Investigation of the High, Finite n Ballooning Mode Limit for Compact Quasi-Axially Symmetric Stellarators

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OUTLINE

- Ballooning stability of QAS
- Global MHD code comparisons: CAS3D, TERPSICHORE
- Kink and vertical stability robustness
- QAS stability without a nearby conducting wall
- Global MHD calculations near the ballooning limit
- Conclusions

MHD STABILITY OF NCSX

- Candidate designs for the National Compact Stellarator Experiment (NCSX) being examined
- QAS3_C82 (EPS'99):

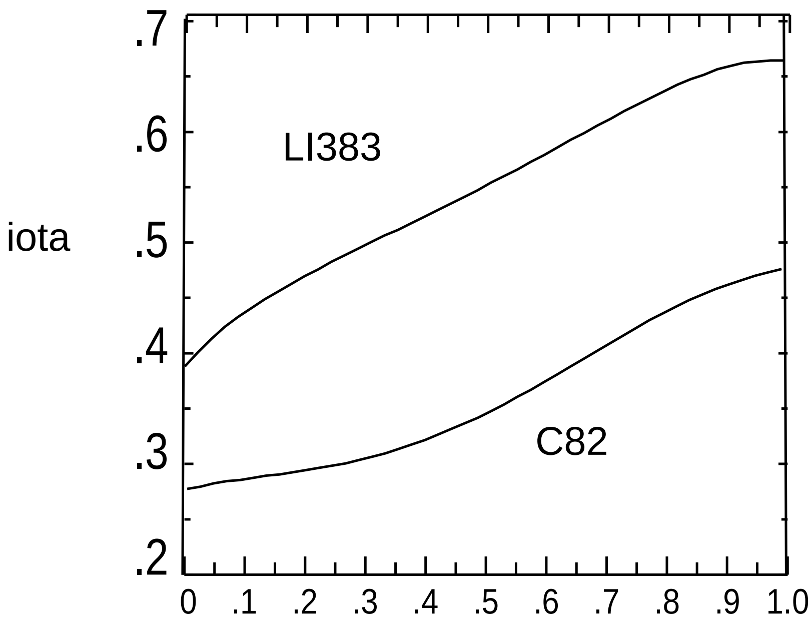
  Good physics properties

  but high coil currents => difficult engineering
- QAS3_LI383 (IAEA'00)

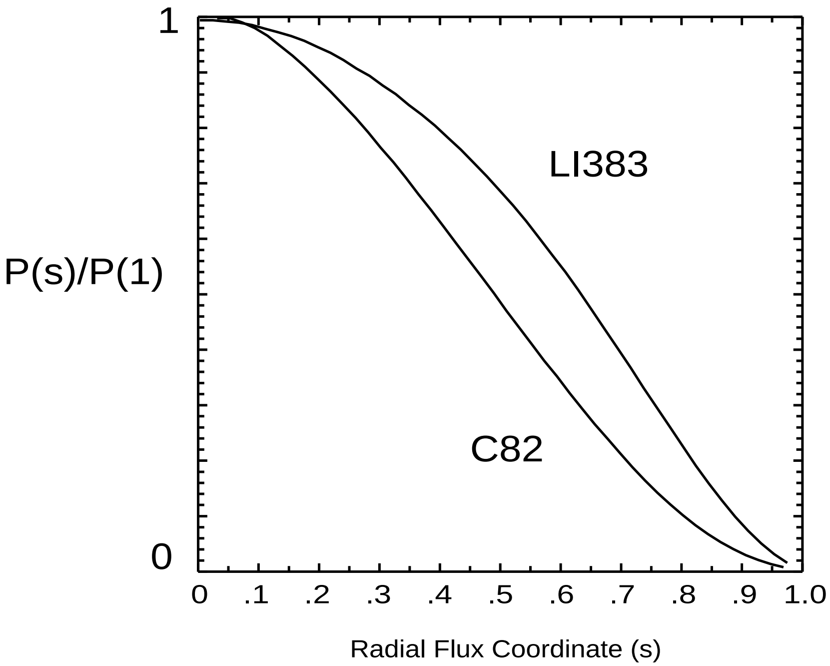
  Better physics properties and several good, improved machine coil designs

Iota is higher with less shear and the pressure profile is broader in LI383 than in the earlier design C82

Higher iota also denotes lower q so that particle loss rates will be lower.

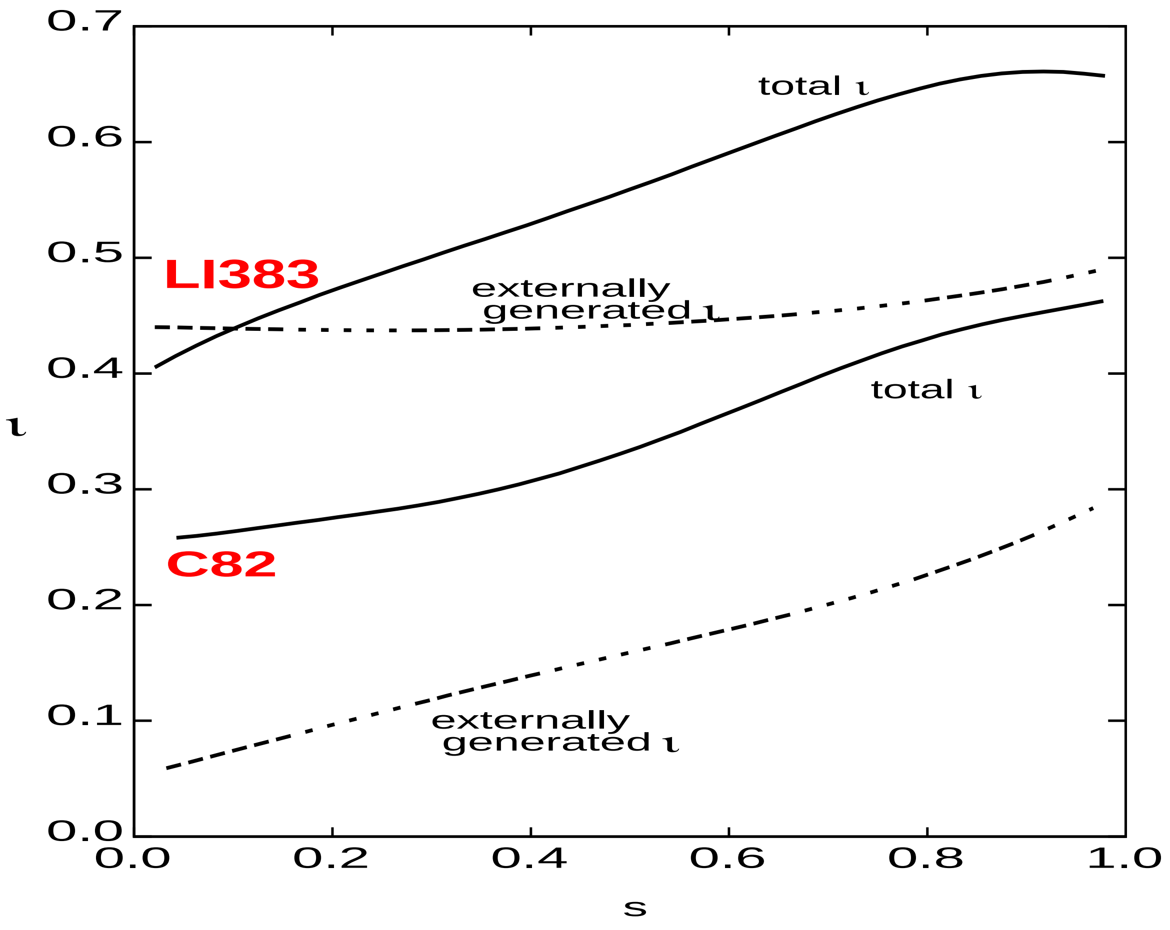


Present stellarator experiments (CHS,LHD) have broader pressure profiles than C82.

