Tutorial on Stellarator Transport

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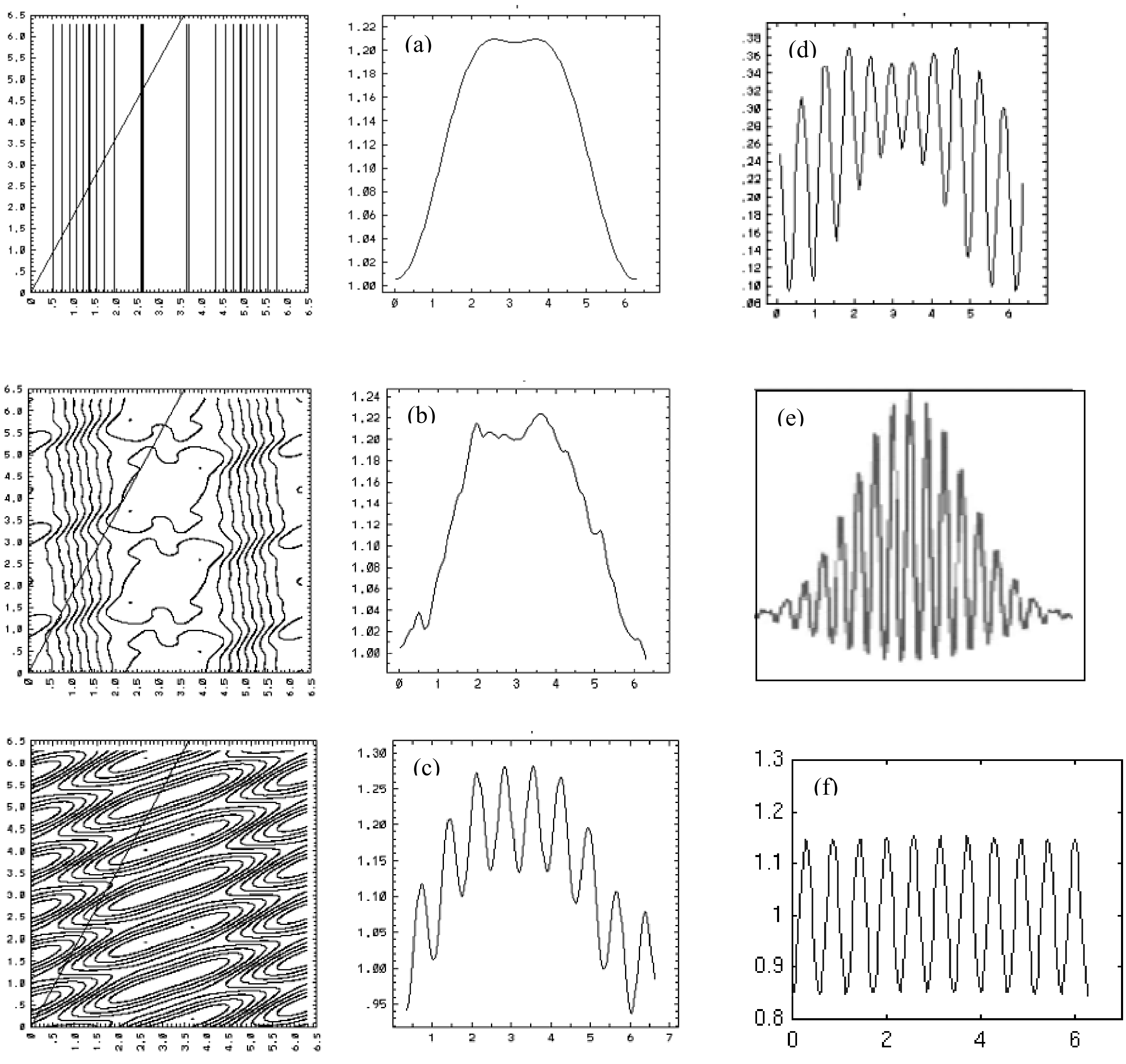
-Magnetic field Structure:

