Anderson Localization of Ballooning Modes, Quantum Chaos and the Stability of Compact Quasiaxially Symmetric Stellarators

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American Physical Society Division of Plasma Physics 43rd Annual Meeting
Oct. 29-Nov. 2, 2001
Long Beach, CA

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Motivation

- Investigate ballooning stability of National Compact Stellarator Experiment (NCSX)
  - Quasiaxially symmetric stellarator (QAS)
    - Predicted to have good stability and particle transport
      - Ballooning or high *n* (>7) kink limited at 4% beta
- Is there global stability above the design point, due to kinetic stabilization ?

OUTLINE

I. Method:
Ballooning stability calculations above 4% β

II. Results:
Eigenvalue isosurfaces too complex for WKB ray tracing
Must use 3D codes for global stability

NCSX National Compact Stellarator Experiment

- Major developments in 3D computation in last 5 years:

  New and extended codes for 3D configurations:
  ORBIT3D, CAS3D, TERPSICHORE, COBRA, STELLOPT, PIES

- Use of new tools

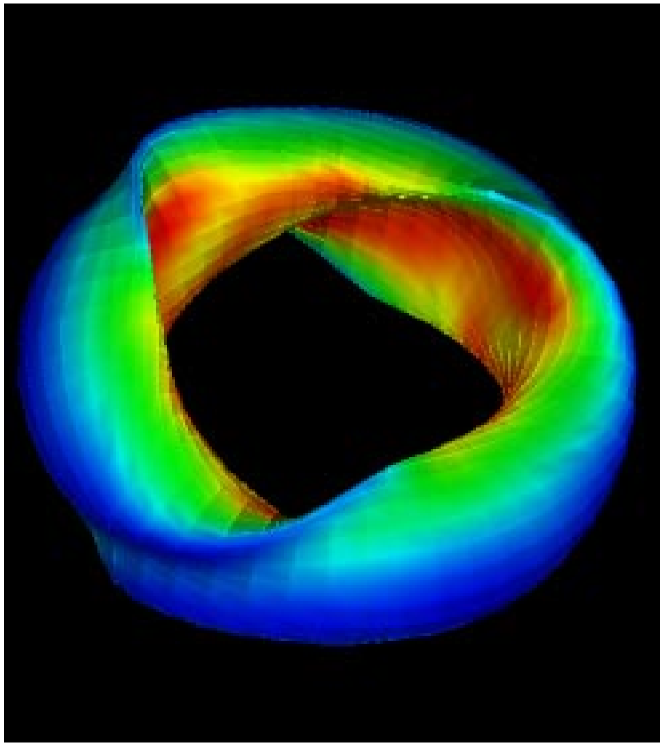
  Led to good physics and engineering properties in a QAS
  R=1.42 m, R/<a> = 4.4, B=1.2-1.7 T,
  6MW NBI, H and D fuelling

- Good confinement:

  Quasisymmetry, drift orbit optimized
  (Nührenberg (1994), Garabedian)

- MHD Stable

  Passively stable at 4% beta to
  kink, vertical, Mercier, ballooning modes
  without feedback control or conducting wall



Method: Ballooning Mode Calculations and WKB Analysis

- Aim: characterize global high n MHD instability
  from properties of 1-d ballooning solutions

- Equilibrium beta scan with VMEC
  Ballooning mode calculations: VVBAL module of TERPSICHORE

- Construct datacube of eigenvalues $\lambda(s,\theta_k,\alpha)$
  radial variable s, ballooning parameter θ_k, field line variable α
  grid (126×21×101)

  Isosurfaces of the eigenvalues
  Constrained by symmetry: configuration and eikonal equation

- WKB Ray tracing
  Can characterize global eigenfunction, Dewar and Glasser (1983)
  Semiclassical quantization or quantum chaotic scattering method
  to estimate unstable maximum toroidal n.

Ballooning Mode Calculations

- Linear, ideal MHD energy principle:

  Plasma energy responds to deformation of plasma flux surfaces, $\xi(s)$
  Energy minimization, radially local limit: local to a given field line.
  Large $k_{\perp}$. Small $k_{//}$. 2D space perp to field line.
  Asymptotically, for toroidal mode number $n \rightarrow \infty$, => a second order ODE

- Ballooning equation for eigenvalue in Boozer magnetic coordinates

  $$\partial/\partial\theta\,[(C_p + C_s\,(\theta - \theta_k) + C_q\,(\theta - \theta_k)^2)\partial\xi/\partial\theta] + (1-\lambda)[d_p + d_s\,(\theta - \theta_k)]\xi = 0$$

  Coefficients $\{C_p,\ C_q,\ d_p,\ d_s\}$ depend on equilibrium magnetic geometry

- Displacement of the flux surface increases with a notional growth rate

  $\xi \propto \exp(\sqrt{\lambda t})$; instability if positive: $\lambda > 0$

  Instability depends on field line curvature,
  driven by strong pressure gradients, stabilized by field shear

