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Investigation of islands in stellarator designs.

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Abstract

Islands are a crucial issue in designing stellarators such as NCSX. The presence of islands is associated with radial magnetic fields and results in poor plasma confinement. It is important to identify what is the physical cause of the islands, and whether the islands may be controlled using boundary perturbations, provided by trim coils.

Computationally, equilibria with islands are calculated using PIES [1], and island widths and resonant perturbations may be determined using [2].

Perturbations to the plasma boundary can be used in an attempt to manipulate island formation. A numerical procedure is described, and the results of island optimization are presented.

OUTLINE

- PIES is used to calculate stellarator MHD equilibrium.
- The resonant fields responsible for causing islands are measured.
- The change in resonant fields as the boundary is varied is determined, and represented with a coupling matrix.
- Boundary variations are used to reduce the size of major islands in NCSX designs.

PIES is used to compute stellarator equilibria

- **P**rinceton **I**terative **E**quilibrium **S**olver finds MHD equilibrium solutions for general (stellarator) geometry using an iterative technique

$$\nabla p = \mathbf{J}^{n+1} \times \mathbf{B}^{n}, \qquad \nabla \cdot \mathbf{J} = 0 \tag{1}$$

$$\mathbf{J}^{n+1} = \nabla \times \mathbf{B}^{n+1}, \qquad \nabla \cdot \mathbf{B} = 0 \tag{2}$$

- Islands are allowed to develop. A magnetic field diagnostic determines regions of good flux surfaces and regions of islands and chaotic field lines.
- On good flux surfaces, magnetic coordinates are used to solve a magnetic differential equation for the parallel current.
- In islands and chaotic regions, the pressure and current profiles are flattened.
- On convergence, the equilibrium condition $\nabla p = \mathbf{J} \times \mathbf{B}$ is satisfied. VMEC is used to initialize the fields.

Quadratic Flux Minimizing Surfaces [3, 4] are used to calculate resonant fields at rational surfaces

- The action integral, S, is the integral of the magnetic vector potential, $\mathbf{A}$, along a curve :

$$S = \oint \mathbf{A} \cdot \mathbf{dl} = \oint (A_\rho \dot{\rho} + A_\theta \dot{\theta} + A_\zeta) d\zeta, \tag{3}$$

- The quadratic-flux functional is a surface integral of the action gradient,

$$\varphi_2 = \frac{1}{2} \int \int \left[\frac{\delta S}{\delta \theta} \right]^2 d\theta d\zeta. \tag{4}$$

- An Euler-Lagrange equation is derived for extremal surfaces,

$$(B^\theta \partial_\theta + B^\zeta \partial_\zeta) \frac{\delta S}{\delta \theta} = 0, \tag{5}$$

- Quadratic-flux minimizing surfaces are comprised of a family of periodic pseudo field lines of the field tangential to the surface, along which the action gradient is constant.

Resonant Fields are related to boundary variations with a Coupling Matrix

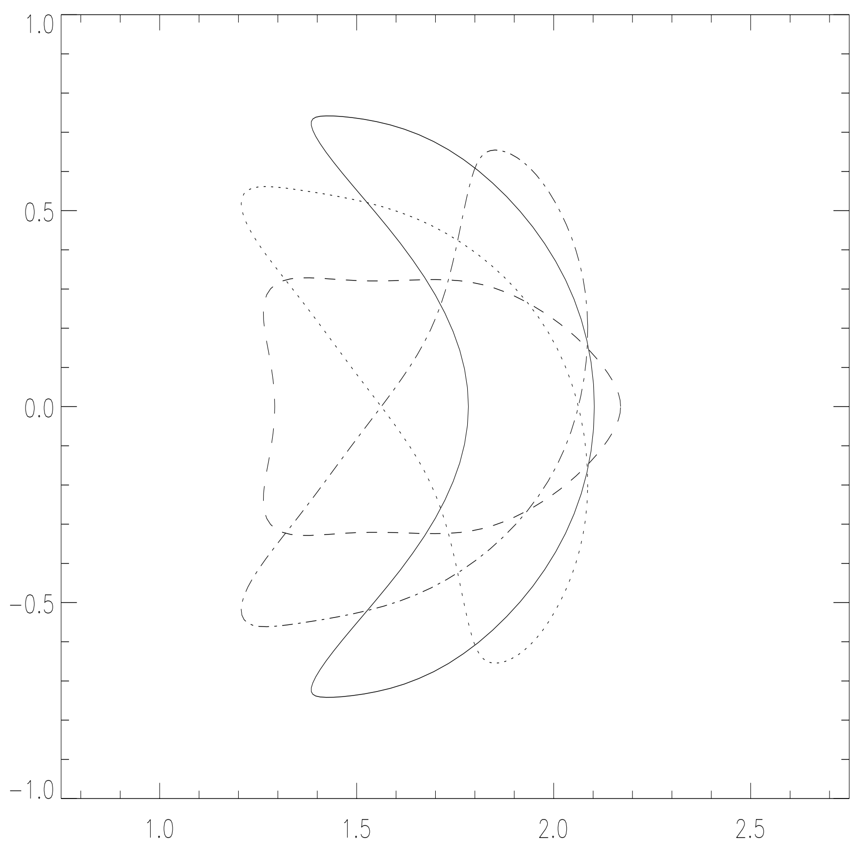
- For small variations $\delta B_{m,n}$ in resonant field amplitudes $B_{m,n}$ at rational ($\iota = 3m/n$) quadratic-flux-minimizing surfaces are related to variations in boundary harmonics $\delta_{n,m}$ via a coupling matrix C to which SVD method is applied