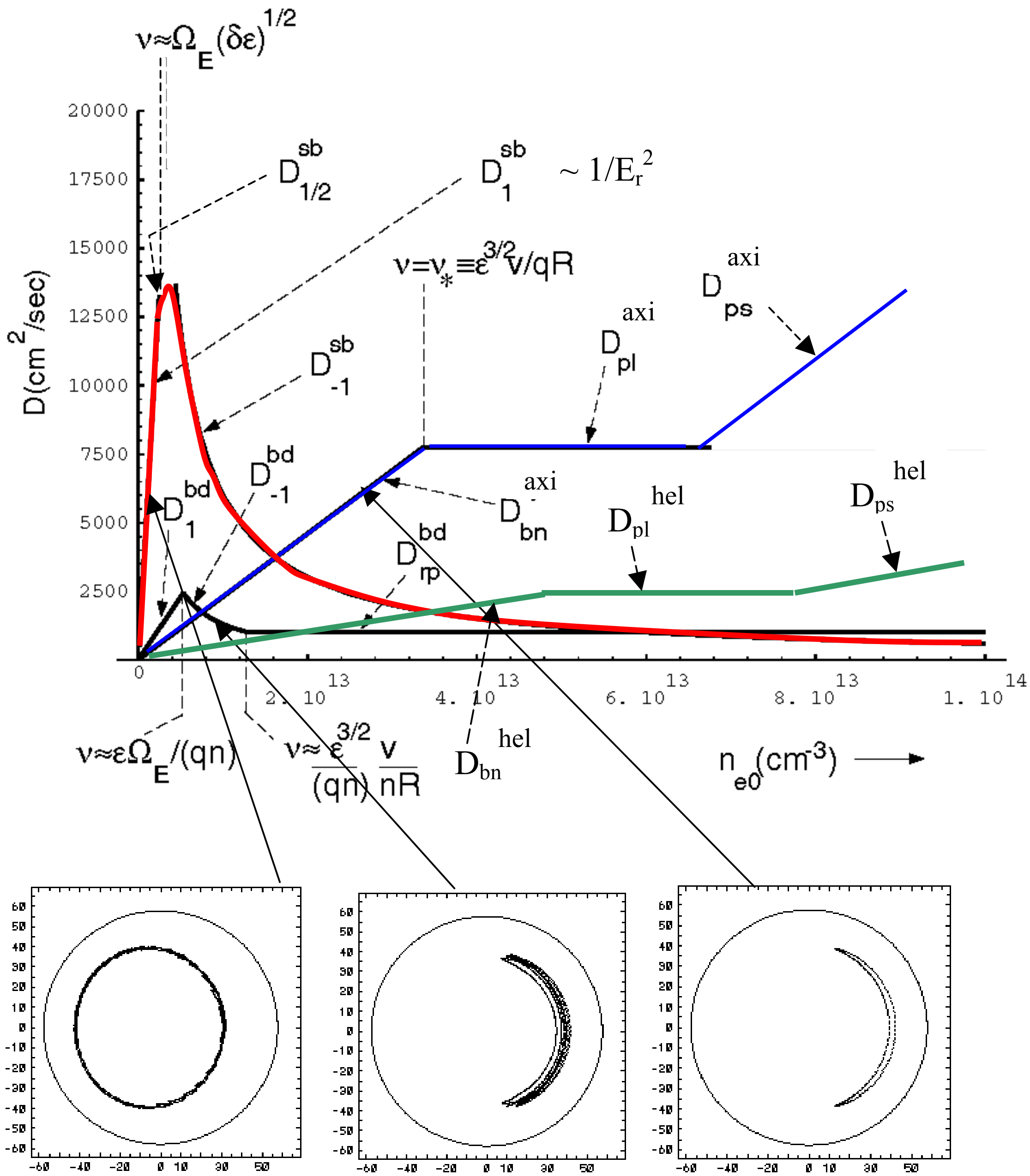


-B-field Model:

$$B(\mathbf{x})=B_0[1-\varepsilon(r)\cos\theta-\delta_h(r,\theta)\cos\eta], \qquad (1)$$

-parameter $p\equiv\delta_h/\varepsilon$ =measure of distance of stellarator from symmetric limits $\delta_h=0$, $\varepsilon=0$

-Neoclassical Transport - Overview:



-Radial diffusion:

$D \approx F\tilde{\nu}\Delta^2$, with

$F \equiv$fraction of particles contributing,

$\tilde{\nu} \equiv$ freq of taking step in random walk

$\Delta \equiv$radial step size.

