stellarator theory teleconference
March 22, 2007, PPPL

Development of Moment Approach for Neoclassical Transport in Stellarators

Presented by

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Acknowledgements:

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Outline

Introduction

Developments of the stellarator moment equation approach.

From Hirshman and Sigmar (1981), Shaing and Callen (1983) to Sugama and Nishimura (2002)

Three mono-energetic viscosity coefficients

*M** (parallel viscosity against flow), *N**(driving force for the flows), *L**(radial diffusion)

Although we do not show today any calculation examples on actual existing or planned helical/stellarator devices, many application examples (QPS, HSX, NCSX, W7X, LHD) by Dr.D.A.Spong were already reported by the QPS group.

Fusion Sci.Technol **45**, 15(2004), **46**, 215(2004), in APS 2004,
Phys.Plasmas **12**, 056114(2004), *Nucl.Fusion* **45**, 918 (2005),
in 15th ISW 2005, *Sellarator News* **11** (Nov. 2005), in past teleconferences

But the DKES code were used in these examples to obtain *M**, *N**, *L**

Analytical methods to obtain *M**, *N**,*L** for applications in an integrated simulation system

please see, S.Nishimura, et al., *Fusion Sci.Technol*. **51**, 61(Jan. 2007)

(1) Physical meaning of a constant H_2 introduced by Shaing, et al. in their bootstrap (BS) current theories.

(2) A role of numerical solvers for the DKE with the pitch-angle-scattering (PAS) collision operator as benchmark tools to test the analytical formulas for the neoclassical viscosities.

(3) A role of bounce- or ripple-averaging codes in the integrated simulation.

Outline (2)

To include the Pfirsch-Schlüter transport in general 3-D configurations

Extension of tokamak P-S transport theory based on the moment approach.

→ Impurity transport studies.

Some suggestions given by CHS/LHD experimental results for this extension.

(1) spontaneous parallel flows of collisional impurity

(2) poloidal variation of the plasma density

electrostatic potential being a flux surface quantity.

This kind of measurements have to be done also in future advanced stellarators (NCSX, QPS)

Steps toward this development

discussions on the collaboration plan for the benchmarking using configuration datum of NCSX, QPS, and other devices in the U.S.

(1) mono-energetic viscosity coefficients M^*, N^*, L^* (DKES, MonteCarlo)

(e.g., $1/\nu^{1/2}$ diffusion in CHS-qa)

(2) total neoclassical fluxes Γ_a, q_a, J_{BS}

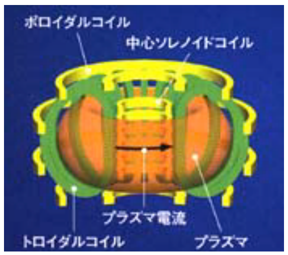
for arbitrary multi-ion species plasmas (other kinetic codes in the U.S.)

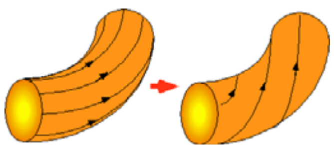
Summary, Concluding Remarks

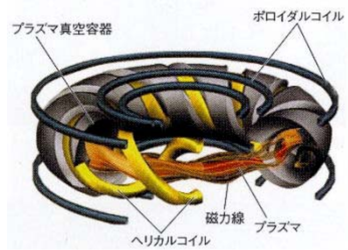
History of the “neoclassical” theory

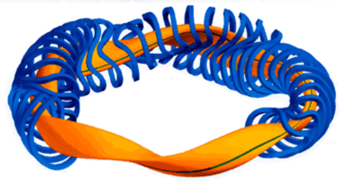
A.A.Galeev, R.Z.Segdeev, H.P.Furth, and M.N.Rosenbluth, *Sov.Phys.JETP* **26**(1968)
PRL **22**, 511 (1969)

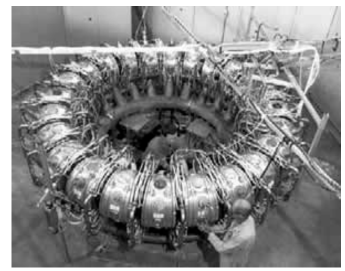
Tokamaks | Helical/Stellarators | Bumpy torus | Mirror











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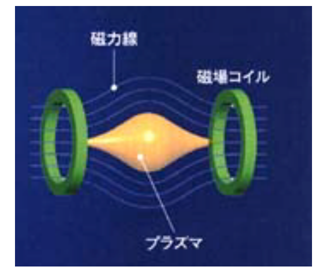


Linked mirror
Without I_p,Without ι

With magnetic flux surfaces with finite ι

S.P.Hirshman and D.J.Sigmar,
Nucl.Fusion **21**, 1079 (1981)

$\langle \mathbf{B}\bullet\nabla\boldsymbol{\pi}_a\rangle - n_a e_a\langle BE_{//}\rangle = \langle BF_{//a1}\rangle$
$\langle \mathbf{B}\bullet\nabla\boldsymbol{\theta}_a\rangle = \langle BF_{//a2}\rangle$

→NCLASS code
W.A.Houlberg, K.C.Shaing,
S.P.Hirshman, and M.Zarnstorff,
Phys.Plasmas **4**, 3230 (1997)

K.C.Shaing and J.D.Callen,
Phys.Fluids **26**, 3315 (1983)

N.Nakajima and M.Okamoto,
J.Phys.Soc.Jpn. **61**, 833 (1992)

H.Sugama and W.Horton,
Phys.Plasmas **3**, 304 (1996)

C.L.Hedrick, D.A.Spong,
D.E.Hastings, J.S.Tolliver, et al. in 1980’s

The bounce-averaged DKEs for the bounce-averaged motion of ripple-trapped particles

$$-\frac{v_D^a m_a}{B_0}\frac{\partial}{\partial\mu}\mu J\frac{\partial}{\partial\mu}G_{Xa}^{(1/\nu)} = \frac{m_a c}{e_a\psi'}\frac{\partial J}{\partial\theta} \qquad (1/\nu)$$

$\mathbf{b}\bullet\nabla G_{Xa}^{(1/\nu)}=0$

→Shaing-Hokin, FPSTEL, GSRAKE,
GIOTA, NEO, etc.,etc.,..

Ambipolar condition $\Sigma e_a \Gamma_a(E_s)=0$

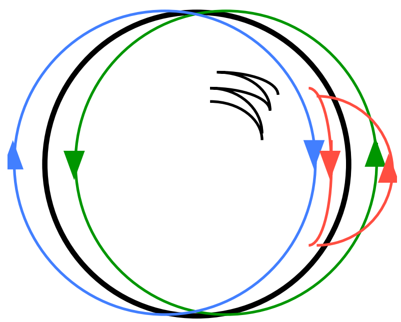
Unified theory!

Why do we discuss now on the moment method by Hirshman and Sigmar?

When the magnetic flux surface is formed….

(1) We have to calculate all of the contributions of circulating, toroidally-trapped, ripple-trapped particles.Non-bounce-averaged motion of these particles make parallel particle and heat flow fluxes such as the bootstrap(BS) currents and the Pfirsch-Schlüter(P-S) currents. And therefore the particle, momentum, and energy balances in these flows have to be taken into account.

(2) Because of a characteristic of the perturbations corresponding to the ripple diffusions $\mathbf{b}\bullet\nabla G_{X\mathrm{a}}^{(1/\nu)}=0$, the theories and/or codes for the ripple diffusions and those for the flows had been constructed independently at least until the former half of 1990's.
But an integration and/or unification of these two types of theories is required now.
(For e.g., BS current calculation under the self-consistent ambipolar radial electric fields)



(3) By the way, when we consider the $\iota\rightarrow 0$ ($\rightarrow$bumpy torus) limit in present B-S currents and the P-S currents formulas, the current $\rightarrow\infty$, since these formulas generally include B_{10}/ι.
This singularity with respect to ι is caused by a fact that circulating, toroidally-trapped particles correspond to the "loss cone" in the bumpy torus.
The unification including the bumpy torus is difficult in resent status.

Fortunately, it seems that researchers of the bumpy tori are not included in the participants of this conference !
Therefore, I present here only treatments of configurations with magnetic flux surfaces with finite rotational transforms. (i.e., helical/stellarators)

H.Sugama and S.Nishimura, *Phys.Plasmas* **9**, 4637 (2002)
"How to calculate the neoclassical viscosity, diffusion, and current coefficients in general toroidal plasmas"
"with magnetic flux surfaces with finite rotational transforms $\iota\neq 0$, and with sub-sonic flows $\mathbf{u}\bullet\nabla\mathbf{u}\approx 0$."

The generalized "correspondence principle" in the "neoclassical" theory

Quantum Mechanics ($h \to 0$) $\to$ Newtonian Mechanics
Theory of Relativity ($c \to \infty$)

Non-symmetric :
Viscous damping of the flows
$\leftrightarrow$ non-ambipolar

$\Leftrightarrow$

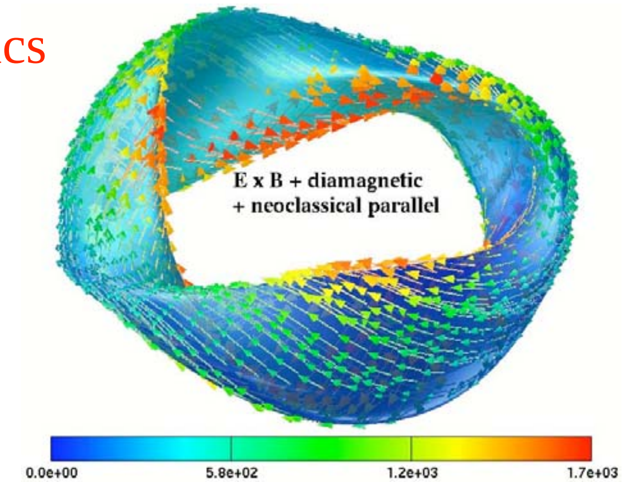
Symmetric (axisymmetric, straight stellarators) :
Rigid rotation in the direction of the symmetry
$\leftrightarrow$ intrinsic ambipolar

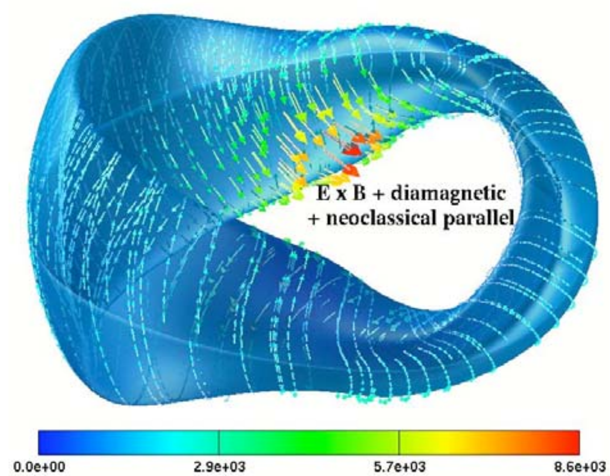
In 1960's and 1970's, this characteristic of the neoclassical diffusions in the symmetric systems was often explained as a conservation of the canonical angular momentum of the systems.