$$\boldsymbol{\delta}\mathbf{B_{m,n}} = C\boldsymbol{\delta} = UwV^T\boldsymbol{\delta} \tag{6}$$

- The coupling matrix is determined by adding a series of small harmonics (about one millimeter) to fixed boundary VMEC runs and running PIES.
- By choosing a perturbation $\boldsymbol{\delta}^R$ according to $\boldsymbol{\delta}^R = Vw^{-1}U^T\mathbf{B}$ where $\mathbf{B}$ are the original resonant fields, a set of island reducing perturbations is obtained.
- If there are more variables than equations more than one solution exists (spanned by the null-space).

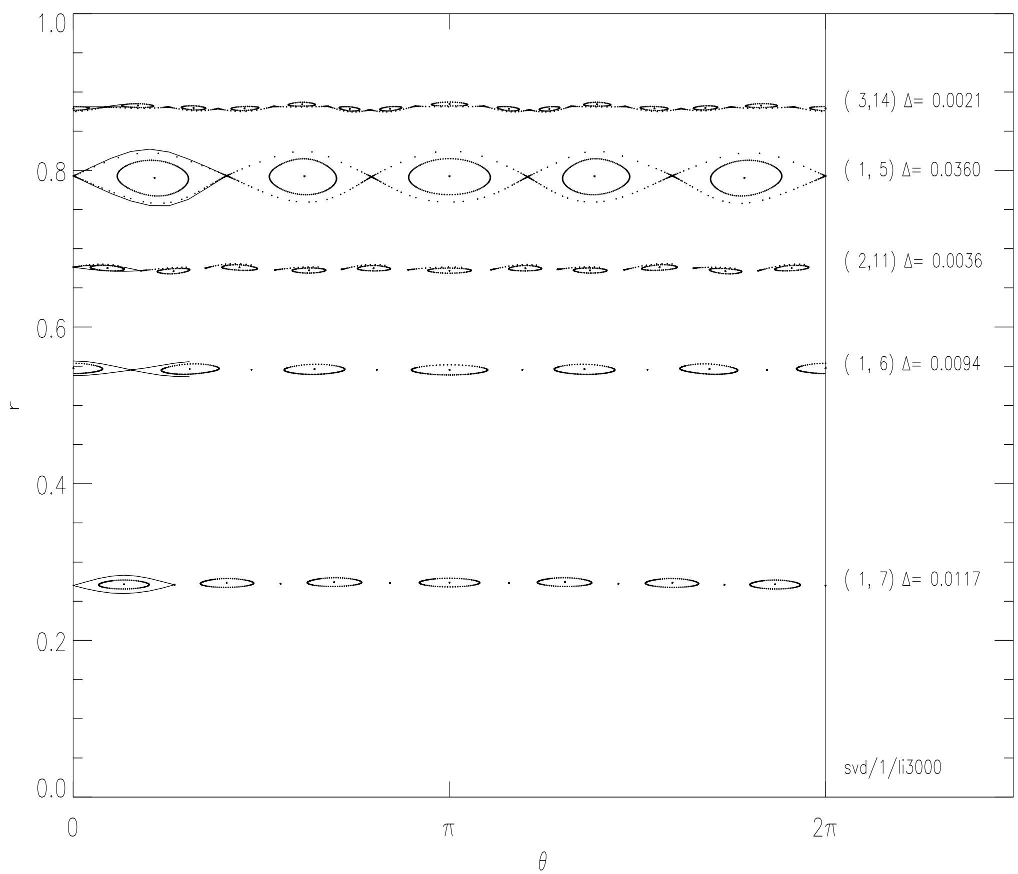
Application to NCSX

- We study candidate designs for the National Compact Stellarator Experiment (NCSX) [5, 6].
- The full pressure full current li383 configuration is considered. The zero pressure full current li383 configuration is also studied.



