Equilibrium Neoclassical Plasma Flows in Stellarators

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Acknowledgements: Hideo Sugama, Shin Nishimura, Jeff Harris, Andrew Ware, Steve Hirshman, Wayne Houlberg, Jim Lyon, Lee Berry, Mike Zarnstorff, Dave Mikkelson, Joe Talmadge

Stellarator Theory Teleconference
January 26, 2006

Plasma flow properties provide an additional dimension to the stellarator transport optimization problem

- Recent stellarator optimizations make cross-field neoclassical losses << anomalous losses
  - Ripple reduction, quasi-symmetry, isodynamicity, omnigeneity
- However, within these devices a variety of parallel momentum transport characteristics are present
  - poloidal/toroidal velocity shearing
  - two-dimensional flow structure within magnetic surfaces

Plasma flow characteristics vary significantly across stellarator configurations - opens opportunities for a range of physics issues to be explored:

- Relevance to turbulence suppression/enhanced confinement regimes
  - Experiments/simulations show importance of E x B shearing

$$\textit{decorrelation time} \propto \frac{1}{1 + C\left(\omega'_{E\times B}\right)^2}$$

  - Reduced neoclassical viscosity lowers zonal flow electric field damping rate

    K. C. Shaing, Phys. Plasmas **12**, (2005) 082508
- Magnetic perturbations shielding - island suppression
  - Toroidal flows shield resonant magnetic perturbations at rational surfaces

    A. Reiman, M. Zarnstorff, et al., Nuc. Fusion **45**, (2005) 360
  - Possible source of “self-healing” mechanism in tokamaks
- Impurity accumulation/shielding analysis

  S. Nishimura, H. Sugama, Fusion Science and Tech. **46** (2004) 77
- New hidden variable in the international stellarator transport scaling database?
- Productive area for experiment/theory comparison
  - stellarator analog of NCLASS code
- Improved bootstrap current/ambipolar electric field analysis

Stellarator flow damping:

- weights viscous stress-tensor components differently than cross-field transport
- removes linear dependencies (from symmetry-breaking effects)

- **Cross-field transport:**

$$\Gamma = \left\langle n\vec{v}_\perp \cdot \vec{\nabla}\rho \right\rangle = \Gamma_{cl} + \Gamma_{bp} + \Gamma_{PS} + \Gamma_{na} + \frac{nc}{\psi'\chi'}\left\langle \vec{B}_X \cdot \vec{E}^A \right\rangle$$

$$\Gamma_{cl} = \frac{c}{e\psi'\chi'}\vec{B}_X \cdot \vec{F}_\perp \qquad \Gamma_{PS} = \frac{c}{e\psi'}\left\langle \frac{F_\parallel}{B}\left(I_X - \langle I_X \rangle\right)\frac{B^2}{\langle B^2 \rangle}\right\rangle$$

$$\Gamma_{bp} = \frac{c}{e\psi'}\frac{\langle I_X \rangle \langle BF_\parallel \rangle}{\langle B^2 \rangle} \qquad \Gamma_{na} = -\frac{c}{e\psi'\chi'}\left\langle \vec{B}_X \cdot \left(\vec{\nabla} \cdot \vec{\Pi}\right)\right\rangle$$

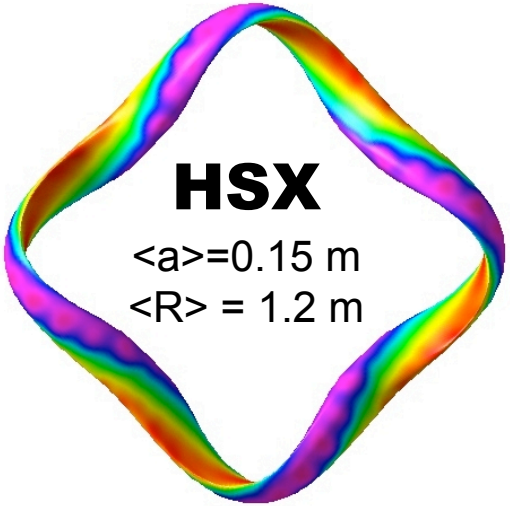
where $X = toroidal, poloidal, \ldots \quad I_X = \vec{B}_X \cdot \vec{B} / \chi'$

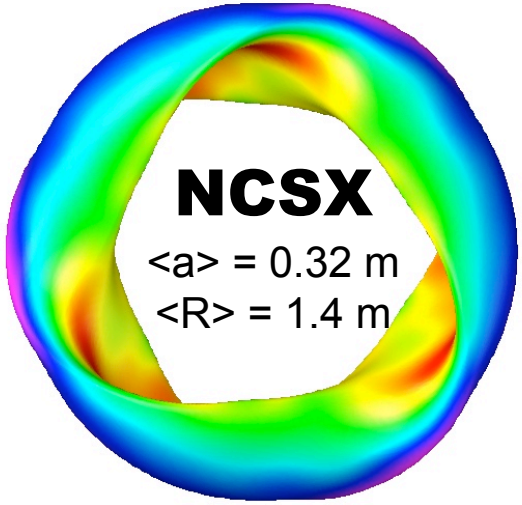
- **Parallel momentum balance -> flow prediction**

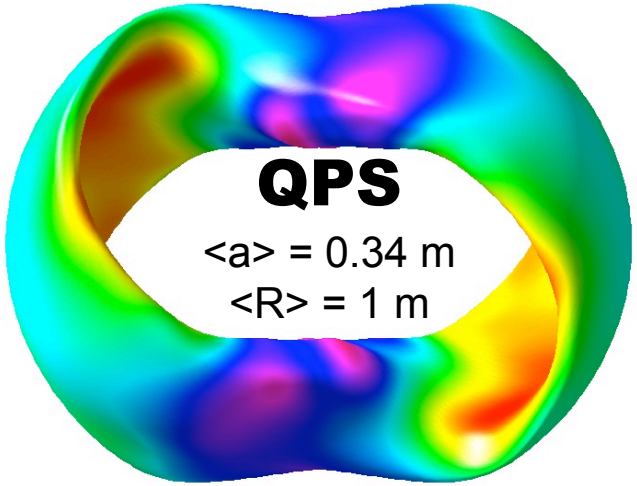
$\left\langle \vec{B} \cdot \left(\vec{\nabla} \cdot \vec{\Pi}\right)\right\rangle = \left\langle BF_\parallel \right\rangle \Rightarrow$ invert viscosity matrix to solve for parallel flow

Reference: K. C. Shaing, J. D. Callen, Phys.Fluids 26, (1983) 3315.

Advances in stellarator optimization have allowed the design of 3D configurations with magnetic structures that approximate: straight helix/tokamak/connected mirrors:

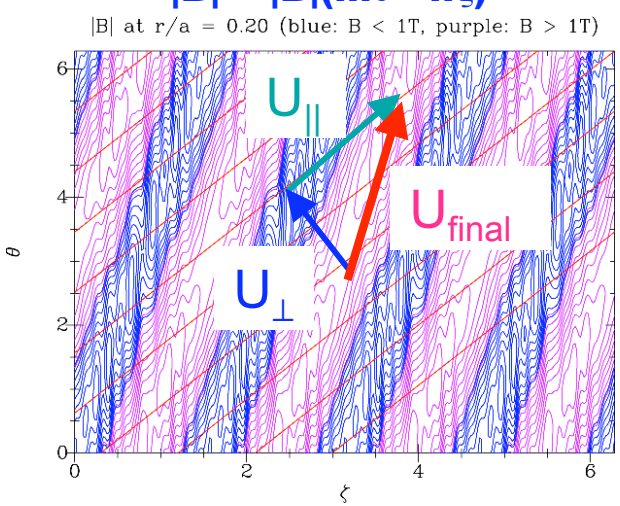






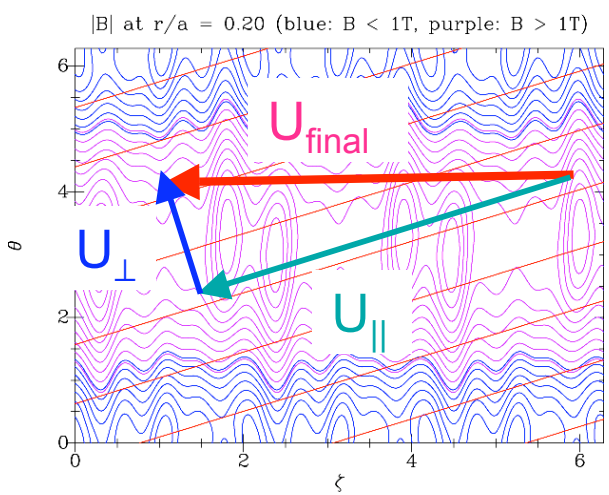
Quasi-helical symmetry

$|B| \sim |B|(m\theta - n\zeta)$



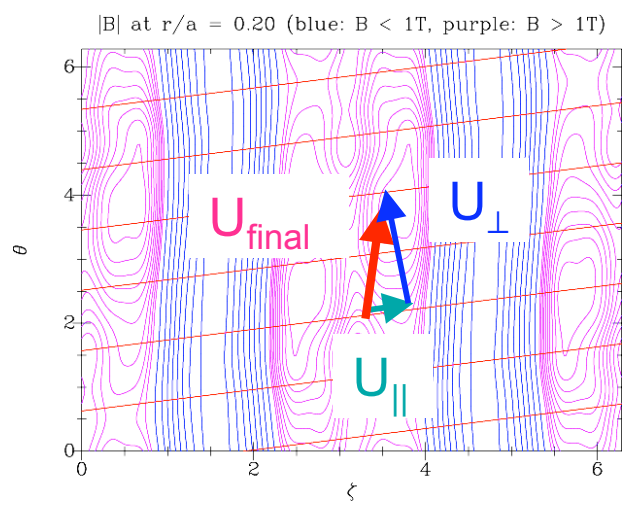
Quasi-toroidal symmetry

$|B| \sim |B|(\theta)$

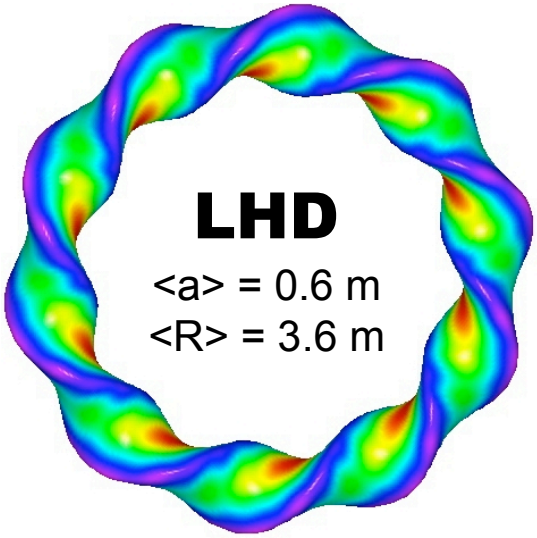


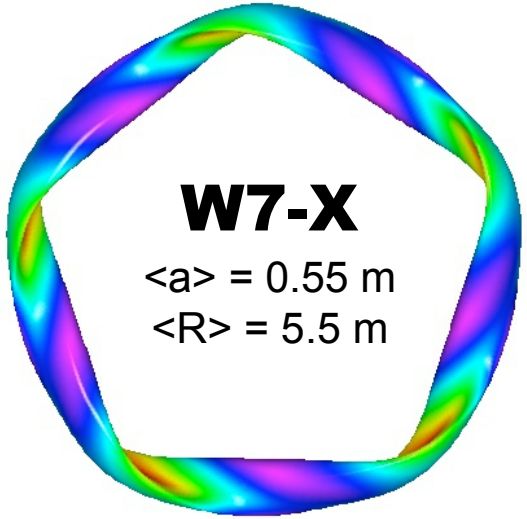
Quasi-poloidal symmetry

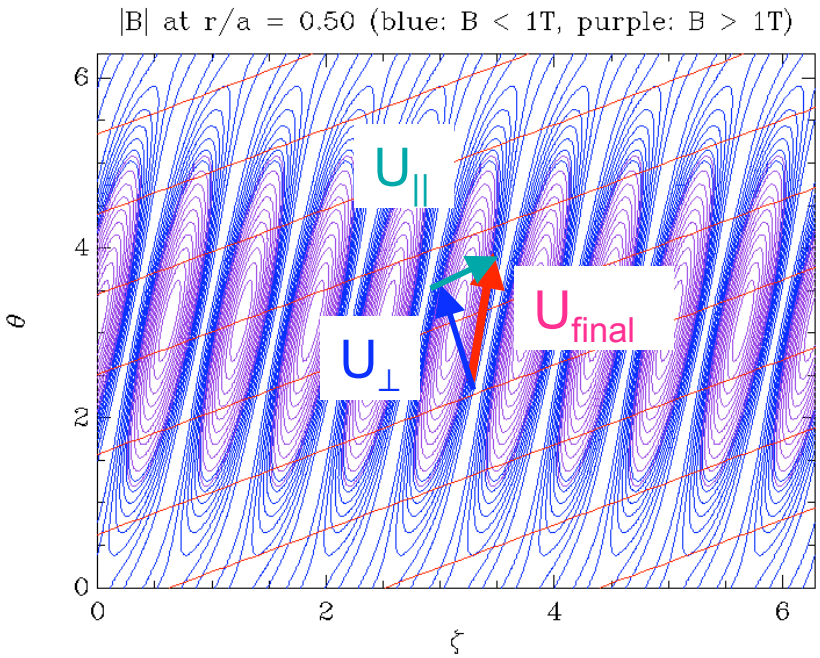
$|B| \sim |B|(\zeta)$

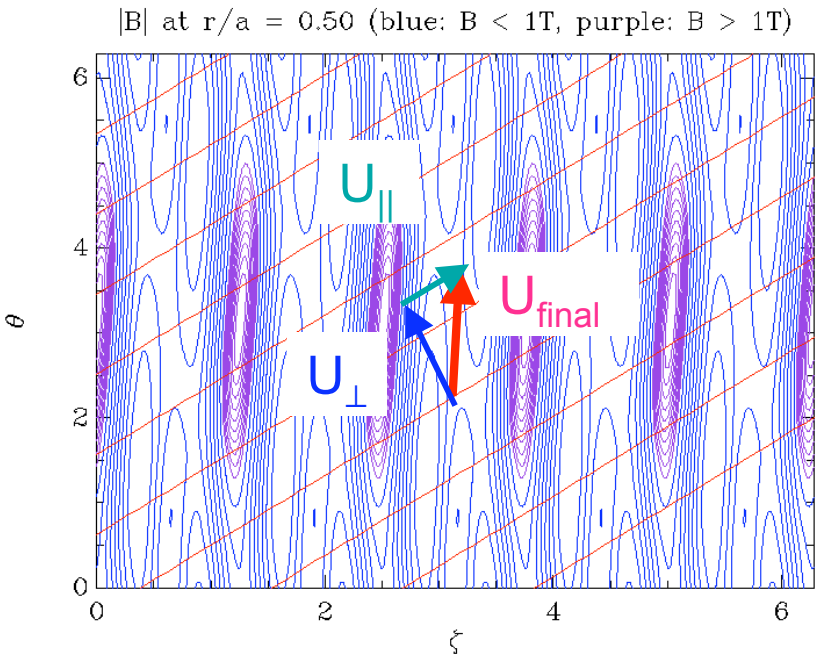


In addition, LHD and W7-X achieve closed drift surfaces by inward shifts (LHD) and finite plasma β effects (W7-X)









Development of stellarator moments methods

- Based on anisotropic pressure moment of distribution function rather than momentum or particle moments
  - Less affected by neglect of field particle collisions on test particles
- Viscosities incorporate all needed kinetic information
  - Momentum balance invoked at macroscopic level rather than kinetic level
  - Multiple species can be more readily decoupled
- Recent work has related viscosities to Drift Kinetic Equation Solver (DKES) transport coefficients
  - Moments method, viscosities related to D_{11} D_{31}, D_{33}

    M. Taguchi, Phys. Fluids B4 (1992) 3638

    H. Sugama, S. Nishimura, Phys. of Plasmas **9** (2002) 4637
  - DKES: D_{11} (diffusion of n,T), D_{13} (bootstrap current), D_{33} (resistivity enhancement)

    W. I.Van Rij and S. P. Hirshman, *Phys. Fluids B*, **1**, 563 (1989)
- Implemented into a suite of codes that generate the transport coefficient database, perform velocity convolutions, find ambipolar roots, and calculate flow components
  - D. A. Spong, Phys. Plasmas **12** (2005) 056114
  - D. A. Spong, S.P. Hirshman, et al., Nuclear Fusion **45** (2005) 918

Moments Method Closures for Stellarators

The parallel viscous stresses, particle and heat flows are treated as fluxes conjugate to the forces of parallel momentum, parallel heat flow, and gradients of density, temperature and potential:

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a/T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \langle u_{\parallel a}B\rangle / \langle B^2\rangle \\ \frac{2}{5p_a}\langle q_{\parallel a}B\rangle / \langle B^2\rangle \\ -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a\frac{\partial \Phi}{\partial s} \\ -\frac{\partial T_a}{\partial s} \end{bmatrix}$$

- Analysis of Sugama and Nishimura related monoenergetic forms of the M, N, L viscosity coefficients to DKES transport coefficients
- Combining the above relation with the parallel momentum balances and friction-flow relations

$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle - n_a e_a \langle BE_\parallel\rangle = \langle BF_{\parallel a1}\rangle$$

$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle = \langle BF_{\parallel a2}\rangle$$

$$\begin{bmatrix} \langle BF_{\parallel a1}\rangle \\ \langle BF_{\parallel a2}\rangle \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{12}^{ab} & l_{22}^{ab} \end{bmatrix} \begin{bmatrix} \langle Bu_{\parallel b}\rangle \\ \frac{2}{5p_b}\langle Bq_{\parallel b}\rangle \end{bmatrix}$$

Leads to coupled equations that can be solved for $\langle u_{\parallel a}B\rangle$, $\langle q_{\parallel a}B\rangle$, Γ_a, Q_a

Using solutions for an electron/ion plasma, the self-consistent electric fields, bootstrap currents and parallel flows can be obtained

(Appendix C - H. Sugama, S. Nishimura, Phys. Plasmas **9** (2002) 4637)

Radial particle flows required for ambipolar condition ➡ self-consistent energy fluxes and bootstrap currents

$$\begin{bmatrix} \Gamma_e \\ q_e / T_e \\ \Gamma_i \\ q_i / T_i \\ J_{BS}^E \end{bmatrix} = \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{11}^{ei} & L_{12}^{ei} & L_{1E}^{e} \\ L_{21}^{ee} & L_{22}^{ee} & L_{21}^{ei} & L_{22}^{ei} & L_{2E}^{e} \\ L_{11}^{ie} & L_{12}^{ie} & L_{11}^{ii} & L_{12}^{ii} & L_{1E}^{i} \\ L_{21}^{ie} & L_{22}^{ie} & L_{21}^{ii} & L_{22}^{ii} & L_{2E}^{i} \\ L_{E1}^{e} & L_{E2}^{e} & L_{E1}^{i} & L_{E2}^{i} & L_{EE} \end{bmatrix} \begin{bmatrix} X_{e1} \\ X_{e2} \\ X_{i1} \\ X_{i2} \\ X_E \end{bmatrix}$$

where $X_{a1} \equiv -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a \frac{\partial \Phi}{\partial s}$, $X_{a2} \equiv -\frac{\partial T_a}{\partial s}$, $X_E = \langle BE_\parallel \rangle / \langle B^2 \rangle^{1/2}$

Parallel mass and energy flows:

$$\begin{bmatrix} \langle Bu_{\parallel i} \rangle \\ \frac{2}{5p_i}\langle Bq_{\parallel i} \rangle \end{bmatrix} = \left[\frac{1}{\langle B^2 \rangle} \begin{bmatrix} M_{i1} & M_{i2} \\ M_{i2} & M_{i3} \end{bmatrix} - \begin{bmatrix} l_{11}^{ii} & -l_{12}^{ii} \\ -l_{12}^{ii} & l_{22}^{ii} \end{bmatrix} \right]^{-1} \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i2} & N_{i3} \end{bmatrix} \begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix}$$

Parallel Environment for Neoclassical Transport Analysis (PENTA)

DKES Transport coefficient Code: D_{11}, D_{13}, D_{33} (functions of ψ, ν/v, E_r/v)

flux surface 1 — processor 1

flux surface 2 — processor 2

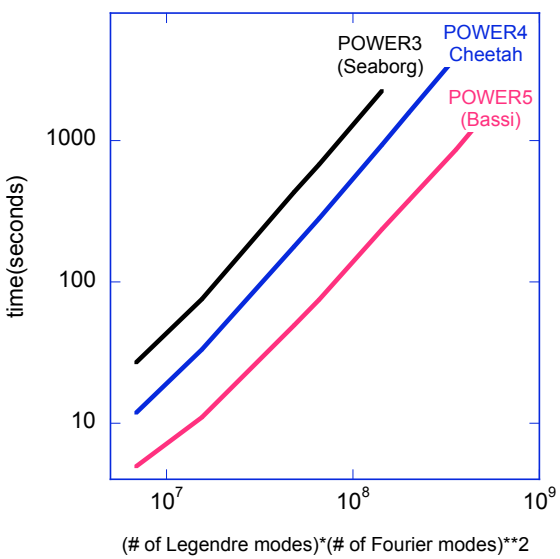
flux surface 3 — processor 3

flux surface 4 — processor 4

...

