INITIAL EVALUATION OF COMPUTATIONAL TOOLS FOR STABILITY OF COMPACT STELLARATOR REACTOR DESIGNS

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SEVERAL IMPORTANT ISSUES ARISE IN EVALUATION OF COMPUTATIONAL TOOLS FOR COMPACT STELLARATOR DESIGN STUDIES

- **Equilibrium:**
  - **Fixed versus free boundary equilibrium**
    - ⇒ **Is the plasma boundary shape imposed or determined by coils?**
  - **Full 3D versus 2D**
    - ⇒ **Is the 'stellarator expansion' (effectively large aspect ratio ordering) utilized?**
    - ⇒ **Is helical symmetry assumed? (i.e infinite aspect ratio and infinite number of field periods)**
    - ⇒ **Is the geometry averaged over field periods? (i.e. axisymmetry)**
    - ⇒ **Is 'stellarator symmetry' assumed (i.e. at least one toroidal plane exists with up-down symmetry)**
  - **Direct or inverse equilibrium**
    - ⇒ **Is $\psi(r,z,\phi)$ determined directly or is $r(\psi,\chi,\phi)$ and $z(\psi,\chi,\phi)$ computed for a set of prescribed flux values ψ, poloidal angles χ and toroidal angles ϕ?**
    - ⇒ **Are nested flux surfaces imposed or not?**
    - ⇒ **If nested flux surfaces are assumed, are they *simply nested*?**
    - ⇒ **Do well defined magnetic islands exist?**
    - ⇒ **If nested flux surfaces are not assumed, do approximate surfaces exist? (i.e. what is the 'flux surface quality')**

SEVERAL IMPORTANT ISSUES ARISE IN EVALUATION OF COMPUTATIONAL TOOLS FOR COMPACT STELLARATOR DESIGN STUDIES (CONT)

- **Global stability:**
  - **Fixed versus free boundary stability**
    - ⇒ **Is the plasma bounded by a wall or by a vacuum region?**
    - ⇒ **Can both internal and external modes be evaluated?**
  - **Nested versus non-nested flux surfaces**
    - ⇒ **Are nested flux surfaces assumed for the equilibrium?**
    - ⇒ **Are non-nested islands allowed in the equilibrium?**
  - **Full 3D versus 2D**
    - ⇒ **Is an equilibrium expansion (e.g. 'stellarator expansion') used?**
    - ⇒ **Is axisymmetry or helical symmetry assumed in the equilibrium?**
    - ⇒ **Is 'stellarator symmetry' assumed in the underlying equilibrium?**
    - ⇒ **Is 'field period' symmetry invoked in the selection of fourier modes? (Fourier modes are coupled within mutually decoupled families that depend on the number of field periods)**
  - **Ideal versus resistive MHD stability**
    - ⇒ **Are plasma and field components incompressible or compressible**
    - ⇒ **Is the plasma resistivity finite?**
    - ⇒ **Are other non-ideal 'Extended MHD' effects (kinetic effects, fast particles, two-fluid or multi-fluid effects) included?**

SEVERAL IMPORTANT ISSUES ARISE IN EVALUATION OF COMPUTATIONAL TOOLS FOR COMPACT STELLARATOR DESIGN STUDIES (CONT)

- **Global stability (continued):**
  - **Primitive or derived MHD equations:**
    - ⇒ **Solution of initial value, dynamic equations, or variational method**
    - ⇒ **Primitive physical variables (δp, δx, δB) or derived quantities**
  - **Linear versus nonlinear stability**
    - ⇒ **Is the underlying equilibrium topology fixed?**

- **Local stability:**
  - **Mercier or ballooning stability**
  - **Is axisymmetry or helical symmetry assumed in the underlying equilibrium?**
  - **Is an equilibrium expansion (e.g. 'stellarator expansion') used?**

A SUITE OF DIRECT AND INVERSE FREE BOUNDARY AND FIXED BOUNDARY EQUILIBRIUM CODES EXISTS

- **VMEC (S. Hirshman, ORNL) :**
  - **Fixed boundary, fully 3D, inverse equilibrium code with simply nested flux surfaces imposed**
    - ⇒ **The plasma boundary surface shape is imposed**
    - ⇒ **Code calculates $r(\psi,\chi,\phi)$ and $z(\psi,\chi,\phi)$ for a set of flux surfaces ψ and angles χ and ϕ?**
    - ⇒ **No assumptions of symmetry except 'stellarator symmetry'**
  - **Industry standard, widely utilized stellarator equilibrium code**
- **HINST (K. Harafuji, NIFT):**
  - **Free boundary, fully 3D, direct equilibrium code**
    - ⇒ **The plasma boundary surface shape is defined by coils**
    - ⇒ **Nested flux surfaces are not assumed**
    - ⇒ **Code calculates ψ (r,z,ϕ) irrespective of whether ψ forms nested flux surfaces**
    - ⇒ **Code can handle islands and ergodic regions**
    - ⇒ **No assumptions of symmetry except 'stellarator symmetry'**