-Eg-1: Banana regime, tokamak:

$F = F_t \equiv (2\varepsilon)^{1/2}$ =frac of toroidally-trapped particles,

$\Delta = \rho_{bt} \equiv$banana width$\approx v_{Bt}/(v_{\|}/qR) \approx q\rho/\varepsilon^{1/2}$,

$\tilde{\nu} = \nu_t \equiv$ toroidal detrapping frequency $= \nu/(2\varepsilon)$,

with $v_{Bt} \approx \rho v/2R$ = toroidally-induced radial drift velocity.

$\Rightarrow D_{bn}^{axi} \approx (2\varepsilon)^{1/2}\nu_t\,\rho_{bt}^2 \approx \nu\, q^2\rho^2/\varepsilon^{3/2}$

-Eg-2: Banana regime, straight stellarator:
[A.Pytte, A.H. Boozer, Phys. Fluids **24**, 88 (1981).]

$F=F_h \equiv (2\delta_h)^{1/2}$ =frac of helically-trapped particles,
$\Delta=\rho_{bh} \equiv$ banana width $\approx v_{Bh}/(v_{\parallel}/L_h) \approx (q_h R/r)\rho\delta_h^{1/2}$,
$\tilde{\nu} = \nu_h \equiv$ ripple detrapping frequency $= \nu/(2\delta)$,
with $L_h \equiv R/n$, $v_{Bh} \approx (\rho v/2)(m\delta_h/r)$ = helically-induced radial drift velocity, $q_h \equiv m/n$.

$$\Rightarrow D_{bn}^{sym} \approx (2\delta_h)^{1/2} \nu_h \rho_{bh}^2 \approx \nu (q_h R/r)^2 \rho^2 \delta_h^{1/2}$$

-Ripple-trapped particles:

-Well-depth parameter
y= 0, deeply ripple-trapped particle
1, marginally-trapped particle
>1, non-ripple-trapped particle.

-For model B-field (1), have
$y = [K/\mu B_0 - 1 + \varepsilon \cos\theta + \delta_h]/(2\delta_h)$, (2)
with $\eta \equiv n\zeta - m\theta$=ripple phase, $K \equiv (E - e\Phi)$= kin.energy

-Diffusion in y due to pitch-angle scattering:
$<(\delta y)^2> \approx \nu_h t$, with $\nu_h \equiv \nu/(2\delta_h)$.

$\Rightarrow$time τ_h to detrap from ripple-well for $\delta y \approx 1$:
$\tau_h \approx 1/\nu_h$.

- “1/ν-regime” ($\nu_h/\Omega_E > 1$):

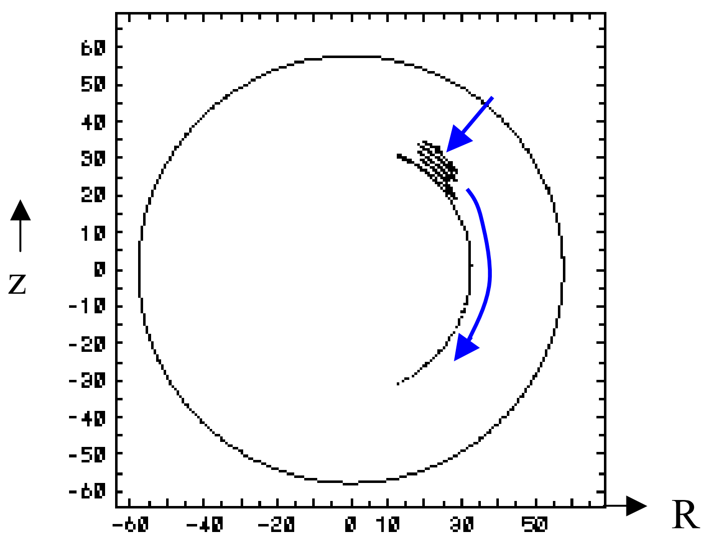
[Galeev, Sagdeev, Furth, Zh.Prikl.Mekh. i Tekhn.Fiz., **3** (1968),
Gibson, Mason, Plasma Phys. **11**, 121 (1969),
Stringer, Nucl. Fusion **12**, 689 (1972),
Connor, R.J. Hastie, Nucl. Fusion **13**, 221 (1973)]