The standard li383 configuration has islands

- A Poincare plot of the PIES field after 32 iterations shows island chains
- Large islands may deteriorate confinement and should be removed.

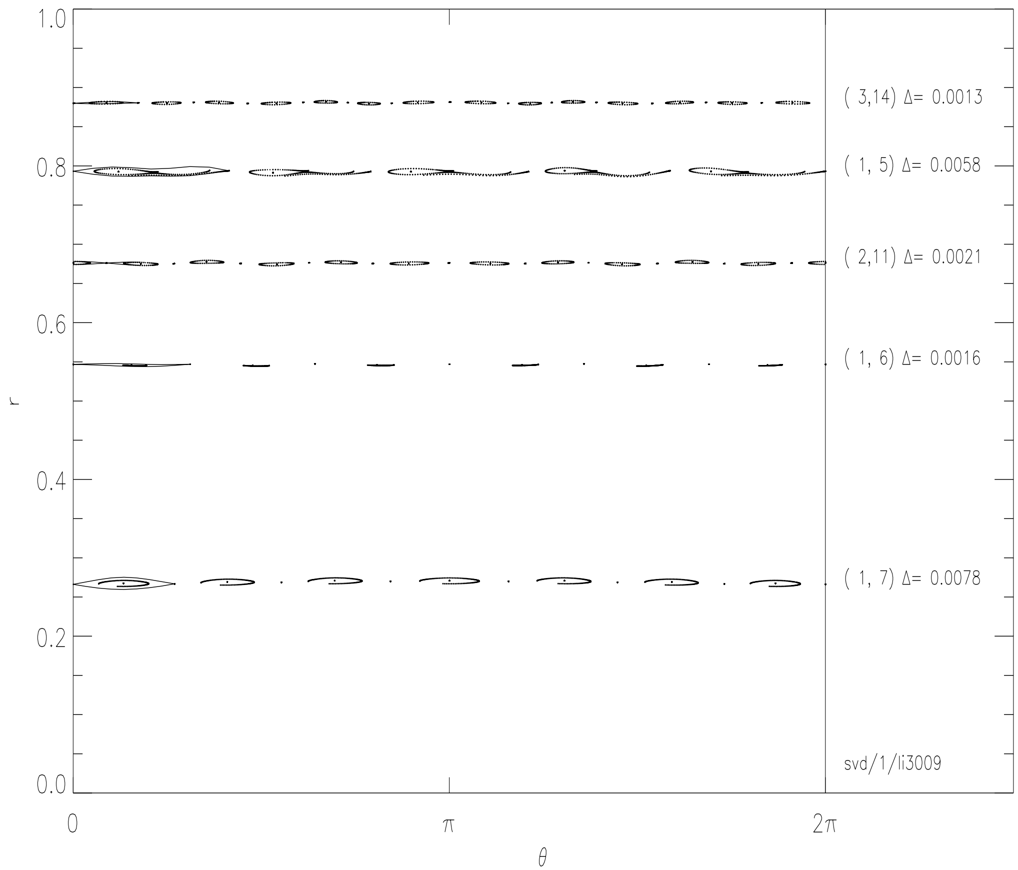


The Coupling Matrix determines island removing boundary variation

- By performing a series of PIES calculations and measuring the variation in resonant fields due to boundary variations the Coupling Matrix is determined.
- In this study the $(1,5),(2,10),(1,6),(2,12)$ resonances are targeted, and the $(1,9),(1,8),(1,7),(1,6),(1,5),(1,4)$ boundary harmonics are varied.
- NOTE there are 2 degrees of freedom.
- Using the coupling matrix, a Newton type procedure is performed to find the island reducing boundary variation.