"correspondence principle" in our 21 century :

$c_1 \partial B/\partial\theta + c_2 \partial B/\partial\zeta \to 0$ $\to$ intrinsic ambipolar rotations minimizing the viscosities

axisymmetry (c_1=0),
helical symmetry ($c_1 c_2 \neq 0$),
poloidal symmetry (c_2=0).



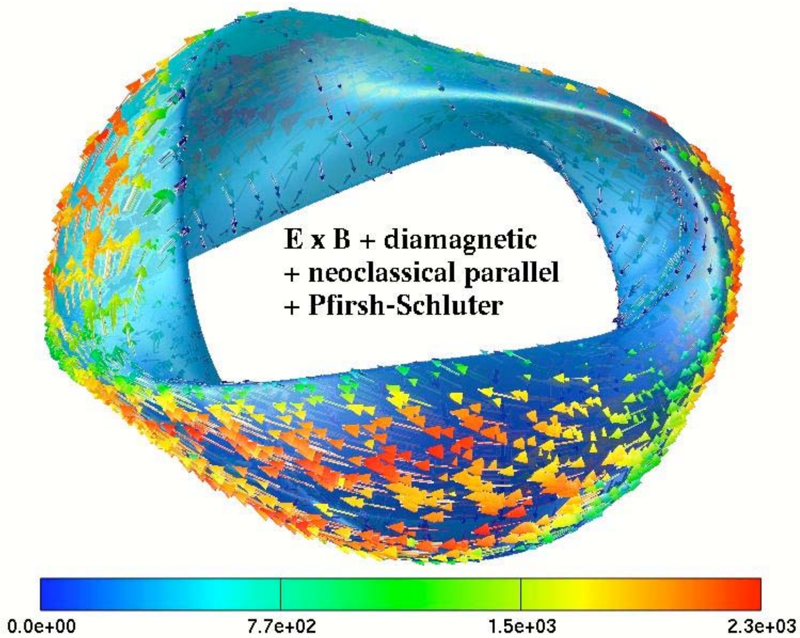


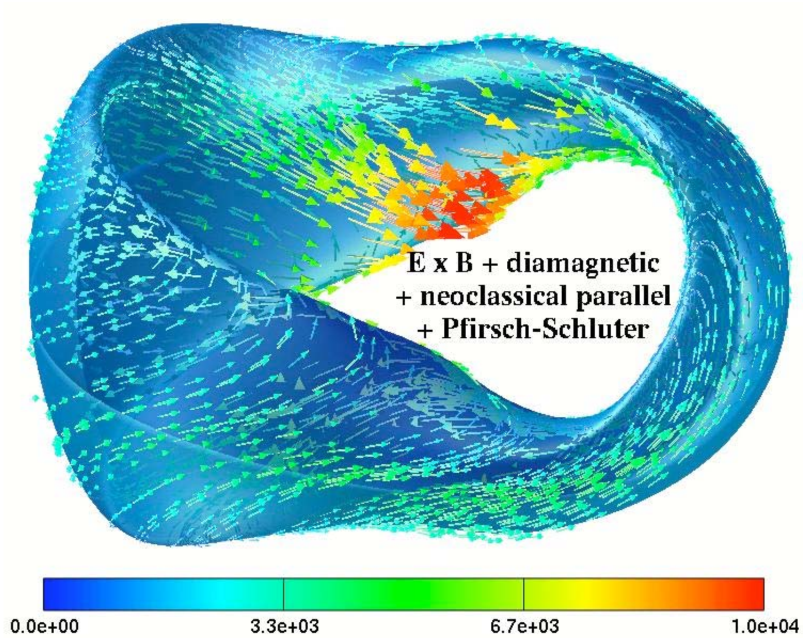
DKES requires B_{mn}, but does not require R_{mn}, Z_{mn}, ϕ_{mn}.

Local structure of the flow pattern
before the flux surface averaging has a winding determined by

$$\nabla\cdot(n_a\mathbf{u}_{//a}) = -\nabla\cdot(n_a\mathbf{u}_{\perp a})$$

(Later,we will consider this structure determining the P-S transport)



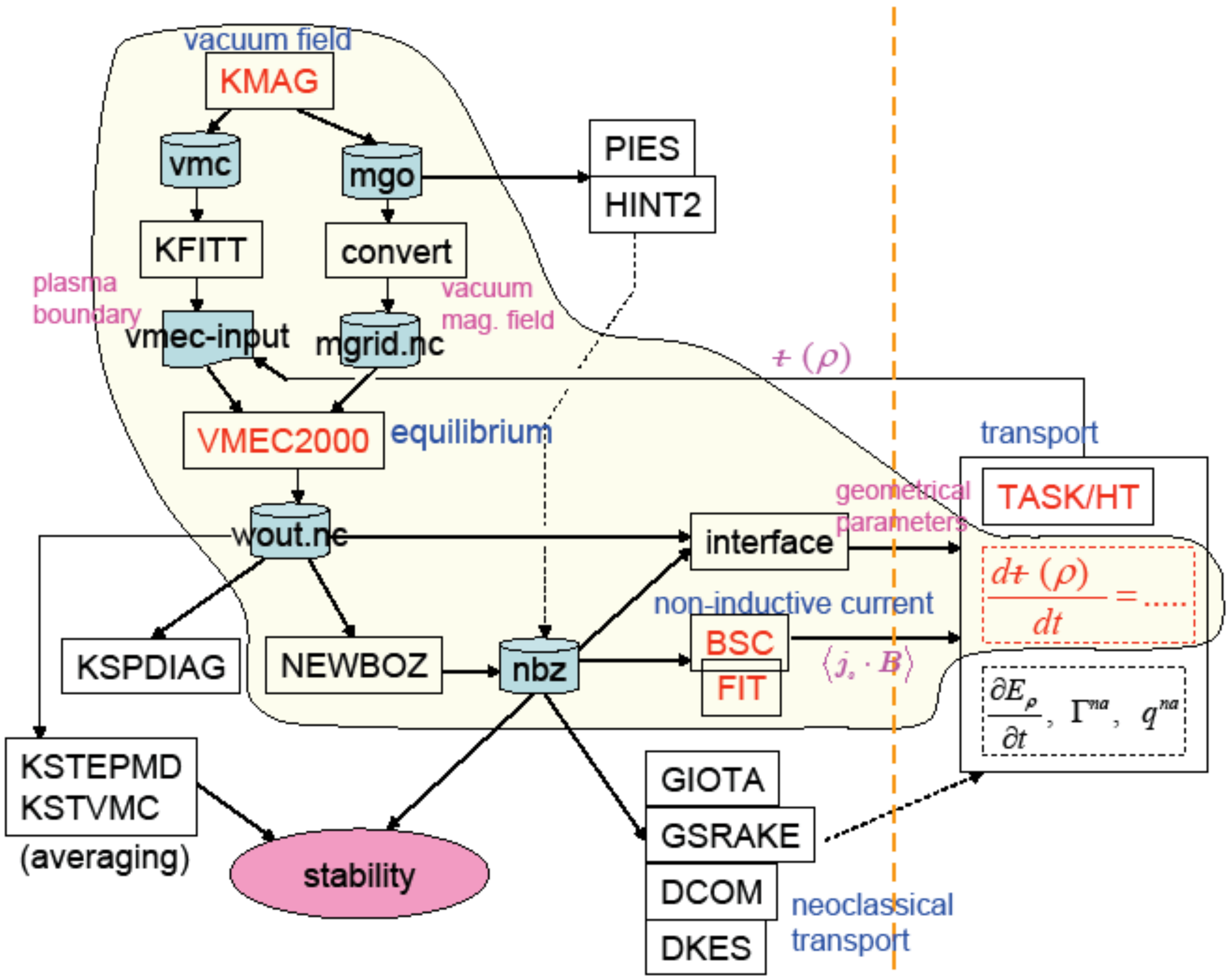


Even though it is well known that the radial diffusions are dominated by the turbulent transport, plasma flows along the flux surfaces will be determined by the neoclassical processes. The momentum balance including friction forces for the flows determines impurity accumulation and/or shielding.

In contrast to toroidally rotating tokamaks, however, this winding structure will not be simply determined by the incompressible condition $\nabla\cdot\mathbf{u}_a=0$, $\nabla\cdot\mathbf{q}_a=0$.

Integrated Simulation System

(Y.Nakamura, et al., in 15th ISW 2005, LHD coordinated research)



Moment equations

(to the MHD equilibrium $\nabla p = \mathbf{J}\times\mathbf{B}$ and a higher order corresponding to the Grad's 13M approximation)

$$\frac{\partial f_a}{\partial t} + \mathbf{v}\bullet\frac{\partial f_a}{\partial \mathbf{x}} + \frac{e_a}{m_a}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\bullet\frac{\partial f_a}{\partial \mathbf{v}} = C_a(f_a) \cong \sum_b C_{ab}(f_{a1}, f_{bM}) + \sum_b C_{ab}(f_{aM}, f_{b1})$$

taking $\int d^3\mathbf{v}\ \mathbf{v}\, v^n$ and $\int d^3\mathbf{v}\ v^n$ moments of distribution function and this equation itself

$$\frac{\partial n_a}{\partial t} = -\nabla\bullet(n_a\mathbf{u}_a)$$

Here we use the particle conservation of $C_a(f_a)$

$$m_a n_a\left(\frac{\partial \mathbf{u}_a}{\partial t} + \mathbf{u}_a\bullet\nabla\mathbf{u}_a\right) = n_a e_a\left(\mathbf{E} + \frac{\mathbf{u}_a\times\mathbf{B}}{c}\right) + \mathbf{F}_{a1} - \nabla p_a - \nabla\bullet\boldsymbol{\pi}_a$$

parallel force balanc neglecting only d/dt

$$\mathbf{b}\bullet\nabla p_a + \mathbf{b}\bullet\nabla\bullet\boldsymbol{\pi}_a - n_a e_a E_{//} = F_{//a1}$$

radial balance and particle and energy conservation

neglecting only higher order with respect to $\delta B/B$ in $\nabla\bullet\boldsymbol{\pi}_a$ and d/dt

$$\nabla\sum_a p_{\perp a} = \frac{\mathbf{J}\times\mathbf{B}}{c},\ \mathbf{J} \equiv \sum_a (n_a\mathbf{u}_a) \quad \because \sum_a \mathbf{F}_{a1} = 0,\quad \sum_a n_a e_a = 0$$

$$\nabla\bullet\mathbf{J}_{//} = -\nabla\bullet\mathbf{J}_\perp = c\,\nabla\bullet\left(\frac{\nabla\sum_a p_{a\perp}\times\mathbf{B}}{B^2}\right)$$

