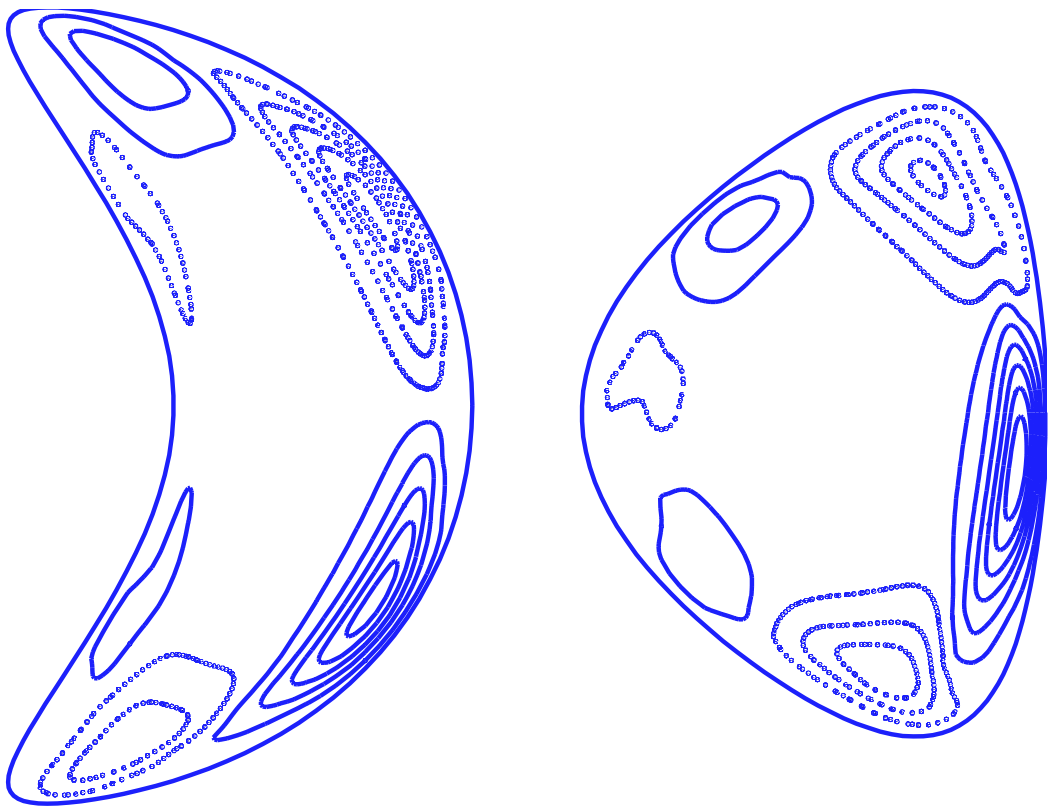


Figure 1: *contour of perturbed pressure at the two symmetric cross-sections*

δW Analysis of 3D Shaping Effects

$$\delta W_p = \frac{1}{2}\int_p d^3x[\delta \mathbf{B}_\perp{}^2 + (\delta \mathbf{B}_\parallel - \mathbf{B}\frac{\xi\cdot\nabla p}{B^2})^2$$
$$+\mathbf{j}_\parallel \cdot \xi \times \delta \mathbf{B} - 2\xi\cdot\nabla p\ \xi\cdot\kappa]$$

	vacuum	bending	kink	ballooning
c3m	1.00	4.05	-3.98	-1.72
c82	1.00	4.51	-3.87	-1.64

- The ballooning term contributes significantly to the kink drive.
- The effects of the 3D shaping mainly enhance the field line bending energy. Our results show that the local magnetic shear on the outboard side of the plasma is enhanced by the 3D shaping. This suggests that the enhanced local magnetic shear is the main mechanism for stabilization via 3D shaping.

External Kink Stability in QOS

- The bootstrap current in QOS is typically smaller than in tokamaks and can be either sign.
- The bootstrap current may be used to control the kink stability.
- Initial results for a $R/a = 3.6$, $\beta = 3.7\%$ QOS with self-consistent bootstrap current:

  The external kink modes are stable. This is consistent with the fact that the bootstrap current is small and negative.