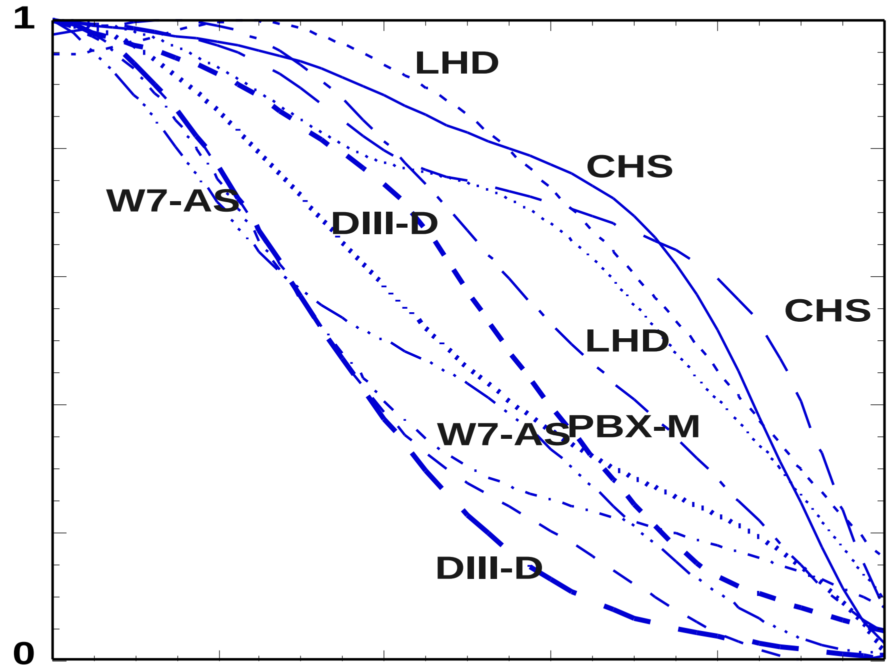


Iota Profiles of Previous and Present Designs for National Compact Stellarator Experiment

High fraction of field transform from stellarator coils



Published Pressure Profiles for Advanced Tokamak and Stellarator Experiments



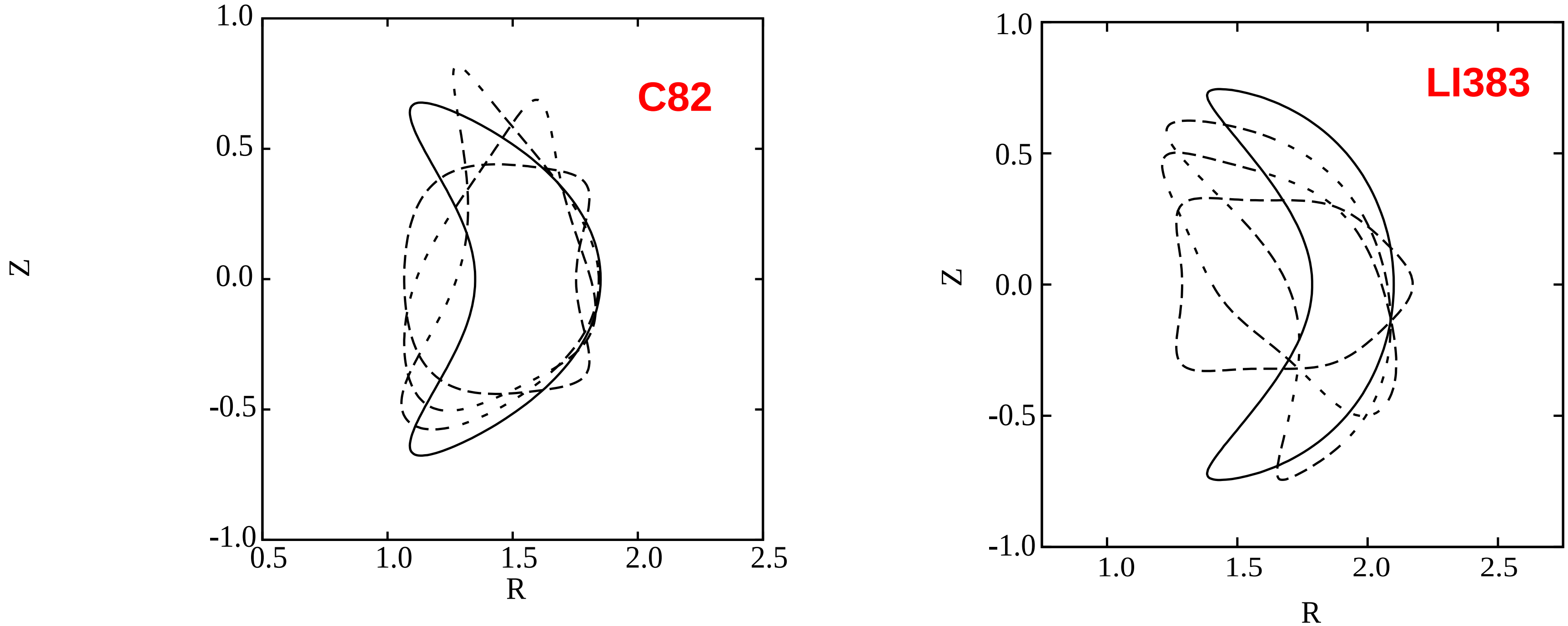
Compiled by David Mikkelson, PPPL

Peaked central pressures found for DIII-D and W7-AS with ECH heating

Broad pressures characteristic of CHS and LHD and as for LI383 design

Boundary Shapes of Previous and Present Designs for National Compact Stellarator Experiment

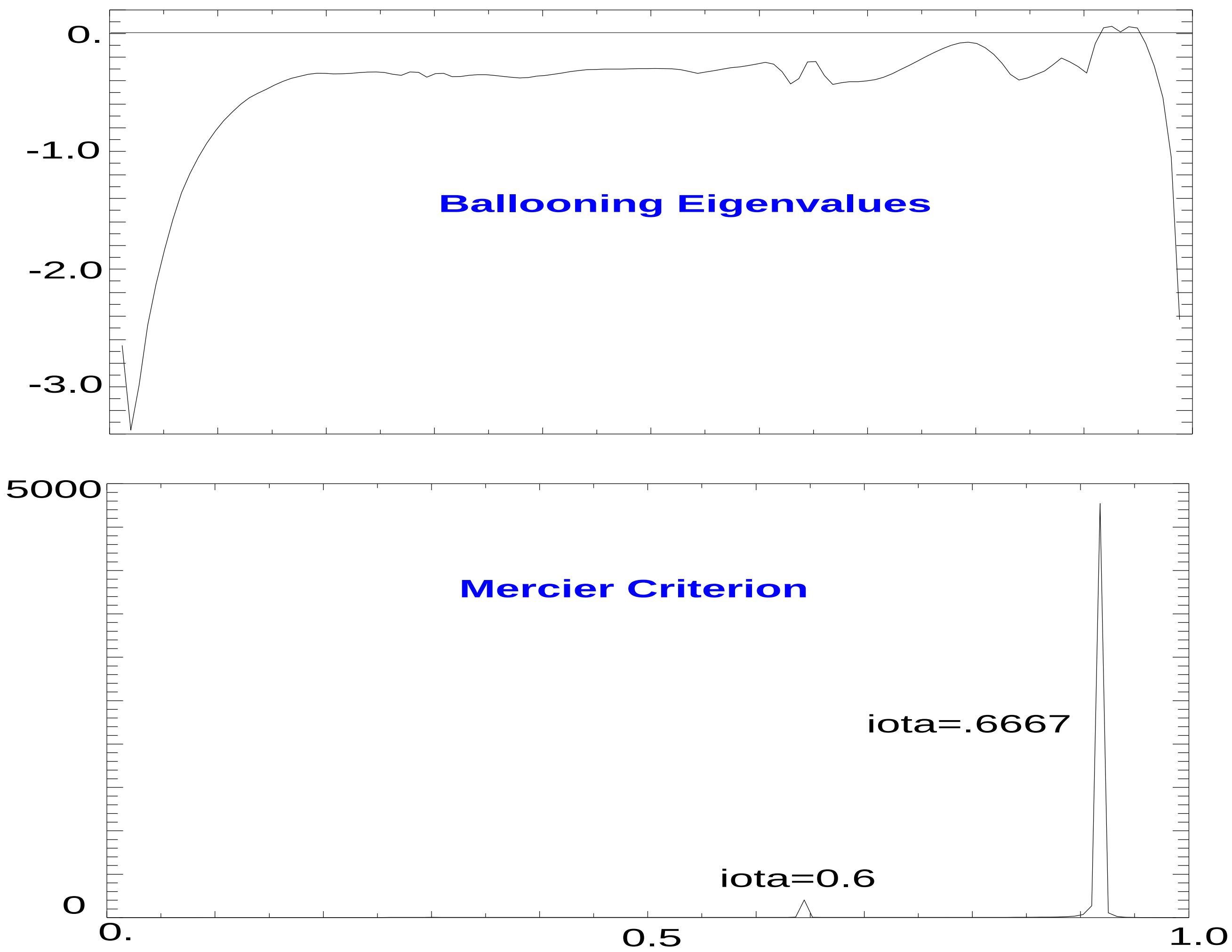
Three field period quasiaxially symmetric stellarator, views spaced 20 degrees apart



