STELLARATOR THEORY AT COLUMBIA

Allen Boozer
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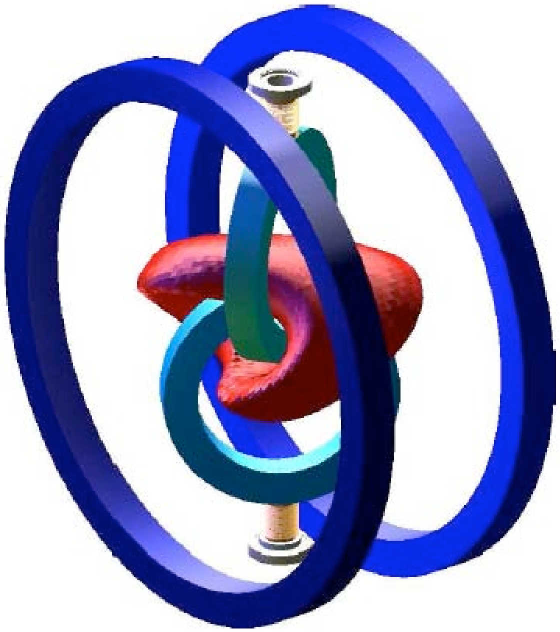
Columbia Non-neutral Torus (Thomas Pedersen)
- Equilibrium (Remi Lefrancois)
- Stability (Allen Boozer)
- Neo-classical Transport (*Harry Mynick*)

Interactions of Coils and Plasma
- Efficient Coils (Ron Schmidt)
- Separation of $\vec{B}$ into parts produced inside and outside a torus (David Lazanja)

Equilibrium Issues
- Perturbed Equilibria (Allen Boozer and *Carolin Nührenberg*)
- Definition of a stochastic plasma edge (Allen Boozer)

Thomas Pedersen's CNT Experiment and Supporting Theory

http://www.ap.columbia.edu/CNT/



THE COLUMBIA NON-NEUTRAL TORUS

Major radius (average)	0.3 m
Minor radius (average)	0.15 - 0.2 m
Magnetic field (on axis)	$\approx 0.3\,\mathrm{T}$
Rotational transform (ι)	0.15 - 0.63
Expected electron density	10^{12} - $10^{14}\,m^{-3}$
Expected electron temperature	1 - 100 eV
Neutral pressure	$< 3 \times 10^{-10}$ Torr

Objective is to study pure electron plasmas of 10's of Debye lengths in scale confined on magnetic surfaces.

Purposes: (1) basic physics, (2) trap for positrons, (3) effect of $\vec{E}$ on confinement

Equilibrium of Pure Electron Plasmas on Magnetic Surfaces

If electrons reach other points on a magnetic surface more rapidly by motion along $\vec{B}$ than by *EXB* motion, then electron density and temperature obey:

$T(\psi)$ and $n(\vec{x}) = N(\psi)\exp(e\phi/T)$.

$\vec{B}$ not modified by presence of plasma due to low density and temperature.

Electric potential obeys $\nabla^2\phi = \dfrac{e}{\epsilon_0} N(\psi)\exp\left(\dfrac{e\phi}{T}\right)$

In an equilibrium of a quasi-neutral plasma: (1) Change in $\vec{B}$ due to $\vec{\nabla} p = \vec{j} \times \vec{B}$ main difficulty. (2) Charge imbalance $e\delta n = \epsilon_0 \nabla^2 \phi$ usually so irrelevant that it is not calculated. (3) $e\phi \sim T$ set by ambipolarity,

For pure electron plasmas: (1) Change in $\vec{B}$ effectively zero. (2) Calculation of electric potential and $n(\vec{x})$ are major difficulties. (3) $e\phi/T \sim (a/\lambda_D)^2$. (3) Force balance $m_e n\vec{v}\cdot\vec{\nabla}\vec{v} + \vec{\nabla} p + en(\vec{E} + \vec{v}\times\vec{B}) = 0$ gives *ExB* drift for $n << n_B \equiv \epsilon_0 B^2/2m_e$, Brillouin limit.