  When beta is increased to 5%, the plasma is unstable to an interchange-like mode with weak ballooning feature. The effects of bootstrap current and free boundary are small.

IV. Stability of Vertical Modes

- It is known that tokamaks with high elongation suffer from vertical instability which results in disruptions.
- We show analytically that the vertical modes are much more stable in QAS than in tokamaks due to stabilizing effects of vacuum rotational transform.
- The configuration c82 is found to be robustly stable to vertical mode. The stability has been confirmed by CAS3D calculations with wall at infinite.

The vertical mode is stabilized by nonaxisymmetric shape

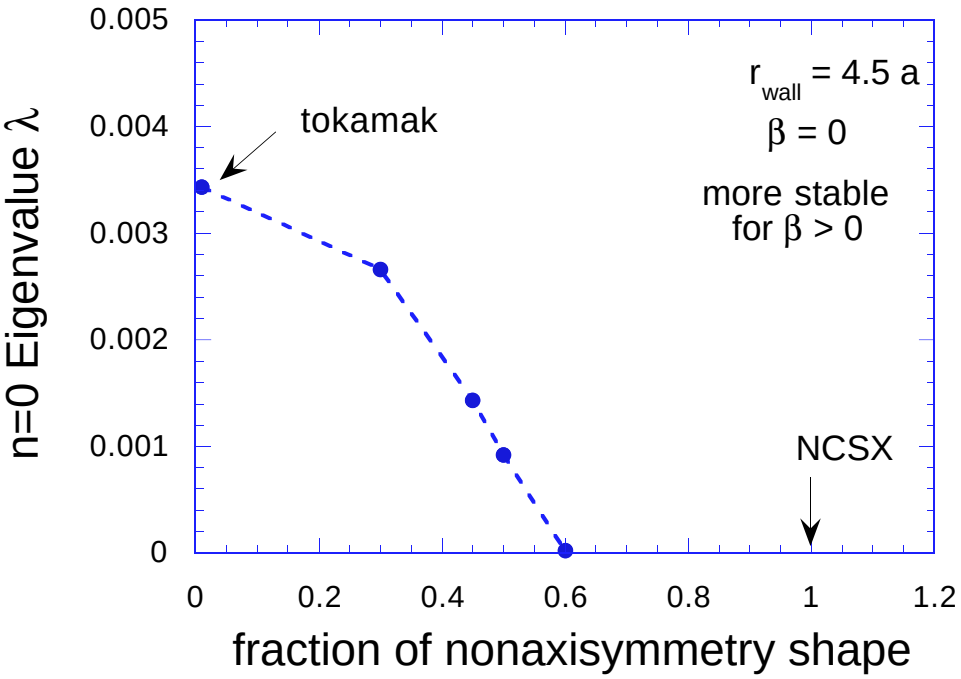


Figure 2: *The eigenvalue of the vertical mode versus fraction of c82's nonaxisymmetric shape*

Analytic Stability Criterion for Vertical Mode

- We have derived an analytic stability criterion for vertical mode in a large aspect ratio QAS with uniform current density and vacuum rotational transform profiles.

- The vacuum rotational transform needed for stability is given by:

$$f = \frac{\kappa^2 - \kappa}{\kappa^2 + 1} \tag{1}$$

where $f = \iota_{vac}/\iota_{total}$ is the fraction of vacuum rotational transform and κ is the axisymmetric elongation. This criterion has been confirmed by the Terpsichore code.

The Terpsichore calculations confirm the analytic stability criterion of the vertical mode in a stellarator