(well-known Pfirsh-Schlueter(P-S) current)

$$n_a \equiv \int f_a\, d^3\mathbf{v}$$

$$n_a\mathbf{u}_a \equiv \int \mathbf{v} f_a\, d^3\mathbf{v}$$

$$p_a \equiv \frac{m_a}{3}\int |\mathbf{v}-\mathbf{u}_a|^2 f_a\, d^3\mathbf{v}$$

$$T_a \equiv p_a/n_a$$

$$\mathbf{Q}_a \equiv \int \mathbf{v}\, v^2 f_a\, d^3\mathbf{v}$$

$$\mathbf{F}_{a1} \equiv m_a\int \mathbf{v}\ C_a(f_a)\, d^3\mathbf{v},\ \sum_a \mathbf{F}_{a1} = 0$$

(momentum conservation)

$$\mathbf{F}_{a2} \equiv m_a\int \mathbf{v}\left(\frac{m_a v^2}{2T_a} - \frac{5}{2}\right) C_a(f_a)\, d^3\mathbf{v}$$

$$\boldsymbol{\pi}_a \equiv m_a\int \left[(\mathbf{v}-\mathbf{u}_a)(\mathbf{v}-\mathbf{u}_a) - |\mathbf{v}-\mathbf{u}_a|^2\mathbf{I}/3\right] f_a\, d^3\mathbf{v}$$

Moment equations for non-symmetric plasmas

($\mathbf{E}\times\mathbf{B}\bullet\nabla n_a$ and $\mathbf{E}\times\mathbf{B}\bullet\nabla T_a$ are retained)

$$\nabla\bullet(n_a\mathbf{u}_{a//}) = -\nabla\bullet(n_a\mathbf{u}_{a\perp}) \cong c\nabla\bullet\left(n_a\frac{\nabla\phi\times\mathbf{B}}{B^2}\right) + \frac{c}{e_a}\nabla\bullet\left(\frac{\nabla p_{a\perp}\times\mathbf{B}}{B^2}\right)$$

neglecting $\nabla\times\mathbf{B}\bullet\nabla\phi=(4\pi/c)\mathbf{J}\bullet\nabla\phi$, $\nabla\times\mathbf{B}\bullet\nabla p_a=(4\pi/c)\mathbf{J}\bullet\nabla p_a$ by following usual transport ordering

$$\nabla\bullet(n_a\mathbf{u}_{a//}) \cong \frac{c}{e_a}\left(\nabla p_{a\perp} + \langle n_a\rangle e_a\nabla\phi\right)\times\mathbf{B}\bullet\nabla\frac{1}{B^2} + \frac{c\nabla\phi\times\mathbf{B}}{\langle B^2\rangle}\bullet\nabla n_a$$

Here,we used $\delta B/B<<1$ approximation in parts including $\langle\bullet\rangle$.

This relation still retains the P-S current as a consequence of the equilibrium condition.

1st term corresponds to the flow divergence given by $\mathbf{v}_{da}\bullet\nabla f_{aM}$ in DKE as shown later, and 2nd term is a part of $\mathbf{E}\times\mathbf{B}\bullet\nabla f_{a1}$

When neglecting $\frac{c\nabla\phi\times\mathbf{B}}{\langle B^2\rangle}\bullet\nabla n_a$, this equation corresponds to $\nabla\bullet\mathbf{u}_a=0$ (2002)

But this assumption was not esssential.

with a local transport anzatz of $\frac{\partial f_{a1}}{\partial s}\cong 0$ and resulting $\nabla p_{a\perp}\cong\frac{\partial\langle p_a\rangle}{\partial s}\nabla s$, $\nabla\phi\cong\frac{\partial\langle\phi\rangle}{\partial s}\nabla s$

$$n_a\mathbf{u}_{a//} = -\widetilde{U}\frac{c}{e_a}\left(\frac{\partial\langle p_a\rangle}{\partial s} + \langle n_a\rangle e_a\frac{\partial\langle\phi\rangle}{\partial s}\right) + B\int^l \mathrm{d}l\,\frac{c\nabla\phi\times\mathbf{B}}{\langle B^2\rangle B}\bullet\nabla n_a \text{ +integration constant}$$

Even in this approximation, the Pfirsch-Schlüter current $\Sigma e_a n_a\mathbf{u}_a$ is still unchanged. However, flow velocities $\mathbf{u}_a$ of individual species a is now not generally incompressible ($\nabla\cdot\mathbf{u}_a\neq 0$, $n_a\neq$const). Although we previously neglected this effect in our explanation(2002) for simplicity, it is not negligible in impure plasmas with high collision frequencies.

I skip today the derivations of moment equations in higher orders and drift kinetic equation, although they are important and essential. (I show only a final result here.)

Linearized approximations for the Vlasov and the collision operators

(Here is an essential reason to use viscosity and friction coefficients)

collision operator in (v, ξ, φ) or $(v_\alpha, v_\beta, v_\gamma)$ coordinates in the velocity space

$$C_{\rm ab}(f_{\rm a1}, f_{\rm bM}) = \frac{\nu_{\rm D}^{\rm ab}(v)}{2}\left[\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi} + \frac{1}{1-\xi^2}\frac{\partial^2}{\partial\varphi^2}\right]f_{\rm a1} + v^{-2}\frac{\partial}{\partial v}\left[v^3\left(\frac{m_{\rm a}}{m_{\rm a}+m_{\rm b}}\nu_{\rm s}^{\rm ab}(v)f_{\rm a1} + \frac{\nu_{//}^{\rm ab}(v)}{2}v\frac{\partial f_{\rm a1}}{\partial v}\right)\right]$$

pitch-angle-scattering (PAS) and energy scattering (ES)

$$C_{\rm ab}(f_{\rm aM}, f_{\rm b1}) = -\frac{2\pi e_{\rm a}^2 e_{\rm b}^2 \ln\Lambda}{m_{\rm a}}\frac{\partial}{\partial v_\alpha}\int {\rm d}^3\mathbf{v}'\left[\frac{f_{\rm aM}(\mathbf{v})}{m_{\rm b}}\frac{\partial f_{\rm b1}(\mathbf{v}')}{\partial v_\beta'} - \frac{\partial f_{\rm aM}(\mathbf{v})}{\partial v_\beta}\frac{f_{\rm b1}(\mathbf{v}')}{m_{\rm a}}\right]U_{\alpha\beta}(\mathbf{v}-\mathbf{v}')$$

$$= -\frac{4\pi e_{\rm a}^2 e_{\rm b}^2 \ln\Lambda}{m_{\rm a}^2}\left[\frac{\partial}{\partial v_\alpha}\left(f_{\rm aM}\frac{\partial h_{\rm ab}(f_{\rm b1})}{\partial v_\alpha}\right) - \frac{1}{2}\frac{\partial^2}{\partial v_\alpha \partial v_\beta}\left(f_{\rm aM}\frac{\partial^2 g_{\rm ab}(f_{\rm b1})}{\partial v_\alpha \partial v_\beta}\right)\right]$$

$$U_{\alpha\beta}(\mathbf{x}) \equiv x^{-3}(x^2\delta_{\alpha\beta} - x_\alpha x_\beta),\ h_{\rm ab}(f_{\rm b1}) \equiv (1+m_{\rm a}/m_{\rm b})\int {\rm d}^3\mathbf{v}'\frac{f_{\rm b1}(\mathbf{v}')}{|\mathbf{v}-\mathbf{v}'|},\ g_{\rm ab}(f_{\rm b1}) \equiv \int {\rm d}^3\mathbf{v}' f_{\rm b1}(\mathbf{v}')|\mathbf{v}-\mathbf{v}'|$$

$$\int {\rm d}^3\mathbf{v}\left[C_{\rm ab}(f_{\rm a1}, f_{\rm bM}) + C_{\rm ab}(f_{\rm aM}, f_{\rm b1})\right] = 0$$

$$\sum_{\rm a,b}\int {\rm d}^3\mathbf{v}\ m_{\rm a}\mathbf{v}\left[C_{\rm ab}(f_{\rm a1}, f_{\rm bM}) + C_{\rm ab}(f_{\rm aM}, f_{\rm b1})\right] = 0$$

$$\sum_{\rm a,b}\int {\rm d}^3\mathbf{v}\ m_{\rm a}v^2\left[C_{\rm ab}(f_{\rm aM}, f_{\rm bM}) + C_{\rm ab}(f_{\rm a1}, f_{\rm bM}) + C_{\rm ab}(f_{\rm aM}, f_{\rm b1})\right] = 0$$

$$C_{\rm ab}\left(f_{\rm a1} \propto Y_{lm}(\xi,\varphi),\ f_{\rm bM}\right) \propto Y_{lm}(\xi,\varphi),\ C_{\rm ab}\left(f_{\rm aM}, f_{\rm b1} \propto Y_{lm}(\xi,\varphi)\right) \propto Y_{lm}(\xi,\varphi)$$

$$Y_{lm}(\xi,\varphi) \propto P_l^{|m|}(\xi)\exp(im\varphi)$$

Linearized approximations for the Vlasov and the collision operators

Legendre expansion of the gyro-phase averaged distribution functions

$$F(\mathbf{x},v,\xi) = \sum_{l=0}^{\infty} F^{(l)}(\mathbf{x},v,\xi)$$

$$F^{(l)}(\mathbf{x},v,\xi) \equiv P_l(\xi)\,\frac{2l+1}{2}\int_{-1}^{1} d\eta P_l(\eta)\, F(\mathbf{x},v,\eta)$$

$$P_0(\xi)=1,\ P_1(\xi)=\xi,\ P_2(\xi)=\frac{3}{2}\xi^2-\frac{1}{2},\ P_3(\xi)=\frac{5}{2}\xi^3-\frac{3}{2}\xi, \ldots$$

How to treat the Legendre orders of

$l=0$ ($\rightarrow n_a,\ T_a,\ p_a,\ r'_a$)

$l=1$ ($\rightarrow \mathbf{u}_a,\ \mathbf{q}_a \equiv \mathbf{Q}_a - \frac{5}{2}p_a\mathbf{u}_a$)

$l=2$ ($\rightarrow \boldsymbol{\pi}_a,\ \boldsymbol{\theta}_a \equiv \frac{m_a}{T_a}(\mathbf{r}'_a - r'_a\mathbf{I}) - \frac{5}{2}\boldsymbol{\pi}_a$),

$l \geq 3, \ldots$

For $l \geq 2$, test/field ratio $\propto l^2$. Furthermore, PAS/ES ratio $\propto l^2$ ($l \rightarrow \infty$) in transport applications.
For $l=0,1$, full parts of the collision operator are comparable and indispensable in viewpoint of the cons ervation laws

$F^{(l)}(\mathbf{x},v,\xi)$ is expressed by a Laguerre series of $F^{(l)}(\mathbf{x},v,\xi) = P_l(\xi) v^l f_{aM} \sum_j F_{l,j} L_j^{(l+1/2)}(x_a^2)$ especially for $l=0,1$

$$x_a^2 \equiv \frac{m_a v^2}{2T_a}$$

$$L_0^{(1/2)}(x_a^2)=1,\ L_1^{(1/2)}(x_a^2)=\frac{3}{2}-x_a^2,\ L_2^{(1/2)}(x_a^2)=\frac{x_a^4}{2}-\frac{5}{2}x_a^2+\frac{15}{8}, \ldots.$$

$$L_0^{(3/2)}(x_a^2)=1,\ L_1^{(3/2)}(x_a^2)=\frac{5}{2}-x_a^2,\ L_2^{(3/2)}(x_a^2)=\frac{x_a^4}{2}-\frac{7}{2}x_a^2+\frac{35}{8}, \ldots.$$

