Computational Study of LHD Equilibrium

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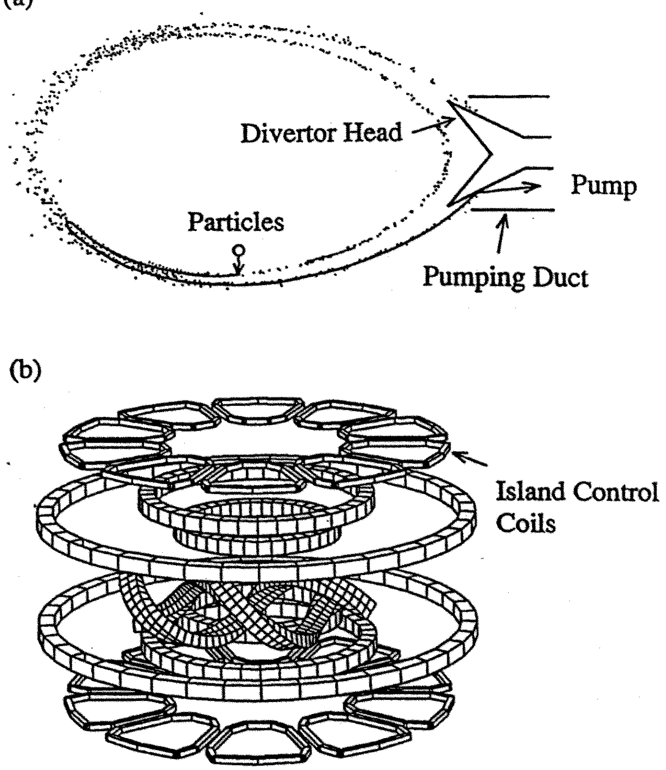
* LHD Equilibrium with m/n=1/1 island
* Standard scheme of HINT Code
* Modification of the Code
* Summary

Outline

- A modification of the three dimensional MHD equilibrium code, HINT, is proposed to analyze a Large Helical Device (LHD) equilibrium with an m/n=1/1 island.
- An LHD equilibrium with the m/n=1/1 island is required in the local island divertor (LID) experiment.
- In a calculation of the equilibrium, there is a numerical problem caused from a fact that a derivative of pressure at the edge of the island is not well-behaved; i.e. it is diverged.
- The modified scheme has the ability to resolve the problem, and achieves a numerically appropriate analysis of the equilibrium with the island.

Local Island Divertor (LID) using m/n=1/1 island

(a)



(b)

* The LID is a divertor that uses an m/n=1/1 island formed at the edge region.
* The island is generated by island control coils.
* Particles crossing the island separatrix flow along field lines to target plates.
* The particles recycled there are pumped out by a pumping system.

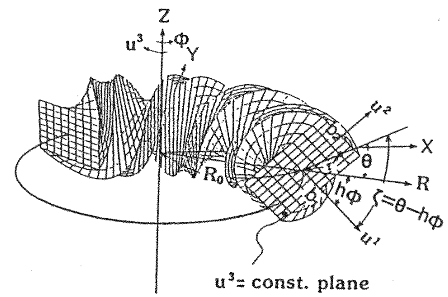
A.Komori *et al.*, Plasma Physics and Controlled Nuclear Fusion Research 1994 (IAEA, Vienna, 1995, vol.2) 773.
N.Ohyabu *et al.*, J. Nucl. Mater. **220-222** (1995) 298.

Standard Scheme of HINT Computation

vacuum field | **initial pressure**

(A) pressure relaxation: $p \to \bar{p} = \dfrac{\oint \frac{\mathrm{d}l}{B} p}{\oint \frac{\mathrm{d}l}{B}}$ $\quad \to \mathbf{B} \cdot \nabla p = 0$

helical coordinate (u^1, u^2, u^3)



(B) estimate net toroidal current:

$$\mathbf{j}_{\mathrm{net}} = \frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{net}}}{\langle \mathbf{B}^2 \rangle} \mathbf{B},$$

$\langle \cdots \rangle$ flux average by using contour lines of pressure

(C) relaxation of magnetic field:

$$\frac{\partial \rho \mathbf{V}}{\partial t} = f_{\mathrm{b}} \{ -\nabla p + (\mathbf{j} - \mathbf{j}_0) \times \mathbf{B} \}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{V} \times \mathbf{B} - \eta(\mathbf{j} - \mathbf{j}_{\mathrm{net}} - \mathbf{j}_0)]$$

MHD equilibrium:

$$\nabla p \approx (\mathbf{j} - \mathbf{j}_0) \times \mathbf{B}$$

Treatment of net toroidal and coil currents

(C) relaxation of magnetic field:

$$\frac{\partial \rho \mathbf{V}}{\partial t} = f_{\mathrm{b}}\left\{-\nabla p + (\mathbf{j}-\mathbf{j}_0)\times \mathbf{B}\right\}$$

$$\text{where}\quad f_{\mathrm{b}} = \begin{cases} 1 & \text{for } B \le B_c \\ (B_c/B)^2 & \text{for } B > B_c \end{cases}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} = \nabla\times\left[\mathbf{V}\times\mathbf{B} - \eta\left(\mathbf{j}-\mathbf{j}_{\mathrm{net}}-\mathbf{j}_0\right)\right]$$

where $\mu_0 \mathbf{j} = \nabla\times\mathbf{B}$

$\mu_0 \mathbf{j}_0 = \nabla\times\mathbf{B}_0$ $\qquad$ $\mathbf{B}_0$: vacuum field

