Neoclassical transport in NCSX

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Methods for calculating neoclassical transport in the National Compact Stellarator Experiment (NCSX) are discussed, with particular attention to developing computationally inexpensive predictions of neoclassical transport. Neoclassical transport is expected to set a lower bound on transport, and many actual stellarator plasmas are at, or not far from, this lower bound. The high degree of quasi-axisymmetry and the ambipolar radial electric field greatly reduce neoclassical transport due to helical ripple so axisymmetric neoclassical transport is dominant. It is hoped that the reduction in viscosity will lead to the suppression of microturbulence by permitting the development of zonal flows, large-scale sheared flows driven by neutral beam injection, and locally highly-sheared flows associated with internal and edge transport barriers.

In spite of its small contribution to particle and thermal fluxes, helical neoclassical transport is still important because it is expected to determine the viscosity and the non-ambipolar component of the radial particle flux. It therefore plays a large role in determining the radial electric field that, in turn, is needed to reduce the helical neoclassical transport to a small level. In the absence of unbalanced torque from neutral beam injection the helical transport can be expected to be small, but an accurate calculation of the viscosities and diffusivities is needed in order to estimate whether unbalanced torques could change E_r enough to lead to more significant ion particle and heat fluxes.

The magnetic geometry of NCSX can be modified significantly by changing the internal bootstrap currents and the independently driven external currents in the modular field coils. This makes it essential to develop *efficient* methods of estimating the helical neoclassical transport to avoid time-consuming 'first principles' re-calculation of the neoclassical transport as experimental operations explore the multi-dimensional configuration space.

Transport simulations for NCSX [1] have been based on solutions of the power balance equations using thermal diffusivities are made up of three parts: neoclassical axisymmetric transport, neoclassical helical ripple transport described below, and an anomalous transport model. The resulting estimated neoclassical ripple transport is negligible compared to the neoclassical axisymmetric transport. The self-consistent ambipolar radial electric field is in the 'ion-root' regime everywhere. A radially constant anomalous diffusivity is adjusted in the predictions to match a target thermal $\langle \square \rangle$ =3%, and this determines how much anomalous transport can be tolerated in the plasma core. We find that the anomalous transport exceeds the neoclassical transport in the outer 2/3 of the plasma and the two are comparable in the core.

The analytic model for neoclassical ripple transport used in these simulations began with the Shaing-Houlberg model [2, 3], which is based on asymptotic theory for two collisionality regimes ($1/\nu$ and ν) in a 'single helicity' magnetic configuration. Crume, et al., [4] improved the representation of the ν regime (confirmed in Ref. 5), and the $1/\nu$ regime was changed to more accurately reflect the original asymptotic theory and to use the effective helical ripple described below. The resulting model is identical to the one described in detail in Ref. 6.

The analytic model for the $1/\nu$ regime has been extended from single helicity to more complex magnetic configurations [7] and benchmarked against two other calculation methods [8]. The calculation of the transport is considerably simplified by assuming that particle motion within a flux surface is purely along field lines, ultimately allowing one to express the transport across the flux surface as a weighted integral of the geodesic curvature along a field line of 'infinite' length (i.e. sufficiently long to sample the entire magnetic surface). Evaluating the integral numerically provides an efficient means of determining the $1/\nu$ regime radial transport for any non-symmetric magnetic configuration. The validity of the result is confined to this regime, however, as a consequence of the assumption of negligible cross-field drift *within* a flux surface. While the $1/\nu$ regime is strongly influenced by the details of the magnetic-field geometry, the in-surface E_rxB drifts are more decisive in the $\sqrt{\nu}$ and ν regimes.

The effective helical ripple for three NCSX configurations is shown in Figure 1, where the variation in magnitude and shear of the rotational transform is also illustrated. The radial profile of the effective helical ripple exhibits a large range in magnitude and shape as a result of the operational flexibility of the device. For comparison we note that the effective helical ripple of W7-X is close to 0.01 at all radii, and that of ATF ranged from 0.3 near the edge to ~0.1 deep in the core. For typical magnitudes of the ambipolar radial electric field the electrons are in the $1/\nu$ regime so their neoclassical fluxes can be efficiently calculated, but the ions are in the $\sqrt{\nu}$ or ν regimes and more complex calculations are needed.