$(l,j)=(0,0)$: $n_{a1}(\theta,\zeta)$
$(l,j)=(0,1)$: $T_{a1}(\theta,\zeta)$
$(l,j)=(1,0)$: $u_{//a}(\theta,\zeta)$
$(l,j)=(1,1)$: $q_{//a}(\theta,\zeta)$

Linearized approximations for the Vlasov and the collision operators

Friction forces as the Legendre (order l=1) and Lagurre moments of the collision operator

$$\mathbf{F}_{a1} \equiv m_a \int \mathbf{v}\ C_a(f_a)\ d^3\mathbf{v},\ \mathbf{F}_{a2} \equiv m_a \int \mathbf{v}\left(x_a^2 - \frac{5}{2}\right) C_a(f_a)\ d^3\mathbf{v},\ \mathbf{F}_{a3} \equiv m_a \int \mathbf{v}\left(\frac{x_a^4}{2} - \frac{7}{2}x_a^2 + \frac{35}{8}\right) C_a(f_a)\ d^3\mathbf{v}$$

friction-flow relation

$$\begin{bmatrix} \mathbf{F}_{a1} \\ \mathbf{F}_{a2} \\ \mathbf{F}_{a3} \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} & l_{13}^{ab} \\ -l_{21}^{ab} & l_{22}^{ab} & -l_{23}^{ab} \\ l_{31}^{ab} & -l_{32}^{ab} & l_{33}^{ab} \end{bmatrix} \begin{bmatrix} \frac{1}{\langle n_b \rangle}(n_b \mathbf{u}_b) \\ \frac{2}{5\langle p_b \rangle}\mathbf{q}_b \\ \mathbf{u}_{b2} \end{bmatrix}$$

Energy exchange or scattering coeeficients for l=0 components

$$\begin{bmatrix} \nabla \bullet (n_a \mathbf{u}_a) \\ \nabla \bullet \mathbf{q}_a \\ \nabla \bullet \mathbf{u}_{2a} \end{bmatrix}_{(\text{collision})} = \sum_b \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & P_{ab}^{12} \\ 0 & -\left(P_{ab}^{21} + P_{ab}^{11}\right) & P_{ab}^{22} + P_{ab}^{12} \end{bmatrix} \begin{bmatrix} n_{a1} \\ T_{a1} \\ n_{a2} \end{bmatrix} + \sum_b \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{T_a}{T_b} Q_{ab}^{11} & Q_{ab}^{12} \\ 0 & P_{ab}^{21} + P_{ab}^{11} & Q_{ab}^{22} + Q_{ab}^{12} \end{bmatrix} \begin{bmatrix} n_{b1} \\ T_{b1} \\ n_{b2} \end{bmatrix}$$

S.P.Hirshman and D.J.Sigmar, *Nucl.Fusion* **21**, 1079 (1981)

In the flux surface averaged part of the momentum balance using the 13M approximation, the energy scattering are neglected, and the 3rd row and the 3rd column (Laguerre order of j=2) in the friction-flow relation are truncated.

But we retain them when calculating the Pfirsch-Schlüter transport as show later.

Full neoclassical matrix for general toroidal plasmas (except the bumpy torus)

collisionless regime in non-symmetric configurations
(so-called ripple diffusion)

$$\begin{bmatrix} \Gamma_a^{bn} \\ q_a^{bn} \end{bmatrix} = \begin{bmatrix} (L^{bn})_{11}^{aa} & (L^{bn})_{12}^{aa} \\ (L^{bn})_{21}^{aa} & (L^{bn})_{22}^{aa} \end{bmatrix} \begin{bmatrix} X_{a1} \\ X_{a2} \end{bmatrix}$$

(GIOTA, NEO, GSRAKE, FPSTEL, etc.)

Pfirsch-Schlueter

$$\begin{bmatrix} \Gamma_a^{PS} \\ q_a^{PS} \end{bmatrix} = \sum_b \begin{bmatrix} (L^{PS})_{11}^{ab} & (L^{PS})_{12}^{ab} \\ (L^{PS})_{21}^{ab} & (L^{PS})_{22}^{ab} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix}$$

$$\sum_a e_a \Gamma_a^{PS} = 0$$

(intrinsic ambipolar condition
due to the momentum conservation)

banana-plateau + ripple

$$\begin{bmatrix} \Gamma_a^{bn} \\ q_a^{bn} \end{bmatrix} = \sum_b \begin{bmatrix} (L^{bn})_{11}^{ab} & (L^{bn})_{12}^{ab} \\ (L^{bn})_{21}^{ab} & (L^{bn})_{22}^{ab} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix} + \begin{bmatrix} (L^{bn})_{E1}^{a} \\ (L^{bn})_{E2}^{a} \end{bmatrix} \langle BE_{//} \rangle$$

$$J^{BS} = \sum_b \begin{bmatrix} (L^{bn})_{E1}^{b} & (L^{bn})_{E2}^{b} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix} + (L^{bn})_{EE} \langle BE_{//} \rangle$$

symmetric configurations

$$\sum_a e_a \Gamma_a^{bn} = 0$$

(intrinsic ambipolar cpndition
due to the momentum conservation)

a,b = electron, ion, impurities

$$X_{a1} \equiv -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a \frac{\partial \phi}{\partial s}, \quad X_{a2} \equiv -\frac{\partial T_a}{\partial s}$$

M,N,L matrix and flux surface averaged parts of the parallel momentum balance determining $\langle n_a u_{//a} B\rangle$, $\langle q_{//a} B\rangle$ as the integration constants of $\nabla\bullet(n_a u_{//a})$, $\nabla\bullet q_{//a}$

(H.Sugama and S.Nishimura, Phys.Plasmas 9, 4637(2002))

$$\begin{bmatrix} \langle \mathbf{B}\bullet\nabla\boldsymbol{\pi}_a\rangle \\ \langle \mathbf{B}\bullet\nabla\boldsymbol{\theta}_a\rangle \\ \Gamma_a^{bn} \\ q_a^{bn} \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \frac{1}{\langle n_a\rangle}\langle n_a u_{//a} B\rangle/\langle B^2\rangle \\ \frac{2}{5\langle p_a\rangle}\langle q_{//a} B\rangle/\langle B^2\rangle \\ -\frac{1}{\langle n_a\rangle}\frac{\partial\langle p_a\rangle}{\partial s} - e_a\frac{\partial\langle\Phi\rangle}{\partial s} \\ -\frac{\partial\langle T_a\rangle}{\partial s} \end{bmatrix}$$

Given by an approximated DKE
(numerically and/or analytically)
(with energy integrations)

In symmetric cases
$L_{aj} \propto N_{aj} \propto M_{aj}$

combined with
the friction-flow relation

$$\sum_b \left(\frac{\delta_{ab}}{\langle B^2\rangle} \begin{bmatrix} M_{a1} & M_{a2} \\ M_{a2} & M_{a3} \end{bmatrix} - \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{21}^{ab} & l_{22} \end{bmatrix} \right) \begin{bmatrix} \frac{1}{\langle n_b\rangle}\langle n_b u_{//b} B\rangle \\ \frac{2}{5\langle p_b\rangle}\langle q_{//b} B\rangle \end{bmatrix}$$

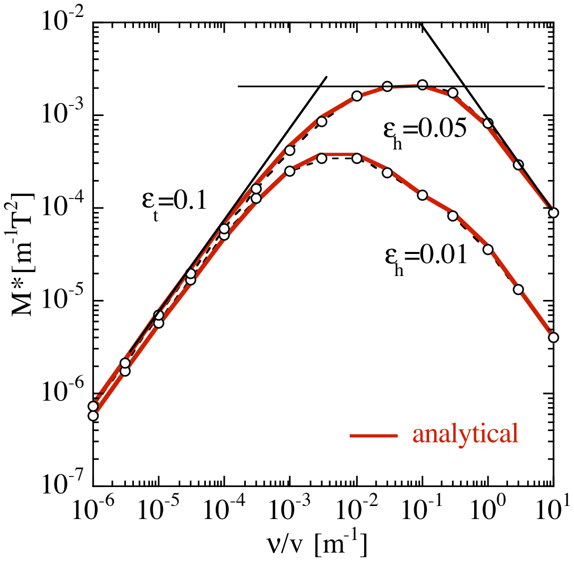
$$= -\begin{bmatrix} N_{a1} & N_{a2} \\ N_{a2} & N_{a3} \end{bmatrix} \begin{bmatrix} -\frac{1}{\langle n_a\rangle}\frac{\partial\langle p_a\rangle}{\partial s} - e_a\frac{\partial\langle\Phi\rangle}{\partial s} \\ -\frac{\partial\langle T_a\rangle}{\partial s} \end{bmatrix} + \begin{bmatrix} n_a e_a \langle B E_{//a}\rangle \\ 0 \end{bmatrix}$$

$a, b = e, D^+, T^+, He^+, He^{2+}, Li^+, Li^{2+}, Li^{3+}, \ldots$

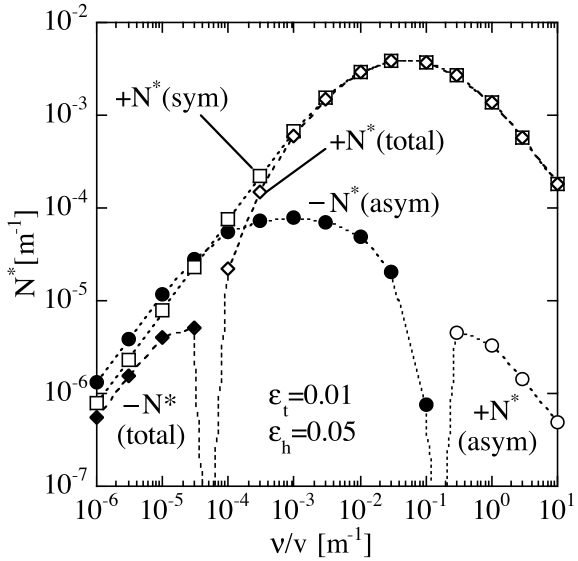
A non-diagonal coupling between particle species is introduced in this step.