CFL condition:

$$\Delta t \le \frac{\Delta u}{V_{\mathrm{A}}} = \frac{\Delta u\sqrt{\mu_0 \rho}}{f_{\mathrm{b}} B}$$

Ohm's law: $(\mathbf{j_0} = 0)$

$$\mathbf{E} + \mathbf{V}\times\mathbf{B} = \eta\left(\mathbf{j} - \mathbf{B}\langle \mathbf{j}\cdot\mathbf{B}\rangle_{\mathrm{net}} \big/ \langle B^2\rangle\right)$$

$$\Rightarrow \langle \mathbf{E}\cdot\mathbf{B}\rangle = 0 = \eta\left(\langle \mathbf{j}\cdot\mathbf{B}\rangle - \langle \mathbf{j}\cdot\mathbf{B}\rangle_{\mathrm{net}}\right)$$

LHD equilibrium with/without m/n=1/1 island ($\beta \approx 2\%$)

- LHD equilibrium with the island:

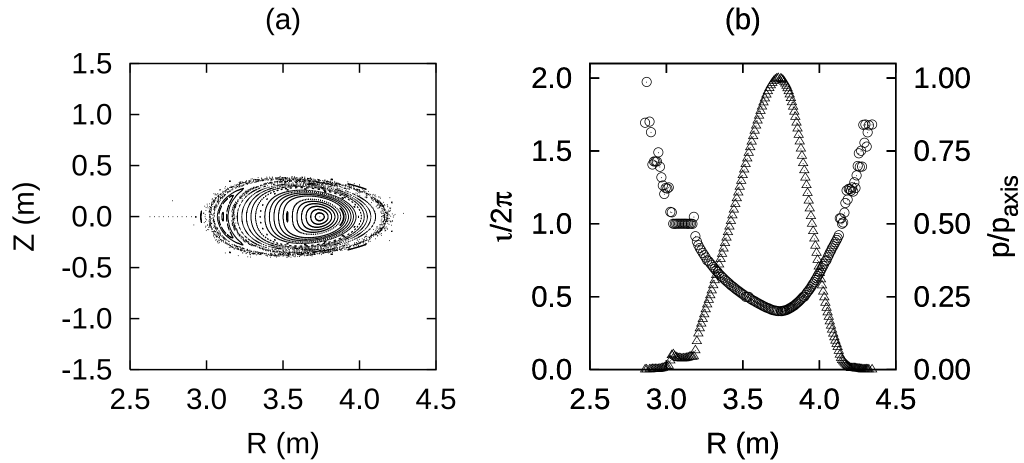
magnetic axis: $R_{ax} = 3.74\,\mathrm{m}, Z_{ax} = 0\,\mathrm{m}$

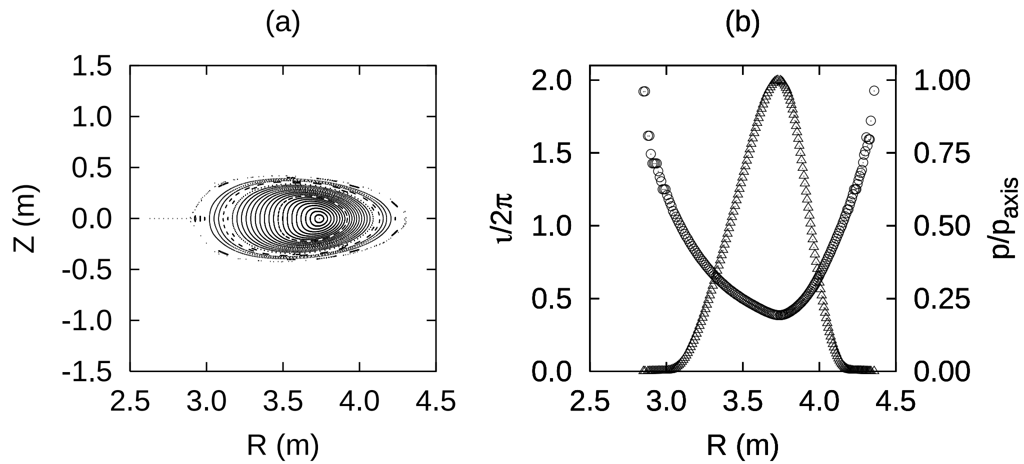
O-point of island: $R_{is} = 3.105\,\mathrm{m}, Z_{is} = 0\,\mathrm{m}$

island width: $w_{is} = 12.6\,\mathrm{cm}$

- LHD equilibrium without the island:

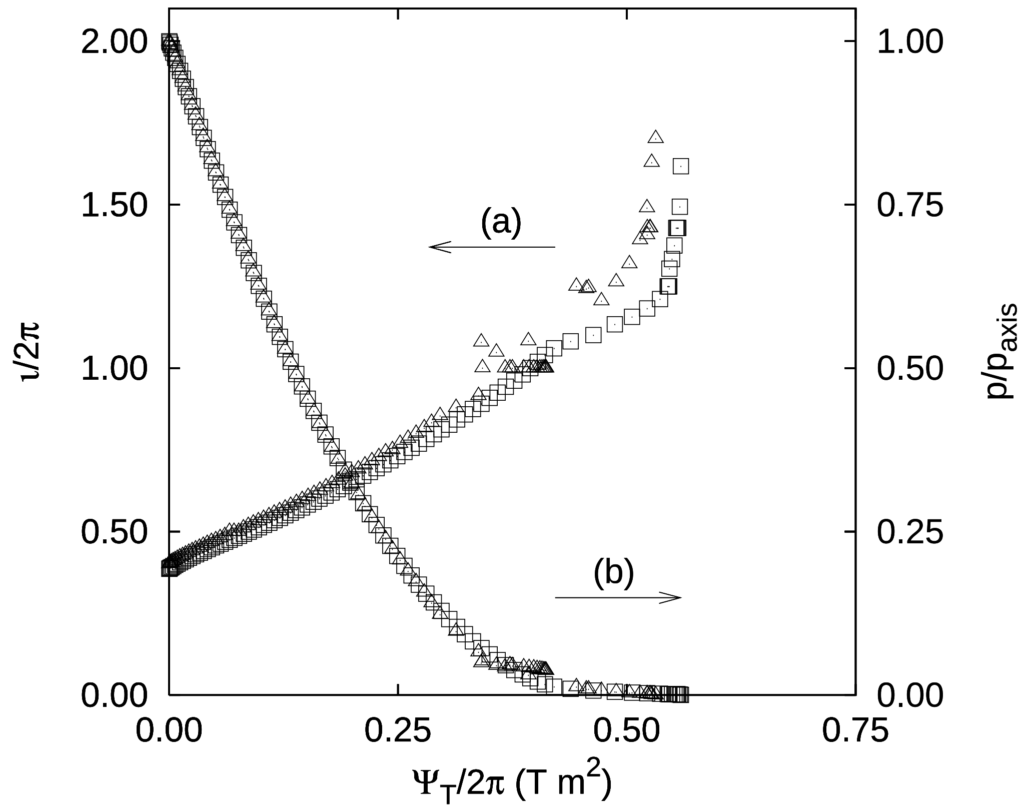
magnetic axis: $R_{ax} = 3.74\,\mathrm{m}, Z_{ax} = 0\,\mathrm{m}$





Comparison for rotational transform and pressure

- Profiles of rotational transform and pressure as functions of toroidal flux are similar.
- The toroidal flux is defined by using contour lines of pressure.



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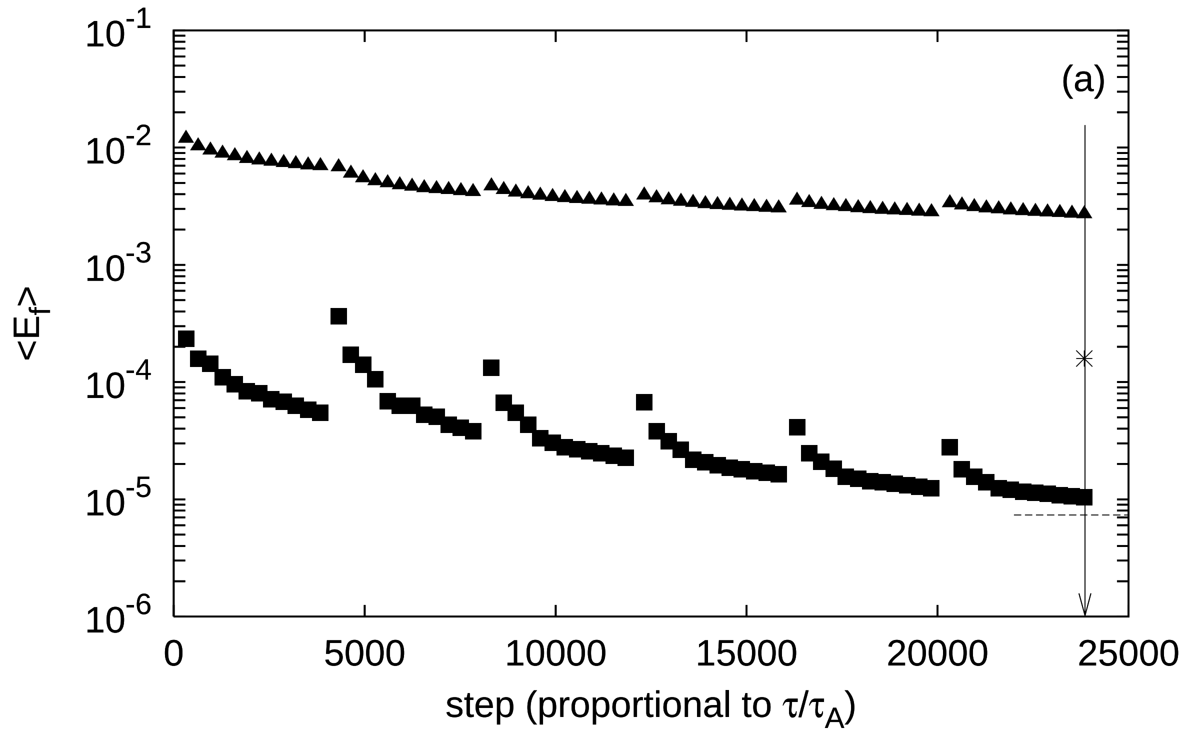
Numerical analysis of the equilibrium with the island seems to succeed outwardly… ??

open triangle: with island
open square: without island

Error for equilibrium having islands without toroidal period of helical coils (LHD)



$$\langle E_f \rangle = \frac{\int dV \left| \nabla p - \mathbf{j} \times \mathbf{B} \right|^2}{\int dV \left\{ \left| \nabla p \right|^2 + \left| \mathbf{j} \times \mathbf{B} \right|^2 \right\}}$$

solid triangle: with island
asterisk: with island (core region)
solid square: without island
dash line: level of error in a previous study
(W7X, 73x73x41; half pitch)

The equilibrium with the island is not numerically appropriate as compared with the case without the island.