  Eigenfunction ξ of axisymmetric case maximum at $\theta \sim \theta_k$

Ballooning equation

$$\partial/\partial\theta\,[(C_p + C_s\,(\theta - \theta_k) + C_q\,(\theta - \theta_k)^2)\partial\xi/\partial\theta]$$
$$+ (1-\lambda)[d_p + d_s\,(\theta - \theta_k)]\xi \;= 0$$

$C_p = g_{ss}/\sqrt{g} - B_s^2/(\sqrt{g}\,B^2)$

$C_s = 2q'(s)/q(s)\,[\,\mathcal{J}(s)B_s/\sqrt{g}\,B^2 - g_{s\theta}/(\sqrt{g}\,B^2)]$

$C_q = [q'(s)\;\psi'(s)]^2\,|\nabla s|^2 / (\sqrt{g}\,B^2)$

$d_p = [p'(s)/\psi'(s)][\sqrt{g}\,p'(s)/B^2 + [\mathcal{J}(s)\;\psi''(s) - \mathcal{I}(s)\Phi''(s)]/B^2 +$
$\quad -\partial\sqrt{g}/\partial s + \sqrt{g}\,B\bullet\nabla(B_s/B^2)]$

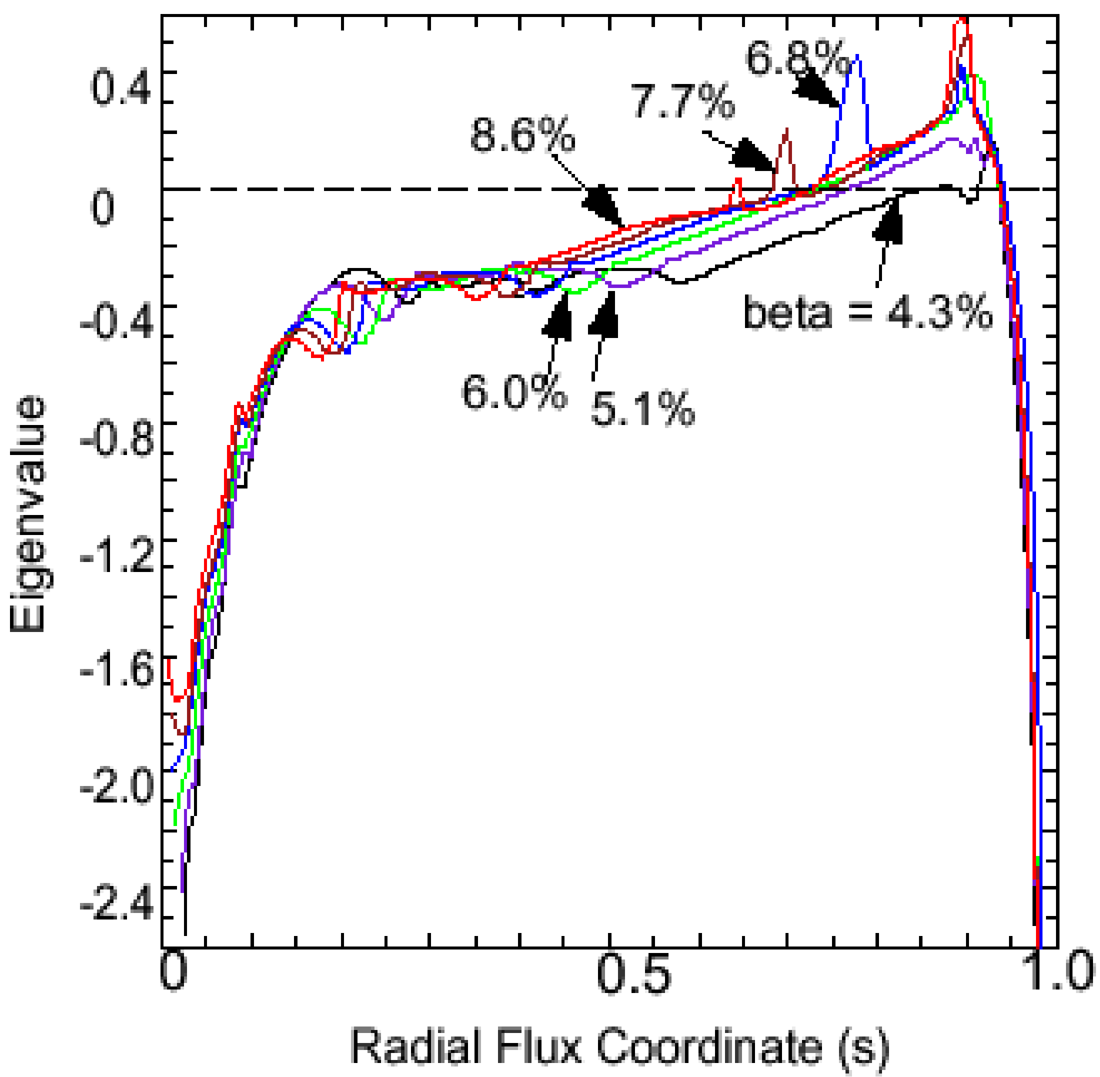
$d_s = -\,[q'(s)/\psi'(s)]\,\sqrt{g}\,B\bullet\nabla(j\bullet B/B^2)$

