Tutorial on Stellarator Transport-3
Transport Optimization

H.E. Mynick (PPPL)

May 12, 2005

-Thanks to: A. Boozer, S. Gerhardt, L.-P. Ku,
D. Mikkelsen, M. Redi, G. Rewoldt, D.Spong

-Transport Optimization:

-Approaches:

-Neoclassical:

-Quasi-Helical (QH)	(HSX)
-Quasi-Axisymmetric (QA)	(NCSX)
-Quasi-Poloidal	(QPS)
-Quasi-Omnigenous (QO)/ Quasi-Isodynamic (QI)	(W7-X, inward-shifted LHD)
-Isometric/ Approximately Omnigenous	
-Pseudo-Symmetric (PS)	

-Turbulent optimization:

- -Internal transport barriers via root-jumping
- -Turbulence modifications from shaping

-Particle motion:

-Magnetic field:

$\mathbf{B}=\nabla\psi_t\times\nabla\theta+\nabla\zeta\times\nabla\psi_p = \nabla\psi_t\times\nabla\alpha_p$ (1)

with $\alpha_p \equiv \theta-\iota\zeta$, $\psi\equiv\psi_t\equiv B_0 r^2\equiv$toroidal flux.

-Drift eqns:

$\mathbf{v}_D= \mathbf{v}_B+\mathbf{v}_E=\frac{\hat{B}}{\mathbf{M\Omega}}\times\nabla V$, with $V\equiv\mu B+e\Phi$. (2)

$$\Rightarrow \dot{\psi}=\nabla\psi\,.\,\mathbf{v}_D=\frac{c}{e}\partial_{\alpha_p}V,\quad \dot{\alpha}_p=\nabla\alpha_p\,.\,\mathbf{v}_D=-\frac{c}{e}\partial_\psi V \qquad (3)$$

-Bounce average, using bounce action

$J(\psi,\alpha_p|\mu,E)=(2\pi)^{-1}\oint ds\, Mv_\|(s)$: (4)

$$\Rightarrow \partial_{\psi,\alpha_p}J=\oint\frac{ds}{2\pi}\partial_{\psi,\alpha_p}Mv_\| = -\oint\frac{ds}{2\pi v_\|}\partial_{\psi,\alpha_p}V=-\overline{\partial_{\psi,\alpha_p}V}/\Omega_b, \qquad (5)$$

with $1/\Omega_b\equiv\oint\frac{ds}{2\pi v_\|}=\partial_E J$ = bounce time/(2π).

$$\Rightarrow \bar{\dot{\psi}}=-\frac{c}{e}\frac{\partial_{\alpha_p}J}{\partial_E J}=\frac{c}{e}\partial_{\alpha_p}H,\quad \bar{\dot{\alpha}}_p=\frac{c}{e}\frac{\partial_\psi J}{\partial_E J}=-\frac{c}{e}\partial_\psi H, \qquad (6)$$

with $E=H(\mathbf{x},\rho_\|,\mu)=\frac{1}{2}M\rho_\|^2\Omega^2+V(\mathbf{x},\mu)$ the Hamiltonian.

-Early ideas (assume Φ=0 for simplicity) :

-Isodynamic (Palumbo) condition:
[Palumbo, Nuovo Cimento **X53B**, 507 (1968).]

-If can create config with $B=B(\psi)$, (7)

then $\dot{\psi} = \frac{c}{e}\partial_{\alpha_p} V = 0$, ie, $\dot{\psi} = \nabla\psi . v_{\mathbf{B}} \sim \nabla\psi . \mathbf{B}\times\nabla B = 0$.

-However, an expansion sol'n of the equilibrium eqs around the magnetic axis yields [eg, Garren, Boozer, Phys. Fluids –B **3**, 2805 (1991)]

$B(\psi,\theta,\zeta)=B_0(\zeta)[1-\kappa(\zeta)x]+ O(x^2)$, (8)

where $x \equiv -\mathbf{r}.\hat{\kappa}(\zeta)=\psi^{1/2}x_1(\zeta)\cos(\theta-\alpha_1(\zeta))$,

$\boldsymbol{\kappa} \equiv$ axis curvature$=\hat{\kappa}\,\kappa(\zeta)$, $\psi \sim O(x^2)$.