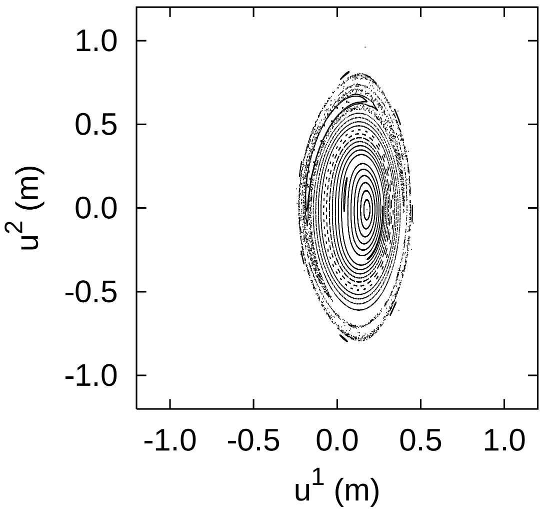
Error concentrates on edge of island

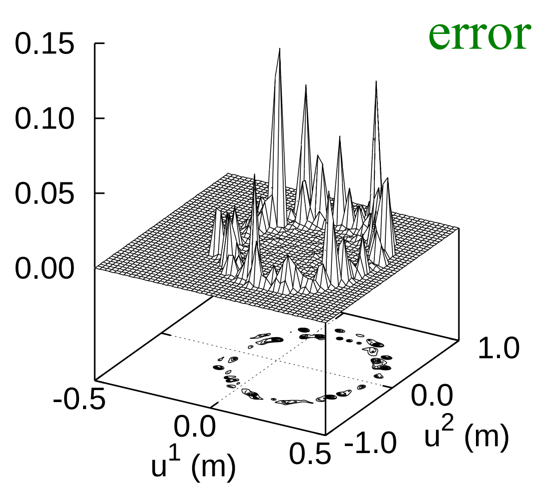
- Error of force balance in the core region is negligibly small.

→ The error in the core seems to be sufficiently reduced.

- Error of force balance concentrates extremely on the edge of the island.

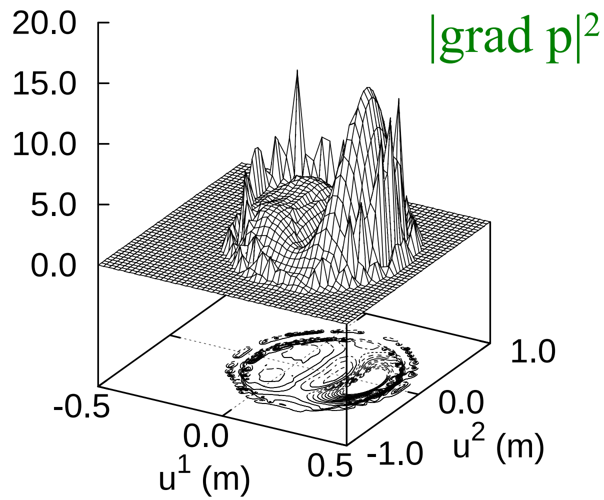
→ Source of the error is fixed on an error of grad p calculation.