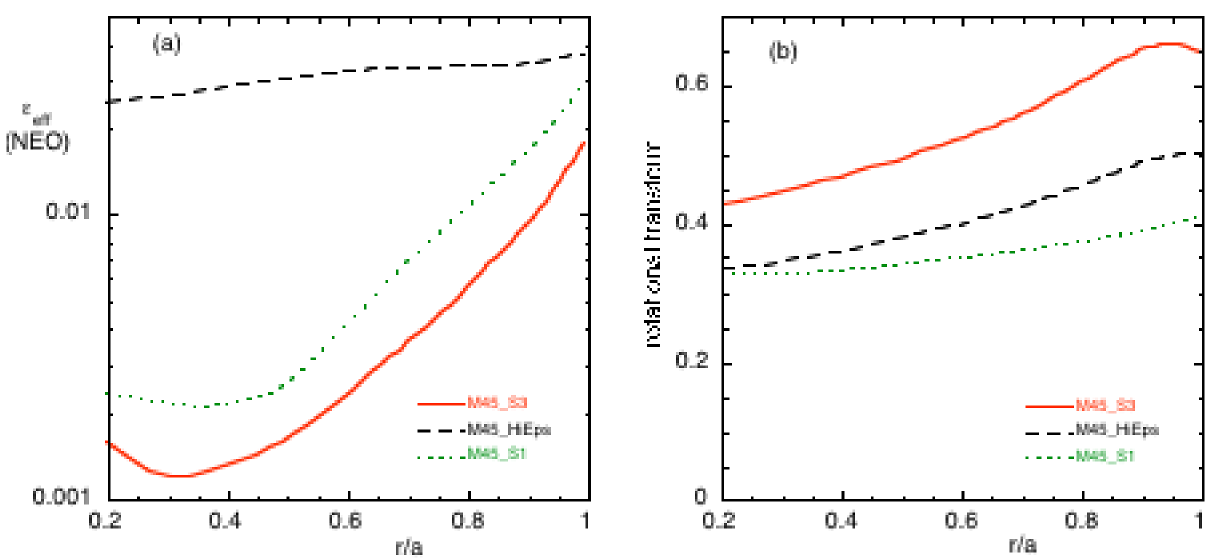

Figure 1. a) the NEO code's effective helical ripple for NCSX vs the square root of the normalized toroidal flux; b) the rotational transform for the same configurations in a).

'First-principles' neoclassical transport theories are based on a few fundamental diffusivities. The local ansatz underlying neoclassical transport theory allows an ordering of the drift kinetic equation so that the minor-radius and energy coordinates appear only as parameters, reducing a nominally 5D problem to a more manageable 3D. It also becomes possible to characterize neoclassical effects in terms of three monoenergetic coefficients describing the radial transport, the bootstrap current, and the parallel conductivity. The full neoclassical transport matrix is obtained by appropriate convolutions of the monoenergetic coefficients with the Maxwellian particle distributions.

Analytic theory and numerical simulation support each other: Mynick [9] approximately verified transport expressions for a single helicity magnetic configuration, and Beidler has shown that numerical simulations of neoclassical transport exhibit the expected collisionality dependences in a number of stellarator configurations [5, 10, 11,]. It is interesting to note, however, that the normalization of the analytic scalings is sometimes not correct even for simple model magnetic configurations (see Ref. 5 for details), and the normalization should be checked with numerical transport calculations for specific magnetic configurations and collisionalities.

Monoenergetic transport coefficients calculated by the DKES [12, 13] code and the Monte Carlo code MOCA [14] are shown in Figure 2 for r=0.9a. In the low collisionality regime, the normalized particle transport coefficient, Γ_{11}^*, approaches the equivalent axisymmetric result as the electric field is increased; and the expected magnitude of E_r/Bv_{th} for ions is greater than the $3x10^{-3}$ level shown in the figure. With the radial electric field required for ambipolar flux (as determined above) the transport should be close to the axisymmetric result. MOCA confirms the DKES results and is able to extend them to lower collisionality where DKES convergence is problematic.

To obtain an inexpensive calculation of neoclassical transport we represent the monoenergetic diffusivities in terms of physically motivated basis functions and seek methods of easily calculating (or estimating) the fit coefficients. A semi-analytic representation of the D_{11}^* coefficient has been developed as part of the international collaboration on neoclassical transport in stellarators [8]. This physics-based representation is derived from a semi-analytic model for D_{11}^* originally developed for classical stellarators. The model characterizes the transport as a sum of three terms, $D_{11} = D_{axi} + D_{lmfp} + D_{add}$, where D_{axi} contains the usual Pfirsch-Schlüter, plateau and banana regimes expected for the axisymmetric field $B = B_o(1 + b_T \cos\theta)$ (for a stellarator, these regimes are only relevant when the mean free path is short), D_{lmfp} describes the stellarator-specific long-mean-free-path regime and D_{add} is an "additional" contribution relevant only when $D_{axi} \sim D_{lmfp}$.