Coefficients d_s and C_s of the linear secular terms
are proportional to the (global magnetic) shear
Coefficient of the quadratic secular term, C_q,
is proportional to the square of the shear.
Radial coordinate, s: edge normalized toroidal flux, proportional to r^2.
θ_k: related to the direction of the mode wave vector
Solutions parametrized by field line variable, $\alpha = \zeta - q\theta$

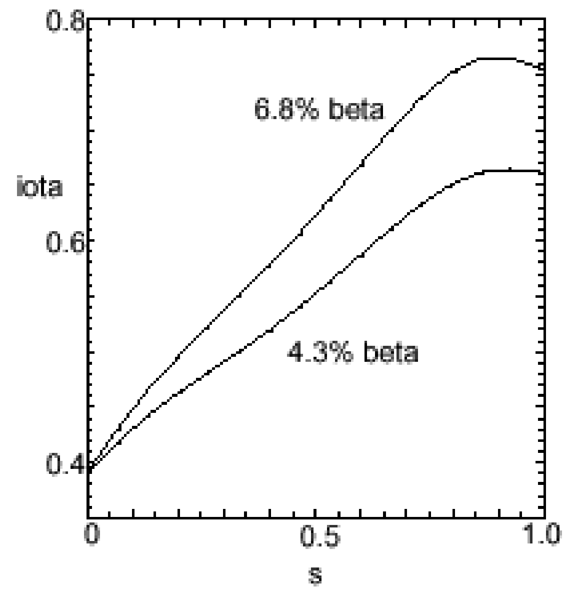
Secular terms cause localization when the shear is nonzero:
very roughly the eigenfunction is localized around $\theta \sim \theta_k$

β Scan of NCSX, with fixed boundary shape

Ballooning Eigenvalues ($\theta_k,\alpha=0$)