A SUITE OF DIRECT AND INVERSE FREE BOUNDARY AND FIXED BOUNDARY EQUILIBRIUM CODES EXISTS (CONT)

- **HINST (continued):**
  - **Iterates to an equilibrium by relaxation of dissipative MHD equations to steady state**
  - **Code is apparently extremely time consuming**
  - **This is not a widely utilized stellarator equilibrium code outside of Japan. The reason is not presently known but may be due to the preceding comment**
- **PIES (A. Rieman, PPPL):**
  - **Free boundary, fully 3D direct equilibrium code**
    - $\Rightarrow$ **The plasma boundary surface shape is defined by coils**
    - $\Rightarrow$ **Nested flux surfaces are not assumed**
    - $\Rightarrow$ **No assumptions of symmetry except 'stellarator symmetry'**
    - $\Rightarrow$ **Code calculates ψ (r,z,ϕ) irrespective of whether ψ forms nested flux surfaces**
    - $\Rightarrow$ **Code can handle islands and ergodic regions**
  - **Integrates field line equations to determine field line structure**
  - **Time consuming but determines true free boundary magnetic structure and is routinely used for NCSX**

A SUITE OF FULL 3D LINEAR GLOBAL MHD STABILITY CODE TOOLS EXISTS

- **TERPSICHORE (W.A. Cooper, CRPP):**
  - **Free boundary, fully 3D, linear ideal MHD stability code with simply nested flux surfaces assumed:**
    - ⇒ **The equilibrium plasma boundary surface shape is bounded by a vacuum region and can be perturbed (i.e. external modes)**
    - ⇒ **The vacuum formulation (so-called 'pseudo-vacuum) has numerical problems except in LHD and quasi-symmetric cases**
    - ⇒ **No expansions or assumptions of symmetry in underlying equilibrium except 'stellarator symmetry'**
    - ⇒ **Takes advantage of 'field period' symmetry**
  - **Simply nested inverse equilibrium input from VMEC**
  - **Variational energy principle formulation**
  - **Compressible and incompressible ideal MHD plasma versions exist**
  - **Industry standard, widely utilized 3D stability code:**
    - ⇒ **benchmarked for tokamak cases**
    - ⇒ **extensively used in design studies for LHD and NCSX, and in exploratory stability studies**
  - **Code runs on NEC-SX5 and porting to Pentium-4 looks promising**
    - ⇒ **Code has also recently been ported to SGI Octane (GA)**

A SUITE OF FULL 3D LINEAR GLOBAL MHD STABILITY CODE TOOLS EXISTS (CONT)

- **CAS-3D (C. Nuhrenberg, MPI):**
  - **Free boundary, fully 3D, global, linear ideal MHD stability code with simply nested flux surfaces assumed**
    - ⇒ **The equilibrium plasma boundary surface shape is imposed and is assumed bounded by a perfectly conducting wall**
    - ⇒ **An option with a vacuum does not appear to be available (i.e. internal global modes only)**
    - ⇒ **No expansions or assumptions of symmetry in underlying equilibrium except 'stellarator symmetry'**
    - ⇒ **Takes advantage of 'field period' symmetry**
  - **Simply nested inverse equilibrium input from VMEC**
  - **Variational energy principle formulation**
  - **Widely utilized 3D stability code benchmarked for tokamak cases and used extensively in the design of W7AS and W7X**
  - **Code consists of a double family of codes:**
    - ⇒ **Compressible and incompressible ideal MHD plasma and compressible and incompressible toroidal field options**
    - ⇒ **Explicit extraction of high toroidal and poloidal mode numbers**

A SUITE OF FULL 3D LINEAR GLOBAL MHD STABILITY CODE TOOLS EXISTS (CONT)

- **SPECTOR-3D (R.G. Storer, Flinders U.):**
  - **Free boundary, fully 3D, global, resistive MHD stability code with simply nested flux surfaces assumed**
    - ⇒ **The equilibrium plasma boundary surface shape is imposed and is assumed bounded by a perfectly conducting wall**
    - ⇒ **An option with a vacuum does not appear to be available (i.e. internal global modes only)**
    - ⇒ **No expansions or assumptions of symmetry in underlying equilibrium except 'stellarator symmetry'**
  - **Simply nested inverse equilibrium input from VMEC**
  - **Initial value code formulation in terms of magnetic and velocity potentials**
  - **Not yet widely utilized code but the only linear resistive MHD code available**
    - ⇒ **Code has some difficulties with accuracy of VMEC equilibria near the axis**
  - **Code is presently incompressible but plans are underway for a compressible option**