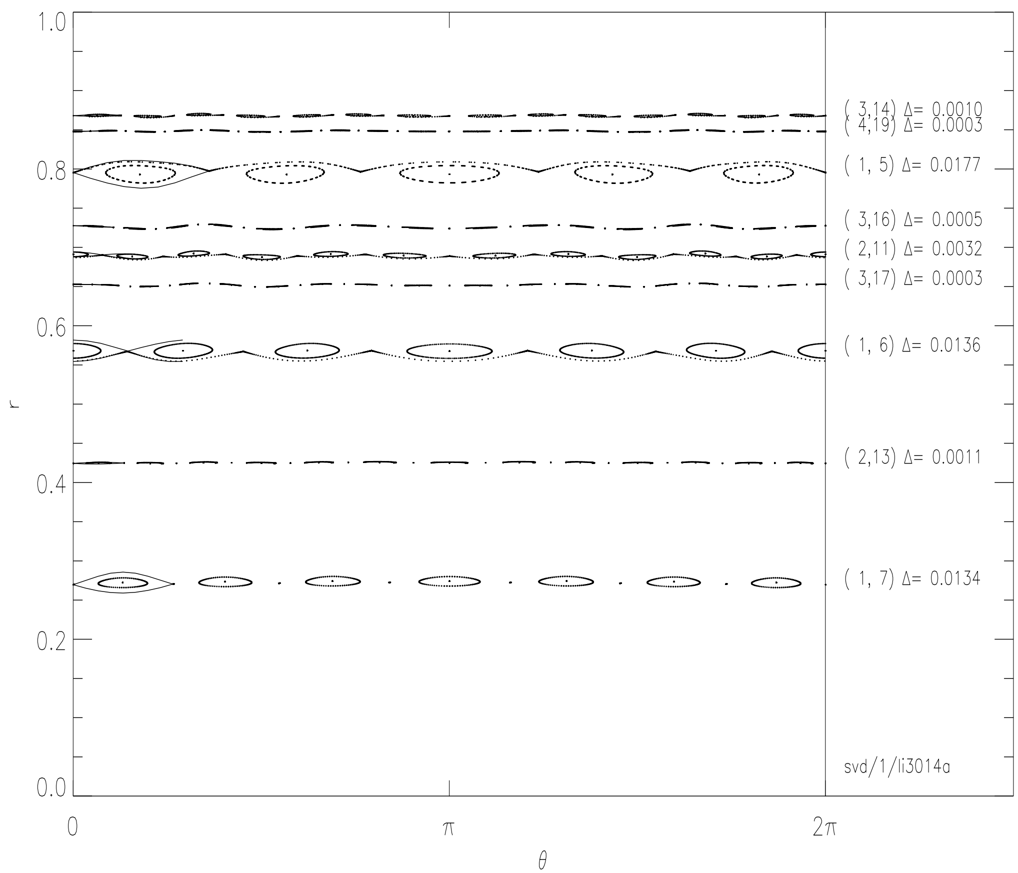
The full pressure li383 configuration islands are reduced.

- The following boundary variation reduces the islands :
  $\boldsymbol{\delta} = 2(-0.00092, -0.00013, 0.00028, 0.00150, 0.00006, 0.00032)^T$



Pressure induced self healing is observed.

- For the healed li383 configuration keeping the boundary fixed, as pressure is lowered the islands reappear.