Basic equilibrium properties:

If $e\phi/T \sim (a/\lambda_D)^2 << 1$: Electron density constant on magnetic surfaces, $n(\psi)$.

If $e\phi/T \sim (a/\lambda_D)^2 >> 1$: $\phi(\vec{x}) = \phi_0(\psi) + \tilde{\phi}(\vec{x})$ with $e\tilde{\phi}/T = \ln(\text{geometric terms})$.

Calculating an equilibrium means finding $\phi(\vec{x})$ and $n(\vec{x})$ given:

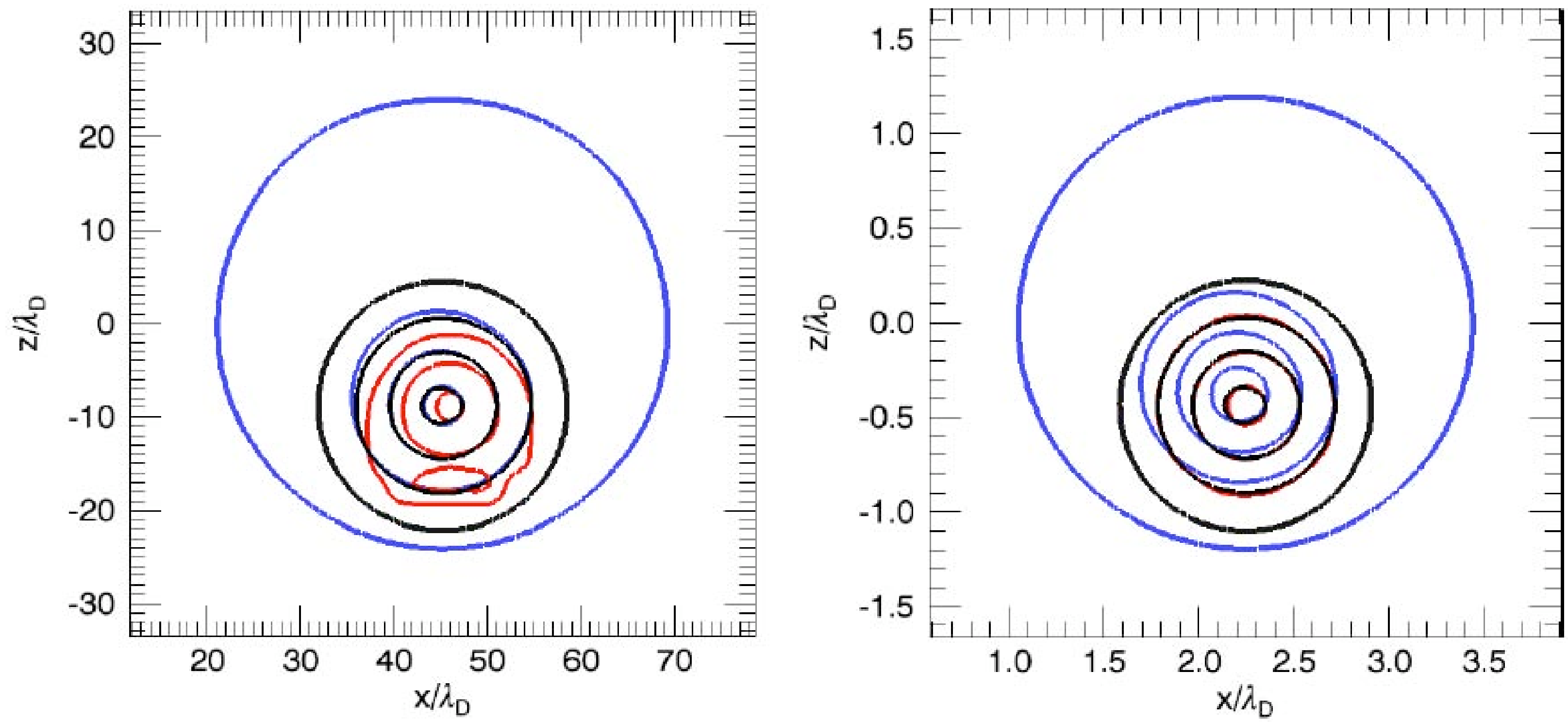
1. the profiles $N(\psi)$ and $T(\psi)$ {Note $n(\vec{x}) = N(\psi)\exp(e\phi/T)$.}