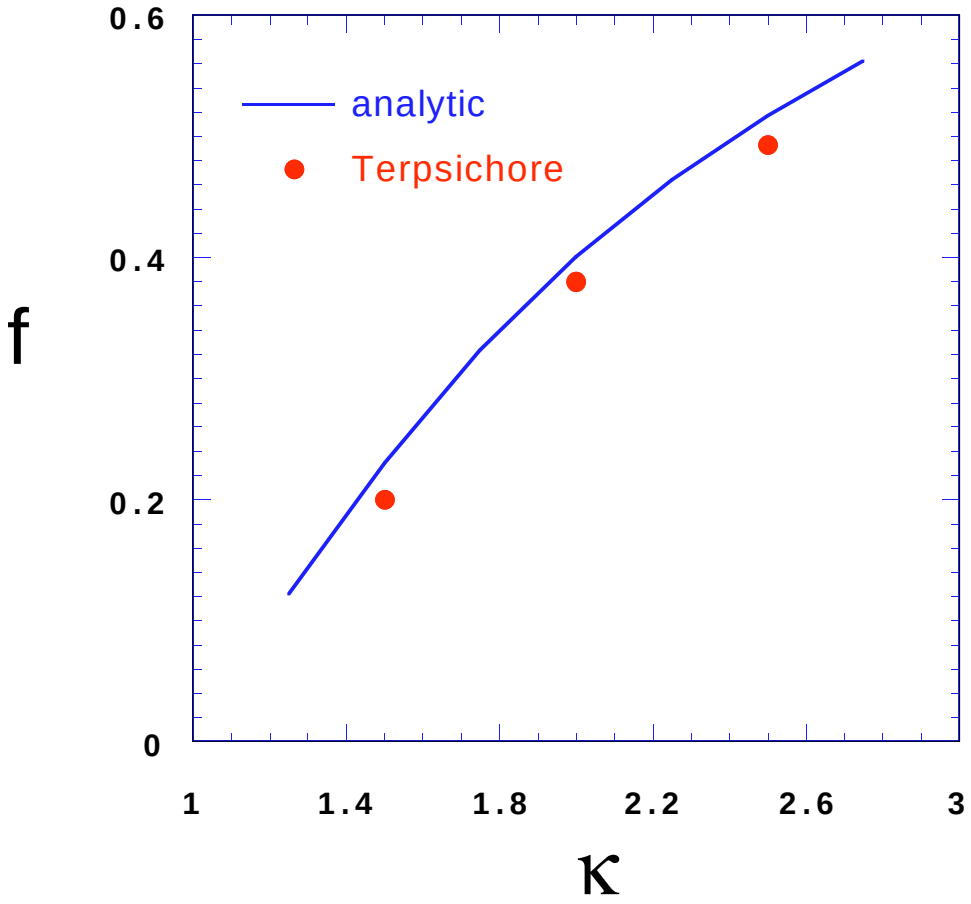


Figure 3: *The critical value of fraction of external transform as function of axisymmetric elongation*

V. Stability of High-n Ballooning Modes

- QAS c82:

  The ballooning mode is stabilized by strong axisymmetric shaping.

  The beta limit is similar to the AT tokamak's at $\beta_{crit} \sim 4\%$.

- QOS:

  The ballooning mode is stabilized by 3D shaping.

  The 3D shaping increases the beta limit from 2% to 4%.

The ballooning modes are stable for the optimized QAS configuration c82.