DKES as a benchmark tool

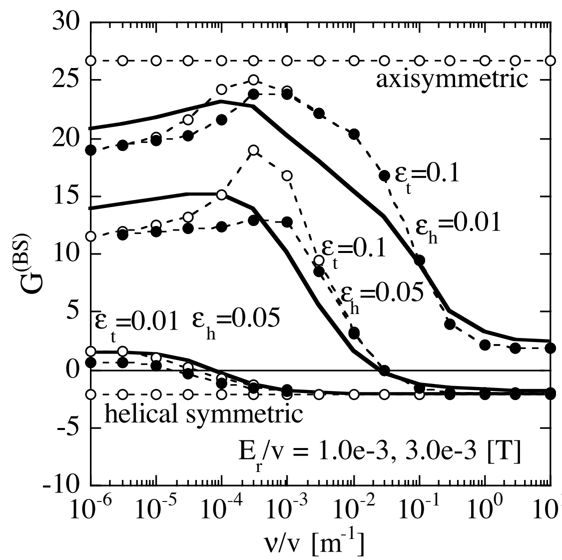
Parallel viscosity against flows M^*



Procedure to obtain off-diagonal coefficients N^* (driving force for BS currents)



Geometrical factor $G^{(BS)} \equiv -\langle B^2 \rangle N^*/M^*$



$$N^{*(\mathrm{sym})} = -M^*\{(\psi' B_\zeta - \chi' B_\theta)/\langle B^2 \rangle + H_2 V'/4\pi^2\}/(2\chi' \psi')$$

(generating the “rigid rotation”)

These coefficients are the function of **B** expressed in the flux surface coordinates as

$M^*=M^*(\chi', \psi', B_\zeta, B_\theta, B_{mn}, \nu/v, E_s/v)$, $N^*=N^*(\chi', \psi', B_\zeta, B_\theta, B_{mn}, \nu/v, E_s/v)$, $L^*=L^*(\chi', \psi', B_\zeta, B_\theta, B_{mn}, \nu/v, E_s/v)$. The formulas are summarized in FS&T **51**, 61(Jan. 2007)

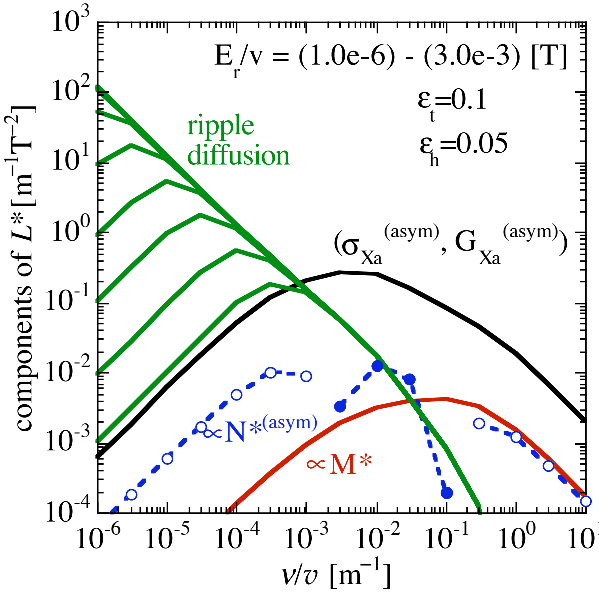
$$B=B_0[1-\varepsilon_{\mathrm{t}} \cos\theta_{\mathrm{B}} + \varepsilon_{\mathrm{h}} \cos(l\theta_{\mathrm{B}} - n\zeta_{\mathrm{B}})],\ l=2,\ n=10,\ B_0=1\mathrm{T},$$

$\chi'=0.15$T•m, $\psi'=0.4$T•m, $B_\theta=0$, $B_\zeta=4$T•m are assumed.

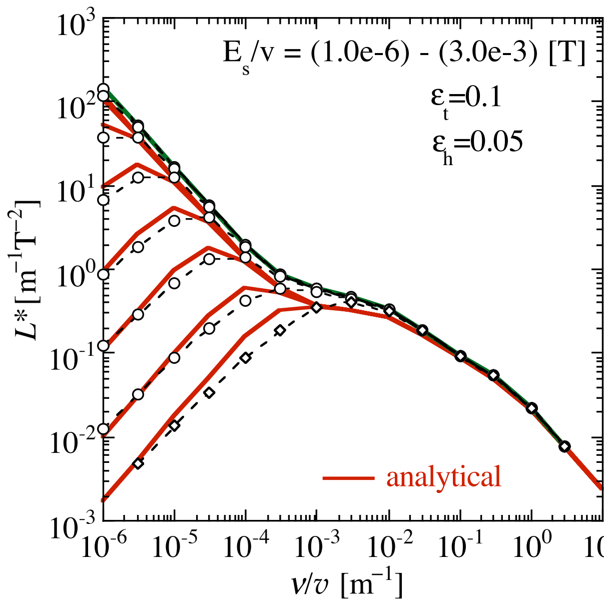
Diagonal radial diffusion coefficients L^* and the boundary layer correction to the parallel viscosity force N^*

(Role of existing bounce- or ripple-averaging codes in the integrated simulation and the analytical calculation of N^*)

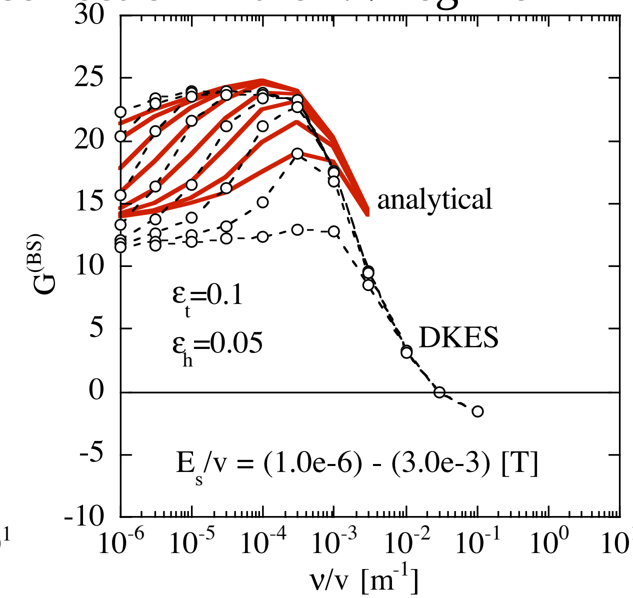
Procedure to obtain diagonal coefficients L^*



Obtained L^*



N^* including the boundary layer correction in the $1/\nu$ regime



mono-energetic coefficients L^* in the LMFP regime

(Importance of $1/\nu^{1/2}$ diffusion in quasi-axisymmetric systems)

$1/\nu^{1/2}$ diffusion coefficient:

extending the theory for rippled tokamaks (K.C.Shaing and J.D.Callen,Phys.Fluids 25,1012(1982))

to multi-helicity stellarators

$$L^{*(-1/2)} = 2.92\,\frac{2}{\pi^2}\left(\frac{v}{\nu_D^a}\right)^{1/2}\frac{2^{3/4}}{(\psi')^2}\left(\frac{V'}{4\pi^2}\frac{B_0}{\chi'}\right)^{1/2}\int_{-\pi}^{\pi}\frac{d\theta}{2\pi}\,\delta_{\rm eff}^{3/4}\left(\frac{\pi-2\sin^{-1}\alpha^*}{N\frac{\psi'}{\chi'}-L-\frac{\partial\gamma}{\partial\theta}}\right)^{1/2}\left\{\left(\frac{\partial\varepsilon_T}{\partial\theta}\right)^2-\sqrt{1-\alpha^{*2}}\,\frac{\partial\varepsilon_T}{\partial\theta}\frac{\partial\varepsilon_H}{\partial\theta}+\frac{2}{9}(1-\alpha^{*2})\left(\frac{\partial\varepsilon_H}{\partial\theta}\right)^2\right\}$$

$1/\nu$ and collisionless detrapping ν regimes diffusion coefficients:

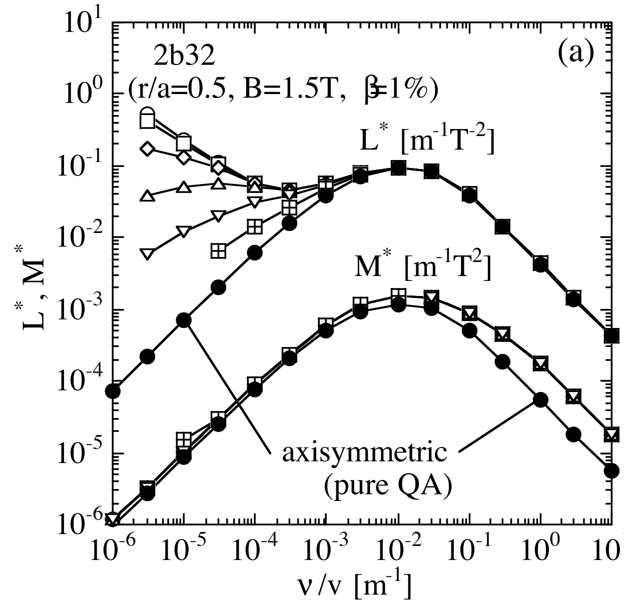
Here we show an example using theory by Shaing and Hokin, and a scaling law by E.C.Crume,Jr. Other advanced bounce averaging codes (FPSTEL, GSRAKE, GIOTA, NEO, etc.) for $1/\nu$, and scaling recently obtained by using DCOM, MOCA, GSRAKE for ν regime are also applicable for this part.

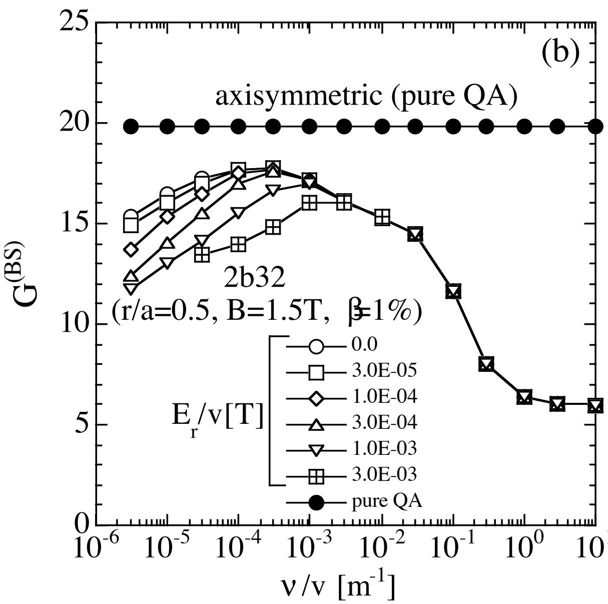
$$L^{*(1/\nu)} = \frac{1}{2\sqrt{2}\pi^2(\psi')^2}\frac{v}{\nu_D^a(K)}\int_0^{2\pi}d\theta_B\{\delta_{\rm eff}\}^{3/2}\left[\frac{16}{9}\left(\frac{\partial\varepsilon_T}{\partial\theta_B}\right)^2-\frac{32}{15}\sqrt{1-\alpha^{*2}}\left(\frac{\partial\varepsilon_T}{\partial\theta_B}\right)\left(\frac{\partial\varepsilon_H}{\partial\theta_B}\right)+0.684(1-\alpha^{*2})\left(\frac{\partial\varepsilon_H}{\partial\theta_B}\right)^2\right]$$

$$L^{*(\nu)} = \frac{\sqrt{\pi}}{6}\,\varepsilon_t^{3/2}\frac{\nu_D^a/v}{(E_s/v)^2}$$

We used these formulas also in the boundary layer correction term in N^*, to determine the boundary collision frequency between $1/\nu$ and ν regimes.