flux surface n — processor n

results vs. ψ, ν/v, E_r/v concatenated together



DKES results supplemented at low/high collisionalities using asymptotic forms

Energy integrations, parallel force balance relations, ambipolarity condition solved, profiles obtained for: E_r, Γ_i, Γ_e, q_i, q_e, $\langle u^\theta_i \rangle$, $\langle u^\xi_i \rangle$, J_E^{BS}

Work in progress:

- delta-f Monte Carlo
- superbanana effects (i.e., limits on $1/\nu$ regime)
- better connection formulas
- DKES extensions
- convergence studies
- E_θ effects
- Magnetic islands

The neoclassical theory provides $\langle u_{\|} B \rangle$ and the ambipolar electric field E_s (from solving $\Gamma_i = \Gamma_e$). The final term needed is U, the Pfirsch-Schlüter flow.

$$\vec{u} = \frac{1}{eB}\left(-\frac{1}{n}\frac{\partial p}{\partial s} + E_s\right)\vec{\nabla} s \times \frac{\vec{B}}{B} + \left[\frac{\langle u_{\|} B \rangle}{\langle B^2 \rangle} + \frac{1}{eB}\left(-\frac{1}{n}\frac{\partial p}{\partial s} + E_s\right) U\right]\frac{\vec{B}}{B}$$

Integrating $\vec{\nabla} \cdot \vec{v} = 0$ leads to an equation for U:

$$\vec{B} \cdot \vec{\nabla}\left(\frac{U}{B}\right) = \vec{B} \times \vec{\nabla}\psi \cdot \vec{\nabla}\left(\frac{1}{B^2}\right) \quad (stellarator)$$

$$U = I(\psi)\left[\frac{B^2}{\langle B^2 \rangle} - 1\right] \quad (tokamak)$$

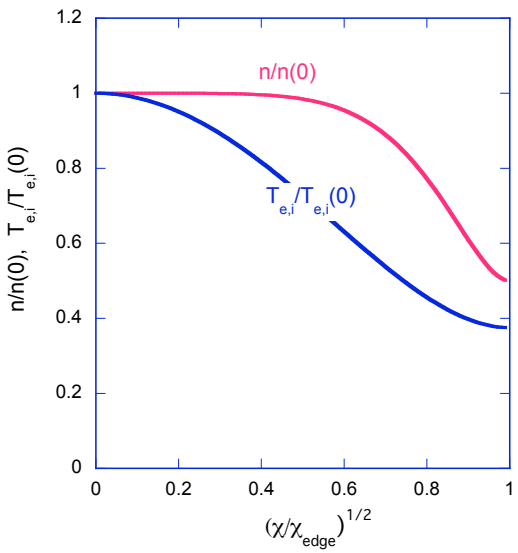
$\langle U^2 \rangle$ can be obtained by:

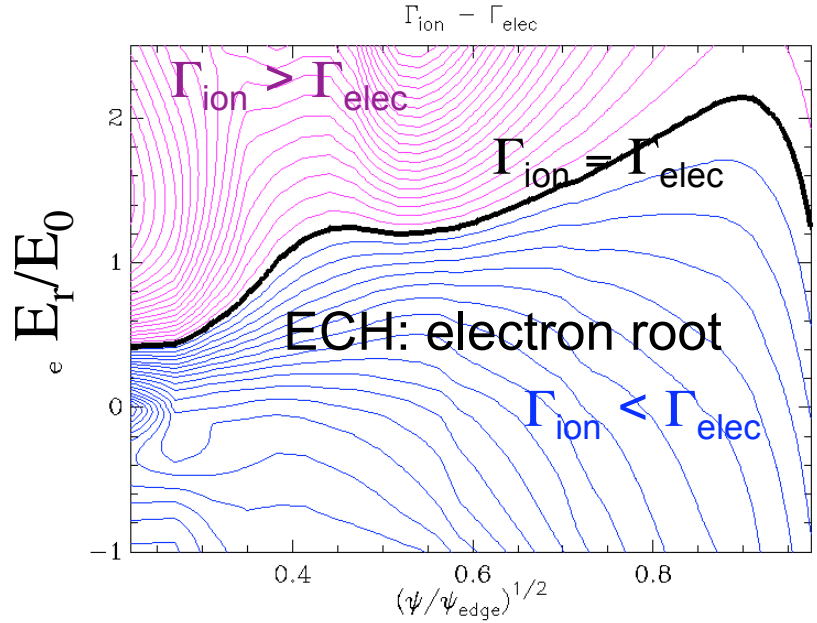
-solving this equation directly (with damping to resolve singularities at rational surfaces)

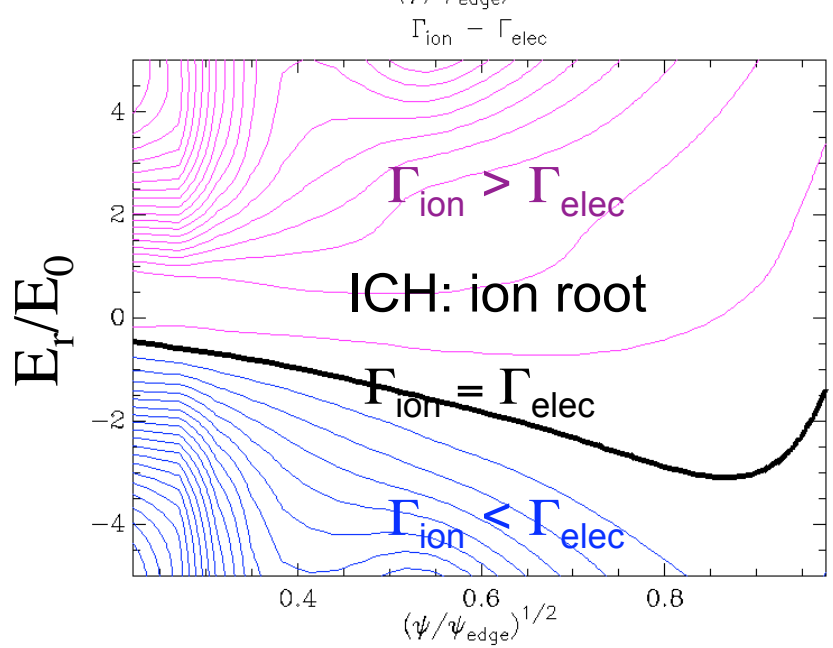
- matching to high collisionality DKES coefficient: $\langle U^2 \rangle = 1.5 D_{11} v/\nu$ (for large ν)

The flow model will be applied to two parameter ranges with radially continuous/stable electric field roots

- ECH regime:
  - $n(0) = 2.5 \times 10^{19}\ m^{-3}$, $T_e(0) = 1.5$ keV, $T_i(0) = 0.2$ keV
- ICH regime
  - $n(0) = 8 \times 10^{19}\ m^{-3}$, $Te(0) = 0.5$ keV, $T_i(0) = 0.3$ keV