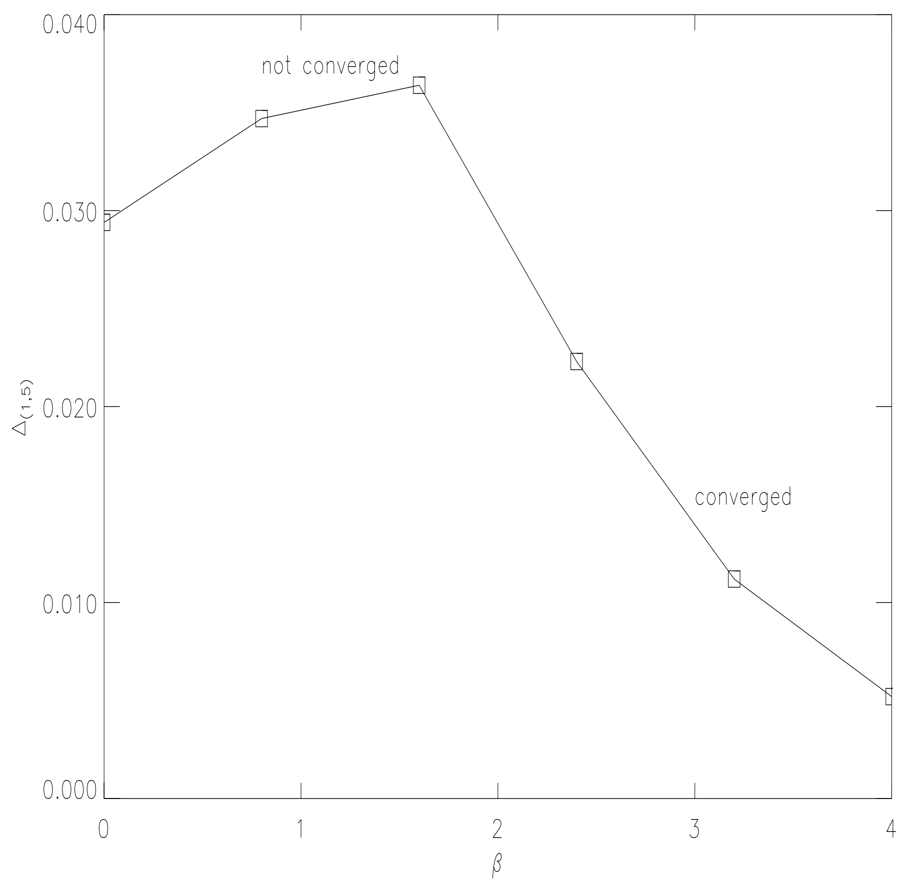
The dependence of island width with pressure will be examined

- Analytic expressions will be used to understand the dependence of island width on pressure. The full island width formula in units of toroidal flux is [7, 8]

$$w = \frac{D}{2} + \sqrt{\frac{D^2}{4} + |C + s w_v^2|} \tag{7}$$

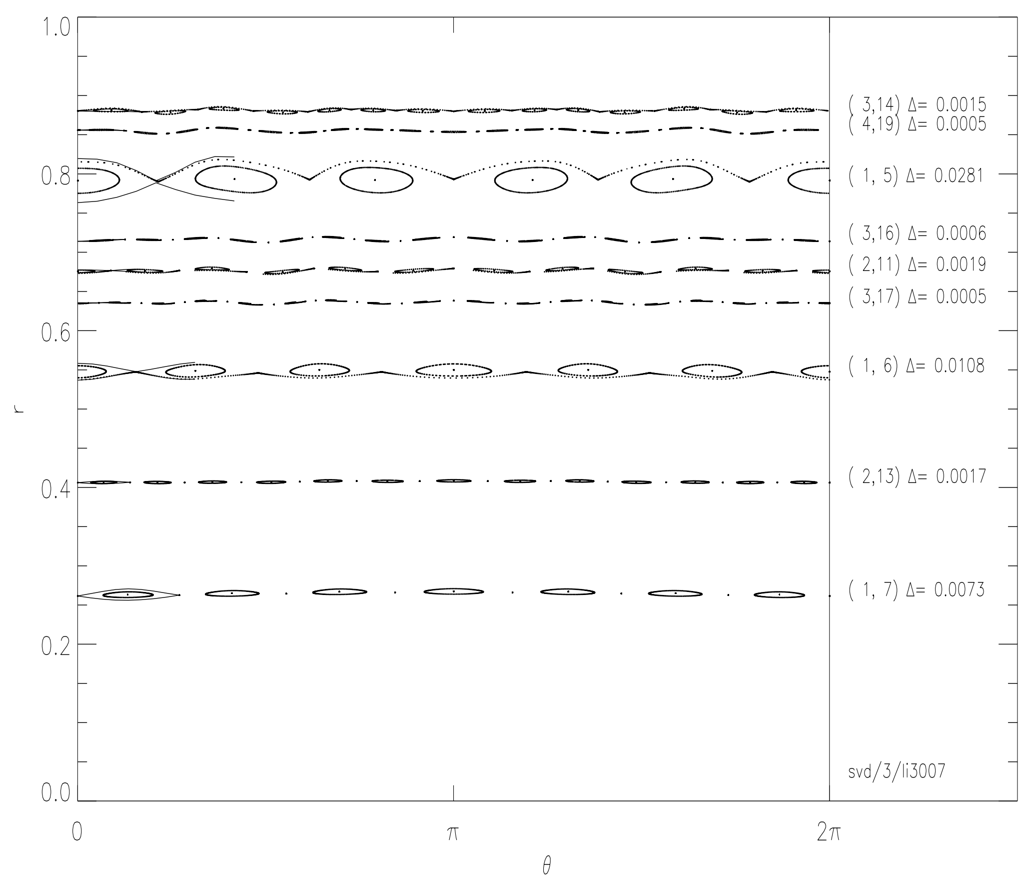
- The width of the $(1, 5)$ island in units of square root of toroidal flux is plotted against pressure. Note that for large islands PIES is not yet converged.