A Numerical Example in CHS-qa





major radius : R =1.5m
minor radius : a =0.47m
magnetic field : $B \leq 1.5$T
toroidal period : N=2
rotational transform: $\iota(r=a)/2\pi=0.4$

(a) mono-energetic radial diffusion coefficients L^*, parallel viscosity coefficients M^*
(b) mono-energetic geometrical factor associated with the bootstrap currents $G^{(BS)}$.

S.Okamura, K.Matsuoka, S.Nishimura, et al., in 19th IAEA (Lyon, 14-19,Oct. 2002)

On poloidally and toroidally varying parts of force balances and flows

In this theory separating the flows and the force balances into two parts,

$\langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_a\rangle - n_a e_a \langle B E_{//}\rangle = \langle B F_{//a1}\rangle$ determining $\langle n_a u_{//a} B\rangle$, $\langle q_{//a} B\rangle$

$\mathbf{b}\bullet\nabla\tilde{p}_a - n_a e_a \tilde{E}_{//} = \tilde{F}_{//a1}$ determining $\widetilde{n_a u_{//a1}}, \widetilde{q_{//a1}}$ ($E_{//}$: by the quasi-neutrality)

where $\nabla\bullet(n_a \mathbf{u}_a)=0$, $\nabla\bullet\mathbf{q}_a$=energy exchange

later part determining the Pfirsh-Schlüter diffusions also must be solved.
(But we didn't discussed about it in 2002.)

Although one simplification of the formulations which often used (Hirshman-Sigmar, 1981, Shaing-Callen, 1983, Sugama-Nishimura, 2002) may be $\nabla\bullet\mathbf{u}_a=0$, $\mathbf{u}_a\bullet\nabla n_a=0$, it is not generally valid in non-symmetric configurations.

Only in the rigid rotation of the symmetric plasmas in the direction of the symmetry, $\mathbf{u}_a\bullet\nabla$ may be $\mathbf{u}_a\bullet\nabla=0$.

The procedure to solve the momentum balance must…

(1) It must automatically include the previous tokamak theory in axisymmetric limits. Particle and energy conservation laws must include not only $\mathbf{E}\times\mathbf{B}\bullet\nabla$ term but also $\mathbf{u}^{E\times B}{}_{//}\bullet\nabla$. $\mathbf{u}^{E\times B}{}_{//}$ is obvious when considering the symmetric plasmas with the rigid rotation.

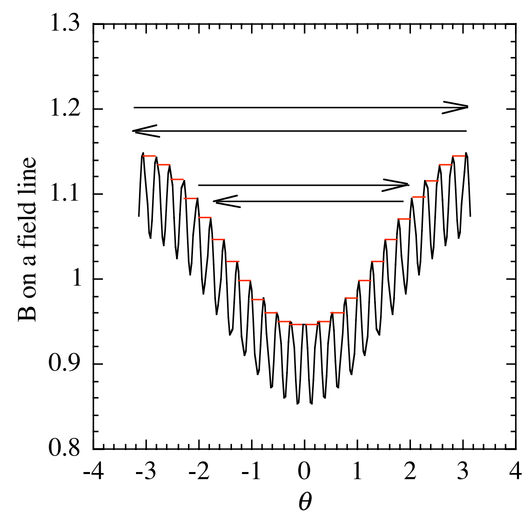
(2) How is it in general non-symmetric configurations ?

The flux surface averaged parallel flows $\langle n_a u_{//a} B\rangle$, $\langle q_{//a} B\rangle$ are composed of two components with contrastive characteristics.

This concept of superposed components is developed in our derivations of analytical formulas for N^* and L^* in 2003-2005.

$$(V_{//}-C_a^{PAS})G_{Xa} = \sigma_{Xa} = \sigma_{Xa}^{(sym)}+\sigma_{Xa}^{(asym)} + \sigma_{Xa}^{(avg)}$$

$$V_{//} \equiv v\xi \mathbf{b}\bullet\nabla - \frac{v}{2}(\mathbf{b}\bullet\nabla \ln B)(1-\xi^2)\frac{\partial}{\partial\xi} \quad C_a^{PAS} \equiv \frac{\nu_D^a}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi}$$



When we divide the radial drift term σ_{Xa} following 3 parts, the solution is given by the linear combination of those for

$(V_{//}-C_a^{PAS})G_{Xa}^{(sym)} = \sigma_{Xa}^{(sym)}$
$(V_{//}-C_a^{PAS})G_{Xa}^{(asym)} = \sigma_{Xa}^{(asym)}$
$(V_{//}-C_a^{PAS})G_{Xa}^{(avg)} = \sigma_{Xa}^{(avg)}$

These parts are solved by different asymptotic expansions ($1/\nu$, banana, plateau, Pfirsch-Schlüter)

$G_{Xa}=G_{Xa}^{(sym)} + G_{Xa}^{(asym)} + G_{Xa}^{(avg)}$, $N^*=N^{*(sym)} + N^{*(asym)} + N^*_{(boundary)}$

The E_r driven parallel flows also include parts corresponding to them.

Characteristics of (sym)/(asym) separation

Bounce averaged motion of trapped particles ($\int(\sigma_{Xa}{}^{(avg)}/v_{//})\mathrm{d}l \neq 0$)

$$\sigma_{Xa}^{(avg)} \equiv -C_a^{PAS} G_{Xa}^{(1/\nu)}$$

Ripple trapped : $1/\nu$ diffusion

Ripple untrapped and boundary layer : $(V_{//}-C_a{}^{PAS})G_{Xa}{}^{(avg)} = 0$

$\rightarrow 1/\nu^{1/2}$ diffusion, parallel viscosity

Remaining non-bounce-averaged guiding center motion is separated into 2 parts.

$\Leftarrow$extension of the theory for the banana regime to collisional regimes.

K.C.Shaing, E.C.Crume, Jr., J.S.Tolliver, et al., Phys.Fluids B**1**, 148 (1989)

K.C.Shaing, B.A.Carreras, N.Dominguez, et al., Phys.Fluids B**1**, 1663 (1989)

$$\sigma_{Xa}^{(sym)} \equiv \frac{m_a c}{2e_a \chi' \psi'}\left\{\frac{\psi' B_\zeta - \chi' B_\theta}{\langle B^2\rangle} + \frac{V'}{4\pi^2}H_2\right\} V_{//}(v\xi B)$$

Local (short L_c)

(sym): E_r driven "rigid rotation" velocity $\mathbf{u}_a$ without friction, viscosity, and heat flow

$$\sigma_{Xa}^{(asym)} \equiv \frac{m_a c}{2e_a \chi' \psi'}\frac{B}{\langle B^2\rangle} v^2 P_2(\xi)\left\{\chi'(1-H_2)\frac{\partial B}{\partial\theta_B} - \psi'(1+H_2)\frac{\partial B}{\partial\zeta_B}\right\}$$
$$+ \frac{m_a c}{e_a}\frac{B}{\langle B^2\rangle} v^2 P_2(\xi)\left(\frac{\partial G}{\partial\zeta_B}\frac{\partial B}{\partial\theta_B} - \frac{\partial G}{\partial\theta_B}\frac{\partial B}{\partial\zeta_B}\right) - \sigma_{Xa}^{(avg)}$$

Global (long L_c)

(asym): E_r driven BS current $e_a n_a \mathbf{u}_a$ (N.Nakajima, et al. *J.Plasma Fusion Res*.68, (1992))

Resulting momentum balance equations for the poloidally and toroidally varying part (→P-S diffusion)

The perturbation functions in the $\nu\to\infty$ limit have to become a shifted Maxwellian. However,this characteristic of the distribution function cannot be automatically obtained by the approximated mono-energetic kinetic equations.

We use moment equations to calculate separated perturbation component (l=0,1) expressed by truncated Laguerre series (j=0,1,2) (corresponding to that in the tokamak P-S transport theory).

$$\langle n_a\rangle \begin{bmatrix} 1 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & 0 & \frac{35}{8} \end{bmatrix} \mathbf{b}\bullet\nabla \begin{bmatrix} n_{a1}^{(j=0)}\langle T_a\rangle/\langle n_a\rangle \\ T_{a1}^{(j=1)} \\ n_{a1}^{(j=2)}\langle T_a\rangle/\langle n_a\rangle \end{bmatrix} - \begin{bmatrix} n_a e_a E_{//} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} F_{//a1} \\ F_{//a2} \\ F_{//a3} \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} & l_{13}^{ab} \\ -l_{21}^{ab} & l_{22}^{ab} & -l_{23}^{ab} \\ l_{31}^{ab} & -l_{32}^{ab} & l_{33}^{ab} \end{bmatrix} \begin{bmatrix} (n_b u_{//b})/\langle n_b\rangle \\ (2/5)q_{//b}/\langle p_b\rangle \\ (n_b u_{//b2})/\langle n_b\rangle \end{bmatrix}$$

Force balance

$$\langle p_a\rangle \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{5}{2} & 0 \\ 0 & 0 & \frac{35}{8} \end{bmatrix} \nabla\bullet \begin{bmatrix} (n_a\mathbf{u}_{//a})/\langle n_a\rangle \\ (2/5)\mathbf{q}_{//a}/\langle p_a\rangle \\ (n_a\mathbf{u}_{//a2})/\langle n_a\rangle \end{bmatrix} + \langle n_a\rangle \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{3}{2} & 0 \\ 1 & -\frac{3}{2} & \frac{15}{8} \end{bmatrix} \left[-\frac{c\nabla\Phi\times\mathbf{B}}{B^2} + u_{//a}^{(\mathrm{rigid})}\mathbf{b} \right]\bullet\nabla \begin{bmatrix} n_{a1}^{(j=0)}\langle T_a\rangle/\langle n_a\rangle \\ T_{a1}^{(j=1)} \\ n_{a1}^{(j=2)}\langle T_a\rangle/\langle n_a\rangle \end{bmatrix}$$