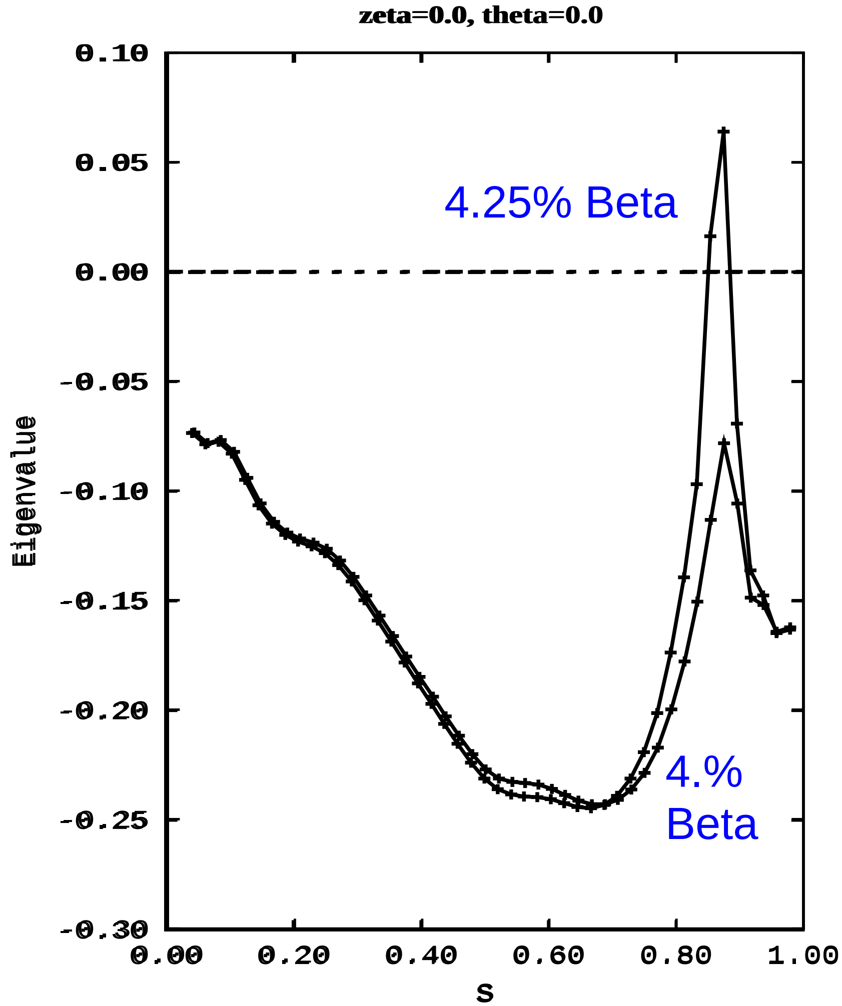
Drift-orbit optimized stellarator design: tokamak particle orbits

Ballooning Stability of QAS

At 4% Beta TERPSICHORE Module VVBAL Predicts Mercier Instability and Marginal Ballooning Stability for QAS3_LI383



COBRA Code Ballooning Stability Calculations for LI383 Show Stability at Beta = 4.0%, Instability at Beta=4.25%



TERPSICHORE Predicts Marginal Stability for QAS3_LI383 at 4% Beta

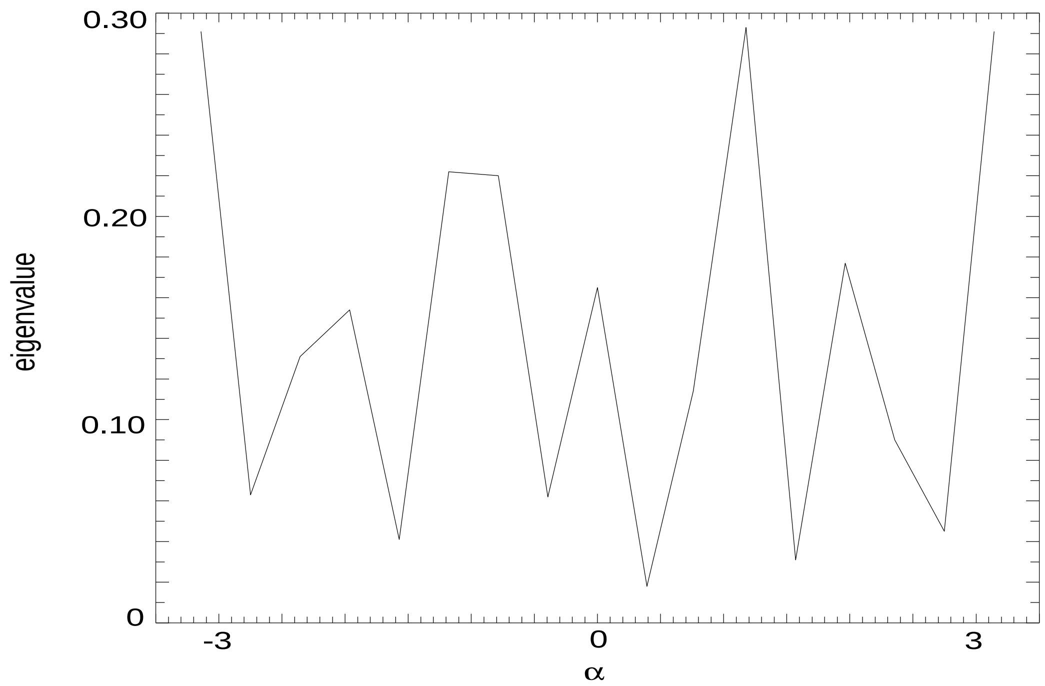
Ballooning Mode Eigenvalues in QAS3_LI383

Show Strong Dependence on Field Line

due to Fully Three-dimensional Symmetry of the Torus

Tokamak Exhibits Monotonic Behavior of the Eigenvalue
with Choice of Field Line, $\alpha = \zeta + q^*\theta$, at 4.3% Beta

=> toroidal localization of ballooning instability: Anderson localization



Maximum eigenvalue versus alpha for flux surfaces between 0.92 and 0.95.

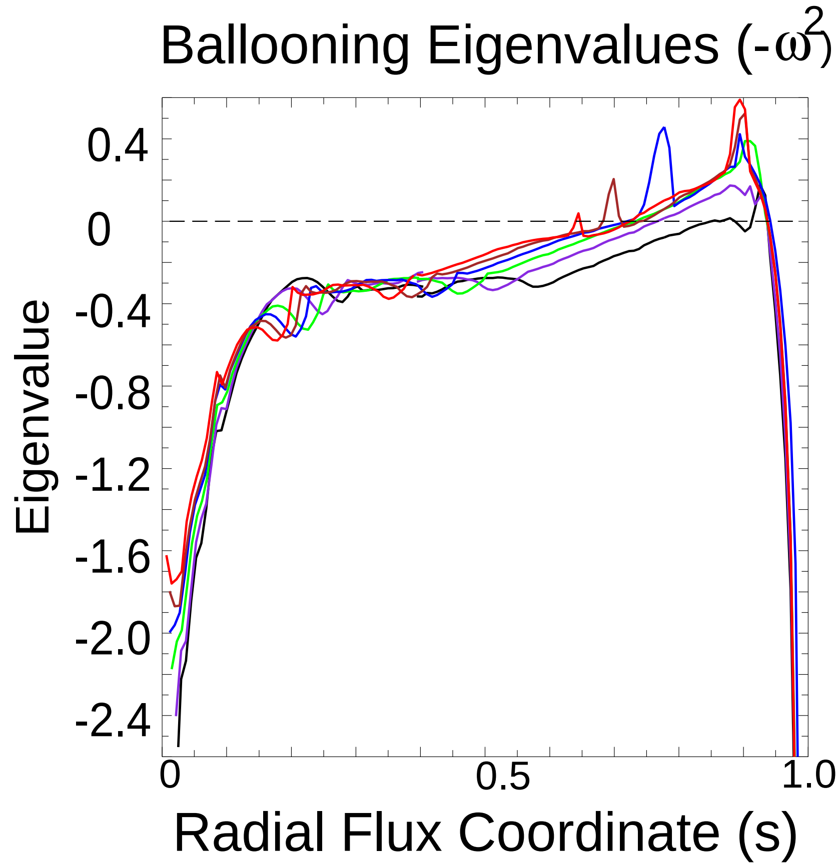
TERPSICHORE ballooning calculations
were made at alpha = pi/2, where eigenvalues are minimal.