So can have $B=B(\psi)$ only if

(a) $B_0(\zeta)=0$ (uninteresting for confinement), or

(b) $\kappa(\zeta)=0$ (can't have for all ζ for toroidal config).

-Omnigenous condition: (originally considered for mirror machines, in
[Hall, McNamara, Phys.Fluids **18**, 552 (1975).])

-Create fields with $J=J(\psi)$. (9)

Then $\overline{\dot{\psi}} = \frac{c}{e}\partial_{\alpha_p} H \propto -\partial_{\alpha_p} J = 0$.

-Meyer-Schmidt (MS) configurations:

[Meyer, Schmidt, Z. Naturforsch. **13A**, 1005 (1958).]

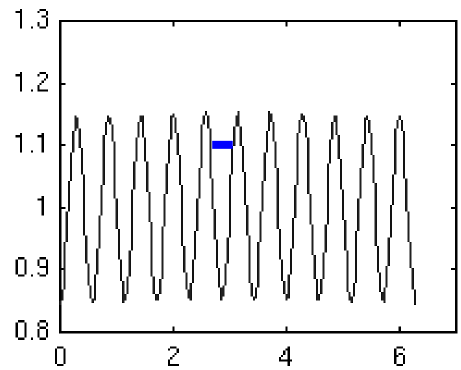
-Sought to improve high-β equilibrium properties by reducing Pfirsch-Schlueter currents J_{PS}. Achieved by localizing ripple to the inside of the torus in such a way that toroidal term $e_t(\psi,\theta)=\varepsilon_t\cos(\theta)$ in field strength

$$B(\mathbf{x})=B_0[1-e_t(\psi,\theta)-e_h(\psi,\theta)\cos\eta], \qquad (10)$$

is approximately eliminated.

-QH (& helically-symmetric) configurations:

[Boozer, Phys.Fluids **26**, 496 (1983),
Nuehrenberg, Zille, Phys. Lett. A **129**, 113 (1988).]



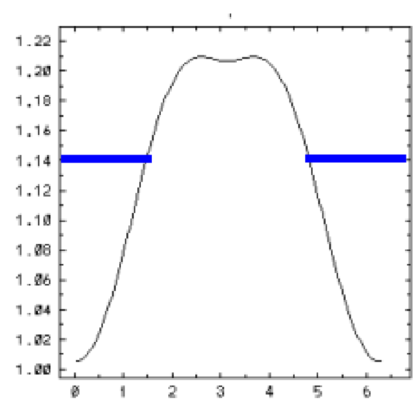
$\Rightarrow B=B(\psi,\eta)$, where $\eta \equiv n\zeta - m\theta$.

$\Rightarrow \int ds.. \rightarrow \int d\eta\, L_h..$, $L_h \equiv (ds/d\eta) \approx R/n$,

$\Rightarrow J=J(\psi)$, and $\bar{\dot{\psi}}=0$.

-QA (& axisymmetric) configurations:

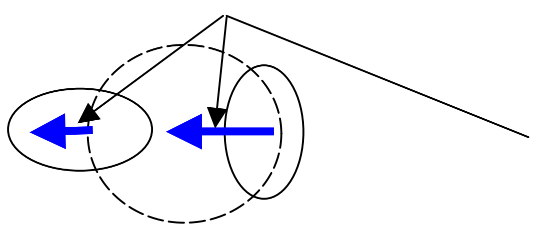
[Nuehrenberg, Lotz, and S. Gori, in *Theory of Fusion Plasmas*, E. Sindoni, F. Tryon and J. Vaclavik eds., SIF, Bologna, (1994),
Garabedian, Phys. Plasmas **3**, 2483 (1996).]



$\Rightarrow B=B(\psi,\theta)$.