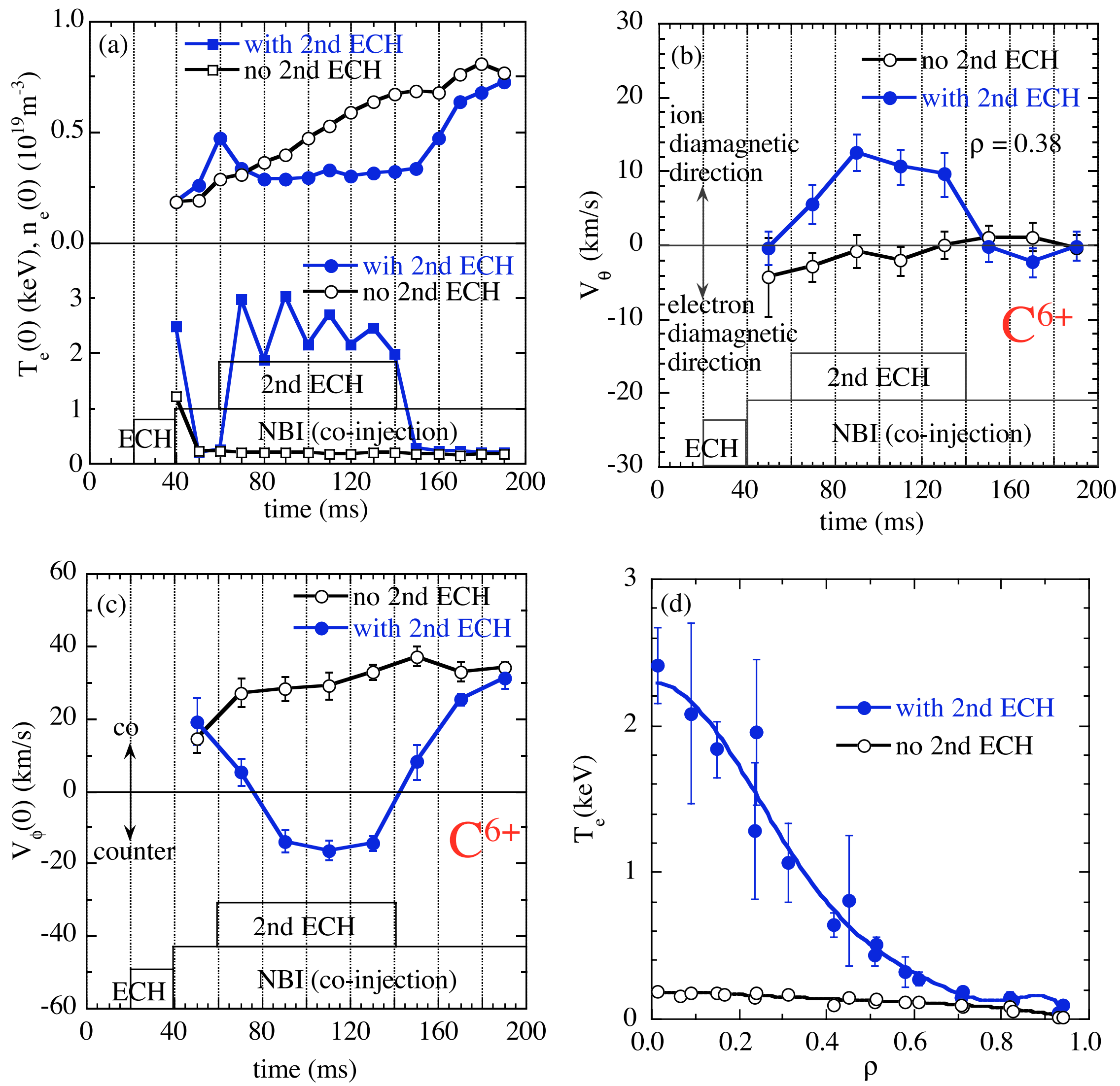
$$-\sum_b \begin{bmatrix} 0 & 0 & 0 \\ 0 & e_{ab}^{11} & 0 \\ 0 & -e_{ab}^{11} & e_{ab}^{22} \end{bmatrix} \begin{bmatrix} n_{b1}^{(j=0)}\langle T_b\rangle/\langle n_b\rangle \\ T_{b1}^{(j=1)} \\ n_{b1}^{(j=2)}\langle T_b\rangle/\langle n_b\rangle \end{bmatrix} = \frac{c}{e_a}\nabla s\times\mathbf{B}\bullet\nabla\frac{1}{B^2} \begin{bmatrix} \langle T_a\rangle\left(\frac{\partial\langle p_a\rangle}{\partial s} + \langle n_a\rangle e_a\frac{\partial\langle\Phi\rangle}{\partial s}\right) \\ \frac{5}{2}\langle p_a\rangle\frac{\partial\langle T_a\rangle}{\partial s} \\ 0 \end{bmatrix}$$

Particle and Energy conservation

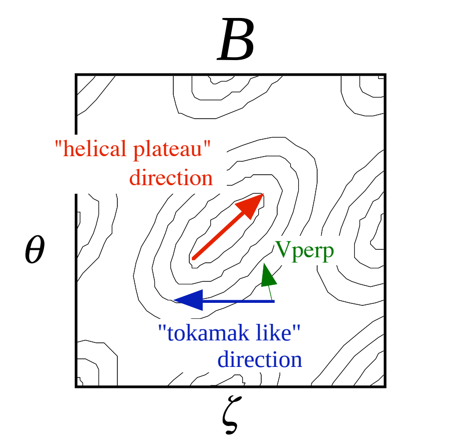
These equations are linear and therefore can be converted to algebraic equations by Fourier expansions $u_{//}/B=(u_{//}/B)_{mn}\exp[i(m\theta-n\zeta)]$, $n=n_{mn}\exp[i(m\theta-n\zeta)]$ and so on.

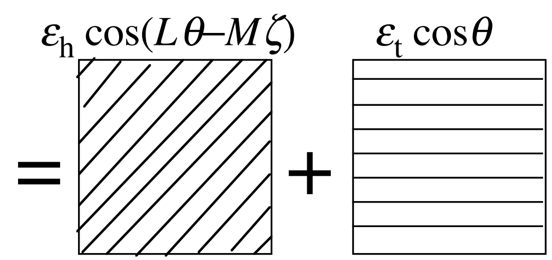
$\mathbf{B}\cdot\nabla(u_{//}/B) \to (V'/4\pi^2)^{-1}(\chi' m-\psi' n)(u_{//}/B)_{mn}$, $\mathbf{b}\cdot\nabla n \to (BV'/4\pi^2)^{-1}(\chi' m-\psi' n)n_{mn}$, …

Suggestions and Supports from experimental results (1)
(spontaneous parallel flows of collisional impurity induced by the positive E_r in the “neoclassical-ITB” operation)

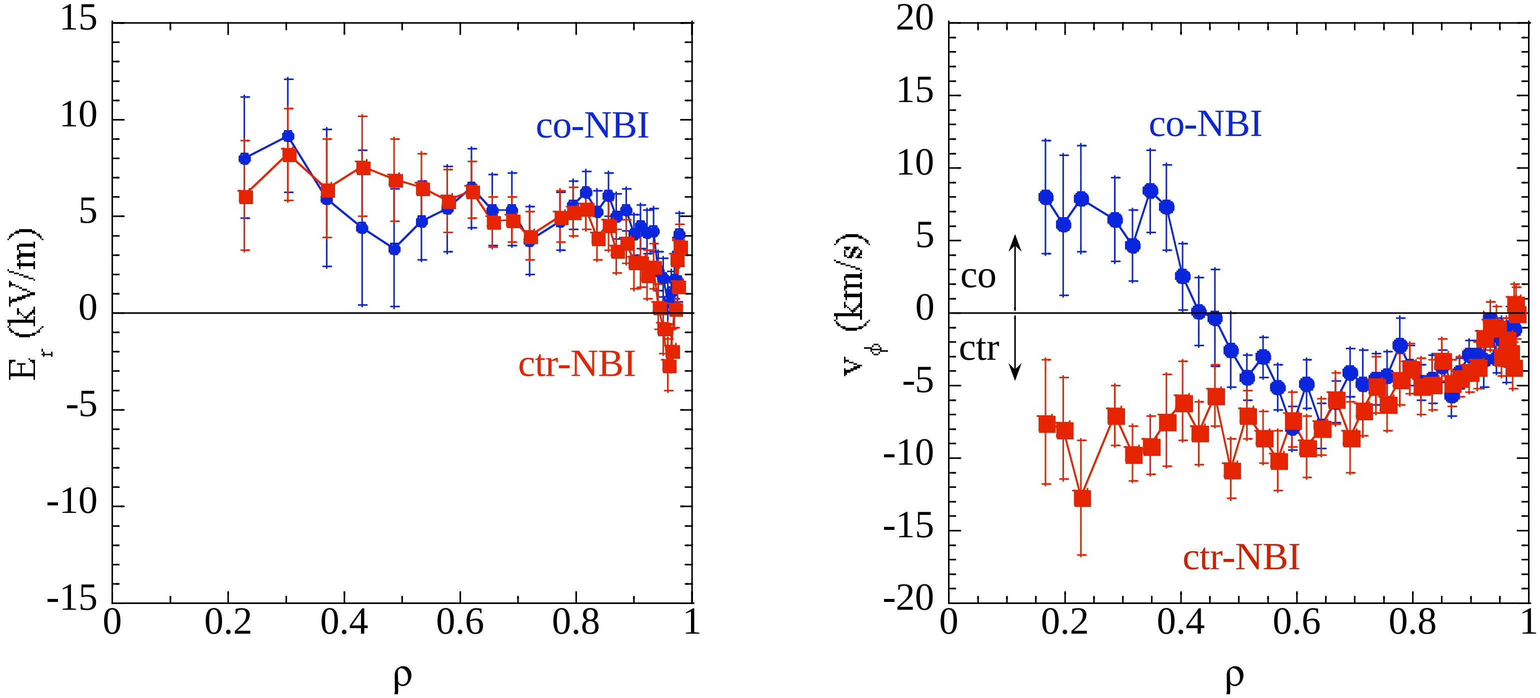


K.Ida, et al., Phys.Rev.Lett.86(2001)3040





An analogous phenomenon was recently observed also in LHD (M.Yoshinuma, et al.)



We previously showed a 2-ion-species model calculation using measured $n_a(r)$, $T_a(r)$ to reproduce these E_r, V_t. (in 14th ISW, 2003)

Next themes:

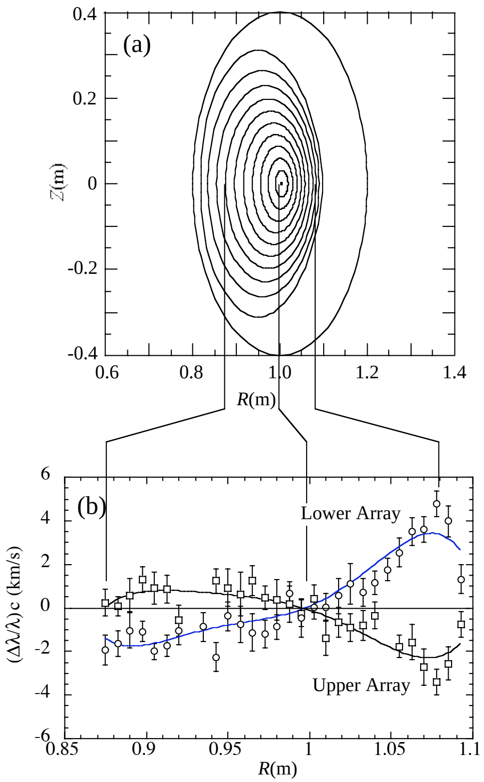
(1) extensions to general multi-species cases