TERPSICHORE ballooning instability for new NCSX design, in range Δs (s = toroidal flux).

VMEC equilibria with pressure and current scaled to increase beta. Shown here are the results calculated at 129 flux surfaces.

QAS3-LI383

Beta (%)	Δs	Color
4.251	.87-.94	black
5.110	.76-.94	purple
5.971	.73-.94	green
6.833	.72-.94	blue
7.695	.68-.94	brown
8.557	.73-.94	red

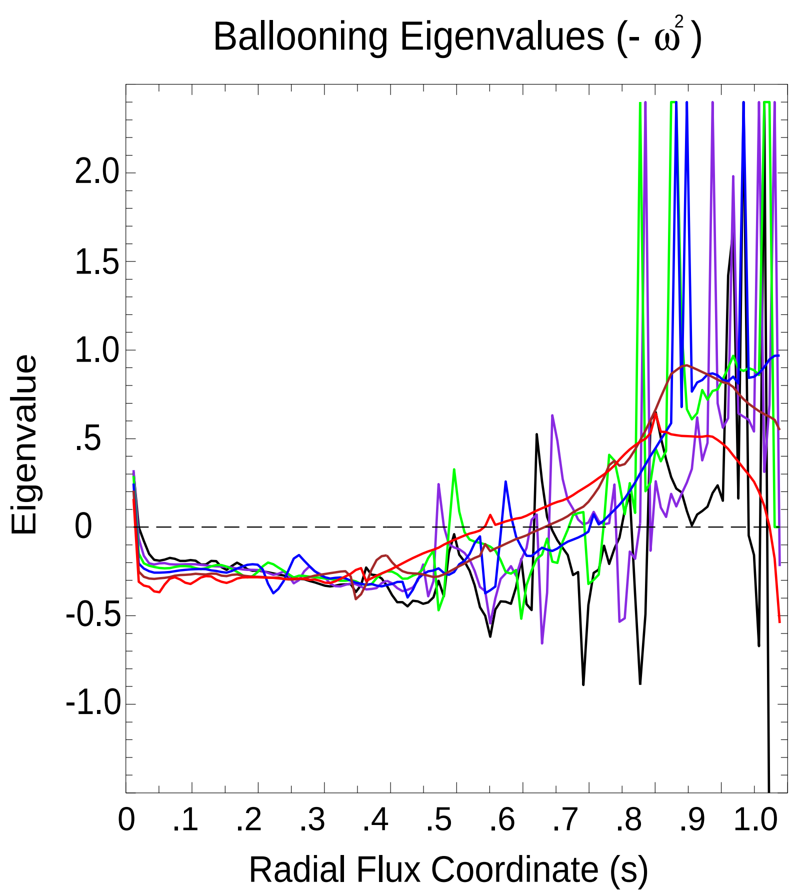


TERPSICHORE ballooning instability for previous NCSX design, in range Δs (s = toroidal flux).

VMEC equilibria with pressure and current scaled to increase beta. Shown here are the results calculated at 129 flux surfaces.

QAS3-C82

Beta (%)	Δs	Color
3.862	.79-.97	■ (black)
4.630	.80-.98	■ (purple)
5.393	.72-.98	■ (green)
6.153	.71-.99	■ (blue)
6.908	.63-.99	■ (brown)
7.659	.54-.99	■ (red)



Global MHD code Comparisons: CAS3D, TERPSICHORE

TERPSICHORE and CAS3D Solve Variational Equation

δWp + δWv - ω^2 δWk = 0.

potential energy in the plasma,

magnetic energy in the vacuum region,

kinetic energy and the eigenvalue of thesystem.

The MHD perturbations evolve as exp (iωt), being unstable if ω <0.

Calculate the normal^2 displacments of the unstable eigenfunction ξ and the plasma potential energy change δW.

CAS3D Code Package Calculations Based on the plasma potential energy

$$W_p = 1/2\iiint d^3r[|C|^2 - A(\xi\bullet\nabla s)^2 + \gamma p(\nabla\bullet\xi)^2]$$

associated with the displacement ξ.

Vector **C**, stabilizes plasma energy integral.
C^1 describes field line bending energy
C^2 depends on local shear and parallel current density
C^3 is field compression energy.

Destabilization is driven by the second term in Wp, with the current density **j** in A

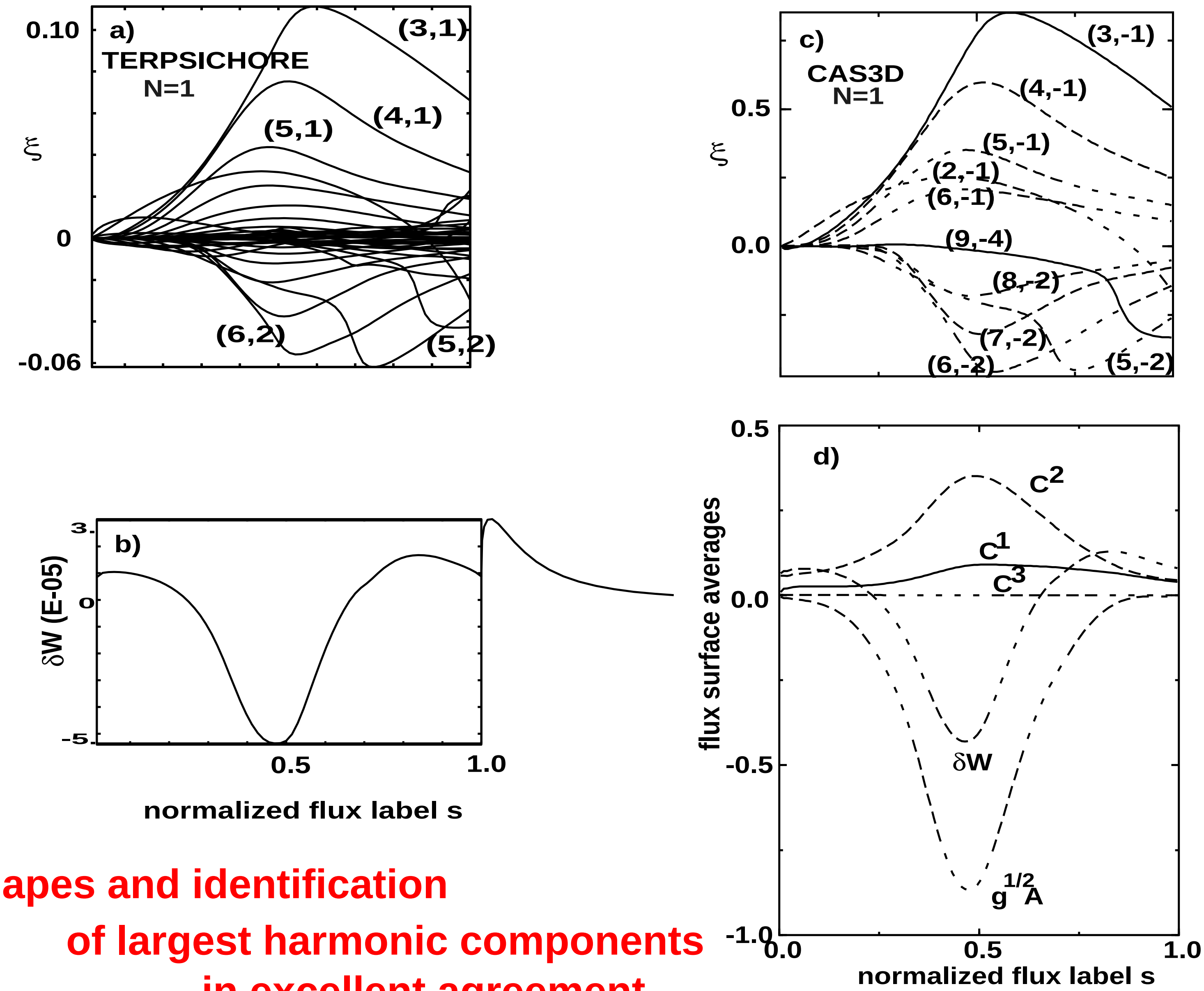
$$A = 2|\nabla s|^{-4}(j\times\nabla s)\bullet(B\bullet\nabla)\nabla s$$