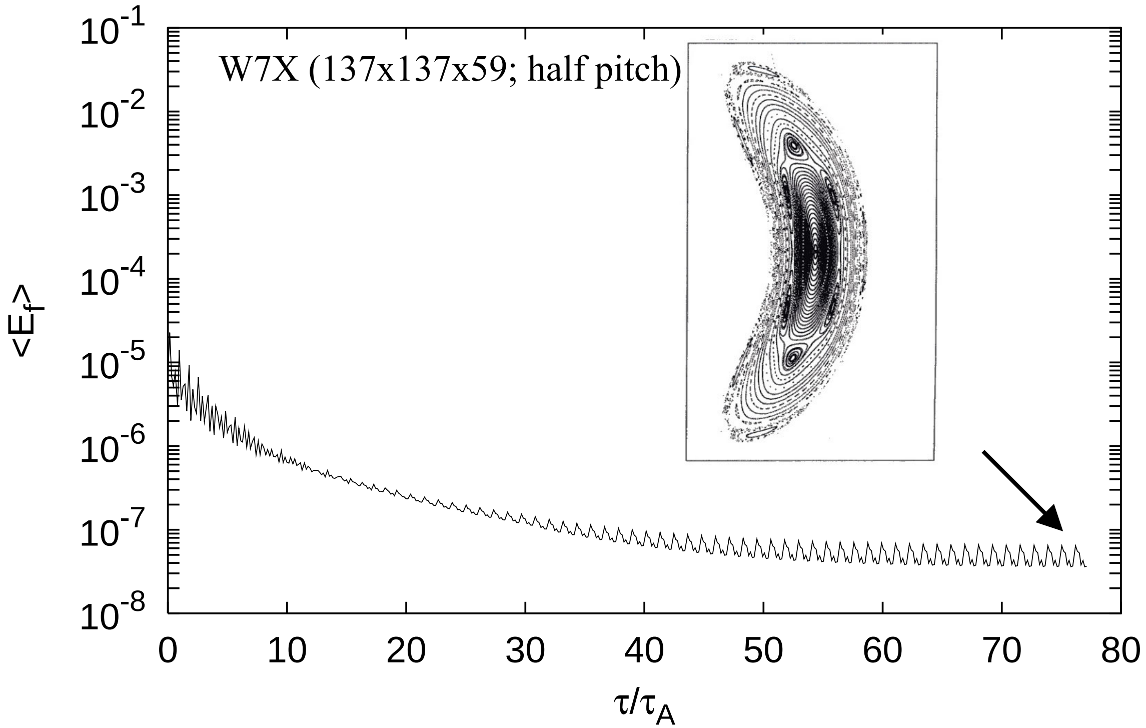


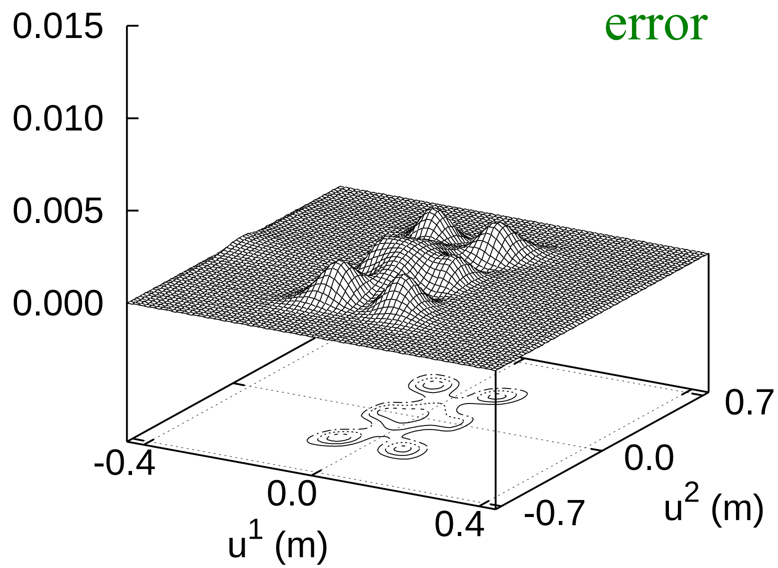
$$p \rightarrow \bar{p} = \frac{\oint \frac{\mathrm{d}\,l}{B}\, p}{\oint \frac{\mathrm{d}\,l}{B}}$$

→ Propagation of information on pressure across a field line is neglected.



Error for equilibrium having islands **with** toroidal period of helical coils (W7X)





When islands yield Fourier components of magnetic fields with toroidal period of helical coils, the problem is not so serious.

Diffusion process of error

(A) pressure relaxation:

$$p \rightarrow \bar{p} = \oint \frac{\mathrm{d}l}{B} p \Big/ \oint \frac{\mathrm{d}l}{B}$$

$$\frac{\partial p}{\partial t} = \kappa \nabla \cdot \left[\nabla p - \left(\mathbf{j} - \mathbf{j}_0 \right) \times \mathbf{B} \right]$$

$$\frac{\partial p}{\partial t} = -\mathbf{V} \cdot \nabla p - \gamma p \nabla \cdot \mathbf{V} + \rho \nabla \cdot \left[\overleftrightarrow{\kappa} \cdot \nabla \left(\frac{p}{\rho} \right) \right]$$

$$\Rightarrow \begin{cases} p \rightarrow \bar{p} = \oint \frac{\mathrm{d}l}{B} p \Big/ \oint \frac{\mathrm{d}l}{B} \\ \frac{\partial \left(p - p_* \right)}{\partial t} = \kappa \nabla^2 \left(p - p_* \right) = \kappa \nabla \cdot \left(\nabla p - \mathbf{j} \times \mathbf{B} \right) \end{cases}$$

where $\mathbf{j_0} = 0$, $\nabla p_* = \mathbf{j} \times \mathbf{B}$ and $\partial p_* / \partial t = 0$

$$\int \mathrm{d}V \left| \mathbf{B} \cdot \nabla p \right| \Big/ \int \mathrm{d}V \left| \mathbf{B} \right| \cdot \left| \nabla p \right| \leq 10^{-7}$$

When grad p is not well-behaved, it is natural to consider the perpendicular diffusion.

Modified Scheme of HINT Computation

vacuum field → **initial pressure**

(A) pressure relaxation:

$$p \Rightarrow \bar{p} = \oint \frac{\mathrm{d}l}{B} p \Big/ \oint \frac{\mathrm{d}l}{B}$$

$$\frac{\partial p}{\partial t} = \kappa \nabla \cdot \left[\nabla p - \left(\mathbf{j} - \mathbf{j}_0 \right) \times \mathbf{B} \right]$$

$\rightarrow$ $\mathbf{B} \cdot \nabla p = 0$

(B) estimate net toroidal current:

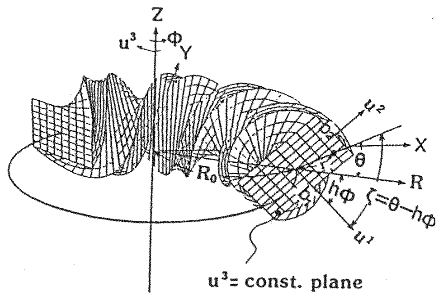
$$\mathbf{j}_{\mathrm{net}} = \frac{\left\langle \mathbf{j} \cdot \mathbf{B} \right\rangle_{\mathrm{net}}}{\left\langle \mathbf{B}^2 \right\rangle} \mathbf{B}$$

(C) relaxation of magnetic field:

$$\frac{\partial \rho \mathbf{V}}{\partial t} = f_{\mathrm{b}} \left\{ -\nabla p + \left(\mathbf{j} - \mathbf{j}_0 \right) \times \mathbf{B} \right\}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{V} \times \mathbf{B} - \eta \left(\mathbf{j} - \mathbf{j}_{\mathrm{net}} - \mathbf{j}_0 \right) \right]$$

helical coordinate (u^1, u^2, u^3)