Vanishing shear
near plasma edge

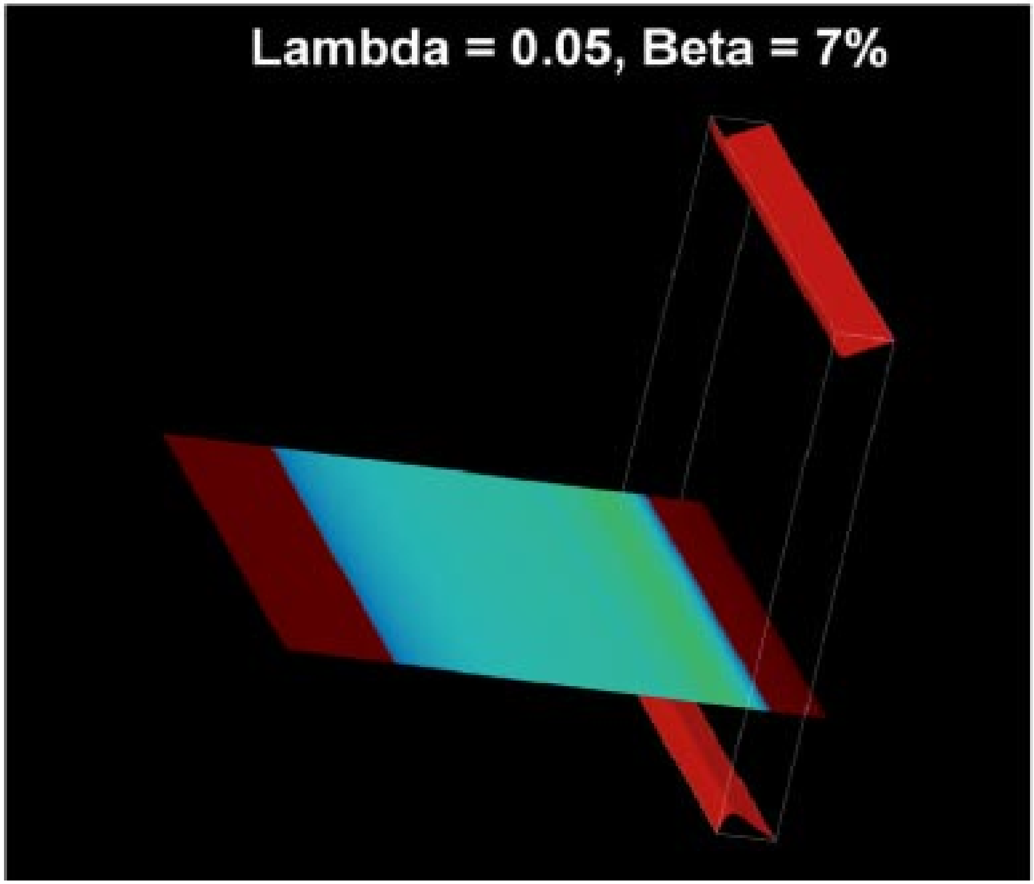


7% β Tokamak

QAS with only n=0 boundary components
No alpha dependence
Isosurfaces and (s,α) plane at $\theta_k \sim \pi$

Stable Isosurface

Unstable Isosurface



Ballooning Mode Eigenvalue Isosurfaces for a Quasiaxially Symmetric Stellarator, National Compact Stellarator Experiment (NCSX), 4.3% beta equilibrium, above the design point (4.1%)

Axes: toroidal flux, s; field line variable alpha, ballooning parameter theta_k

Scan from stable (negative) to unstable (positive) eigenvalues.

Zoom to near plasma edge, s=0.8 to 1.0

Marginal point at eigenvalue = 0.

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Anderson Localization - Condensed Matter

- Schrödinger equation:

$$[d^2/d\theta^2 + E - V(\theta)\,]\Psi(\theta) = 0$$

- Condensed matter example:

  crystal, electron wavefunctions, V modified by "impurity doping"

  Periodic potential => periodic solution

  Small perturbation of potential => nearly periodic eigenfunction?

  No.

  Random disturbance of the potential

  => localization of eigenfunctions,

  exponentially localized in space as disorder increases

- P. W. Anderson discovery, condensed matter (1958, Nobel prize 1977)

  Important also in acoustics, nonlinear optics, field theories

  Broken symmetry => localization

QAS: Anderson Localization at 4.3% β

- If global shear = 0, axisymmetric localization effects vanish

  Broken axisymmetry => localization 3D QAS configuration
  analogous to Anderson localization in condensed matter

- Cast the ballooning equation into Schrodinger equation form:

$$[d^2/d\theta^2 + E - V\,]\mathcal{A}^{1/2}\,\xi = 0$$

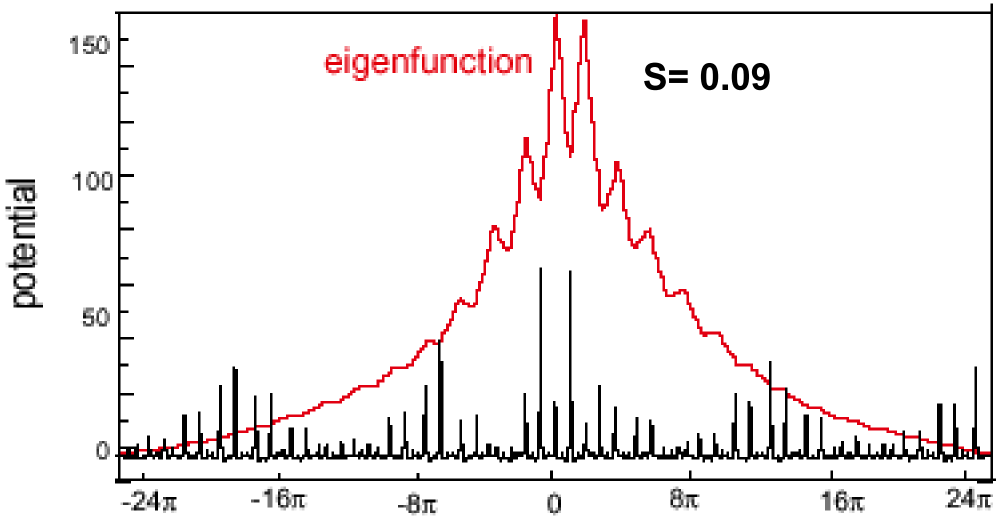
$$\mathcal{A} = 1/J|\nabla\psi|^2 + [|\nabla\psi|^2/J_{ac}\,B^2]\,[\mathcal{R}+(\partial_\psi q)(\theta-\theta_k)]^2$$