driving instability, modulated by the plasma curvature and the local shear

The third term in Wp is stabilizing,proportional to γ;
γ is the ratio of the specific heats and
describes the energy associated with field compression
The code version used here is for incompressible modes
$(\nabla\bullet\xi = 0)$
The stabilizing term proportional to γp does not contribute.

Comparison of TERPSICHORE and CAS3D

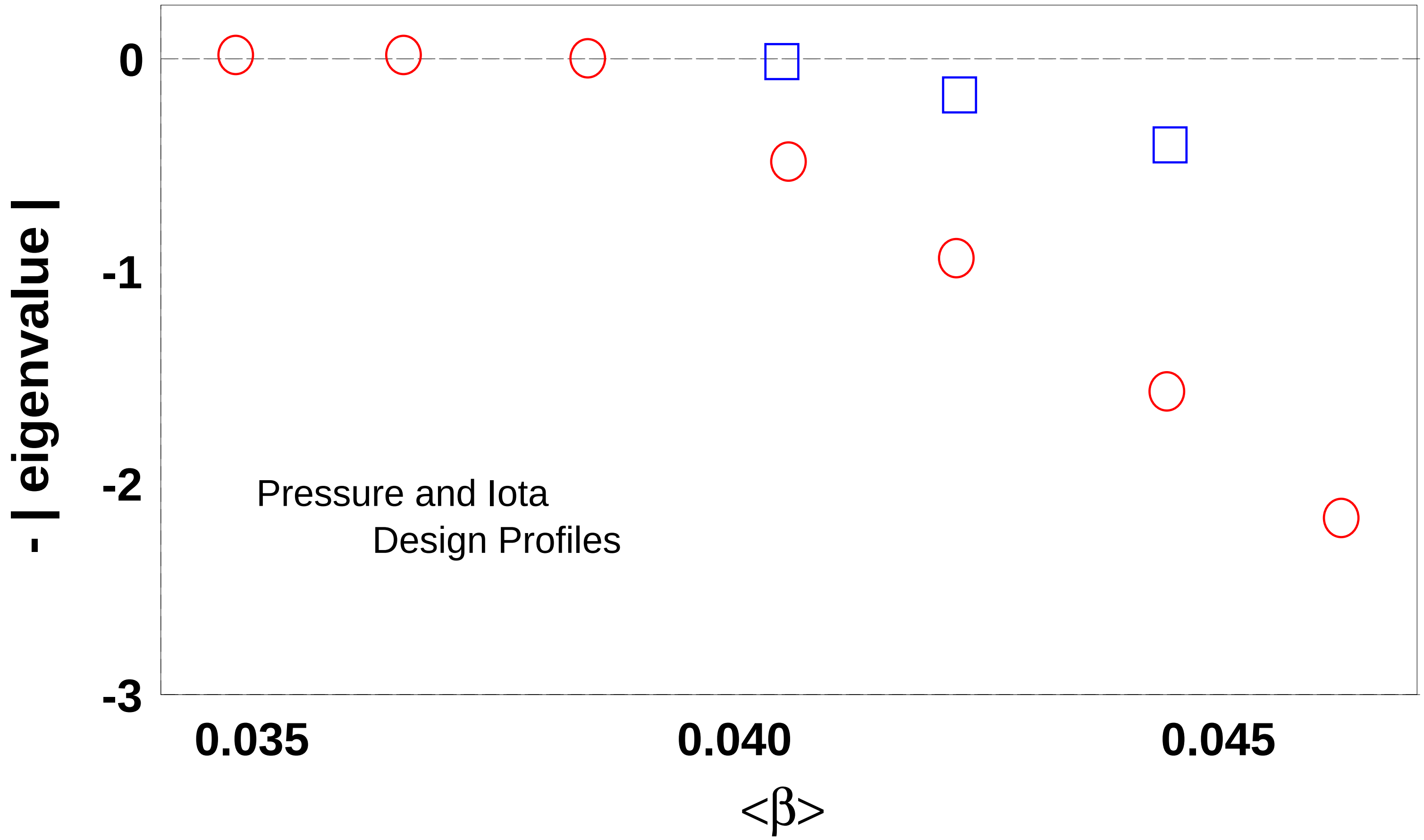
Calculations of Kink Instability for Peaked Pressure in C82



Shapes and identification of largest harmonic components in excellent agreement.

QAS Stability without a Conducting Wall

CAS3D and TERPSICHORE Agree: QAS Kink Stability



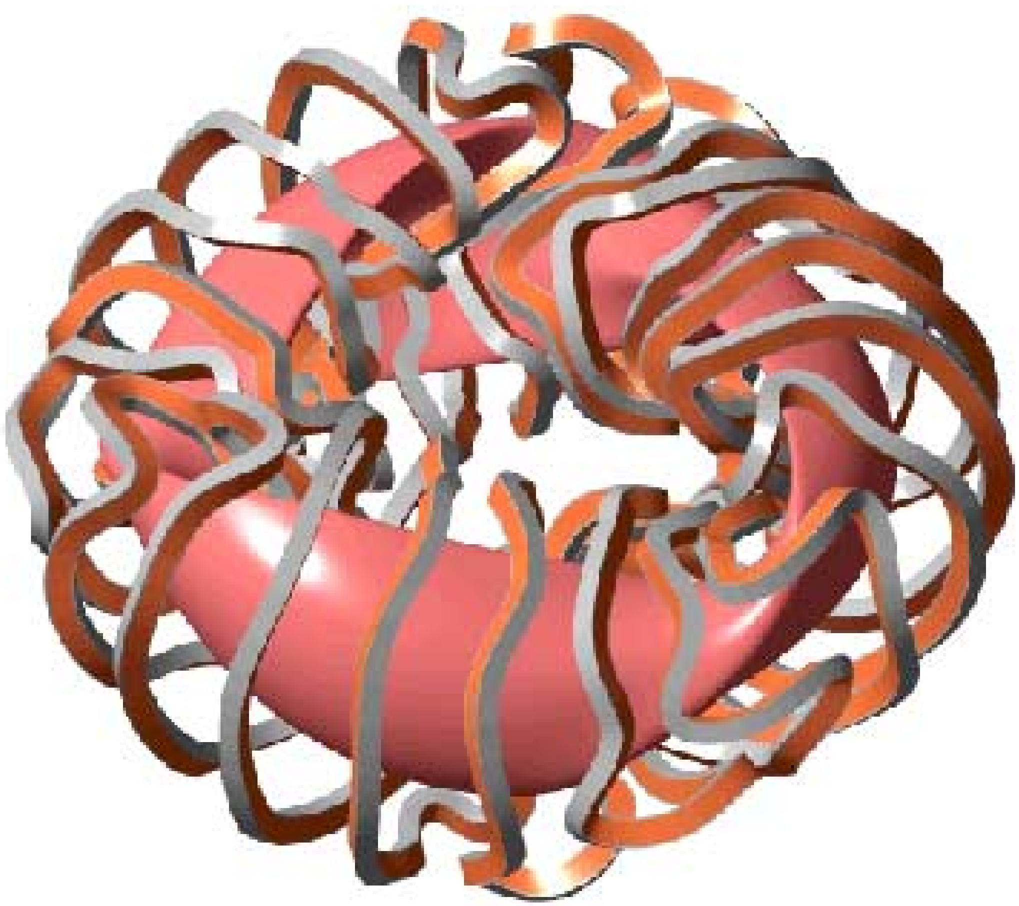
○ Open circles: CAS3D with wall at infinity; stabilization at beta ~ 3.9%.