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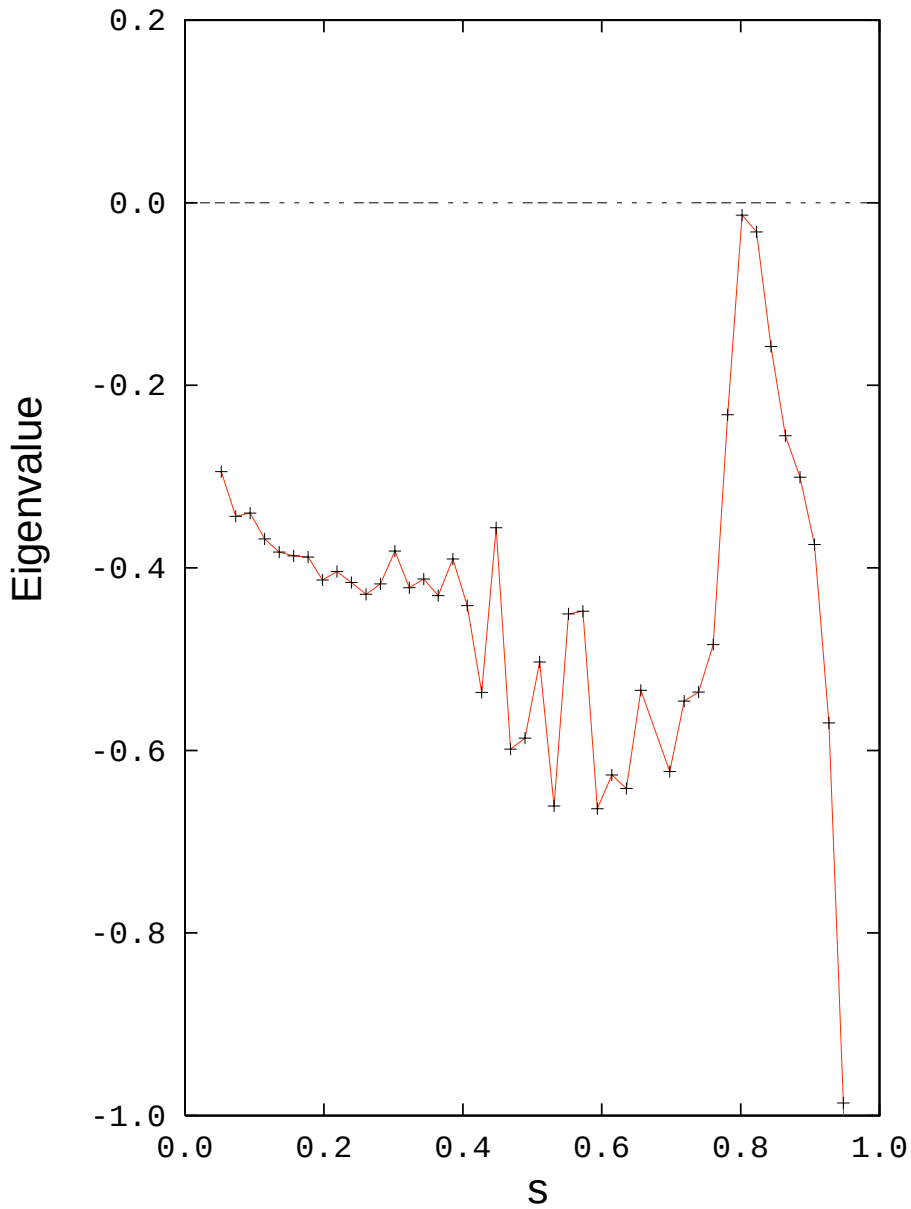

VI. Conclusions

- The MHD stability of quasi-axisymmetric stellarators are investigated using fully 3D calculations.

- The vertical mode in quasi-axisymmetric stellarators is more stable than in tokamaks due to stabilizing effects of vacuum rotational transform.

- The external kink modes in QAS can be stabilized at high beta without conducting wall by 3D shaping via global and local magnetic shear.

- The results found here demonstrate that there exists a new class of stellarators with quasi-axisymmetry, large bootstrap current, high MHD beta limit without conducting wall, and compact size.