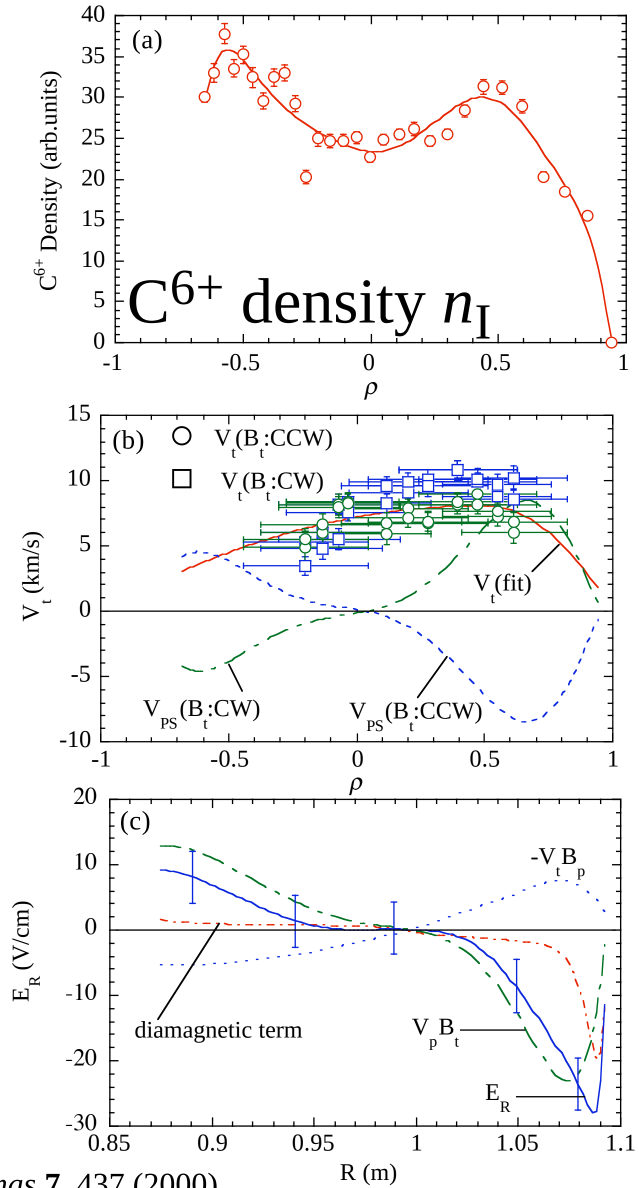
(2) self-consistent determination of $n_a(r)$ by including the P-S diffusion.
(i.e., impurity accumulation/shielding studies)

Suggestions and Supports from experimental results (2)

poloidal variation of the plasma density
under the electrostatic potential being a flux surface quantity.

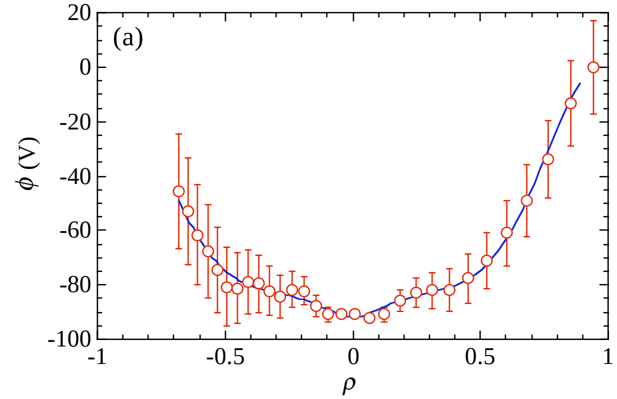
When retaining $\mathbf{u}^{E\times B}\cdot\nabla n_a$, $\mathbf{u}^{E\times B}\cdot\nabla T_a$ in the particle and energy balances,
$\nabla\cdot(n_a\mathbf{u}_a)=0$ but $\nabla\cdot\mathbf{u}_a\neq 0$



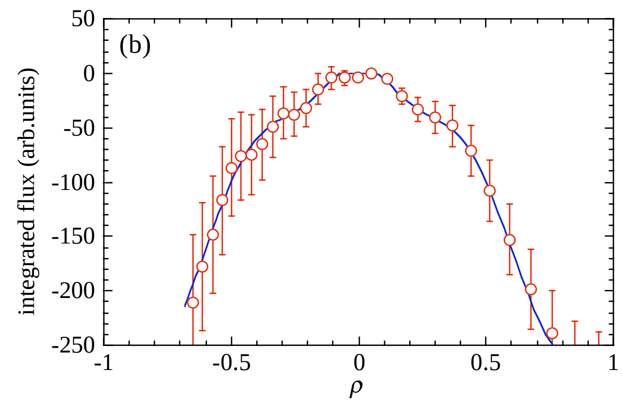


S.Nishimura, K.Ida, M.Okasabe, et al.,
in 12th ISW, 1999(Madison), *Phys.Plasmas* **7**, 437 (2000)

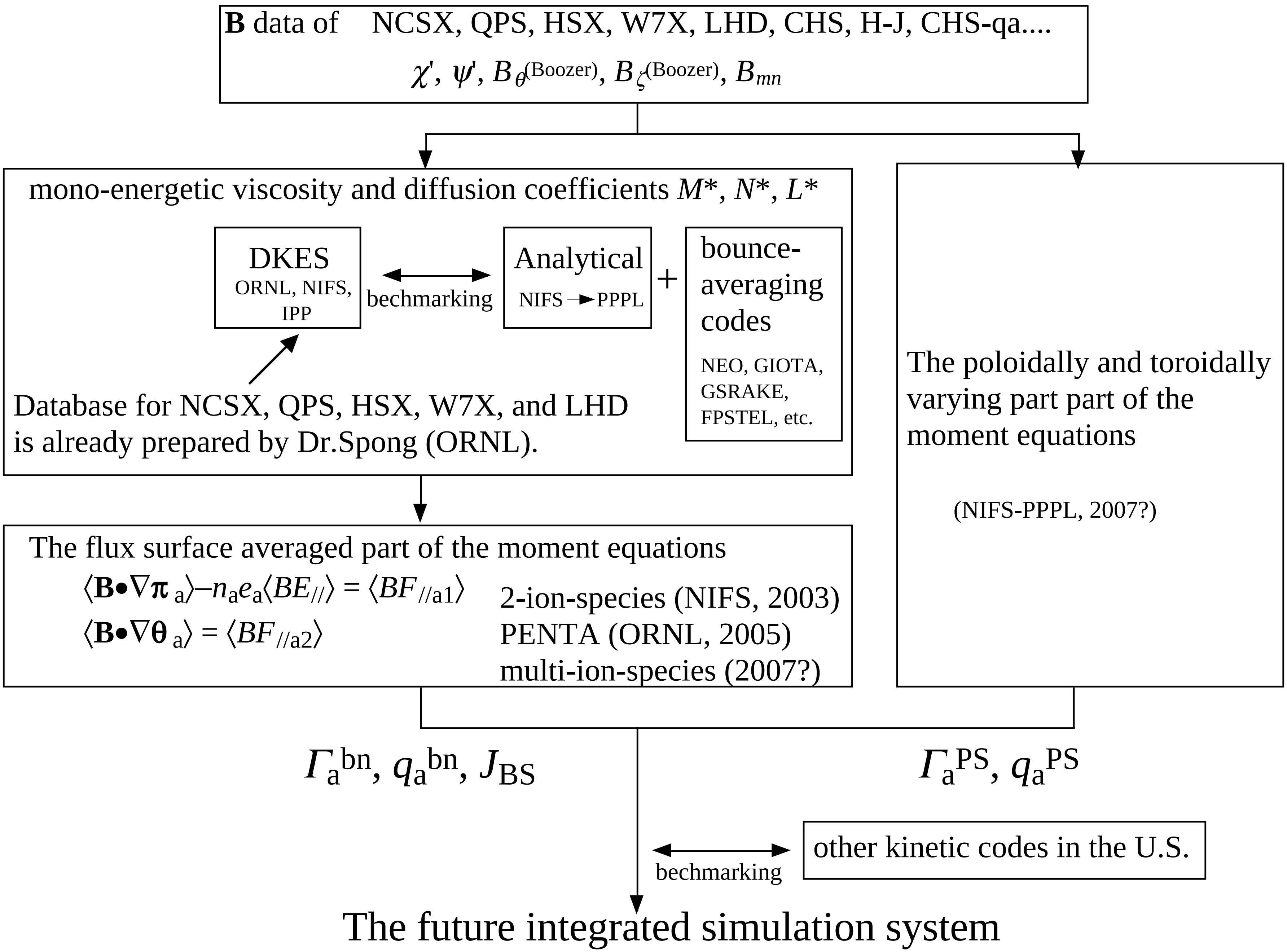
Electrostatic potential



$$\int n_I V_p R dR$$



In this case, $\nabla\cdot(n_a\mathbf{E}\times\mathbf{B}/B^2)\cong 0$
$\therefore\ n_a\propto B^2$


B data of NCSX, QPS, HSX, W7X, LHD, CHS, H-J, CHS-qa....
χ', ψ', $B_\theta^{(\mathrm{Boozer})}$, $B_\zeta^{(\mathrm{Boozer})}$, B_{mn}
mono-energetic viscosity and diffusion coefficients M^*, N^*, L^*
DKES
ORNL, NIFS, IPP
bechmarking
Analytical
NIFS → PPPL
+
bounce-averaging codes
NEO, GIOTA, GSRAKE, FPSTEL, etc.
Database for NCSX, QPS, HSX, W7X, and LHD is already prepared by Dr.Spong (ORNL).
The poloidally and toroidally varying part part of the moment equations
(NIFS-PPPL, 2007?)
The flux surface averaged part of the moment equations
$\langle \mathbf{B}\bullet\nabla\boldsymbol{\pi}_a\rangle - n_a e_a \langle BE_{//}\rangle = \langle BF_{//a1}\rangle$
$\langle \mathbf{B}\bullet\nabla\boldsymbol{\theta}_a\rangle = \langle BF_{//a2}\rangle$
2-ion-species (NIFS, 2003)
PENTA (ORNL, 2005)
multi-ion-species (2007?)
Γ_a^{bn}, q_a^{bn}, J_{BS}
Γ_a^{PS}, q_a^{PS}
bechmarking
other kinetic codes in the U.S.
The future integrated simulation system

Summary

Development of the stellarator moment method :

(1) A difficulty to treat the field particle portion of the collision operator.
$\rightarrow$ An algebraic treatment of them based on the Legendre(l)-Laguerre(j) expansions.

(2) By a characteristic of the DKE, it is better to use only the Legendre order of l=2 component given by the approximated DKE, while we forsake the l=0 component.

(3) Remaining components of the distribution function (l=0,1) are determined by combining the viscosity-flow relation and the friction-flow relation.

(4) In non-symmetric plasmas, 3 mono-energetic viscosity coefficients (M^*, N^*, L^*) are required for this procedure, while the theories for symmetric plasmas use only one coefficient.

(5) A reduction of the computational efforts for these coefficients is required for a planned integrated simulation system, and therefore derivations and tests of analytical expressions for them are now in progress.

(6) Numerical solvers for the DKE in the 3 dimensional phase space(pitch-angle, poloidal-angle, toroidal-angle) are useful as benchmark tools in this study. Bounce- or ripple- averaging codes are also useful for N^* and L^*.