- This is work in progress.


0.040
0.030
0.020
0.010
0.000
$\Delta_{(1,5)}$
0
1
2
3
4
β
not converged
converged

Islands may be removed or created with opposite phase.

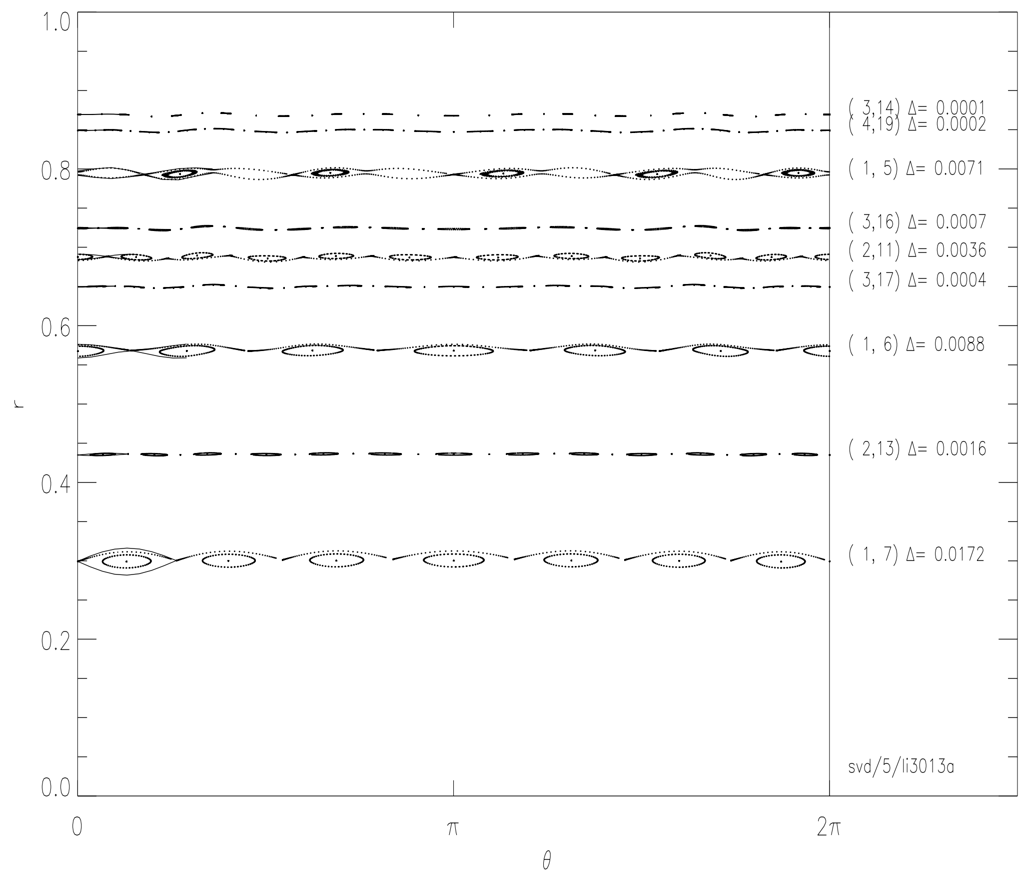
- Using a similar procedure, the boundary variations swapping the phase of the $(1,5)$ island are determined.
- Applying the boundary variation $\boldsymbol{\delta} = 2(-0.00208, -0.00157, 0.00087, 0.00214, 0.00060, 0.00065)^T$ swaps the phase of the $(1,5)$ island.
- This information may be useful for the design of trim coils.