2. the shapes of the magnetic surfaces $\vec{x}(\psi,\theta,\zeta)$

3. the boundary condition on $\phi(\vec{x})$.

The optimal boundary condition is $\phi(\vec{x})$ constant on outermost magnetic surface, which can be enforced by a highly conducting grid.

Thomas Pedersen and Remi Lefrancois have written an iterative code to find 3D equilibria. $\square^2\square = \frac{e}{\square_0} n(\vec{x})$ solved in Fourier space using $\frac{\partial\square_{\vec{k}}}{\partial t} = \square k^2 \square_{\vec{k}} \square \frac{e}{\square_0} n_{\vec{k}}$.



Toroidally symmetric equilibria with downward shifted surfaces: blue is $\square$, n is red, and magnetic surfaces, or $\square$, are black. Left has $a/\square_D \square 10$; right $a/\square_D \square 1$.

Stability of Pure Electron Plasmas on Magnetic Surfaces

Boozer, Phys. Plasmas 11, 4709 (2004) showed plasmas confined on magnetic surfaces are in a minimum energy state for perturbations that maintain:

(1) Particle and entropy conservation;
(2) Force balance;
(3) Constancy of T along $\vec{B}$.

Essential result is that under these assumptions perturbations do not allow electrons to cross magnetic surfaces, so they cannot tap their enormous repulsive energy.

Method: Imagine plasma surface covered by a thin shell by which one can control the potential on the plasma surface $\phi_s(\theta,\varphi) = \sum V_i(t) f_i^*(\theta,\varphi)$. The surface charge on the shell has the form $\sigma_s(\theta,\varphi) = \sum Q_i(t) f_i^*(\theta,\varphi) / \oint da$.

Power required to perturb $V_i(t)$ is $P = \sum \delta V_i^* d\delta Q_i / dt$. Show capacitance is Hermitian, $\delta Q_i = \sum C_{ij} \delta V_j$, so $P = d\delta W / dt$ with $\delta W = \frac{1}{2} \delta\vec{V}^\dagger \cdot \overleftrightarrow{C} \cdot \delta\vec{V} = \frac{1}{2} \int \left(\epsilon_0 (\vec{\nabla}\delta\phi)^2 - \delta\rho\,\delta\phi \right) d^3x$ where $\delta\rho \equiv -\epsilon_0 \nabla^2 \delta\phi$. Charge perturbation, $\delta\rho[\delta\phi]$ is shown to respond so $\delta W > 0$.

Neoclassical Transport

Simple balance of $\vec{B}\times\vec{\nabla}B$ drift and *ExB* drifts has been used to estimate neoclassical confinement, which improves as $\rho_D/a \to 0$.

Harry Mynick is carrying out Monte Carol calculations.