MHD equilibrium:

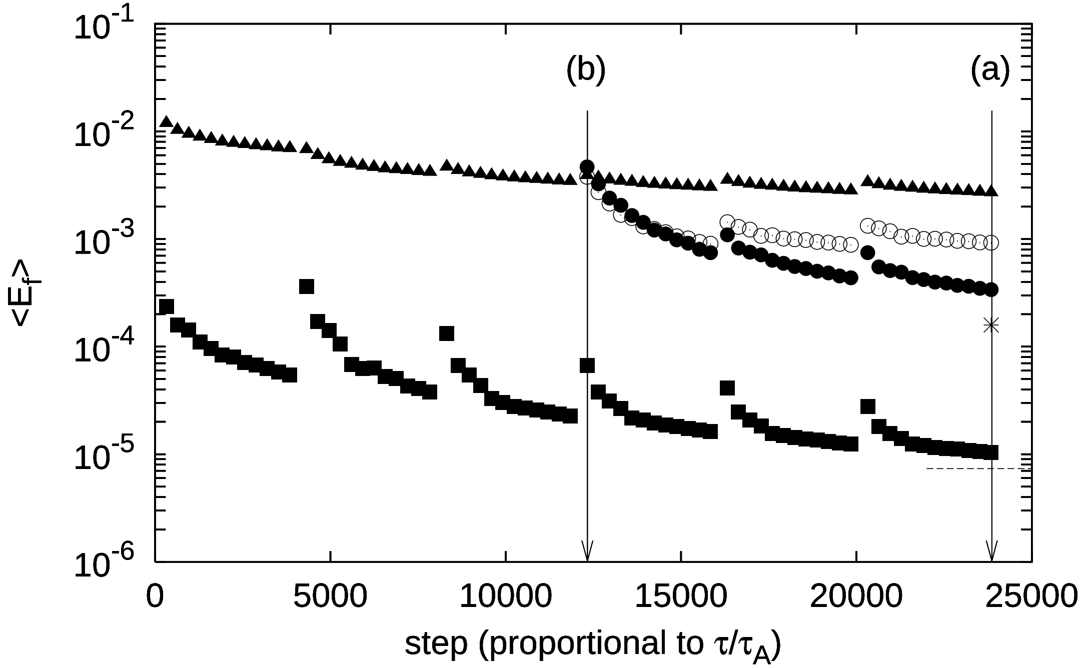
$$\nabla p \approx \left(\mathbf{j} - \mathbf{j}_0 \right) \times \mathbf{B}$$

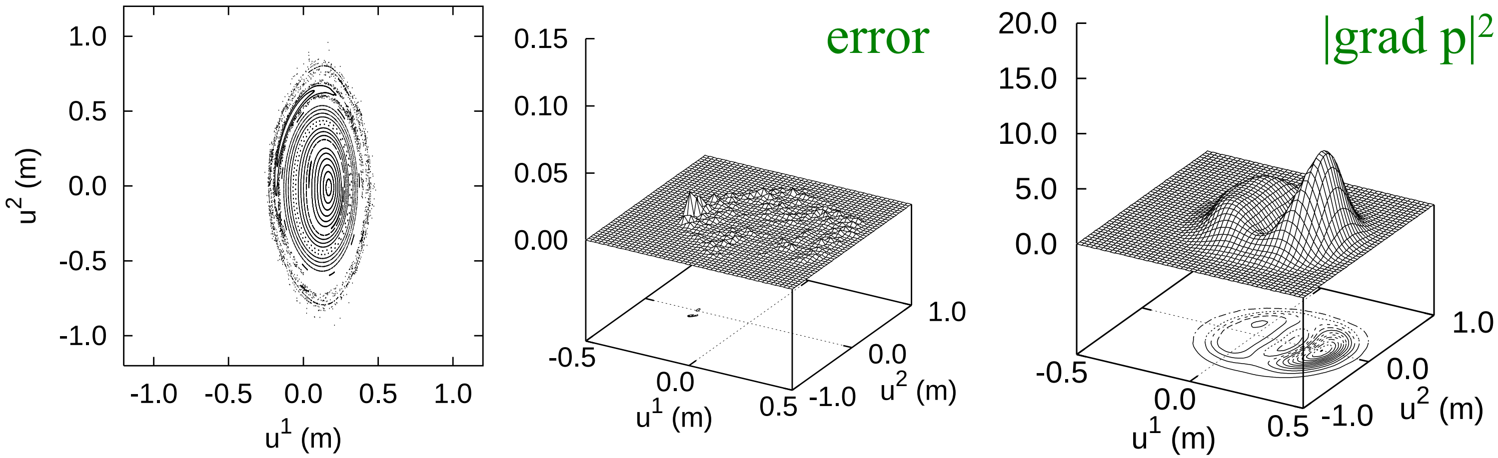
Error of force balance (Modified scheme)

- Error of force balance is improving clearly with reducing the mesh size.

→ Validity of modified scheme is justified.