Contours of Displacement $\xi \cdot \nabla S$ for $n = 0$ Vertical Mode

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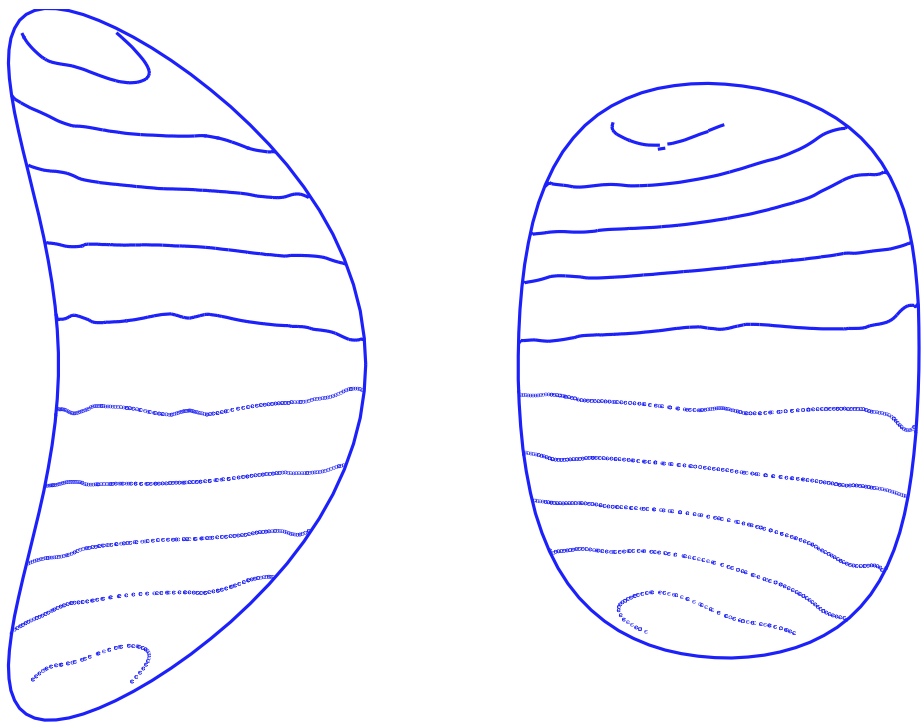
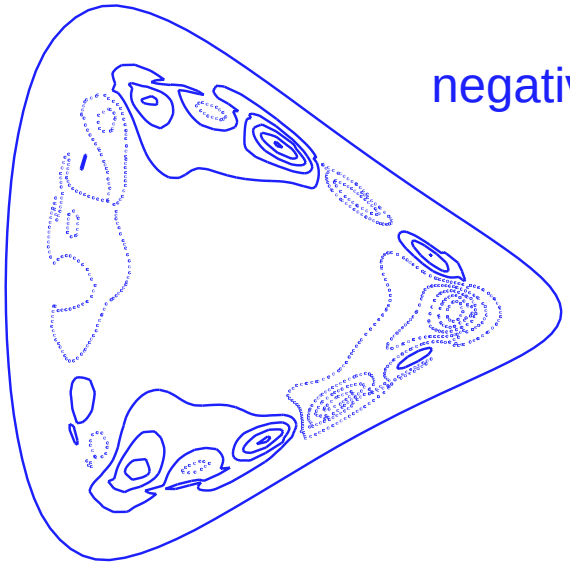
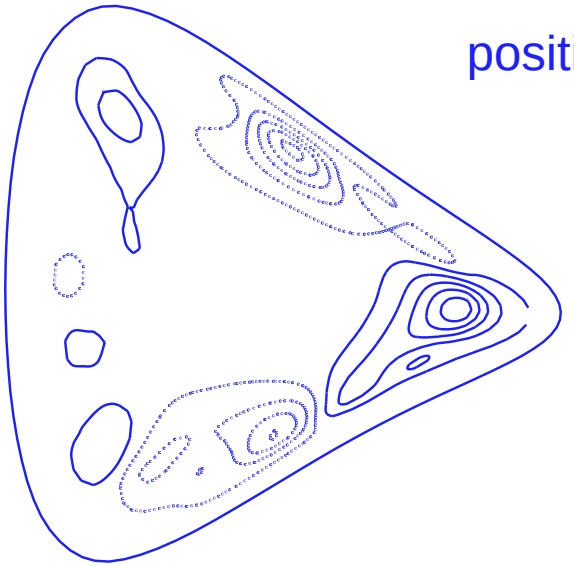