□ Open squares: TERPSICHORE with wall at 2.5 a, marginal beta at 4.03%.

Eigenvalues from CAS3D: $\lambda(10^{-3})$, from TERPSICHORE, $\lambda(10^{-2})$

The value of the marginal beta below which the kink is stable depends on the distance between the plasma and the conducting wall for QAS as for tokamaks.

NCSX: Toroidal Plasma with Helical Coils for Passive Control



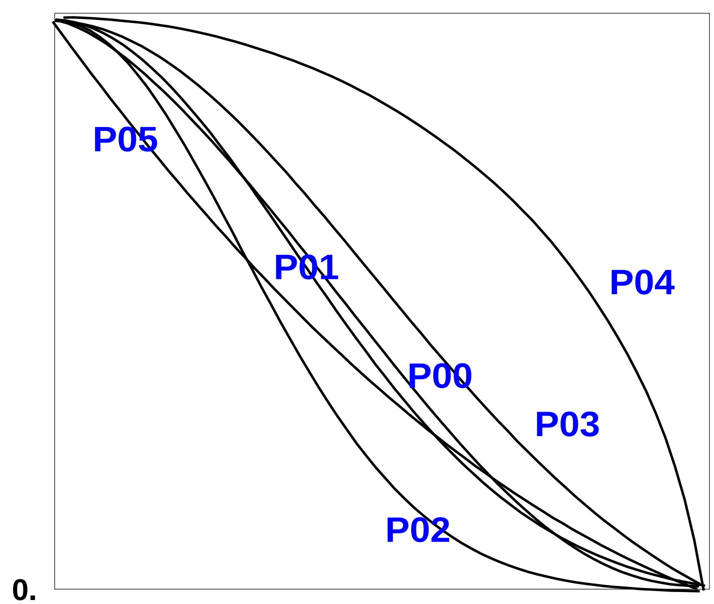
Kink and Vertical Stability Robustness

Calculations for QAS3_C82 with TERPSICHORE

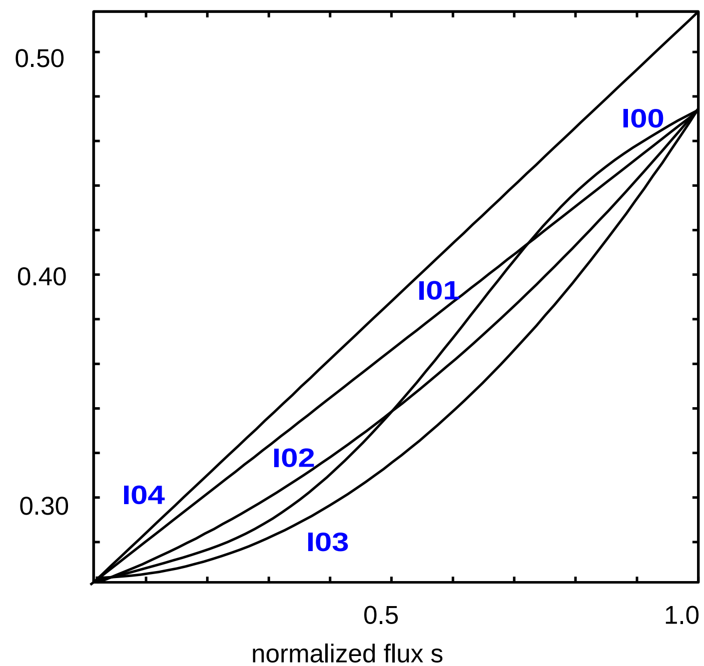
Pressure and Iota Profiles
Varied about the Design Point Configuration