- Error of grad p is also reduced.





solid triangle : standard (97x97x324)

solid square : standard (97x97x324)

solid circle : Modified (97x97x324)

open circle : Modified (49x49x324)

LHD equilibrium with m/n=1/1 island

$\beta \approx 2\%$

- **Modified scheme:**

magnetic axis: $R_{ax} = 3.73\,\mathrm{m}, Z_{ax} = 0\,\mathrm{m}$

O-point of island: $R_{is} = 3.104\,\mathrm{m}, Z_{is} = 0\,\mathrm{m}$

island width: $w_{is} = 13.3\,\mathrm{cm}$

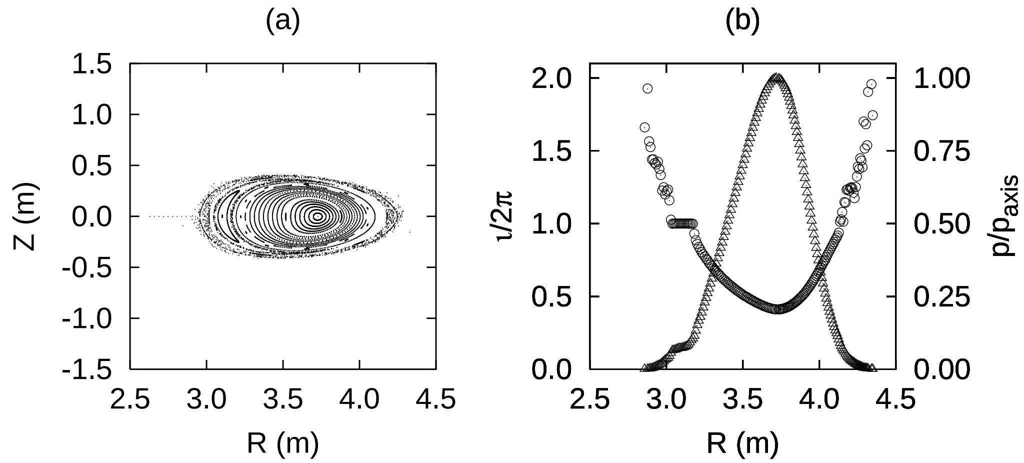
Central beta value decreases slightly due to the diffusion process.