QOS with Negative Current Is More Stable



negative current

Interchange
like mode
λ = 4.9e-4



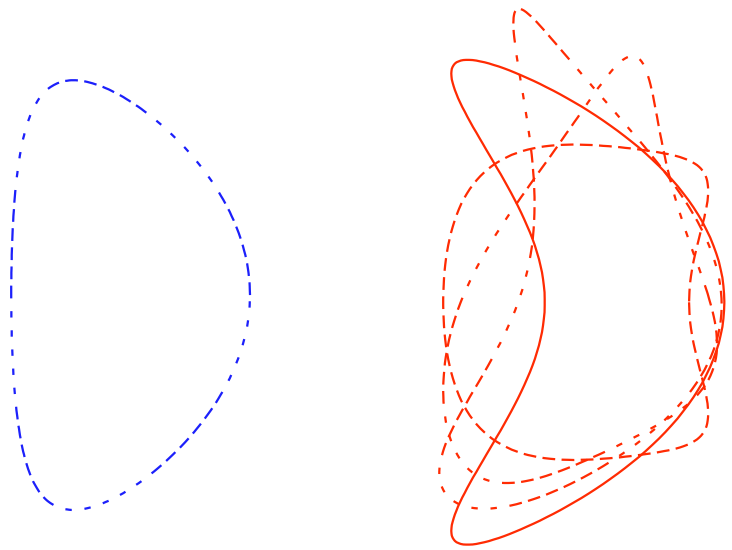
positive current

kink-ballooning
external mode
λ = 1.2e-3

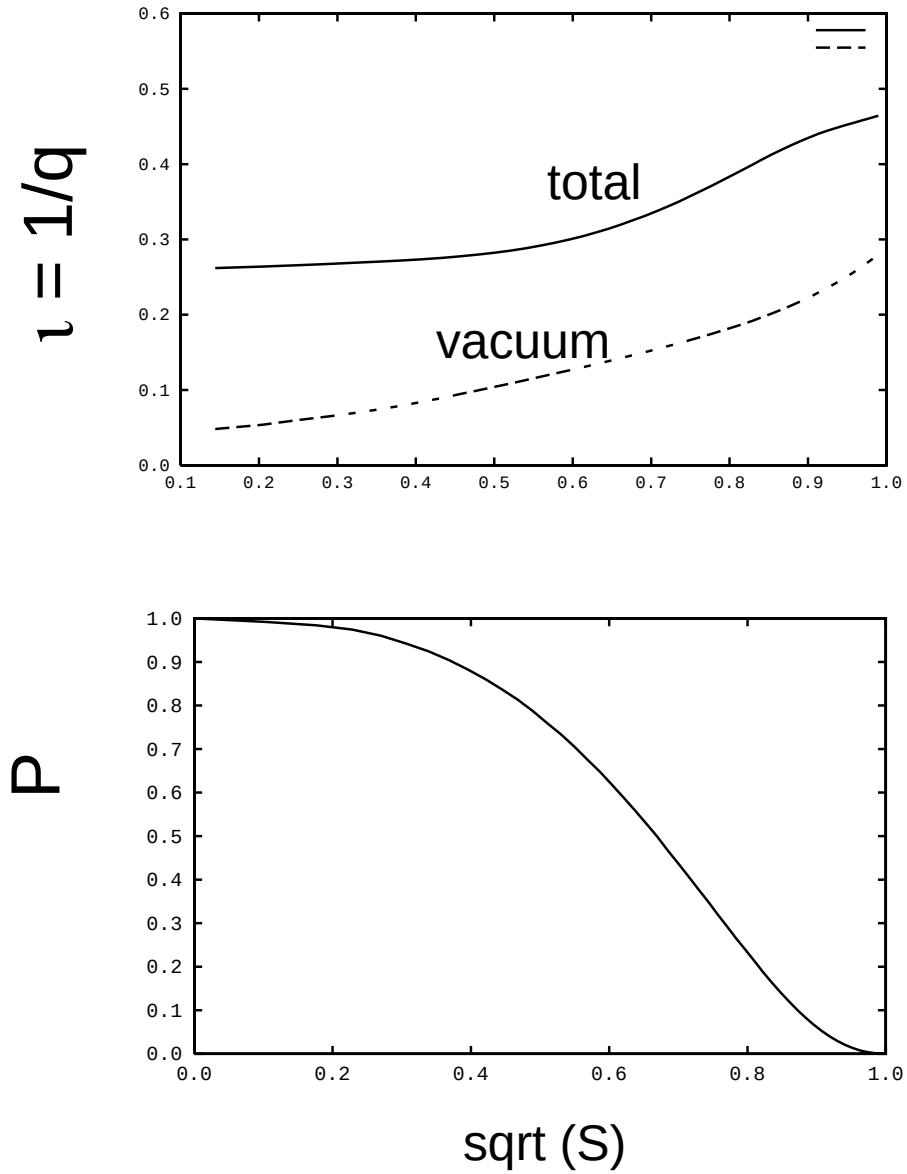
Highlights

3D Shaping

AT Tokamak → QAS C82



ballooning	β_crit ~ 4.5%	β_crit ~ 4.0%
kink	β_crit < 2.0%	β_crit ~ 4.0%
vertical	unstable	stable

QAS C82 Reference Profiles

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