To test robustness of stability and confinement of QAS configuration

Pressure Profiles



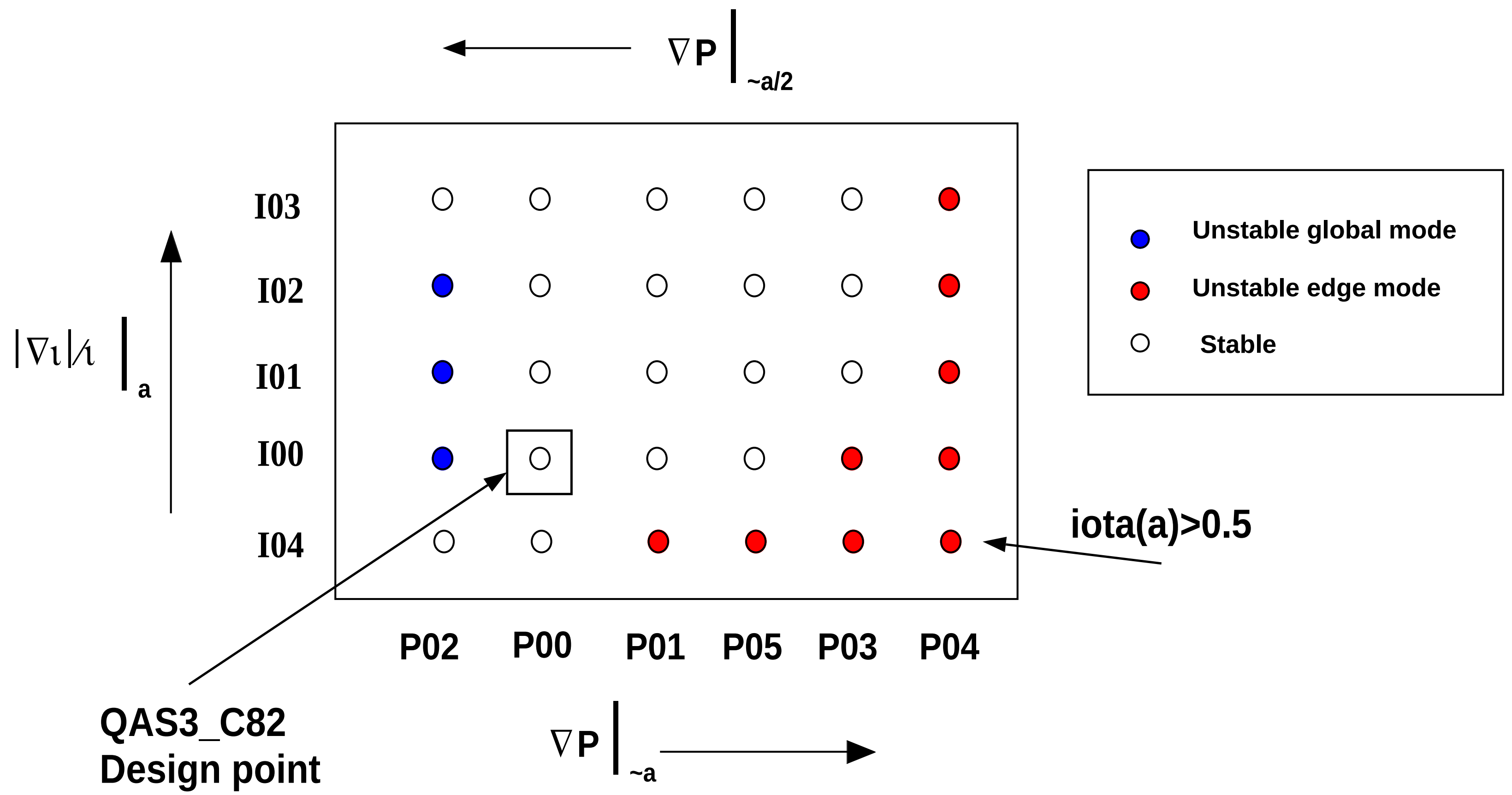
Iota Profiles



Non-Periodicity-Preserving N=1 Mode

Kink Stability Results Summary for

NCSX Maintaining < β >=3.8% and Boundary Shape

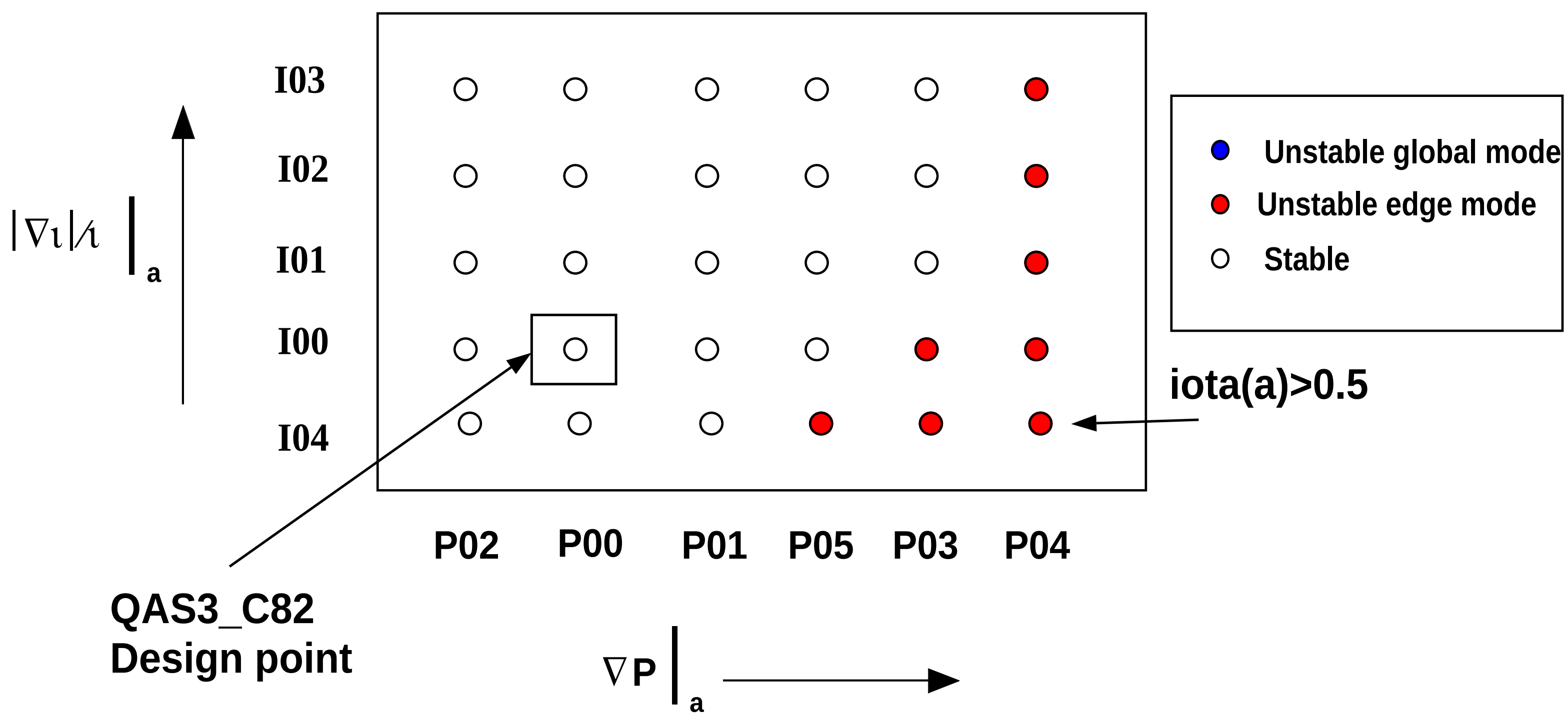


N=1, External kink stability in general, increases with lower pressure gradient and higher magnetic shearat edge.

P02 drives global kink mode for iota(a)>0.5.

Global mode thought to be stabilized by position of the rational q surface with respect to the configuration profiles.

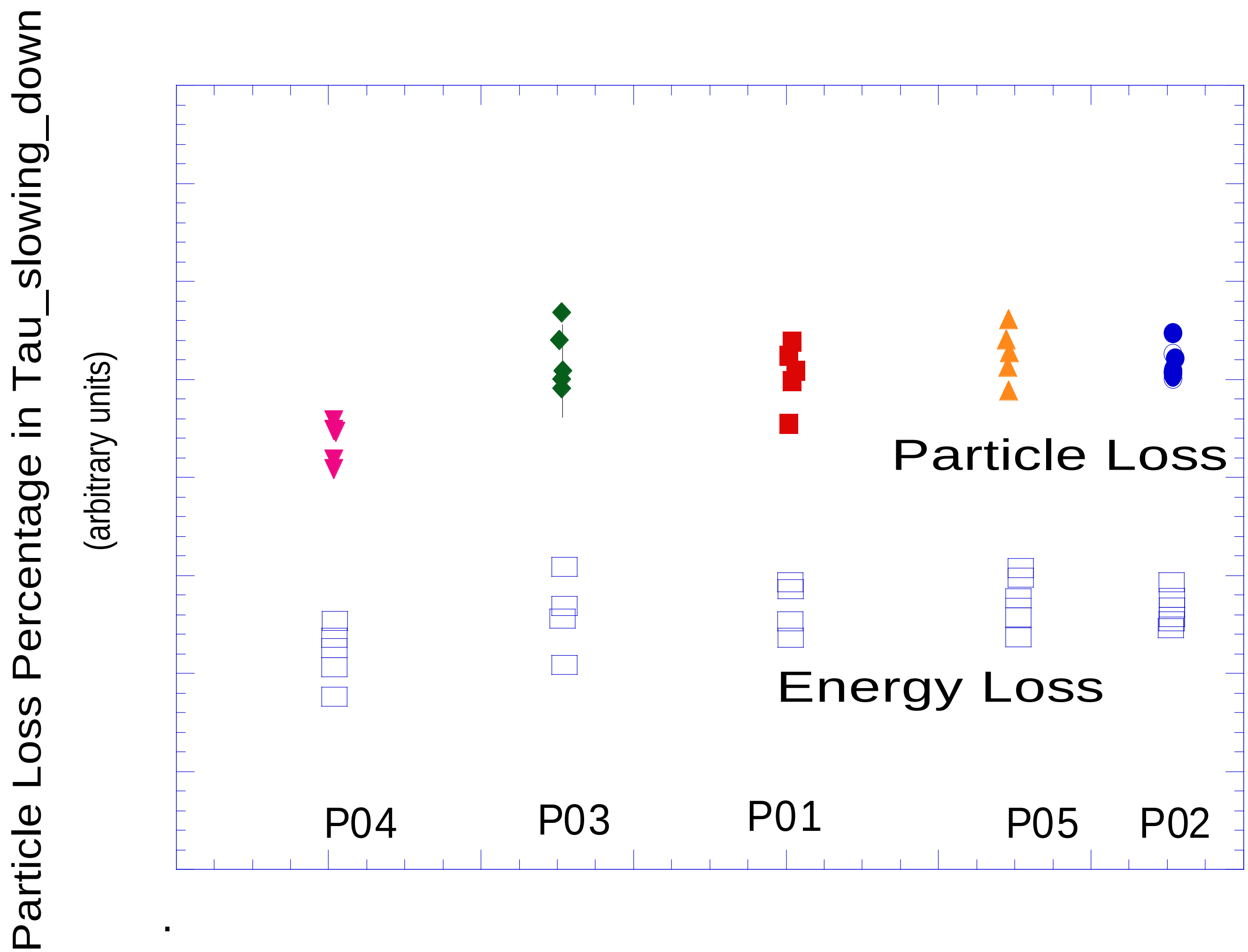
N=0 Periodicity Preserving Kink Stability Results Summary for NCSX, Maintaining< β >=3.8% and Boundary Shape



N=0, Periodicity-preserving mode stability
increases with lower pressure gradient
and higher magnetic shear;
More regions of "vertical" than kink stability.