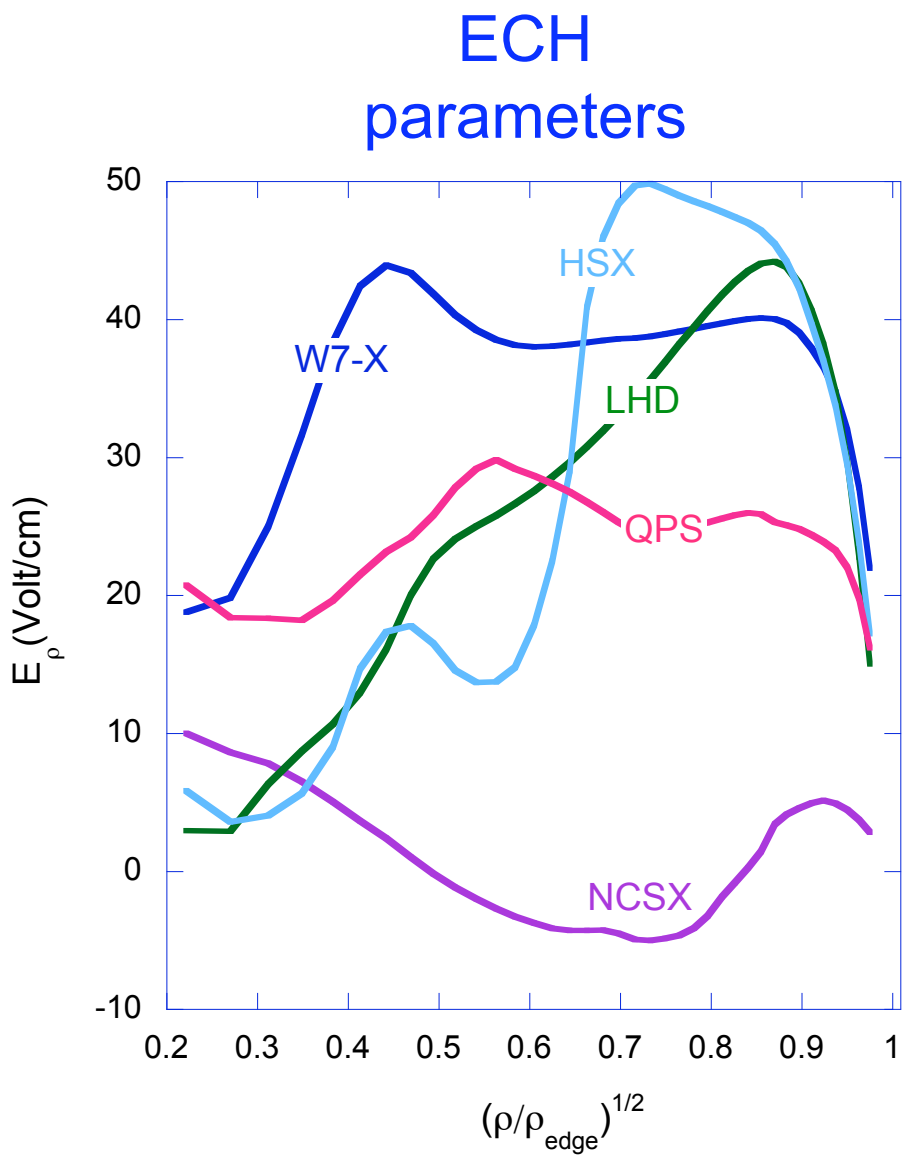


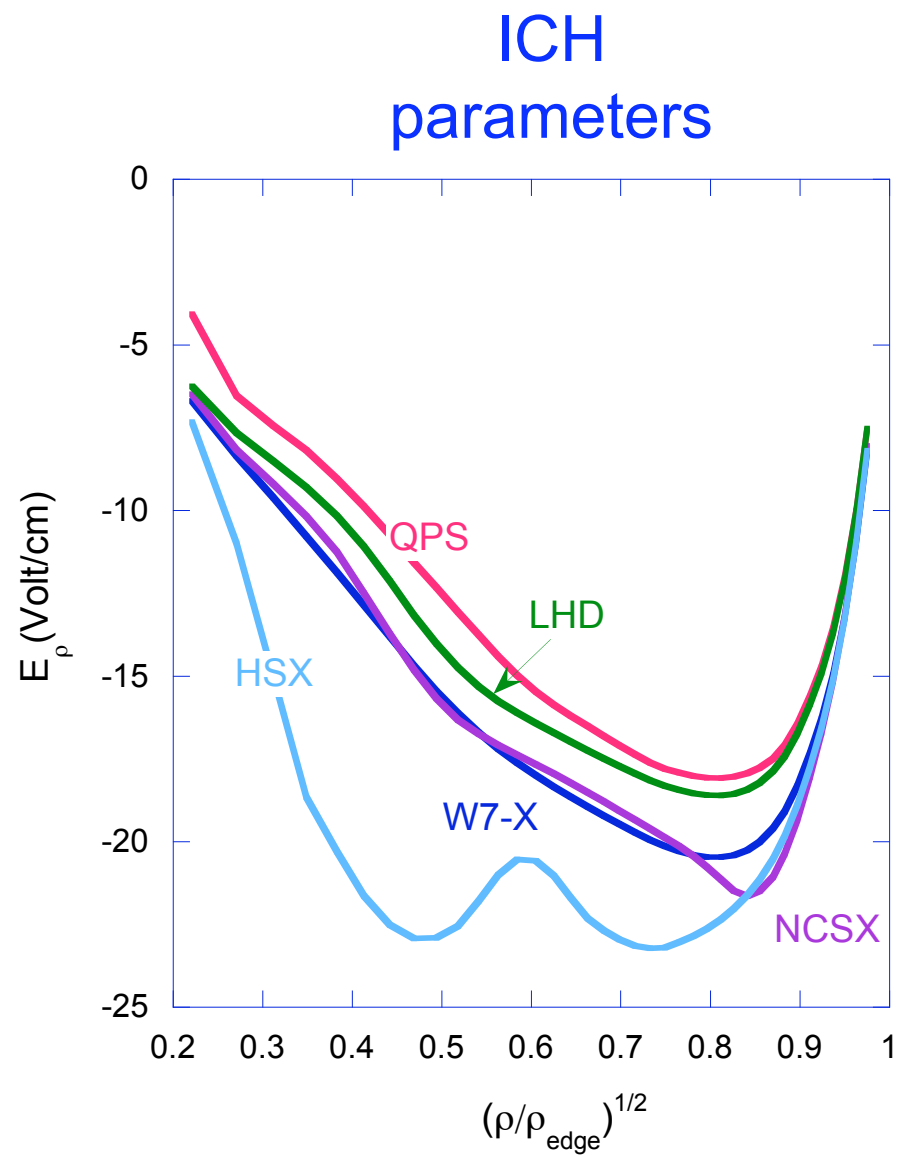


Roots chosen to give stable restoring force for electric field perturbations through:

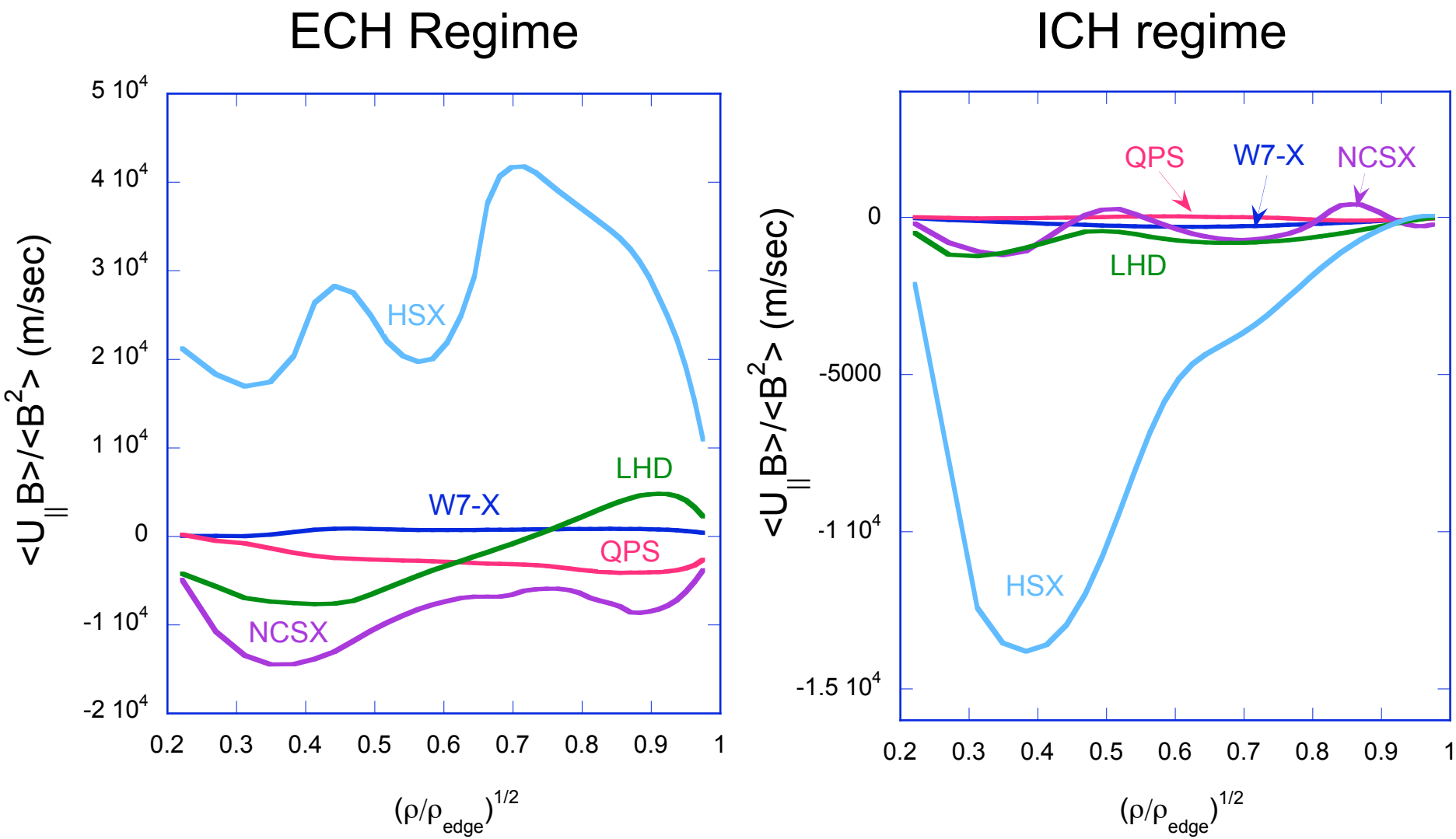
$$\frac{\varepsilon_\perp}{q}\frac{\partial E_r}{\partial t} = \Gamma_e - \Gamma_i$$

Electric field profiles for ICH (ion root) and ECH (electron root) cases





The neoclassical parallel flow velocity can vary significantly among configurations. The lowest levels are present in W7-X. Higher $u_{||}$ flows characterize HSX/NCSX



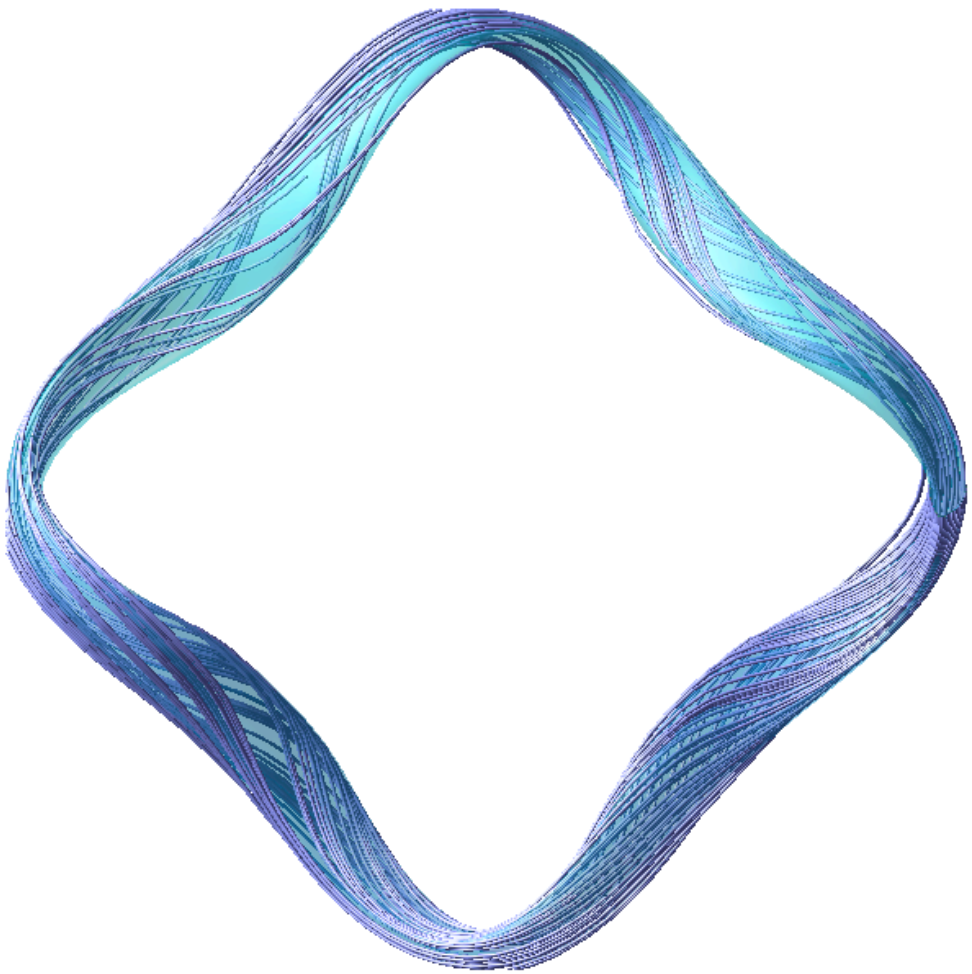
2D flow variation within a flux surface:

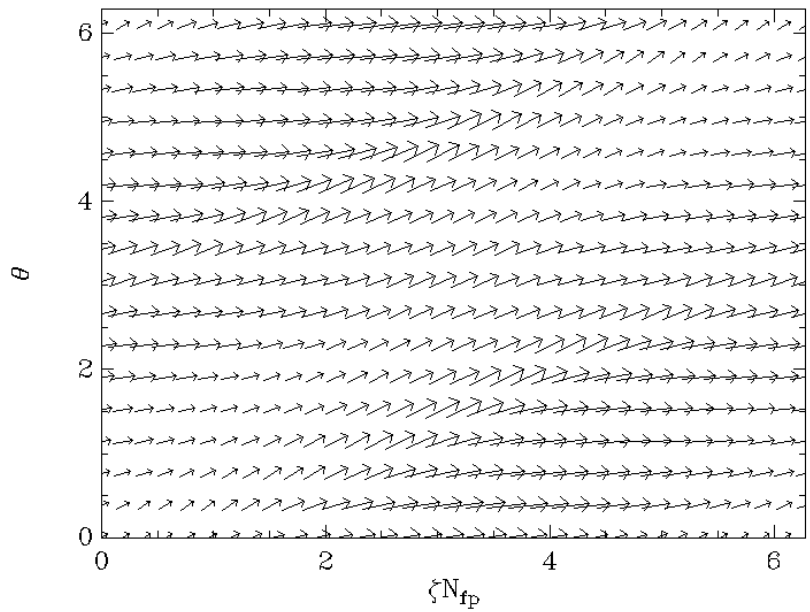
$$\vec{V} = \frac{X_1}{eB^2}\vec{\nabla}s \times \vec{B} + \frac{\langle u_{\parallel} B \rangle}{\langle B^2 \rangle}\vec{B} + \frac{X_1}{e}\tilde{U}\vec{B} \quad \text{where } X_1 \equiv -\frac{1}{n}\frac{\partial p}{\partial s} - e\frac{\partial \Phi}{\partial s}$$

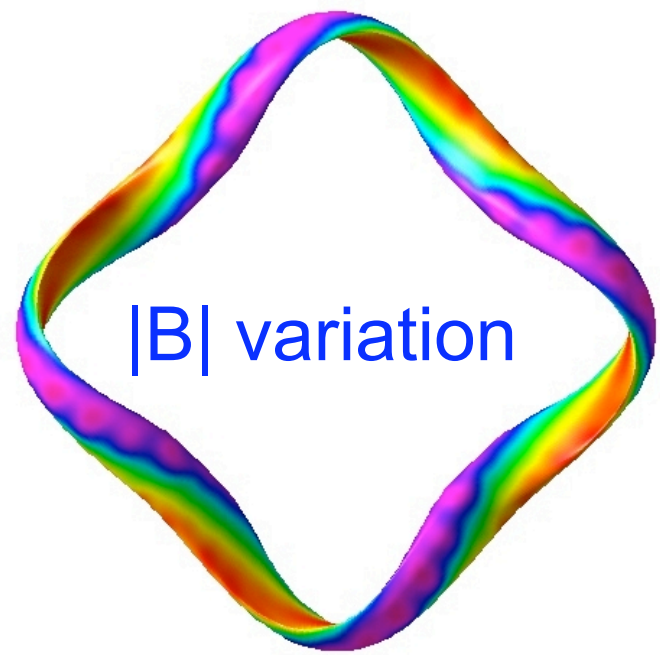
diamagnetic and E x B	neoclassical parallel flow	Pfirsch-Schlüter - for $\vec{\nabla} \bullet \left(\rho \vec{V}\right) = 0$	
$\propto$ 1/B	B	f(θ,ζ)B	(variation within a flux surface)

- Visualization of flows in real (Cartesian space)
- Indicates flow shearing (geodesic) within a flux surface over shorter connection lengths than for a tokamak
  - Could impact ballooning, interchange stability, microturbulence
- Implies need for multipoint experimental measurements and/or theoretical modeling support