$\Rightarrow \int ds.. \rightarrow \int d\theta\, L_t..$, $L_t \equiv (ds/d\theta) \approx qR$,

$\Rightarrow J=J(\psi)$, and $\bar{\dot{\psi}}=0$



Create QA's by modulating $\kappa(\zeta)$ so that $\kappa(\zeta)\, x_1(\zeta)$=const as ζ varies:

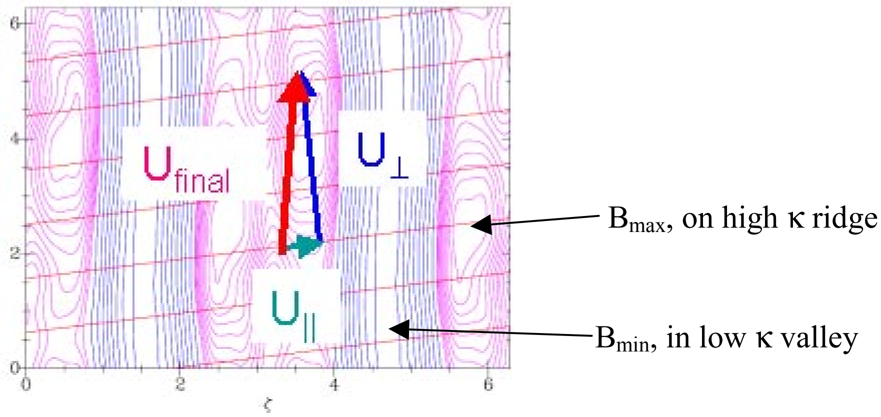
-All QS systems [QA,QH, QP?'s] approximate omnigenity.

-QP configurations:

$B=B(\psi,\zeta)$.

-Optimizers like the one (STELLOPT) used to design NCSX, QPS, have achieved excellent confinement aiming at QP symmetry:

[Spong, Phys. Plasmas 12, 056114 (2005)].



- However, axis-expansion sol’ns

$B(\psi,\theta,\zeta)=B_0(\zeta)[1-\kappa(\zeta)x]+ O(x^2)$

lead to the conclusion [Mikhailov, Shafranov, Subbotin, Isaev, Nuehrenberg, Zille, Cooper, Nucl.Fusion 42, L23 (2002)] :

“The third type of symmetry, poloidal symmetry, cannot be satisfied in toroidally closed stellarator configurations, in particular not in a linear approximation with respect to the distance from the magnetic axis.”

-While the B_{min}-valley is quite flat, there is appreciable variation along the B_{max}-ridge, making QPS a low-A member of the QO family.

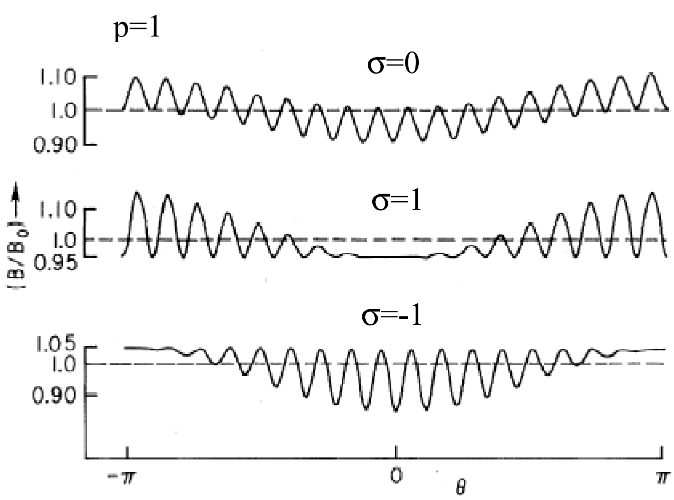
-QO/QI configurations:

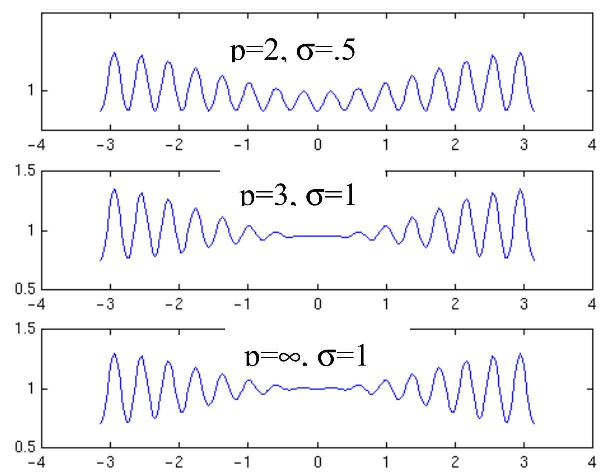
-Discovered while investigating configs related to MS configs. [Mynick, Chu, Boozer, Phys. Rev. Letters **48**, 322 (1982), Nuehrenberg, Zille, Phys. Lett. **114A**, 129 (1986)]

-Relaxes requirement $\bar{\dot{\psi}}=0$ for *all* particles to just $\bar{\dot{\psi}}\approx 0$ for *most troublesome particles*.

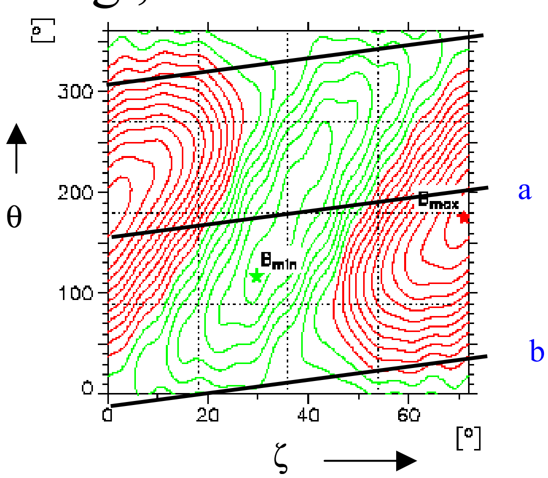
-Makes use of contribution to $\bar{\dot{\psi}}$ from modulation of $e_h(\psi,\theta)$. Take model

$e_h(\psi,\theta)=\varepsilon_h(\psi)(1-\sigma\cos\theta)$, $\varepsilon_h(\psi) \sim \psi^{|m/2|} \sim r^{|m|}$. (11)

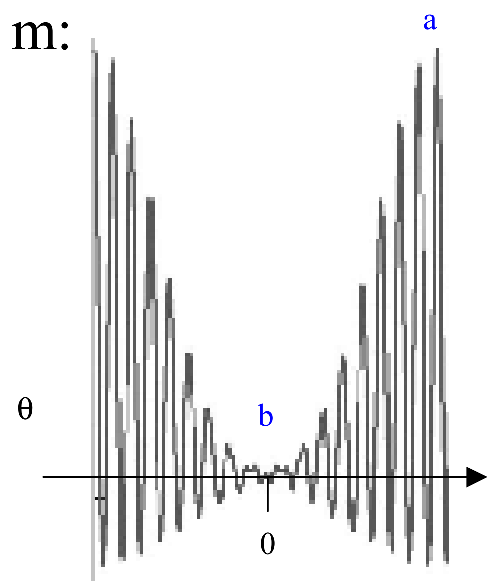




-E.g., for W7-X:



-LHD, R_{ax}=3.53 m:



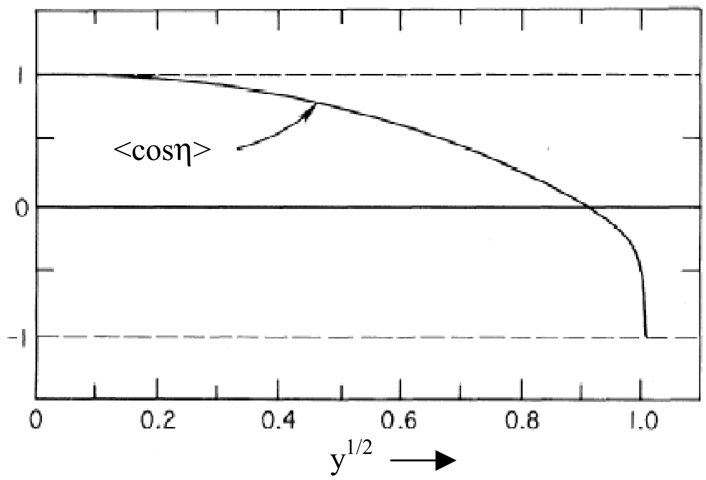
-From (3) or (6), find

$\bar{\dot{r}} = (dr/d\psi)\bar{\dot{\psi}} \;= v_{B0}\sin\theta(\varepsilon_t - \sigma\varepsilon_h \langle\cos\eta\rangle)$,

$\bar{\dot{\theta}} = v_{B0}(e_t + m e_h \langle\cos\eta\rangle)/r$, (12)

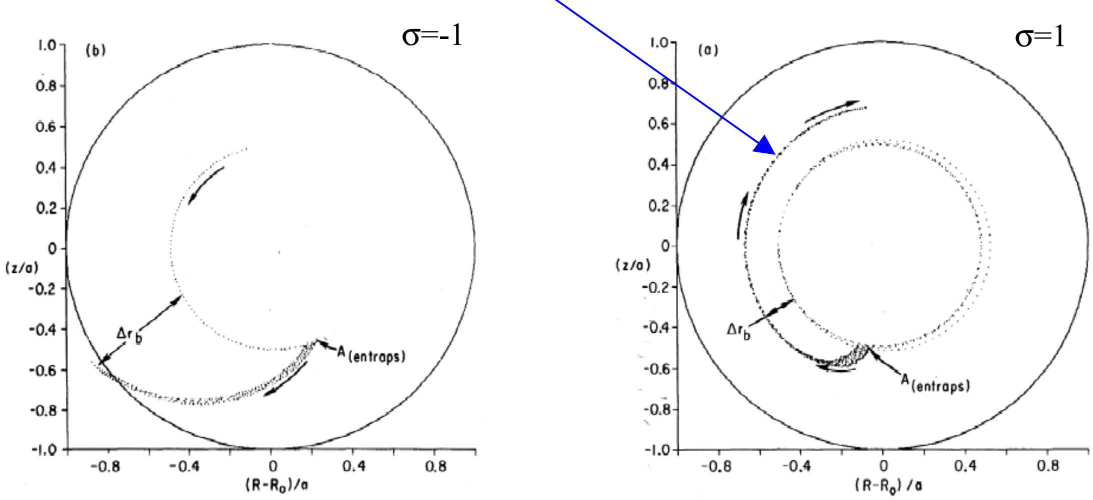
where $v_{B0} \equiv \mu B_0/(M\Omega r)$,

$\langle\cos\eta\rangle(y) = 2E(y^{1/2})/K(y^{1/2}) - 1$:



$\Rightarrow$Can make $\bar{\dot{r}} = 0$ for $1 = \sigma p \langle\cos\eta\rangle$, (13)

where $p \equiv \varepsilon_h/\varepsilon_t$.