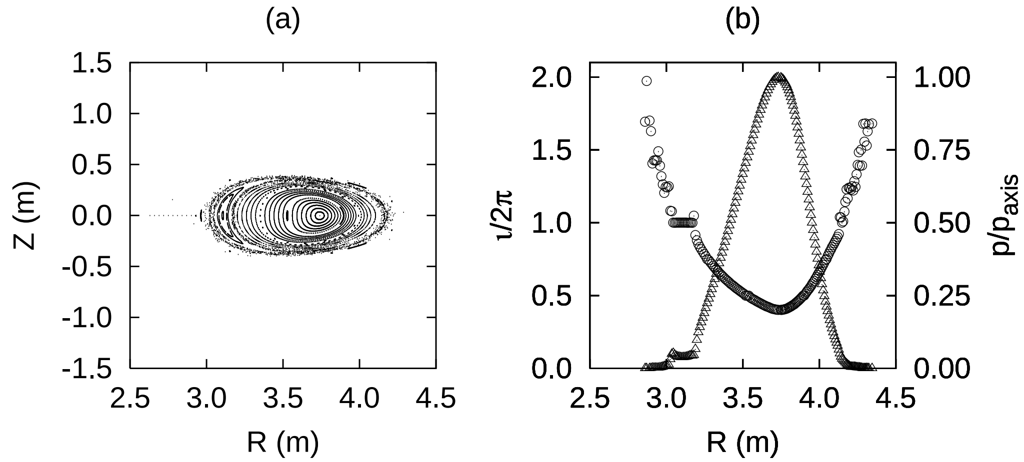
- **Standard scheme:**

magnetic axis: $R_{ax} = 3.74\,\mathrm{m}, Z_{ax} = 0\,\mathrm{m}$

O-point of island: $R_{is} = 3.105\,\mathrm{m}, Z_{is} = 0\,\mathrm{m}$

island width: $w_{is} = 12.6\,\mathrm{cm}$

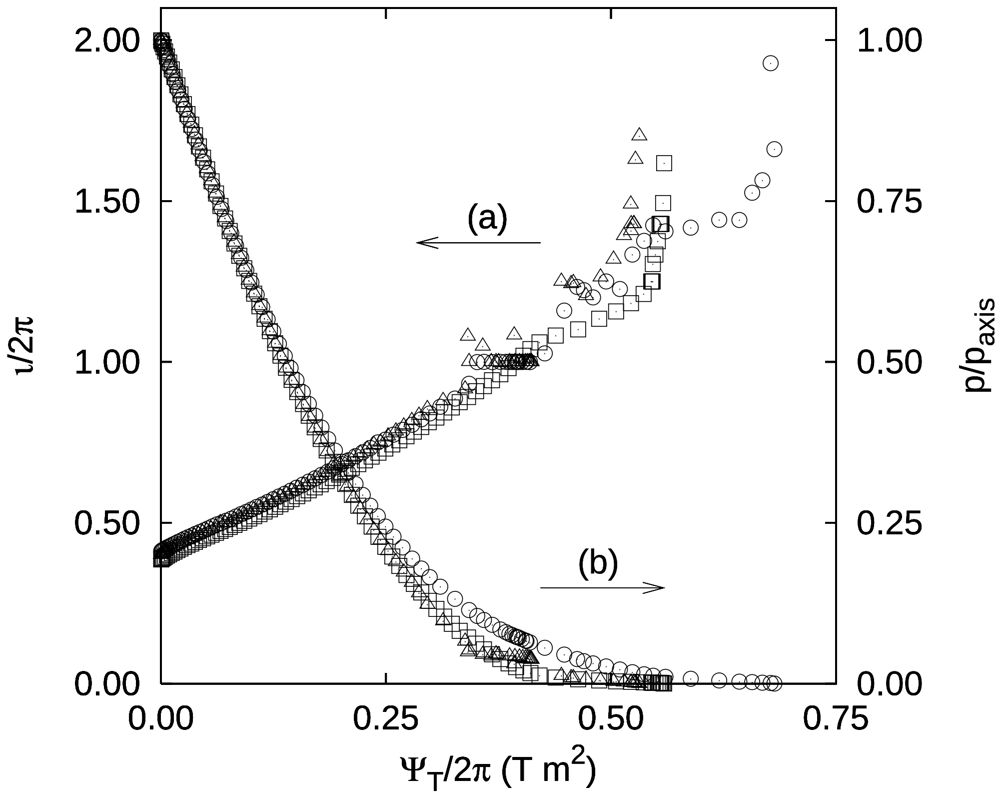




Comparison for rotational transform and pressure

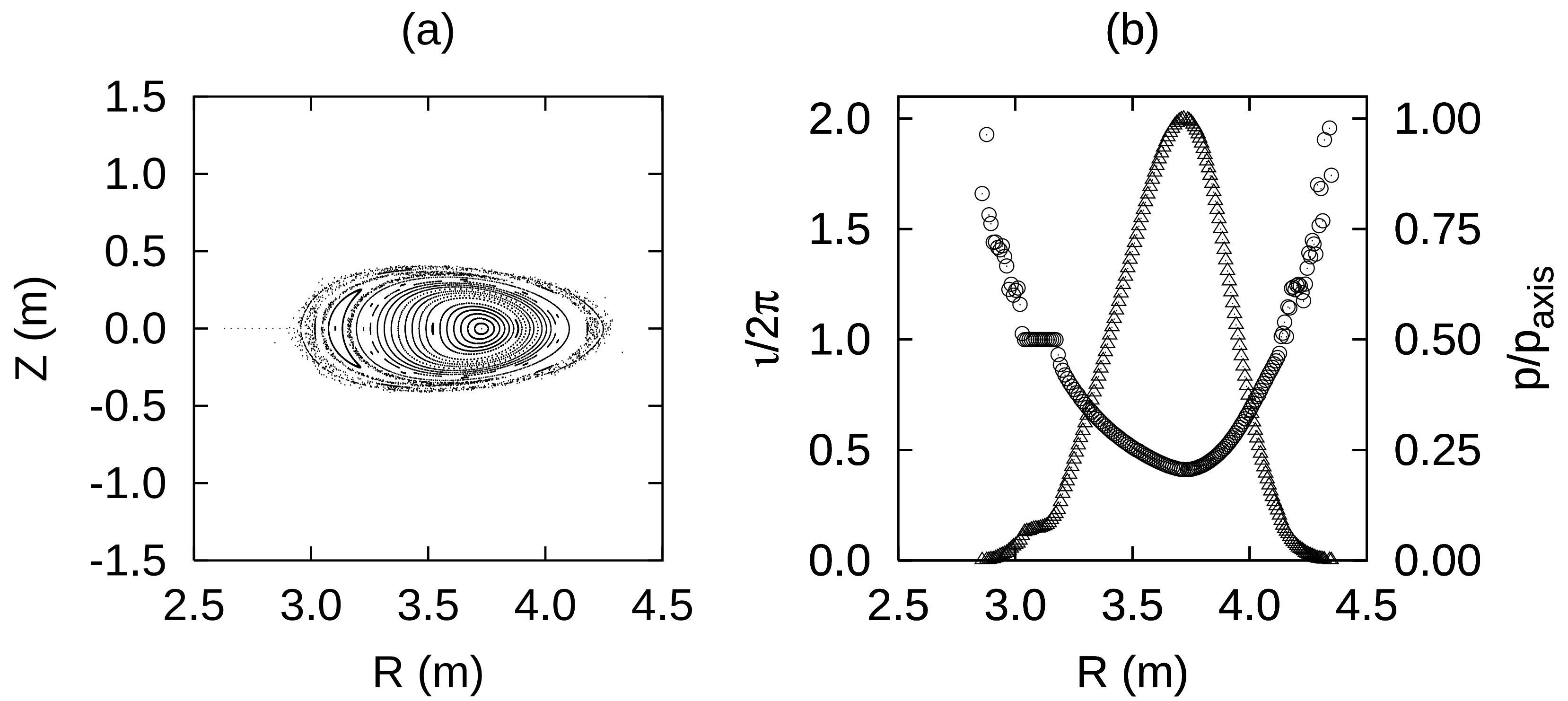
Modified scheme

- Absolute values of pressure around the axis decrease slightly.
- However, shapes of pressure and rotational transform are almost kept except for in and around the island.



open triangle: standard scheme, with island
open square: standard scheme, without island
open circle: modified scheme, with island

LHD equilibrium with m/n=1/1 island
(Modified scheme, $\beta \approx 2\%$)



- The island is surrounded with ergodic field lines.

→ A performance of the island divertor may be deteriorated.

Summary

- We have modified the HINT code to study an LHD equilibrium with m/n=1/1 island.
- The modified scheme solving the diffusion process of the error of force balance has several advantages;

  (a) effect of the diffusion process should be reduced with evolving the relaxation;

  (b) it does not disturb a region already satisfying a sufficiently small error given by the standard scheme.
- In the LHD equilibrium with m/n=1/1 island, good flux surfaces with the island are kept for beta=2%, but the island divertor may be deteriorated because of existence of the ergodic region between the closed surfaces and the island.

Future works

- Difference of the errors of force balance for islands with/without the toroidal period of helical coils will be studied.
- A transport analysis of particles in and around the island is needed to estimate a performance of the island divetor. It is going to be studied and will be reported in near future.