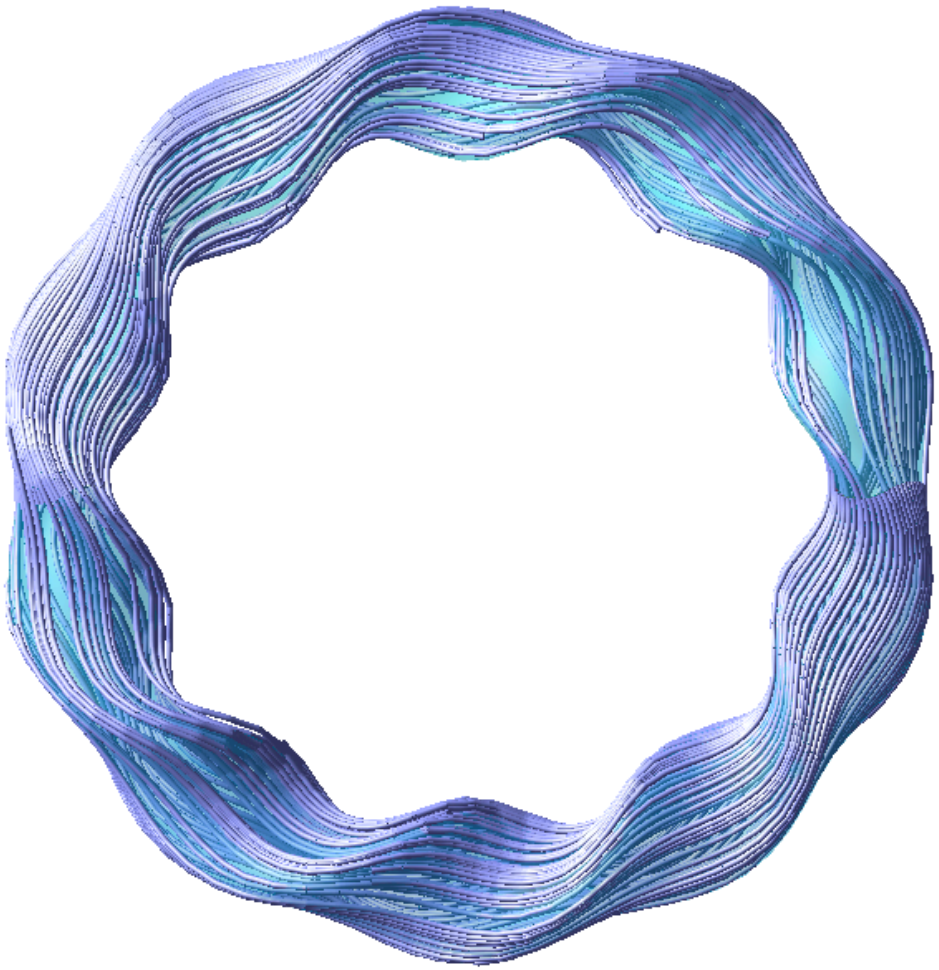
HSX 2D-flow streamlines

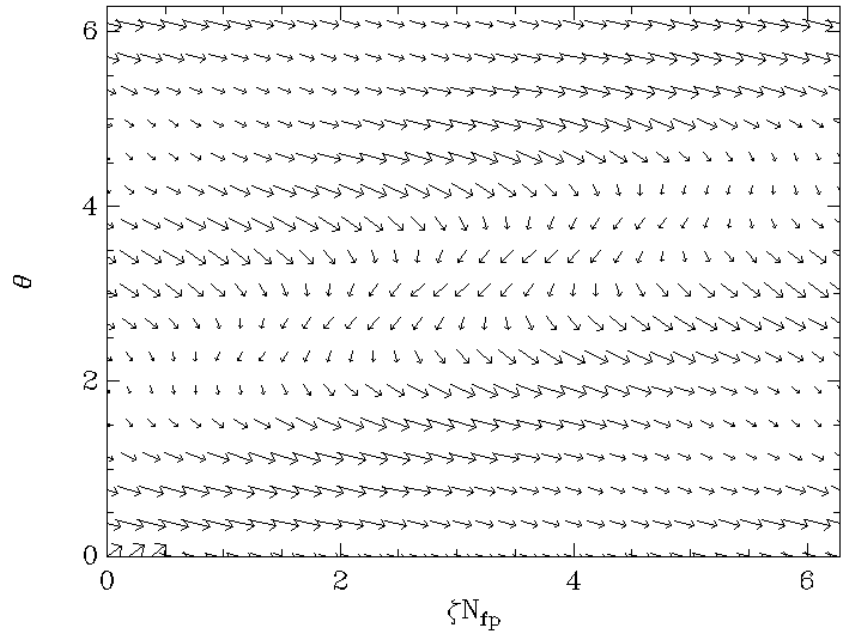


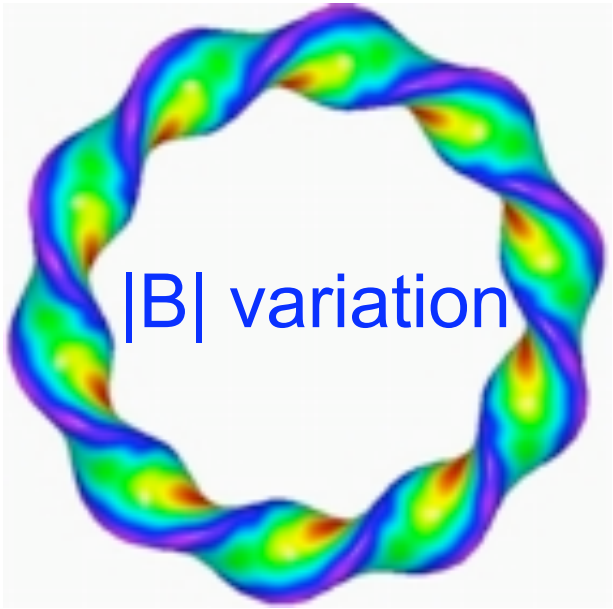




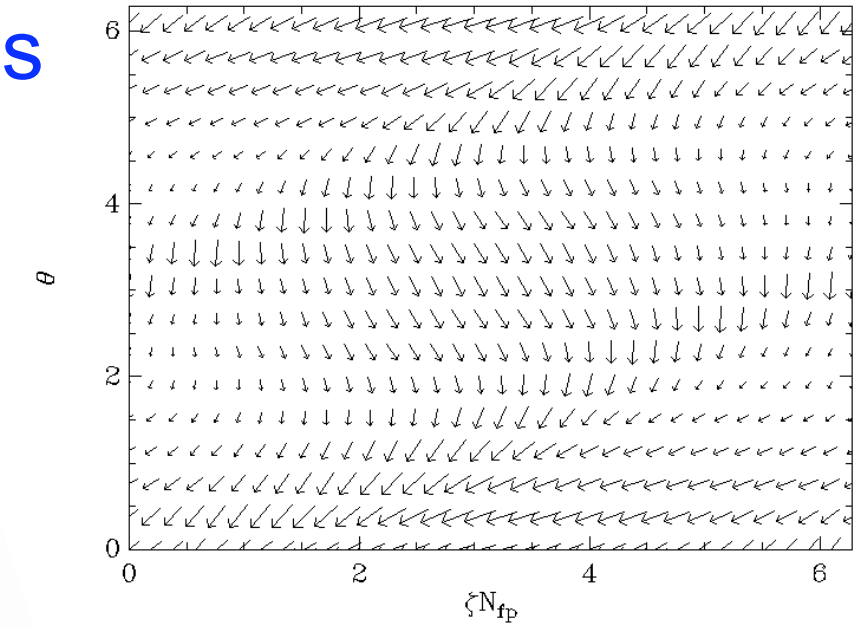
LHD 2D-flow streamlines

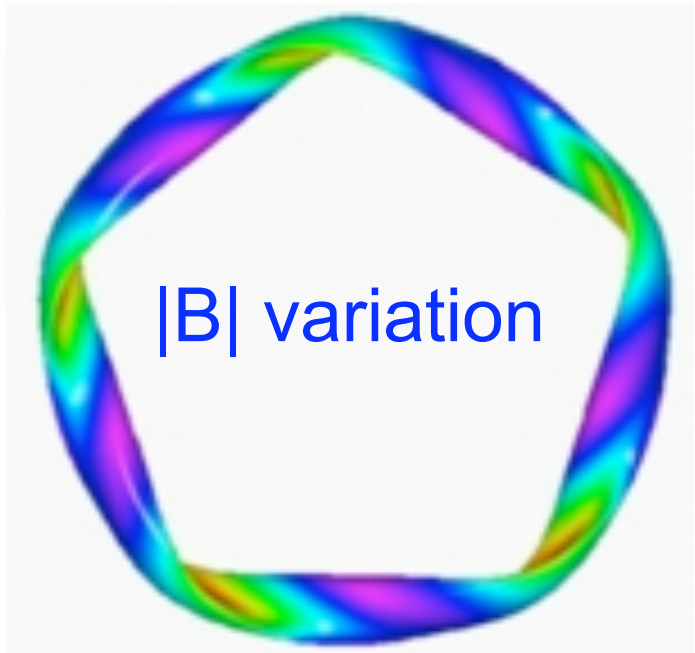




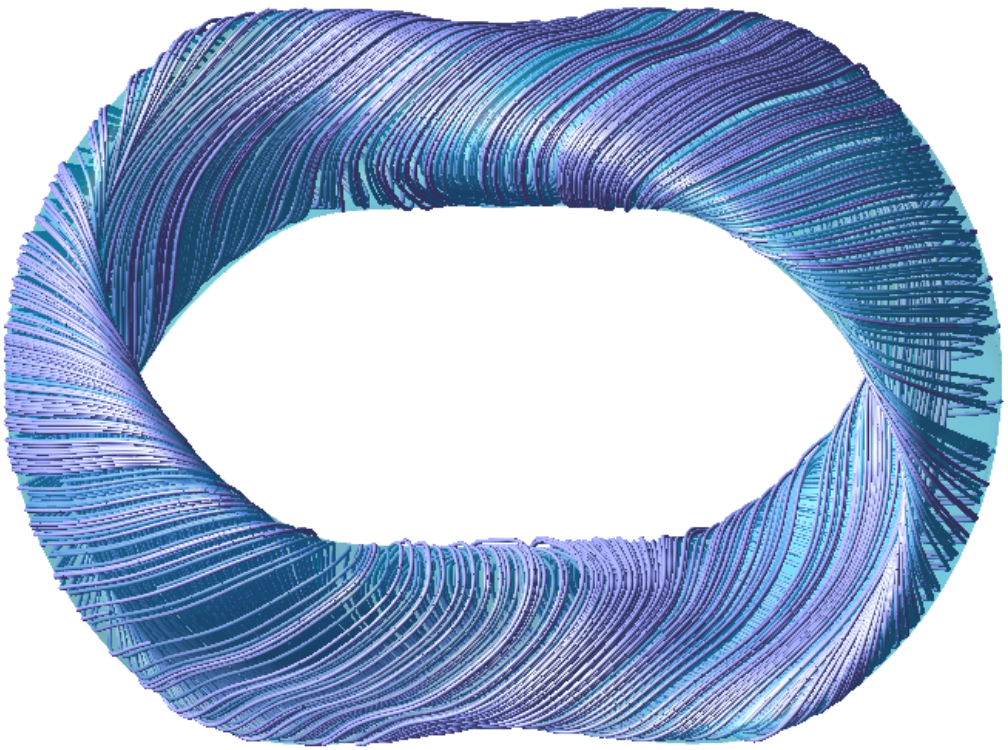


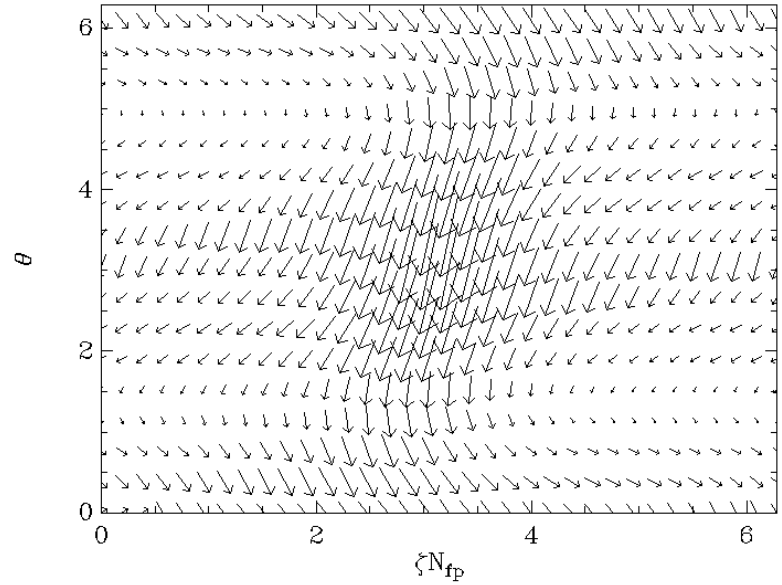
W7-X 2D-flow streamlines

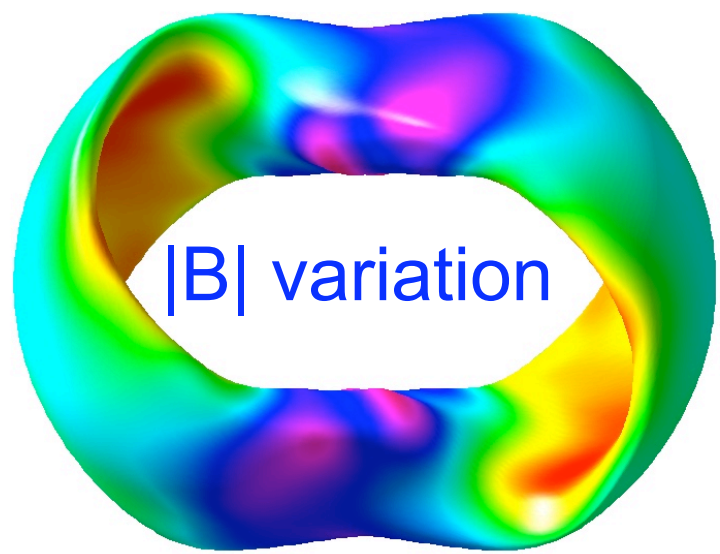




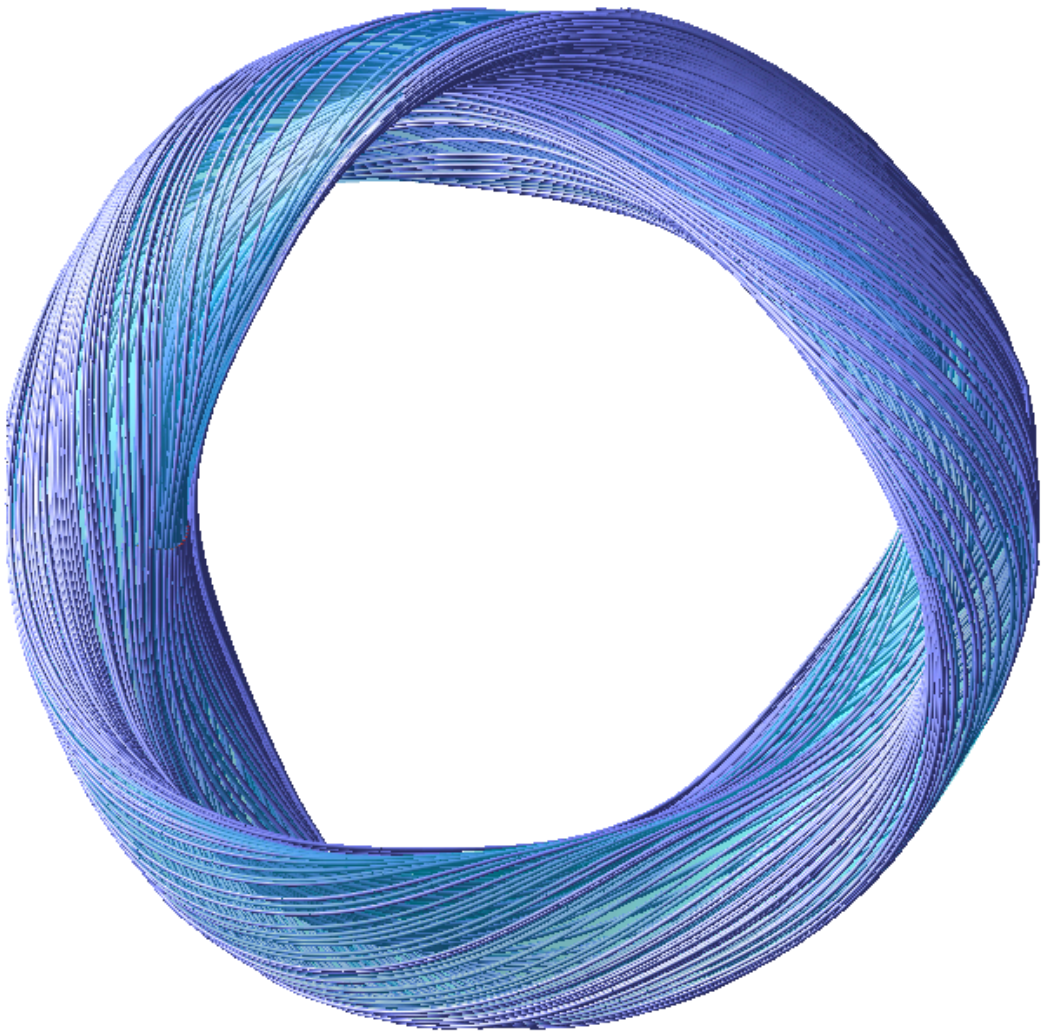
QPS 2D-flow streamlines

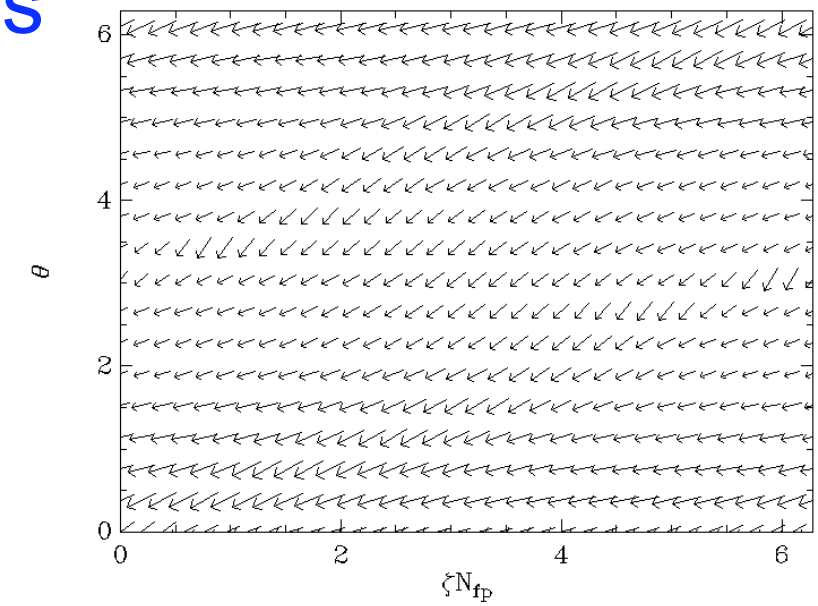


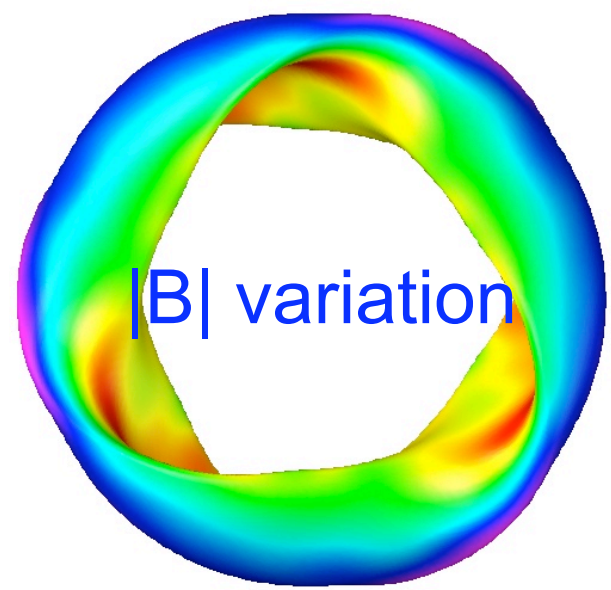




NCSX 2D-flow streamlines







Plasma flow velocity - averages

$$\vec{V} = \frac{X_1}{eB^2}\vec{\nabla}s \times \vec{B} + \frac{\langle u_{\parallel}B\rangle}{\langle B^2\rangle}\vec{B} + \frac{X_1}{e}\tilde{U}\vec{B} \quad \text{where } X_1 \equiv -\frac{1}{n}\frac{\partial p}{\partial s} - e\frac{\partial \Phi}{\partial s}$$

