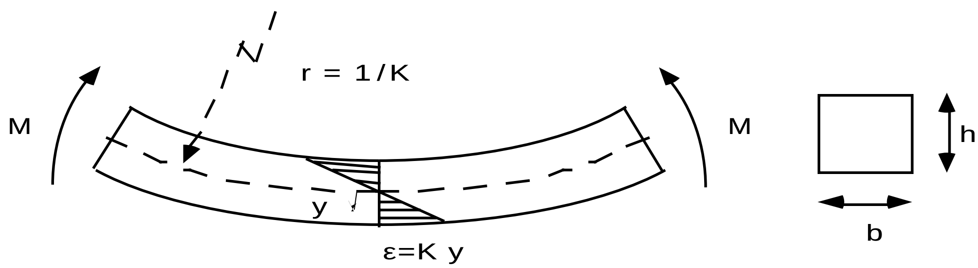
Deformation Process Example: Simple Bending

For this problem we consider the simplest situation: that of pure bending. This implies a constant moment along the length of the material. We also consider the simplest material model: elastic - perfectly plastic, which implies that after yield, stress becomes independent of strain.

The first goal is to determine the stresses (σ) and strains (ε) in the cross section caused by the applied moments (M) or the resulting curvature (K). Note that both stress and strain vary with the distance from the neutral axis (y); thus $\sigma = \sigma(y)$ and $\varepsilon = \varepsilon(y)$.



$$\varepsilon(y) = Ky \qquad (1)$$

$$\sigma = E\varepsilon = E\,K\,y \text{ (for elastic beam only)} \qquad (2)$$

$$M = 2\int_0^{h/2} \sigma\, b\, y\, dy \qquad (3)$$

$$= 2b \int_0^{h/2} E\,K\,y^2\, dy \qquad (4)$$

$$= \frac{2}{3}\, b\, E\, K\, (h/2)^3 \qquad (5)$$

$$= E\,\frac{1}{12}\, b\, h^3\, K \qquad (6)$$

Moment of inertia for a rectangular section

$$I = \frac{1}{12}\, b\, h^3$$

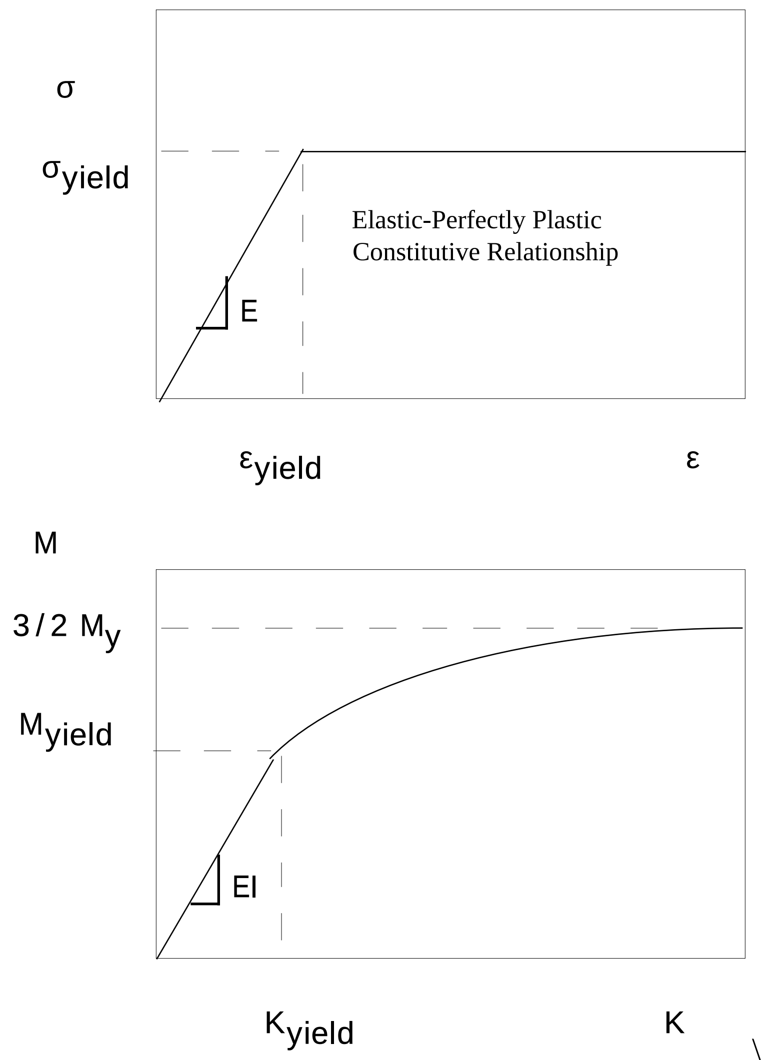
Therefore for the elastic region (when $K<K_{yield}$):

$$M = EI\,K \qquad (7)$$

A similar derivation for the elastic - plastic region, assuming that $\sigma = \sigma_{yield}$ for $\varepsilon > \varepsilon_{yield}$

leads to the expression for the moment curvature relationship for a rectangular beam for $K > K_{yield}$:

$$M = \frac{3}{2} M_{yield} \left[1 - \frac{1}{3} \left(\frac{K_{yield}}{K} \right)^2 \right] \quad (8)$$