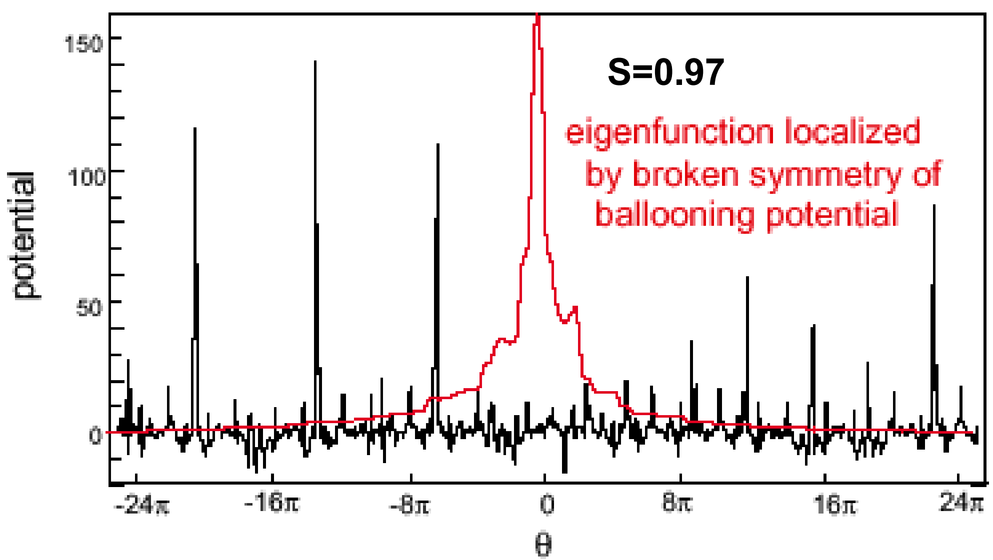
$\mathcal{R} = (q\,\nabla\psi\bullet\nabla\theta - \nabla\psi\bullet\nabla\zeta\,)\,/\,|\nabla\psi|^2$ Integrated residual shear

V and $\mathcal{A}$ depend on local parameters; vary with θ

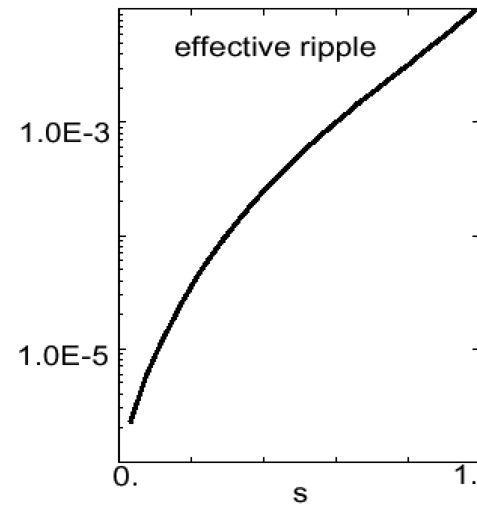
- Toroidal localization in stellarators (does not occur in tokamaks)
  LHD (Chen,Nakajima,Okamoto 1999)
  H1 Heliac (Cuthbert, Dewar 2000) Quasiperiodicity of “potential”
  HSX (Hegna, Hudson 2001)
  Quasiperiodicity of local parameters: incommensurate periods of toroidal and poloidal variations on irrational q field line

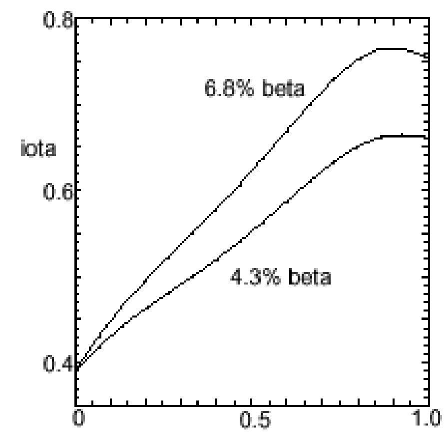
Anderson Localization in QAS





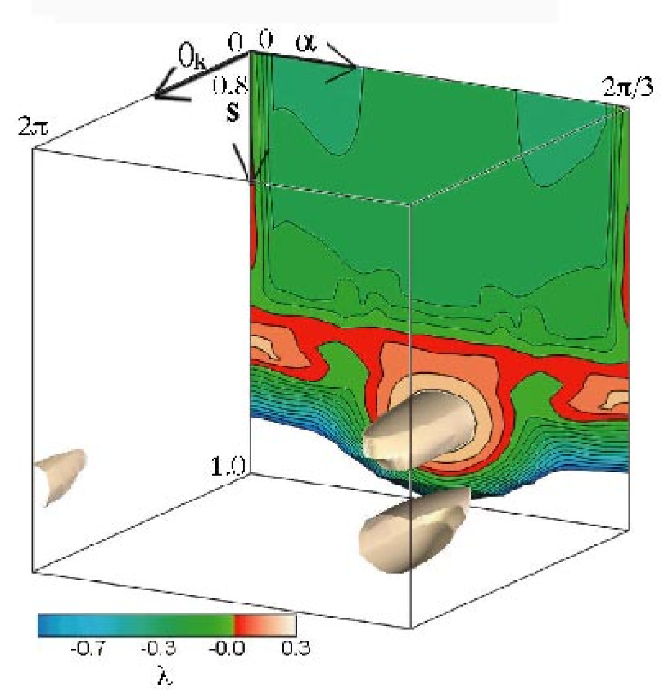
Effective Ripple: increases stochastic losses of fast particles in stellarators; measure of “disorder” - drives eigenfunction localization