diamagnetic and E x B

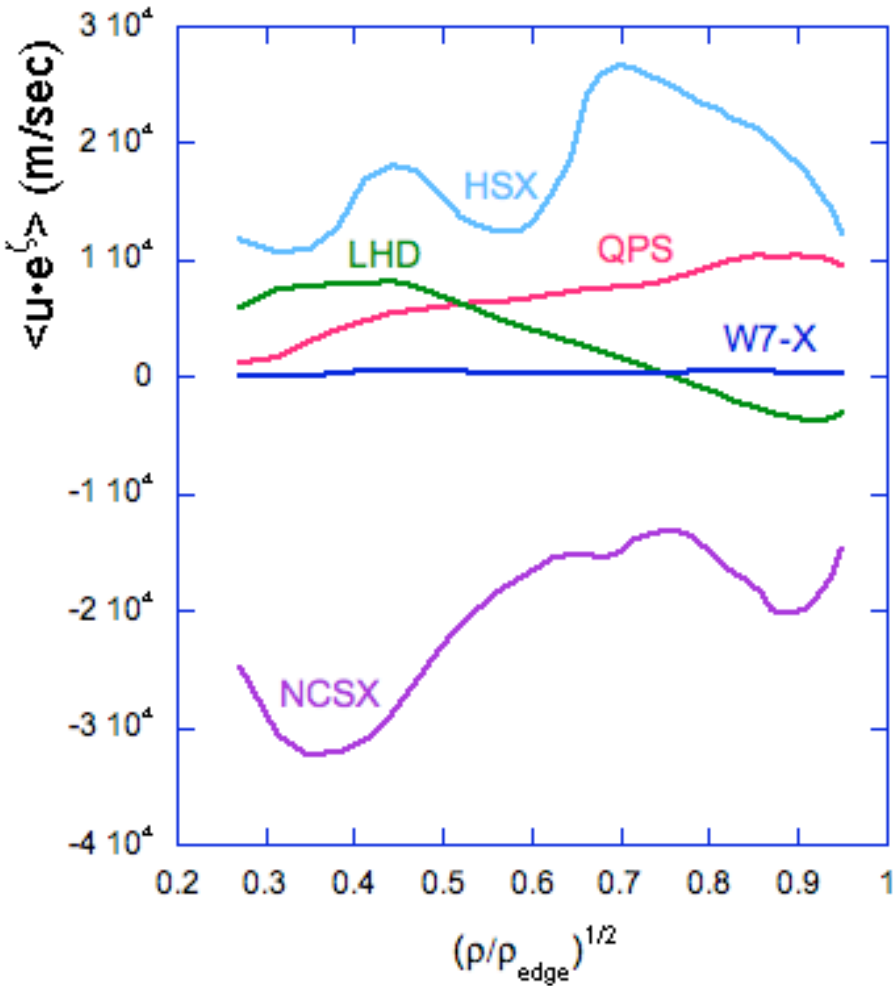
neoclassical parallel flow

Pfirsch-Schlüter - for $\vec{\nabla}\bullet\left(\rho\vec{V}\right) = 0$

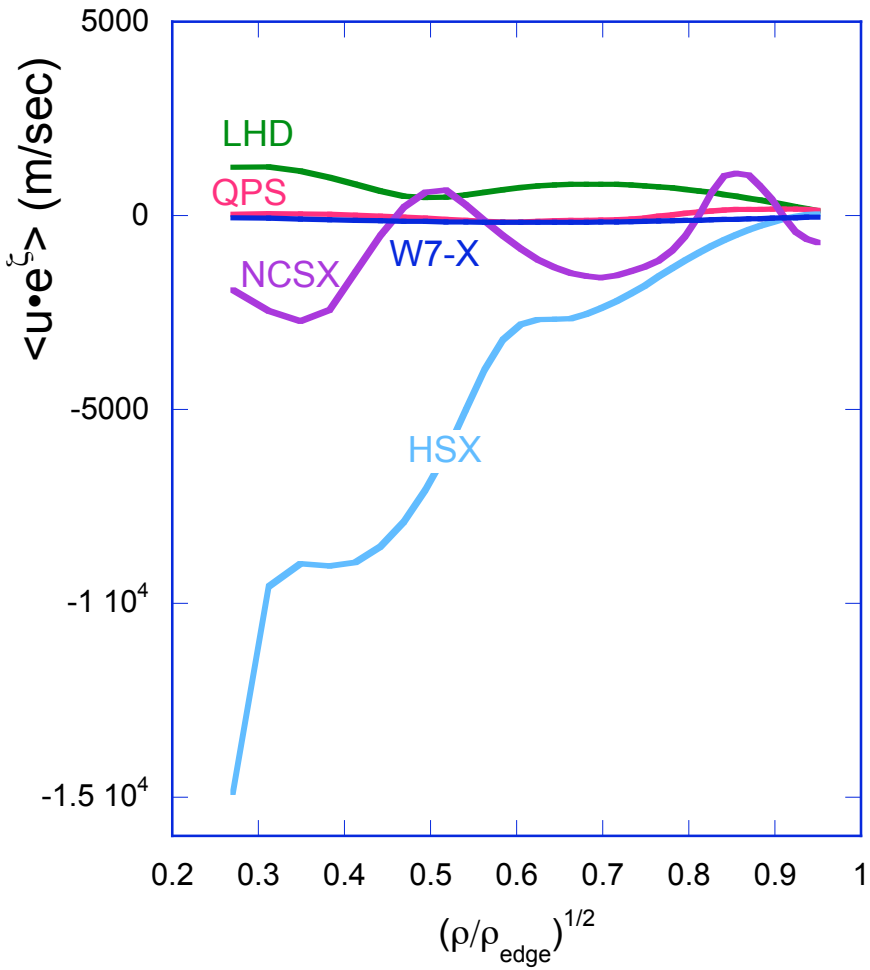
- Components taken: $e^{\theta} = \vec{\nabla}\theta / \left|\vec{\nabla}\theta\right|, \; and \; e^{\xi} = \vec{\nabla}\zeta / \left|\vec{\nabla}\zeta\right|$
- Reduction to 1D - flux surface average: $\langle \ldots \rangle = \frac{1}{V'}\iint d\theta d\zeta \sqrt{g}\left(\ldots\right)$
- Due to $1/B^2$ variation of Jacobian, averages will be influenced by whether $\propto 1/B$ ($E\times B$, diamagnetic) or $\propto B$ (neo. Parallel, PfirshSchlüter) terms dominate