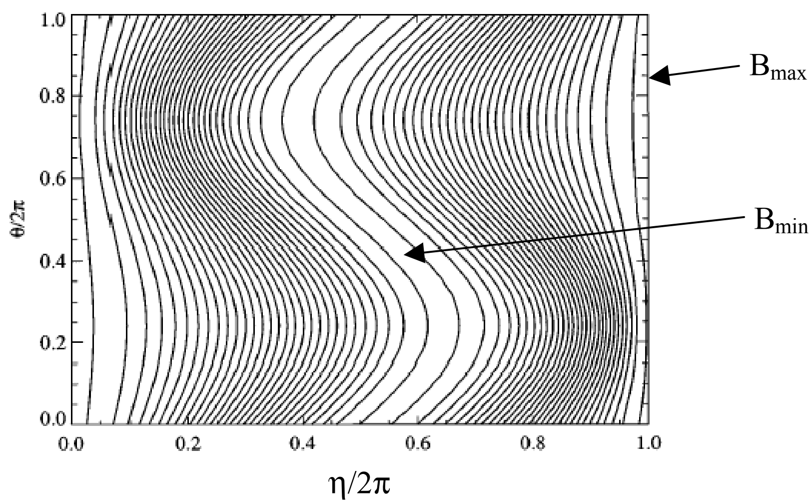


Since $D \sim \bar{\dot{r}}^2$, can reduce D by factors 10-50 by using $\sigma p \sim 1$- 1.5.

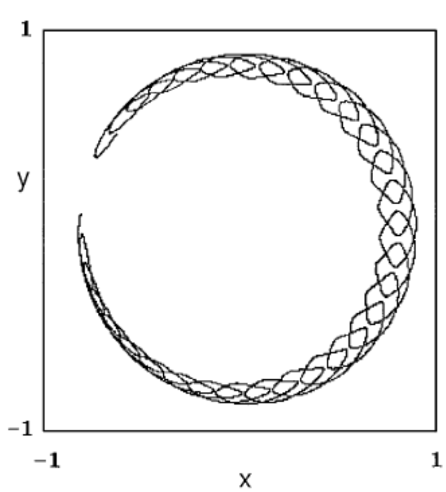
-Isometric/ Approximately Omnigenous Configs:
[Skovoroda, Shafranov, Plasma Physics Reports **21**, 886 (1995),
Cary, Shasharina, Phys.Rev.Lett., **78**, 674 (1997).]

-Set of configurations *properly* containing QS ones, satisfying J≈J(ψ), $\bar{\dot{\psi}} \approx 0$ for *almost all* particles, but permitting variation in shape of ripple-well as particle drifts poloidally.

-J=J(ψ) results in "isometry condition", that lengths along **B** between any 2 contours with constant B=|**B**| are constant.



-The θ-dependence makes the banana-width vary as the s.b. precesses:



-No concrete implementations have yet been attempted.

-Pseudo-symmetric Configurations:

[Shafranov, et al., Proc of Int. Symposium on Plasma Dynamics in Complex Electromagnetic Fields for Comprehension of Physics in Advanced Toroidal Plasma Confinement, Dec.8-11, 1997, Research Report, Inst. of Advanced Energy, Kyoto Univ, March 20 (1998) p.193,
Mikhailov, Shafranov, Suender, Plasma Physics Reports **24** 653 (1998).]

-Widen range of transport-optimized configs by imposing less stringent condition of no ripple-wells.
$\Rightarrow$Removes sb mechanism [no ripple-trapped particles (τ=h)], leaving only the banana-drift branch, due to toroidally-trapped ones (τ=t).

-Energetic vs thermal transport:

-Design features which diminish $\bar{\psi}$ tend to improve confinement for both thermal and energetic ions.

-However, energetic particles (eg, NB ions, α's) have features which differ importantly from thermal:
-ν≈0 transport determined by the full ν=0 orbits, where not just $\bar{\psi}$, but also $\bar{\dot{\theta}}$, are important.
-insensitive to effects of Φ (V≈μB).
-less sensitive to turbulence, due to orbit-avging effects [Mynick, Strachan, Phys.Fluids **24**, 695 (1981)]
-particles τ=t,p *more* sensitive to effects of variations δB in B (stochastic mechanisms).

-Thus, energetic & thermal confinement results can sometimes be uncorrelated. Eg:
-NCSX has much lower ε_{ef} (recall $D_{-1} \sim \varepsilon_{ef}^{3/2}/\nu$) than W7X, but much worse α confinement ($F_{loss}\sim$ .25 vs < .05) at finite β (eg, [Lotz, et al., PPCF **34** 1037 (1992)]), due to adequate $\bar{\dot{\theta}}$ for τ=h orbits in W7X , which doesn't help τ=t particles in QAs.