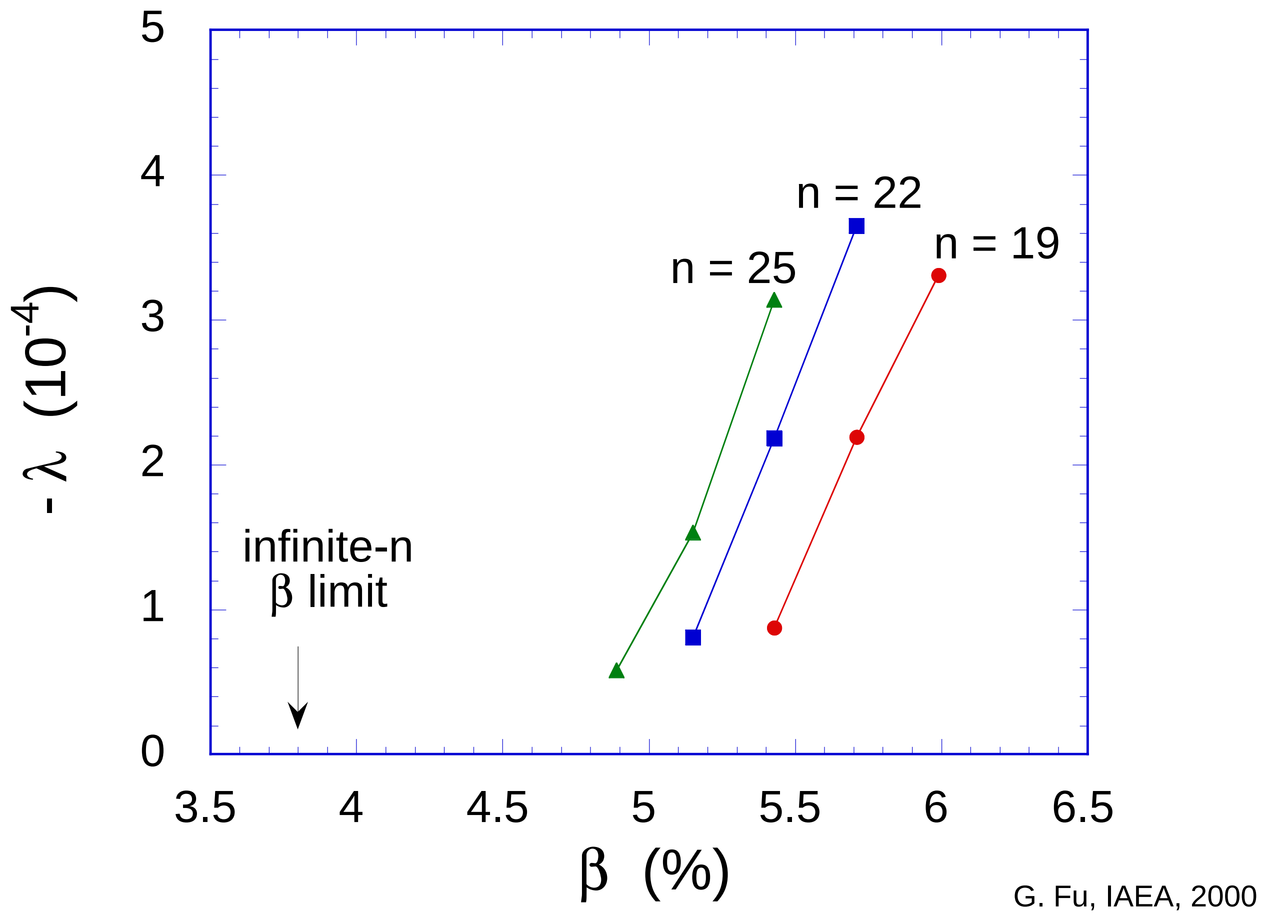
Fast Particle Loss Fraction for QAS3_C82, D Beams, 1T Shows Little Dependence on Pressure and Iota Configuration

At 2T, hydrogen beam loss with full collision model ~25% energy loss.



Global MHD Calculations near the Ballooning Limit

GLOBAL, 3D MHD: REDUCED BETA LIMIT WITH INCREASED MAXIMUM TOROIDAL MODE OF UNSTABLE EIGENFUNCTION



CALCULATIONS WITH TERPSICHORE FOR QAS3_C82

Ballooning Beta Limit

- Beta limit for ballooning mode stability: radially local, infinite n
- Density of states argument based on quantum chaos (R. Dewar see WP1.088)

  Basis functions with maximum n_toroidal ~60

  needed for ballooning limit (for H1 heliac).
- Kinetic effects on the ballooning limit: finite Larmor radius and diamagnetic ion frequency.
- Finite Larmor radius:

  For the NCSX, B~1T, hydrogen plasmas, Te(0)~2kev, Te(a)~0.5kev

  - Larmor radius is 0.7 cm at the center and 0.3cm near the plasma edge.
  - The CAS3D high n Mercier modes at n_toroidal=70 are ~2cm.

  Unless modes at high n have much reduced widths, stabilization by kinetic effects on radial width does not seem likely.
- Diamagnetic ion frequency effects on ballooning limit difficult to estimate, because of CAS3D energy normalization
- Rewoldt has shown that kinetic stabilization can change the mode growth rates, but not the beta limit, for tokamaks.

CAS3D Calculations of Mercier, High n MHD

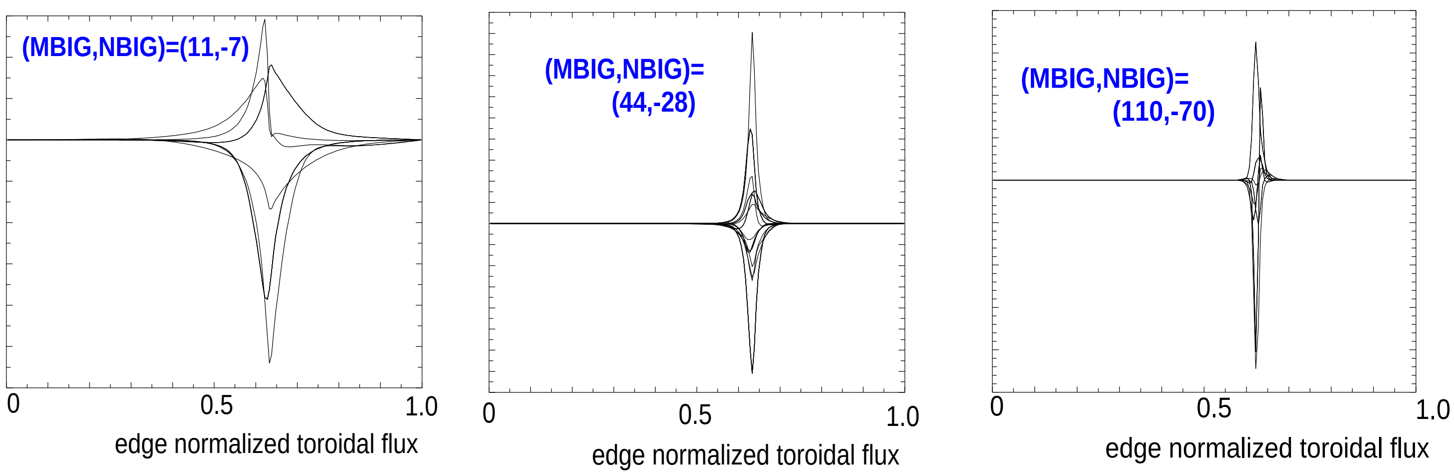
- High n: designate (M_{BIG}, N_{BIG}) centroid of basis functions: (11,-7), (44,-28), (110,-70)
- Initial results for natural resonances: Mercier modes near iota = 0.6
- Radial extent of eigenfunctions decreases as N_BIG increases: radial localization
- Work in progress without natural resonances, for QAS3_LI383

CAS3D CALCULATIONS
FOR QAS3_LI383

AT 4.3% BETA: INCREASING TOROIDAL MODE NUMBER REDUCES RADIAL EXTENT OF MOST UNSTABLE MODE

PRESENT CALCULATIONS, FOR 192 FLUX SURFACES, 70 BASIS FUNCTIONS ABOUT (M_BIG,N_BIG) INCLUDE NATURAL RESONANCES

WORK IN PROGRESS TO EXAMINE BETA LIMIT WITHOUT NATURAL RESONANCES (MERCIER MODES)



Conclusions

- Compact quasiaxial stellarators have been designed to have good particle confinement and good MHD stability
- Ballooning stability is found at 4% beta
- Global MHD calculations are in progress for high n modes, possibly to increase plasma beta
- CAS3D and TERPSICHORE codes are verified to be in good agreement
- Compact QAS designs are kink stable without a nearby conducting wall.

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