"Quantum Chaos"

- At 4.3% beta, highest eigenvalues λ,
  spherical isosurfaces, indicative of strong "quantum chaos"
- Description for the paths of rays of the ballooning equation
  does not mean that the plasma behavior is chaotic
- Mathematics of a quantum chaotic scattering problem
  can be used for instabilities
  far above the marginal point
  of the equilibrium
- Use density of states method
  from quantum chaotic scattering
  physics to estimate n_{max}



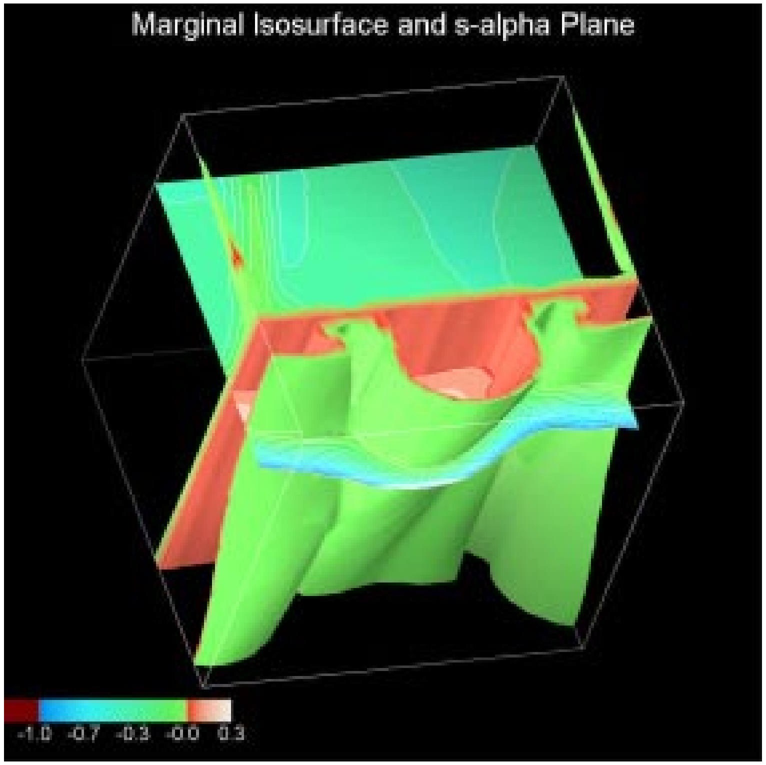
Will the Ballooning Mode Limit QAS Beta?

- Ideal MHD ballooning theory for $k_\perp L_{eq}$>>1, $(k_\perp \rho_i)^2$<<1
  Estimate $k_\perp$ from isosurfaces,
  Beta limit: marginal isosurface, λ=0
  Ray tracing in progress to derive semiclassical quantization conditions

- For QAS: topologically cylindrical surfaces connected by plane (θ_k, α)
  Other stellarator cases different: topologically cylindrical surfaces aligned along s or α, not along θ_k.

- Isosurface complexity of QAS near design point => ballooning codes (COBRA, VVBAL) and WKB method impractical for QAS global stability estimate

- May need fully 3D, ideal MHD codes (CAS3D, TERPSICHORE) to assess high n stability



Conclusions

- Applied hybrid local/global WKB approach to QAS ballooning stability
  - Identified distinctive, unique eigenvalue isosurfaces
    for stable and unstable spectra of QAS and tokamak
  - Spherical isosurfaces at high eigenvalues: quantum chaos
  - Anderson localization of ballooning eigenfunction
    found near zero shear for high effective ripple

- Ballooning calculations of beta limit are always pessimistic:
  local calculations may not reflect full plasma response.
  What will be the experimental beta?

- WKB ray tracing method to identify global internal, high n instability appears unworkable for the QAS

- **Fully 3D MHD codes appear necessary**
  for high n stability calculations of QAS β