$\Delta \approx v_{Bt}/\nu_h$, with $v_{Bt} \approx \rho v/2R$ = toroidally-induced radial drift velocity,

$F \approx (2\delta_h)^{1/2}$ = frac of ripple-trapped particles,

$\tilde{\nu} \approx \nu_h$ = detrapping frequency.

$\Rightarrow D_{-1} \approx (2\delta_h)^{1/2}\, \nu_h\, (v_{Bt}/\nu_h)^2 \approx (2\delta_h)^{3/2}\, v_{Bt}^{\ 2}/\nu$

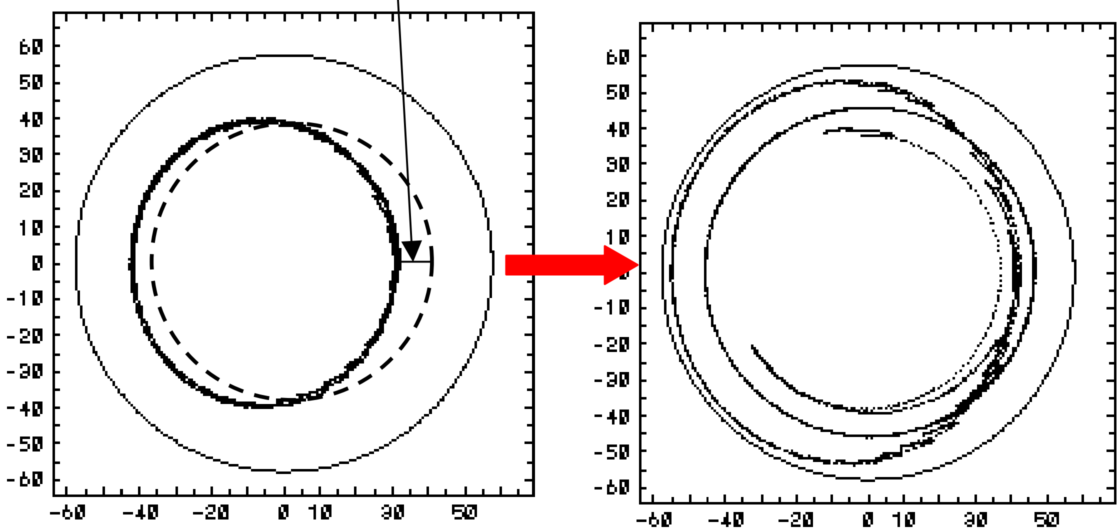


D_{-1} has strong energy dependence, $\sim K^{7/2}$,
and is indep of $\Omega_E \sim E_r = -\partial_r\Phi$.

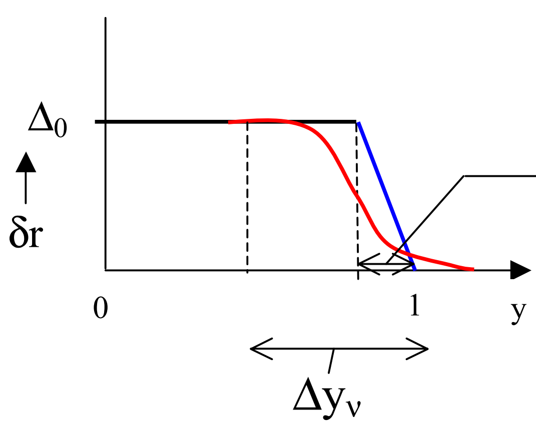
$D_{-1i}/D_{-1e} \sim (M_i/M_e)^{1/2} \gg 1$.