-**Turbulent transport:**

-Methods of mitigation:

-Internal transport barriers via root-jumping:

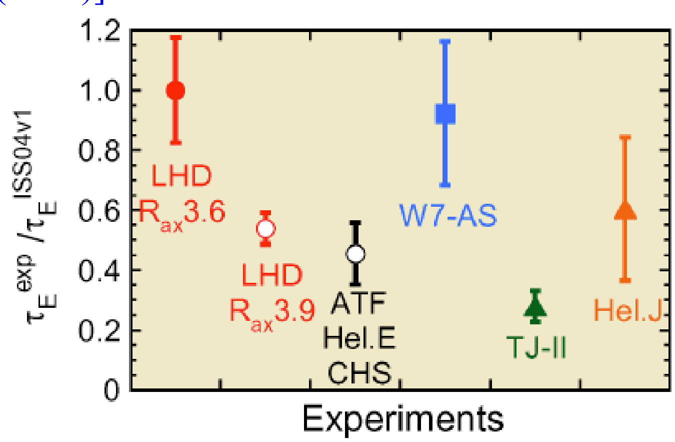
-As discussed last time, root jumps provide rapidly-changing $E_r \Rightarrow$possibility of flow-shear.

-Observed on W7-AS [Stroth, et, al., Phys. Rev. Lett., **86**, 5910 (2001)] , and on LHD [Ida, et al., Phys. Rev. Lett. **91**, (2003).].

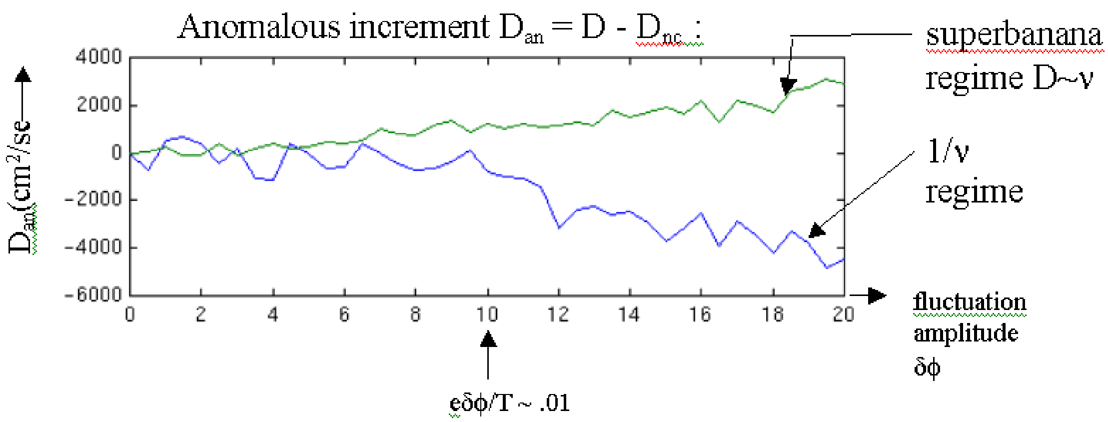
-Turbulence modifications from shaping:

-Factors which affect microstability, like global & local shear, locations of good & bad curvature and trapped particles, and the E_r-profile, will vary across toroidal configurations, and thus one may expect levels of turbulent transport to vary with design.

-Empirical support for this [H.Yamada, EPS **28B**, P-5.099 (2004)]:



-Effect of turbulence on total transport can be counter to our tokamak-based intuitions. Eg [Mynick, Boozer, (to appear in Phys. Plasmas (2005))] :



-Numerical work on this still in early stages. Eg:
[Rewoldt, Ku, Tang, W.A. Cooper,Phys. Plasmas **6** 4705 (1999),
Scott, Phys. Plasmas **7**, 1845 (2000),
Kendl, Scott, Wobig, Plasma Phys. Controlled Fusion **42**, L23 (2000),
Jenko, Kendl, New J. of Phys.**4**, 35.1 (2002)].

-Much remains to be studied.