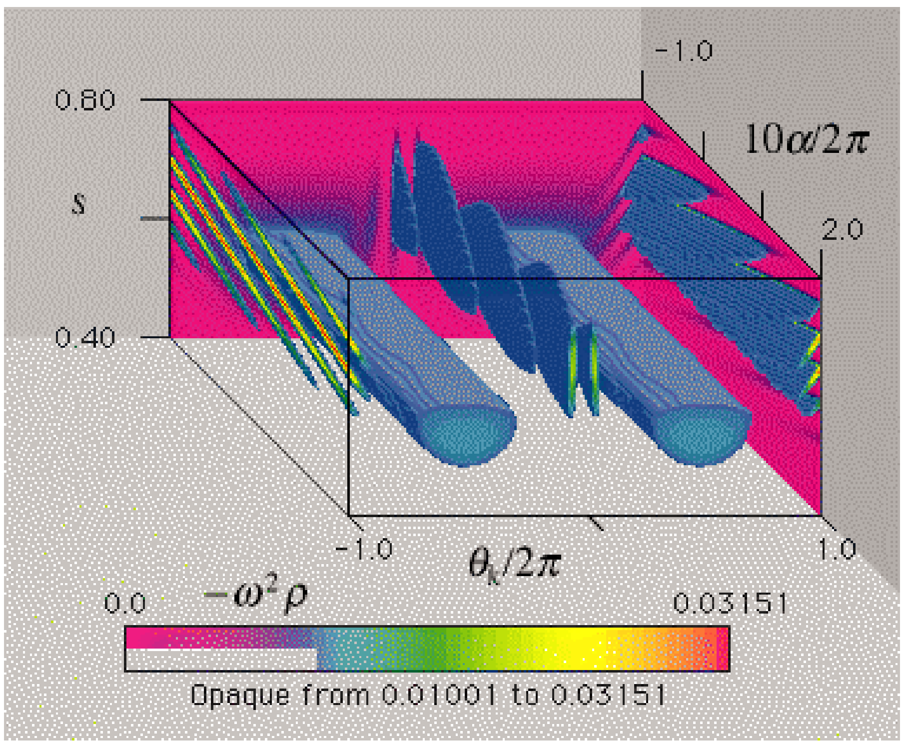
Further Work

- Can ray tracing be carried out with complex marginal isosurfaces, including all possible ray trajectories in a renormalized mathematics?
- Will grid refinement change isosurfaces? Do other energy normalizations of the ballooning equation yield different structures?
- Can semiclassical quantization conditions be formulated?
- How will the topology of the marginal isosurfaces and ballooning eigenfunction localization affect anomalous transport?
- Can optimization of the eigenvalue datacube properties be generalized and extended to improve device performance?