For each configuration the fit parameters are determined by least-squares minimization of the errors. The fitting is done in two steps, beginning with the results obtained for zero electric field. Here, D_{lmfp} is given by the asymptotic result (with ϵ_{eff} most efficiently determined by NEO) and least-squares fitting is used to determine "best" values of b_T and two free coefficients (magnitude and saturation) in D_{add}. With this step completed, D_{lmfp} is then given for arbitrary values of the electric field by means of an extremely efficient solution of the bounce-averaged kinetic equation for a model field of the form $B = B_o(1 + b_T \cos\theta - \epsilon_h(1 - \mu\cos\theta)\cos\phi)$, which is the simplest model field capable of describing strong drift optimization. Least-squares fitting is now carried out to determine values of ϵ_h, μ and a_I, a third quantity appearing in the boundary conditions (physically, this quantity controls the importance of collisionless trapping and detrapping in the local ripples of the magnetic field).

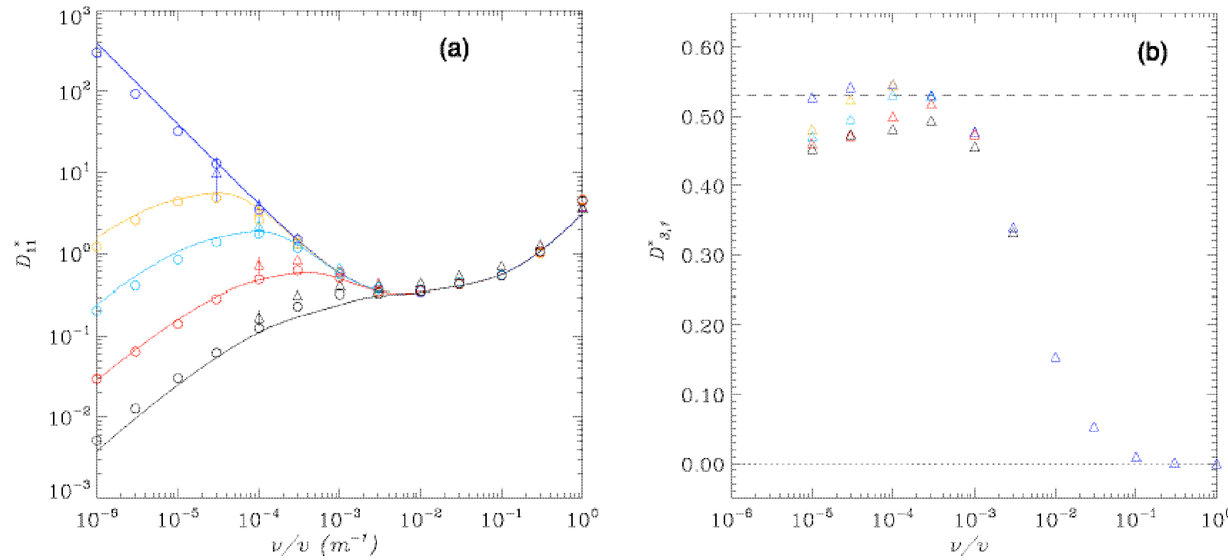

Figure 2. DKES (triangles) and MOCA (circles) results for NCSX: (a) monoenergetic particle transport coefficient at r/=0.9 normalized to the plateau value of an equivalent circular axisymmetric configuration, and (b) the normalized bootstrap current coefficient at r/=0.5. The abscissa is the inverse of the mean free path. Radial electric field values: $E_r/(Bv)$= 0 blue, $3\text{x}10^{-5}$ green, $1\text{x}10^{-4}$ yellow, $3\text{x}10^{-4}$ cyan, $1\text{x}10^{-3}$ red, $3\text{x}10^{-3}$ black.