1.0
0.8
0.6
0.4
0.2
0.0
r
0
π
2π
θ
(3,14) Δ= 0.0015
(4,19) Δ= 0.0005
(1, 5) Δ= 0.0281
(3,16) Δ= 0.0006
(2,11) Δ= 0.0019
(3,17) Δ= 0.0005
(1, 6) Δ= 0.0108
(2,13) Δ= 0.0017
(1, 7) Δ= 0.0073
svd/3/li3007

The extra freedom is used to eliminate the zero pressure islands

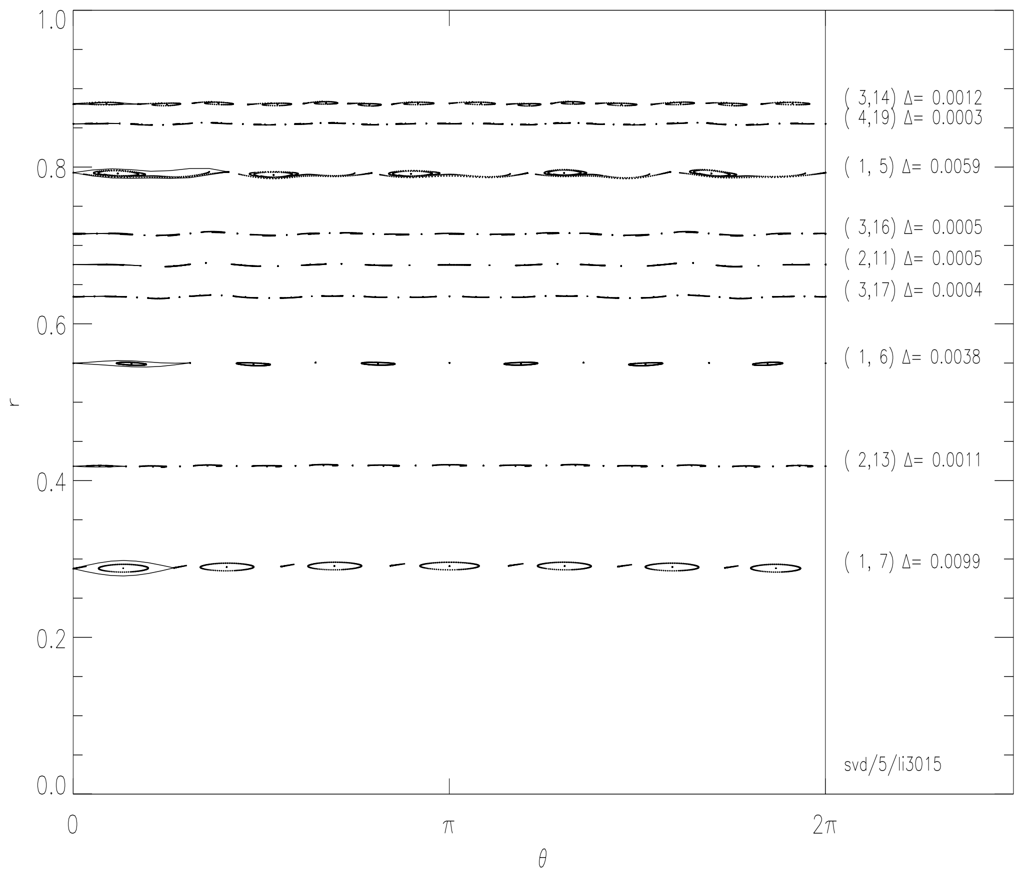
- Variations spanned by the null-space of the coupling matrix are used to eliminate zero pressure islands.
- This will assist in flexibility studies for the NCSX design
- Applying the nullspace variation $\boldsymbol{\delta} = (-0.00076, 0.00086)^T$ reduces the island widths at zero pressure.



r
1.0
0.8
0.6
0.4
0.2
0.0
0
π
2π
θ
(3,14) Δ= 0.0001
(4,19) Δ= 0.0002
(1, 5) Δ= 0.0071
(3,16) Δ= 0.0007
(2,11) Δ= 0.0036
(3,17) Δ= 0.0004
(1, 6) Δ= 0.0088
(2,13) Δ= 0.0016
(1, 7) Δ= 0.0172
svd/5/li3013a

Confirmation that null-space variations do not disturb the full pressure islands

- Another PIES calculation confirms that the zero pressure island healing variation do not disturb the full pressure islands



Conclusions

- Fixed boundary variations enable control of island phase and width.

Future Work

- Fixed boundary variations may be related to normal magnetic fields at the surface. This may help in the design of coils.
- Apply this technique to free boundary calculations, where the independent variables may be related to coil geometry/currents
- Include measures of other physics properties to simultaneously optimize island content and stability etc.

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