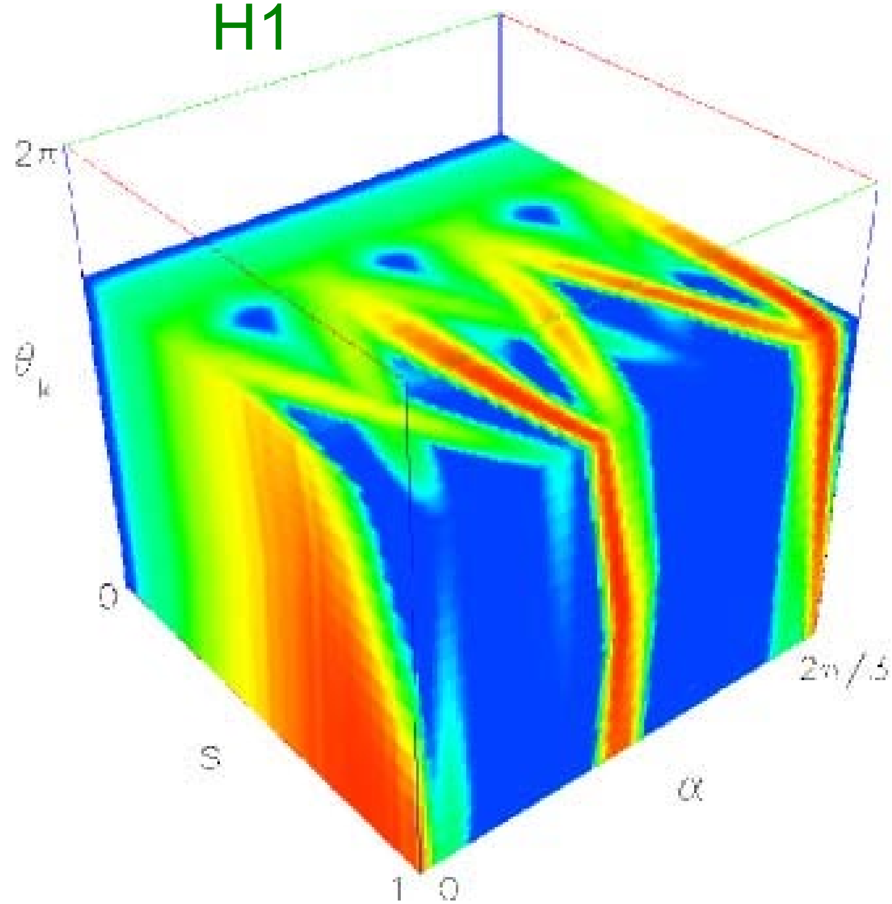
Ballooning eigenvalue datacubes

$\lambda(s, \alpha, \theta_k)$ Dewar,Cuthbert

LHD-like stellarator



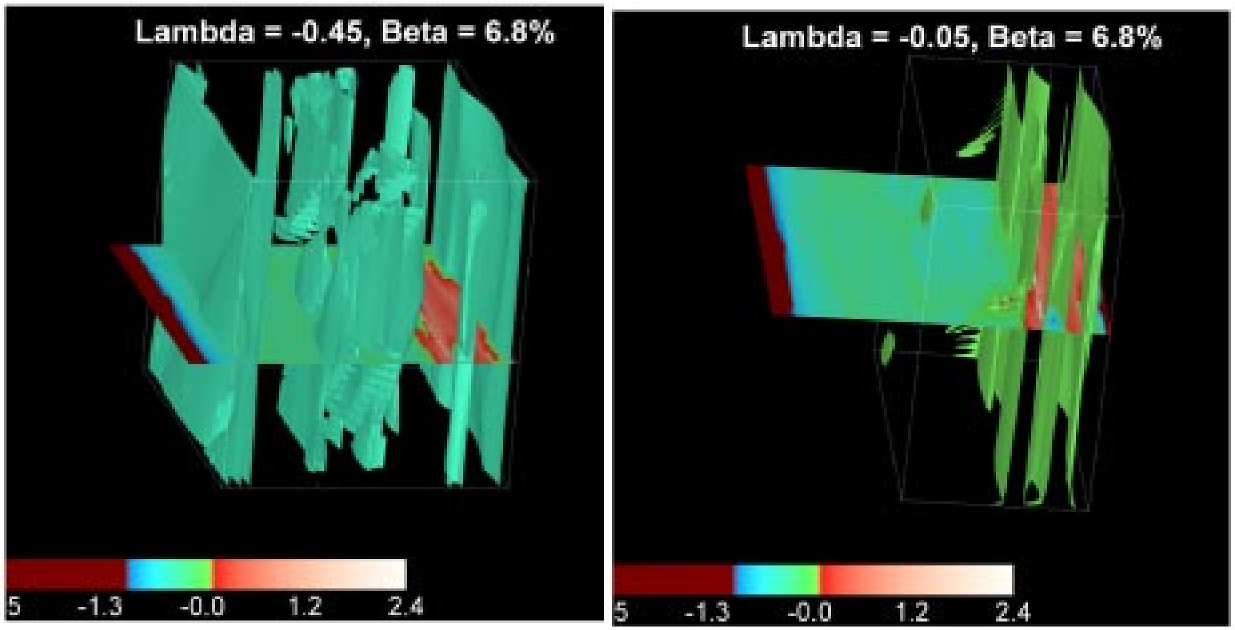
H1



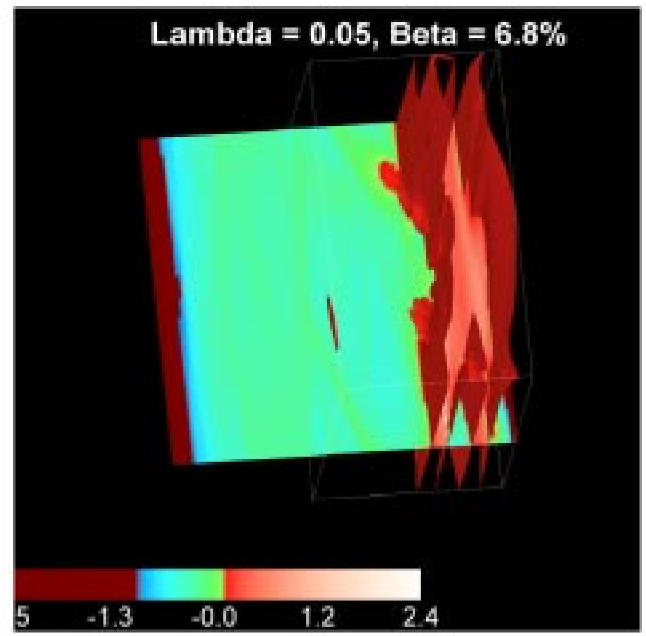
6.8% β QAS

More complex; features similar to 4.3% β

Stable Isosurfaces



Unstable Isosurface



Isosurfaces and (s,α) plane at θ_k~π

Results: Four dimensional data set

Datacube:

Eigenvalue (s,θ_k,α)

Display data:

eigenvalue isosurfaces at stable and unstable energies

3-space axes:

s, the toroidal flux;

θ_k the ballooning parameter

α , field line label

Tokamak: no α dependence in datacube

Stellarators: more complex: H1-NF, LHD

QAS: very complex - movie follows

which scans eigenvalue isosurfaces from stable to unstable energies