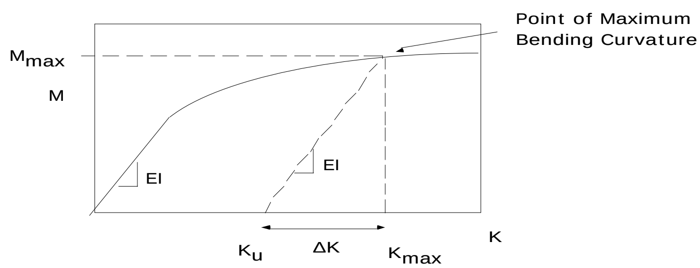
Moment Curvature Relationship for a Rectangular Beam with Elastic - Perfectly Plastic

Springback Estimation

Note from this diagram that K_{Yield} = curvature at which first yielding of the beam occurs. This will occur at the edge of the beam first where $y = h/2$; thus

$$K_{yield} = \frac{\varepsilon_{Yield}}{h/2}$$

The springback (ΔK) can be defined from the *M-K* relationship (Eqn(8)) as shown below:



$$\Delta K = K_{max} - K_u$$

If we know the curvature of the tool, then we know K_{max}, and M_{max} can be estimated from the M-K relationship. This then leads to the springback estimate:

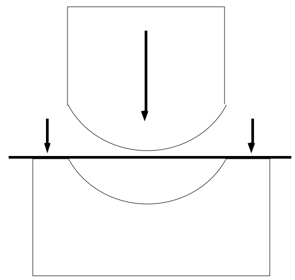
$$\Delta K = \frac{M_{max}}{EI}$$

Thus, to estimate the spring back, we must know:

- The curvature to which the material is formed (K_{max})
- The Moment Curvature Relationship, which in turn requires:
  - The elastic modulus E
  - The Yield Stress σ_Y
  - The Area moment of inertia I
  - An assumption about post-yield behavior (such as perfectly plastic)

Adding Stretch to Bending to Reduce Springback

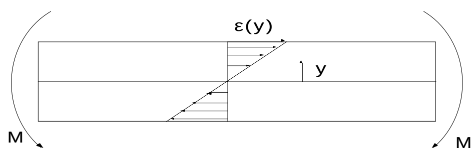
The springback ΔK can be very large, and compensating for it by "over-bending" depends on precise prediction of ΔK. An alternative is to create forming methods that minimize ΔK in the first place. One such method is adding a tensile stress to the bending by stretching the material as it is formed. This is often done by clamping the material at the edges so that it will stretch as it is drawn into the tool:



The effect of this stretch can be analyzed by looking at the stress patterns in the cross section. For simple bending, the strain pattern through the cross section is given by Eqn(1)

$$\varepsilon = Ky$$

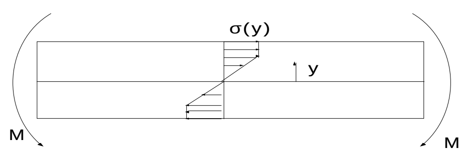
or:



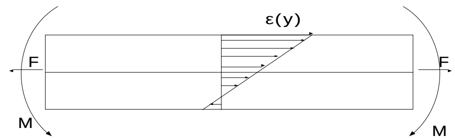
The corresponding stresses, for post - yield, elastic - perfectly plastic behavior is found simply by applying the fact that :

$$\sigma = E\,\varepsilon \qquad \text{for } \varepsilon<\varepsilon_Y$$
$$\sigma = \sigma_Y \text{ for } \varepsilon>\varepsilon_Y$$

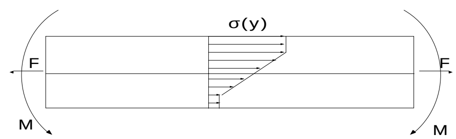
This leads to the pattern:



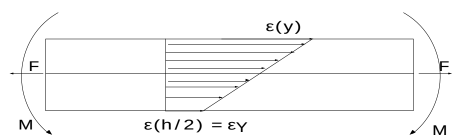
Now assume that in addition to the bending strain, a tensile strain of ε_t is applied. This applied tensile strain will act to bias the bending strains to the positive side as shown below:

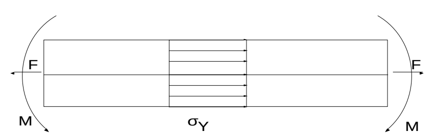


For elastic perfectly plastic behavior, this will lead to the following stress pattern in the cross section:



If the applied strains are such that the lowest value of strain $\varepsilon(-h/2) \geq \varepsilon_Y$ then the corresponding stress at that point will be σ_Y and in fact all points in the cross section will be at that same value σ_Y:





From this state of stress the material will relax elastically uniformly across the cross section and no change in curvature (or springback) will occur.

For materials that strain harder, it will be difficult if not impossible to achieve the uniform stress state, and it may be difficult to do so for any material if the required strains cause failure. If that is the case, lesser strains can be imposed, and some springback will exist, but it will be greatly reduced from the pure bending case.