The radial variation of the fit parameters is shown in Figure 3 for the standard NCSX configuration. Fits to MOCA results for several other configurations will be made and simple prescriptions for the parameters will be sought; it is hoped that this will be successful enough to reduce the need to use DKES or MOCA to calculate D_{11}^* in new NCSX configurations. The parameters related to the model field, ϵ_h, μ, a_I, as well as b_T might be obtained from the Boozer spectrum, but this hypothesis has not been tested. Predicting the parameters A and B is more problematic and a more physically motivated replacement is being developed.

The pitch-angle-scattering collision operator used in these codes does not conserve momentum, so the usual method of generating the transport matrix is incorrect for quasi-symmetric configurations. It is possible, however, to use a correction procedure proposed by Sugama [15] that employs the Hirshman-Sigmar moment method. The fluid momentum balance and friction-flow relations (which include collisional momentum conservation) are used with the viscosity-flow relations derived from the drift kinetic equation solutions. The procedure also takes advantage of the fact that the calculated viscosities are less affected by the defects of the collision operator than the particle and thermal diffusivities. In quasi-symmetric configurations it is *necessary* to use a correction procedure to obtain physically meaningful predictions for radial

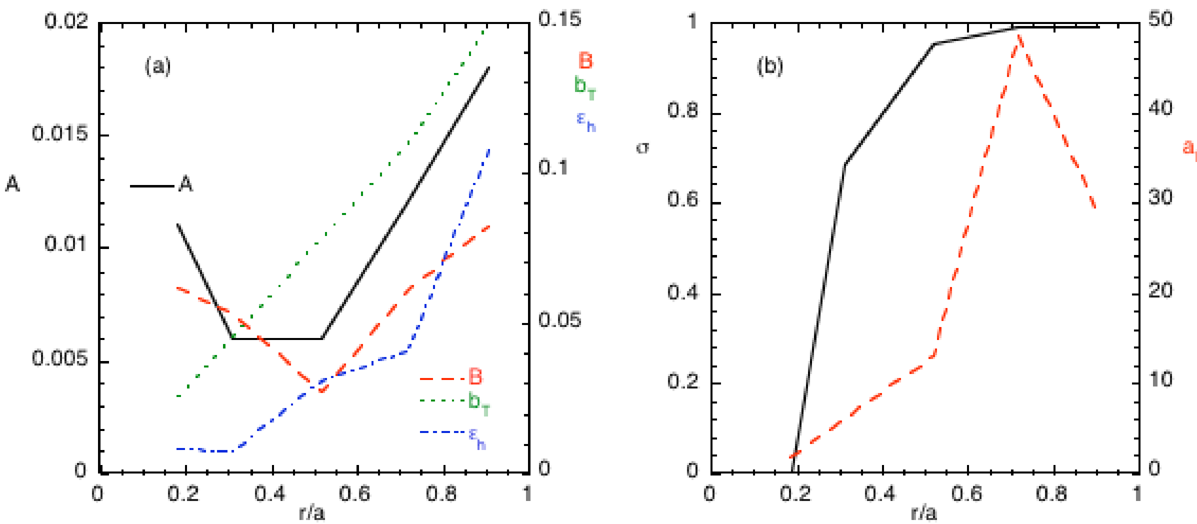

Figure 3. Values of the parameters used in the fits to D_{11}^{*} for several radii in the standard NCSX configuration.

fluxes from the DKES/MOCA diffusivities, and preliminary steps in this direction have been taken. Note that the correction procedure links the D_{13} and D_{33} coefficients to the calculation of the particle and energy fluxes, so reliable representations of these are also needed. In NCSX the D_{33} coefficient has essentially no dependence on E_r, but the dependence on configuration is not known yet.

DKES is the only tool available for determining the bootstrap current coefficient, D_{13}, for arbitrary values of collision frequency and radial electric field in stellarators. A typical result is provided in Figure 2b, in which the monoenergetic bootstrap current coefficient, normalized to the limiting value of the equivalent axisymmetric tokamak (with circular flux surfaces), D_{31}^{*}, is plotted as a function of inverse mean free path for the flux surface at half the plasma radius in the standard NCSX configuration. The theoretical prediction by Shaing and Callen [16] – valid only for vanishing □□ is indicated by the dashed line. The bootstrap coefficient shows much less dependence on E_r than is usual for stellarators but this *is* typical of tokamaks. These results exhibit a rather small dependence of D_{31}^{*} on the value of the radial electric field for experimentally relevant values of □/v, and the theoretical low □ limit is accurate. If the □ dependence could be provided by analytic predictions then DKES would not be required to provide D_{31}^{*}.

Acknowledgements

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