<u>Interactions of Coils and Plasma</u>

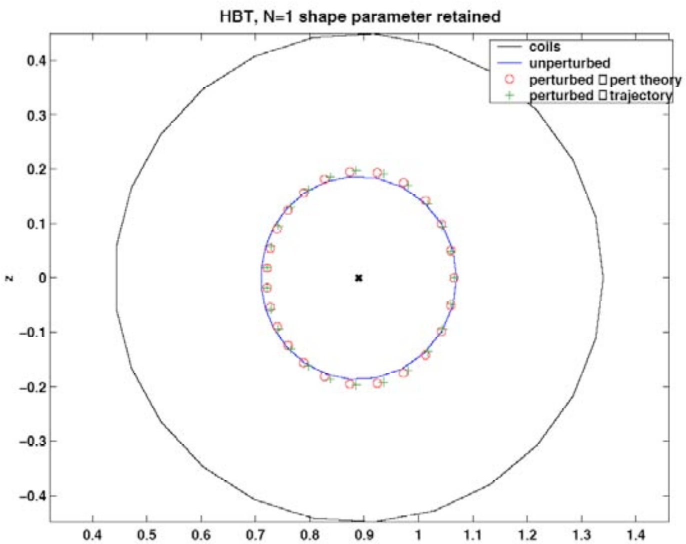
<u>Efficient Coils (Ron Schmidt)</u>

Issue is finding the optimal coils for sustaining features of plasma shape that are known to be important to the physics without otherwise penalizing coils.

Coil/plasma field ratio scales $(b/a)^{(m-1)}=exp\{(m-1)ln(b/a)\}$, so difficulty of coils increases exponentially with the number of constraints the coils must obey.

Plasma surface described as $\vec{x}_s(\theta,\varphi,\vec{s})$ with $\vec{s}$ the set of important shape parameters.

1. Find $\hat{n}\cdot\partial\vec{x}_s/\partial s_i$ with $\hat{n}$ normal to surface. (Gives changes in shape,)
2. Use $\delta\vec{B}\cdot\vec{\nabla}r = \vec{B}\cdot\vec{\nabla}(\vec{\xi}\cdot\vec{\nabla}r)$ to find $(\delta\vec{B}\cdot\vec{\nabla}r)_i$. (Field distribution of a shape change.)
3. When plasma present need permeability matrix of RWM theory to find external field change due to a shape change, $(\delta\vec{B}\cdot\hat{n})_i = \sum P_{ij}(\delta\vec{B}_x\cdot\hat{n})_j$.
4. Let $b_i \equiv (\delta\vec{B}_x\cdot\hat{n})_i = \sum M_{ij}I_j$, then SVD analysis of $\overleftrightarrow{M}\cdot\overleftrightarrow{R}^{-1/2}$ finds surface current distribution $\kappa(\theta,\varphi) = \sum I_j f_j(\theta,\varphi)$ on a coil surface that minimizes $\vec{I}^{\dagger}\cdot\overleftrightarrow{R}\cdot\vec{I}$.


HBT, N=1 shape parameter retained
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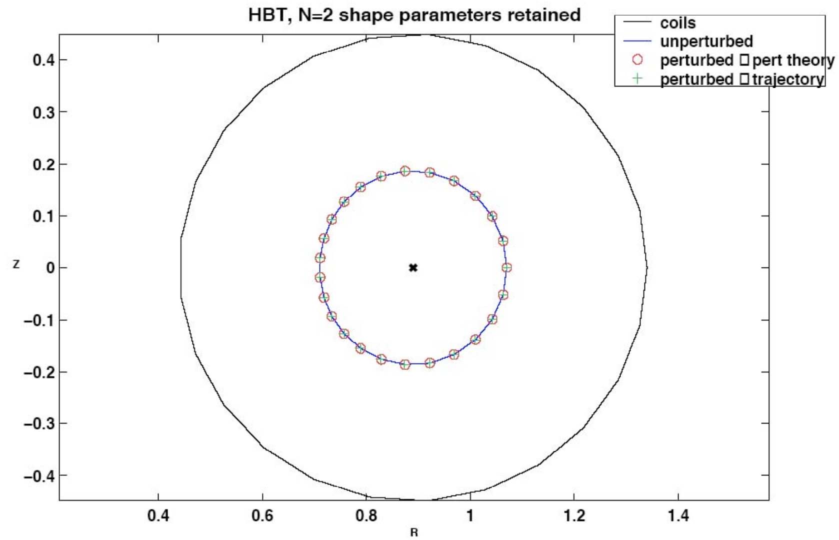
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perturbed trajectory
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Separation of $\vec{B}$ into parts produced inside and outside a torus

David Lazanja

In annulus between a toroidal plasma and external currents $\vec{B} = \vec{\nabla}\phi$ with $\nabla^2\phi = 0$.