- "ν^1, $\nu^{1/2}$ superbanana regimes" ($\nu_h/\Omega_E < 1$) :

-Collisions perturb orbits from ν=0 superbananas, having sb width Δ_0=v_{Bt}/Ω_E .



- sb's within a distance Δy_0=1/p of y=1 detrap collisionlessly, making sb excursion $\delta r(y)$ continuous. ($p \equiv \delta_h/\varepsilon$):



For $\nu_h/\Omega_E > p^{-2}$, a collisional boundary layer is formed, of width $\Delta y_\nu = (\nu_h/\Omega_E)^{1/2}$, swamping Δy_0.

- "$\nu^{1/2}$ sb-regime" ($p^{-2} < \nu_h/\Omega_E < 1$):

[Galeev, Sagdeev, Sov.Phys.Usp. **14**, 810 (1969),
Galeev, Sagdeev, Furth, Rosenbluth, Phys. Rev. Letters **22**, 511 (1969).]

$\Delta = \Delta_0$,

$F \approx F_\nu \equiv (2\delta_h)^{1/2}\, \Delta y_\nu = (\nu/\Omega_E)^{1/2}$,

$\tilde{\nu} \approx \nu_h/(\Delta y_\nu)^2 = \Omega_E$

$\Rightarrow D_{1/2} \approx \nu^{1/2}\, v_{Bt}^2/\Omega_E^{3/2}$

- "ν^1 sb-regime" ($\nu_h/\Omega_E < p^{-2}$):

[Galeev, Sagdeev, Sov. Phys. Usp. **12**, 810 (1970)]

$\Delta = \Delta_0$,

$F \approx F_0 \equiv (2\delta_h)^{1/2}\, \Delta y_0$,

$\tilde{\nu} \approx \nu_h/(\Delta y_0)^2$,

$\Rightarrow D_1 \approx \nu p (2\delta_h)^{-1/2} v_{Bt}^2/\Omega_E^2$

-Banana-drift branch:

- "stochastic regime"

[Goldston, White and Boozer, Phys.Rev. Lett. **47**, 647 (1981).]

- "ν^{1},ν^{-1} bd-regimes"

[Linsker, Boozer, Phys. Fluids **25**, 143 (1982).]

- "banana-plateau regime"

[Boozer, Phys. Fluids **23**, 2283 (1983).]

-Fluxes:

$$\begin{bmatrix}\Gamma_s\\ Q_s\end{bmatrix} = -\frac{2n_s}{\sqrt{\pi}}\int dx\ x^{1/2}\ e^{-x}\begin{bmatrix}1\\ T_s x\end{bmatrix} D_q(x)\left[\frac{n'_s}{n_s}-\frac{e_s E_r}{T_s}+(x-\frac{3}{2})\frac{T'_s}{T_s}\right], \quad (3)$$

with $x \equiv K/T$, $q=-1,1/2,1$ = power of ν in D. Do energy-integration over ν-regimes,

$$\int_0^\infty dx..D_q = \int_0^{x_1} dx..D_{-1} + \int_{x_1}^{x_2} dx..D_{1/2} + \int_{x_2}^{\infty} dx..D_1 , \quad \text{with} \qquad (4)$$

$$D_{-1}(x) = \sigma_{-1}(2\delta_h)^{1/2} V_{Bt}{}^2/\nu \sim x^{7/2}, \qquad (5)$$

$$D_{1/2}(x) = \sigma_{1/2}\nu^{1/2}\ V_{Bt}{}^2/\Omega_E{}^{3/2} \sim x^{5/4},$$

$$D_1(x) = \sigma_1 \nu p(2\delta_h)^{-1/2}\ V_{Bt}{}^2/\Omega_E{}^2 \sim x^{1/2}.$$

-Ambipolarity:

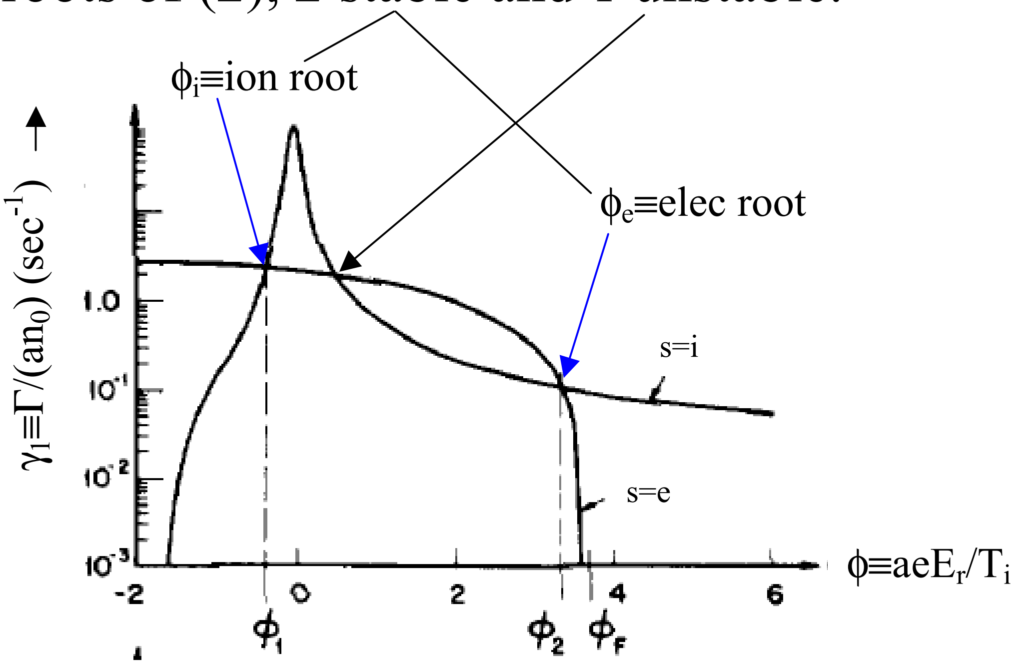
-For $\delta_h>0$, a particle's angular momentum $p_\zeta \equiv MRv_\zeta-(e/c)\psi_p$ is not conserved, even for $\nu=0$, so that one no longer has $\Gamma_i=\Gamma_e$ for arbitrary E_r, as in tokamaks. Instead, E_r is *determined* from the ambipolarity condition

$\Gamma_i(E_r)=\Gamma_e(E_r)$. (6)

-Roots of the ambipolarity condition for E_r :

(1)Galeev, Sagdeev, Furth, Rosenbluth, [Phys. Rev. Letters **22**, 511 (1969)] found $D_{-1,1/2}$, and found a single root, with $E_r<0$, electrons in the $1/\nu$ regime, holding in the ions, which are in the $\nu^{1/2}$ regime (the "ion root").

(2)Multiple roots: Mynick, Hitchon [Nucl. Fusion **23**, 1053 (1983)] using model with $D_{-1,1/2,1}$, and found multiple roots of (2), 2 stable and 1 unstable.



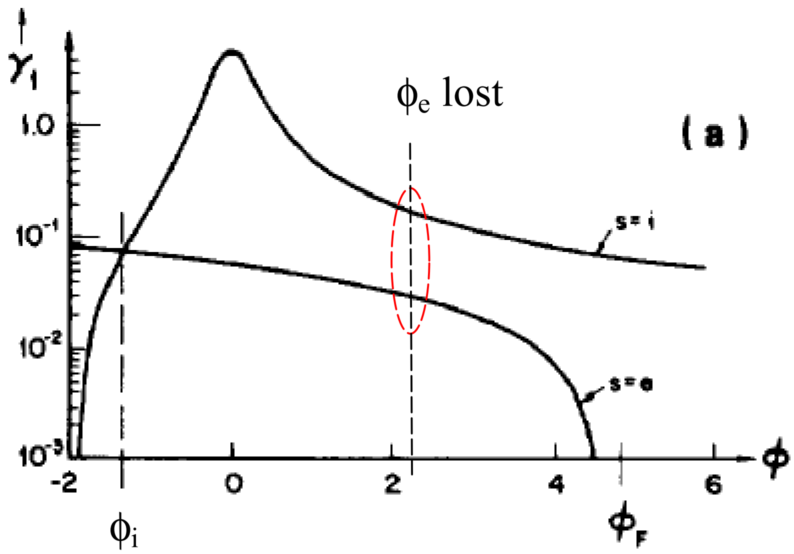
⇒New root: Electron root, with $E_r>0$, ions holding in electrons.