OTHER MHD EQUILIBRIUM AND GLOBAL STABILITY CODE TOOLS ARE ALSO AVAILABLE

- **NEAR (T. Hender, Culham):**
  - **3D MHD Equilibrium code**
- **BETA (O. Betancourt, Courant Inst.):**
  - **3D MHD Equilibrium code**
- **RSTEQ (B. Carreras, ORNL):**
  - **2D Equilibrium from averaging 3D equilibrium over field periods**
- **HERA (R. Gruber, EPFL):**
  - **Helically symmetric 2D linear MHD stability code**
- **TWIST (S. Medvedev, Keldysh Inst.):**
  - **Linear ideal MHD stability code with 3D equilibrium averaged over field periods**
- **RST (B. Carreras, ORNL):**
  - **Linear ideal MHD stability code with 3D equilibrium from RSTEQ averaged over field periods**

LIMITED NONLINEAR 3D GLOBAL EXTENDED MHD STABILITY CODE TOOLS ARE AVAILABLE

- **M3D (W. Park, PPPL):**
  - **Full 3D nonlinear extended MHD code has recently been applied to compact stellarator equilibria**
  - **Presumably assumes nested (not simply nested) flux surfaces, at least to a high approximation:**
    - ⇒ **Otherwise numerical problems arise in accurately resolving perturbations parallel and perpendicular to the equilibrium field**

- **NIMROD (Nimrod Team):**
  - **Full 3D nonlinear extended MHD code can also be applied in principle to compact stellarator equilibria**
  - **Presumably assumes nested (not simply nested) flux surfaces, at least to a high approximation:**
    - ⇒ **Otherwise numerical problems arise in accurately resolving perturbations parallel and perpendicular to the equilibrium field**

MISCELLANEOUS COMMENTS

- **In principle, the equilibrium codes can also be thought of as stability codes since if they find an equilibrium under certain constraints, that equilibrium must be stable unless those constraints can be avoided by a physically valid motion:**
  - **VMEC imposes fixed boundary and simply nested flux surface constraints**
  - **PIES and HINST have essentially no constraints on the equilibrium**

- **VMEC is the state-of-the equilibrium code and is interfaced with TERPSICHORE, CAS-3D, and SPECTOR-3D:**
  - **PIES or HINST are necessary to determine to what extent a nested flux surface configuration actually exists with a specific finite coil set**

- **CAS-3D and TERPSICHORE are almost equivalent**
  - **CAS-3D and TERPSICHORE have been benchmarked for LHD equilibria**
  - **TERPSICHORE is apparently easier to use and does have a working vacuum boundary condition for some cases at least**
  - **CAS-3D has a number of versions within the code family, some of which greatly reduce the computation time**

MISCELLANEOUS COMMENTS (CONT)

- **SPECTOR-3D is still under development but is used for the Heliac at ANU in Canberra**
  - **There is no other comparable code and like most resistive MHD codes it tends to be restricted to lower magnetic Reynolds numbers than are typical of experiments**
  - **An asymptotic matching resistive MHD code suitable for high S does not appear to exist, though a proposal from S. Galkin (UCSD) to develop such a code was withdrawn for lack of support**

- **Local stability criteria are routinely computed by the global linear stability codes**

- **The NCSX Group has developed a procedure for optimizing the design using reverse engineering (S. Hudson, PPPL):**
  - **Design the configuration for specific physics properties using VMEC**
  - **Find a coil set to give a nearby equivalent free boundary equilibrium but possibly with some bad surfaces.**
  - **Heal the bad surfaces using small shifts in coil positions or trim coils**

SUMMARY

- **Equilibrium codes:**
  - **Situation is adequate with VMEC for accurate and fast equilibrium calculations suitable for stability analysis and**
  - **HINST and PIES for evaluating the 'realizability' of the VMEC equilibria**
    - ⇒ **The major deficiency is in the lack of a 3D Stellarator equilibrium fitting code analogous to EFIT for 2D equilibria**

- **Local stability codes:**
  - **Situation is adequate since TERPSICHORE and CAS-3D both compute local Mercier and ballooning criteria**
  - **Calculations with CAS-3D indicate the local stability limits are usually close to the fixed boundary global stability limits**

SUMMARY (CONT)

- **Global linear stability codes:**
  - **Situation is almost adequate for linear ideal stability:**
    - ⇒ **The major deficiency is in a reliable free boundary and vacuum formulation**
    - ⇒ **Also there is no linear stability code capable of handling islands or ergodic regions (except for the sense in which PIES and HINST can guarantee some stability)**
  - **Situation is still inadequate for linear resistive stability:**
    - ⇒ **The SPECTOR-3D code presently suffers from numerical problems when mapping from VMEC near the axis**
    - ⇒ **No linear code exists based on the asymptotic matching method suitable for high S**

- **Global nonlinear stability codes:**
  - **Situation is almost adequate for nonlinear extended MHD stability:**
    - ⇒ **The M3D code appears to be working for stellarator configurations but is extremely time consuming**