Solution consists of a part that is non-singular inside region enclosed by plasma surface, $\phi_x(\vec{x})$, due to external currents and a part non-singular outside the plasma surface, $\phi_p(\vec{x})$, due to currents in the plasma.

By giving ϕ and $\hat{n}\cdot\vec{\nabla}\phi$ on the plasma surface one can find both $\phi_x(\vec{x})$ and $\phi_p(\vec{x})$ throughout the annulus.

A δW code calculates full $\delta\vec{B}$ on plasma surface given $\delta\vec{B}\cdot\hat{n}$ on that surface. This allows one to calculate permeability matrix $(\delta\vec{B}\cdot\hat{n})_i = \sum P_{ij}(\delta\vec{B}_x\cdot\hat{n})_j$.

Equilibrium Issues

Perturbed Equilibria (Allen Boozer & Carolin Nührenberg)

C. Nührenberg and A. H. Boozer, Physics of Plasmas 10, 2840 (2003).

δW Stability Codes can be used to:

1. Find improved equilibria and equilibria with complicated forces
2. Find and eliminate islands through jumps in $[\iota']$
3. Design and interpret magnetic diagnostics
4. Find improved equilibria and find minimal number constraints on coils.

Plasma Effects on the Location of the Outermost Magnetic Surface

Allen Boozer (to be discussed at EPS)

In stellarators a stochastic magnetic field lines can provide the plasma boundary with the open field lines having sufficient length that a significant pressure drop Δp occurs across this stochastic layer.

Small magnetic perturbations $(\delta B/B)_x$ produce these stochastic layers. The $(\delta B/B)_x$ are produced by currents in coils and in the body of the plasma.

Parallel currents, associated with the *jxB* forces required to balance the pressure gradients in the stochastic layer, produce magnetic fields that resonate with the magnetic structure.

The magnetic perturbations $(\delta B/B)_\mathrm{p}$ produced by Δp are relatively simple to calculate if one assumes they are small compared $(\delta B/B)_x$.

Simple picture of the stochastic layer is clearly violated if $(\delta B/B)_p>(\delta B/B)_x$, which occurs for a sufficiently high $\langle\beta\rangle$. That is, for $\langle\beta\rangle>\langle\beta\rangle_\mathrm{c}$.

Calculation of $\Box\Box_c$ requires $p(r,\Box,\Box)$ in differentiable form.

Find $n(r,\Box,\Box)$ using Monte Carlo method $\dfrac{\partial n}{\partial t} + \vec{\mathrm{v}}_{\|} \cdot \vec{\Box} n = \vec{\Box} \cdot (D \vec{\Box} n)$

Basic validity requires $\vec{\Box} \cdot \vec{\mathrm{v}}_{\|} = 0$. Let $\vec{\mathrm{v}}_{\|} = \Box(3C_s / B_0)\vec{B}$ with $3C_s / B_0 = const.$ and the change in $\Box$ over a time step $\Box$ is $\Box_n = (1 \Box \Box\Box)\Box_o \pm \sqrt{(1 \Box \Box_o^2)\Box\Box}$ with ± a random sign.

Each spatial coordinate x changes over a time step $\Box$ due to diffusion $x_n = x_o \pm \sqrt{D\Box}$.

Find expansion $n(\vec{x}) = \Box\, n_i f_i(\vec{x})$ with f_i smooth functions over volume of stochastic layer and $\Box f_i f_j d^3x = \Box_{ij} \Box d^3x$. Then $n_i = \dfrac{1}{N} \sum_{n=1}^{N} f_i(\vec{x}_n) n(\vec{x}_n)$ with $\vec{x}_n = \vec{x}(t = n\Box)$.

Summary

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