-Interesting because Γ_s, Q_s appreciably smaller.

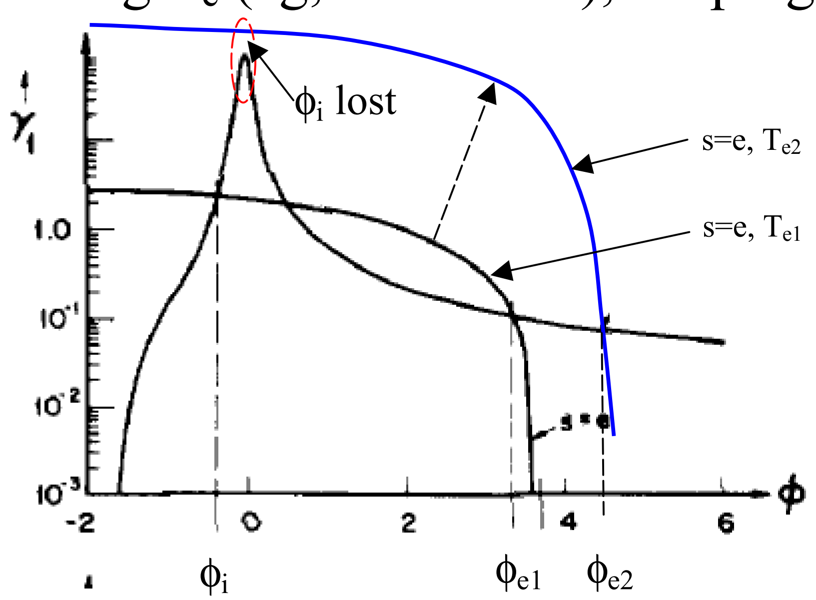
-Root loss:

- As parameters change (eg, as r changes), Eq.(6) can lose its real roots (in pairs), so that one may have only an ion, or an electron root.

-Eg, raising n_0 increases ν, broadens Γ_i peak:



-Raising T_e (eg, via ECRH), keeping T_i, can lose ϕ_i:



-ECRH-heated plasmas provided the 1st access to ϕ_e in CHS ,W7-AS, near the plasma core. [Idei et al., Phys. Rev. Lett **71** 2220 (1993), Maassberg, Beider, Gasparino, M. Rome', et al.,Phys. Plasmas **7**, 295 (2000).]

-Accessing ϕ_e in a plasma $T_e \sim T_i$ has proven more difficult. Achieved more recently on LHD with NBI. [Ida, et. al, Phys. Rev. Letters 86, 5297 (2001).]

-Root jumping:

-Ambipolarity constraint (6) is algebraic, solved at each r. Doesn't answer which root is selected, or what happens if different roots occur at different r. $\Rightarrow$Need a p.d.e. to evolve E_r in (r,t). Done in

[Shaing, Phys.Fluids 27, 1567 (1984),

Hastings, Houlberg, Shaing, Nucl. Fusion, **25**, 445 (1985)]:

$$\partial_t[\varepsilon_0 \frac{c^2}{v_A^2}\frac{m}{nq} E_r] = -\frac{1}{V'}[\partial_r V' D_E \partial_r E_r] + \Sigma_s e_s \Gamma_s \ .$$

$D_E \equiv$ "electric diffusion coef", obtained by solving the kinetic eqn which gives Γ_s to higher order in $\delta r/a$.

-Inducing transport barriers:

-When root-jumps occur, provides a rapidly-changing $E_r \Rightarrow$Possibility of flow-shear, inducing a transport barrier.

-Observed on W7-AS by Stroth, et al. [Phys. Rev. Lett., **86**, 5910 (2001)] , and on LHD by Ida, et al. [Phys. Rev. Lett. **91**, (2003).] , both with ECRH.