Comparison of flux-averaged toroidal flow components (contra-variant) among devices indicates NCSX and HSX have the largest toroidal flows

ECH parameters

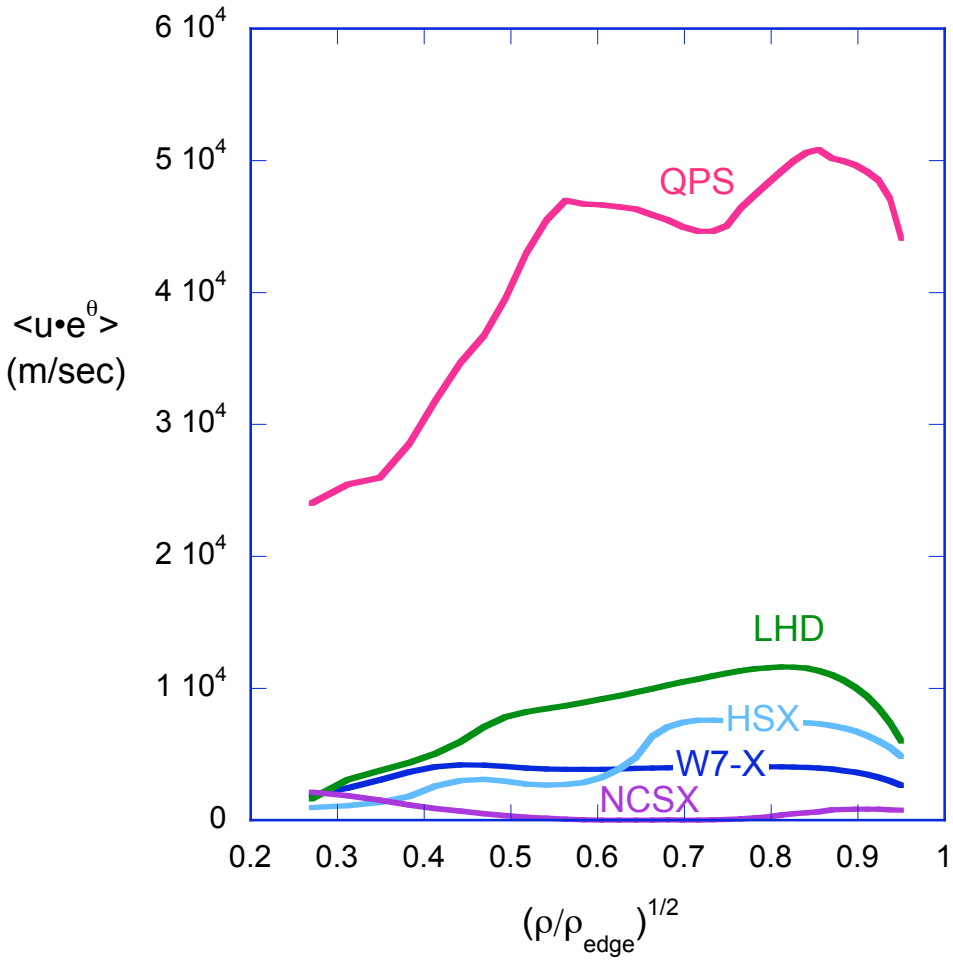


ICH parameters

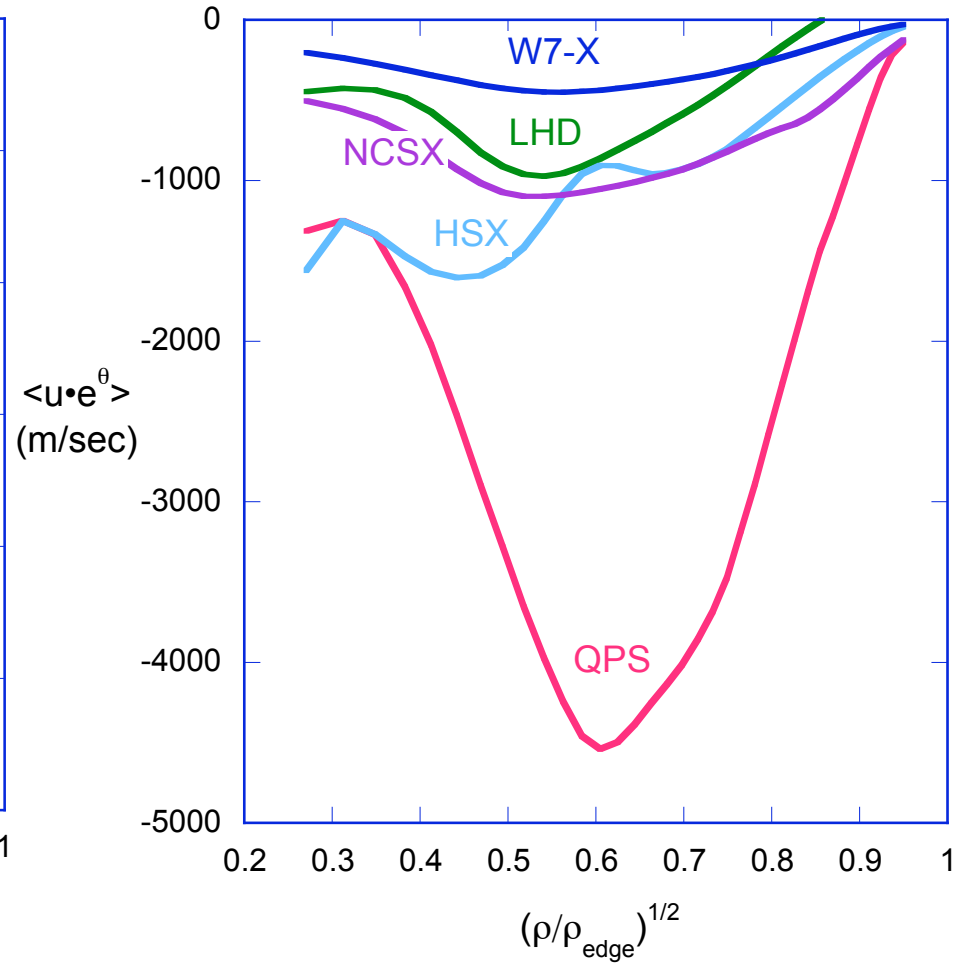


Comparison of flux-averaged poloidal flow components (contra-variant) among devices indicates QPS has largest poloidal flows

ECH parameters



ICH parameters



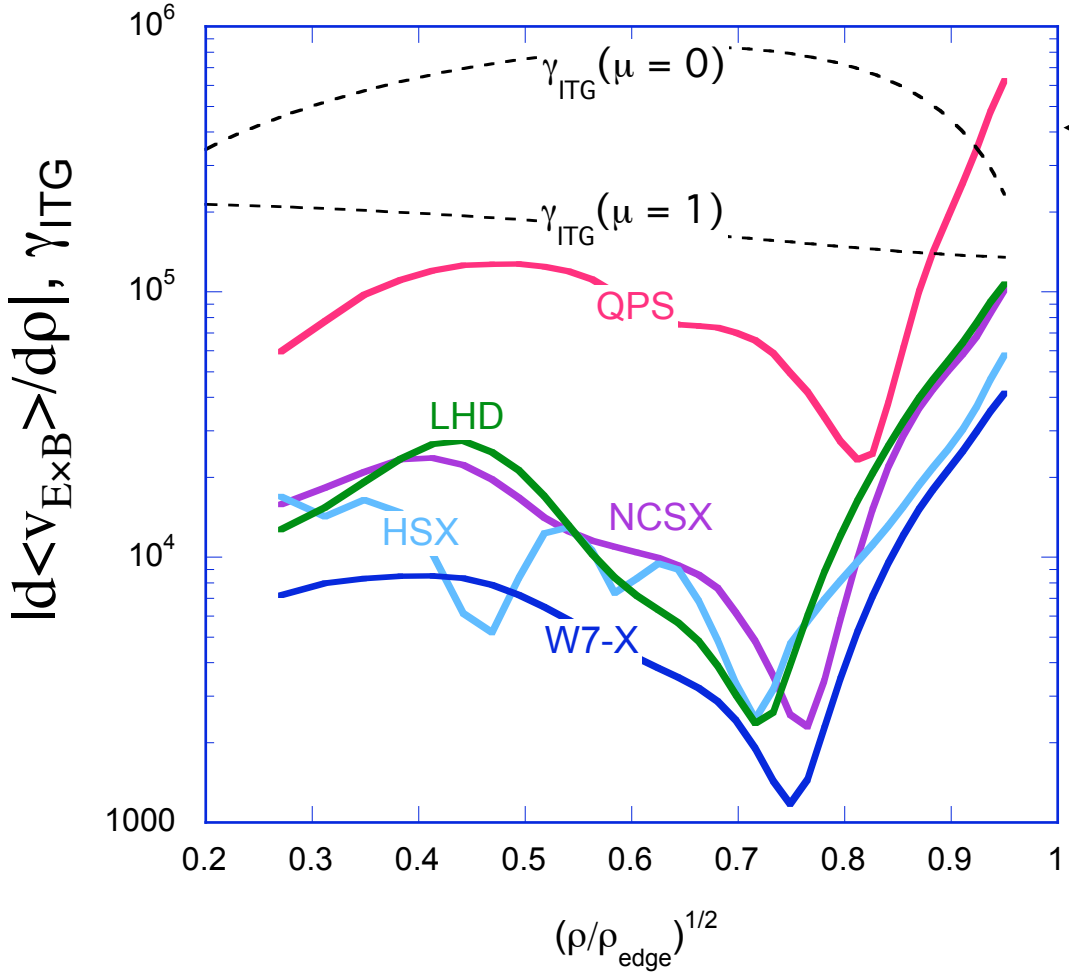
Comparison of shearing rates from ambient flows with ITG growth rates:

Transport barrier condition: shearing rate > γ_{ITG}

$\gamma_{ITG} = (C_S / L_T)(L_T/R)^{\mu}$

where $0 < \mu < 1$, C_S = sound speed

from J. W. Connor, et al., Nuclear Fusion **44** (2004) R1



Recent stellarator DTEM-ITG growth rates

from G. Rewoldt, L.-P. Ku, W. M. Tang, PPPL-4082, June, 2005

HSX

$\gamma = 4 \times 10^5$ sec^{-1}

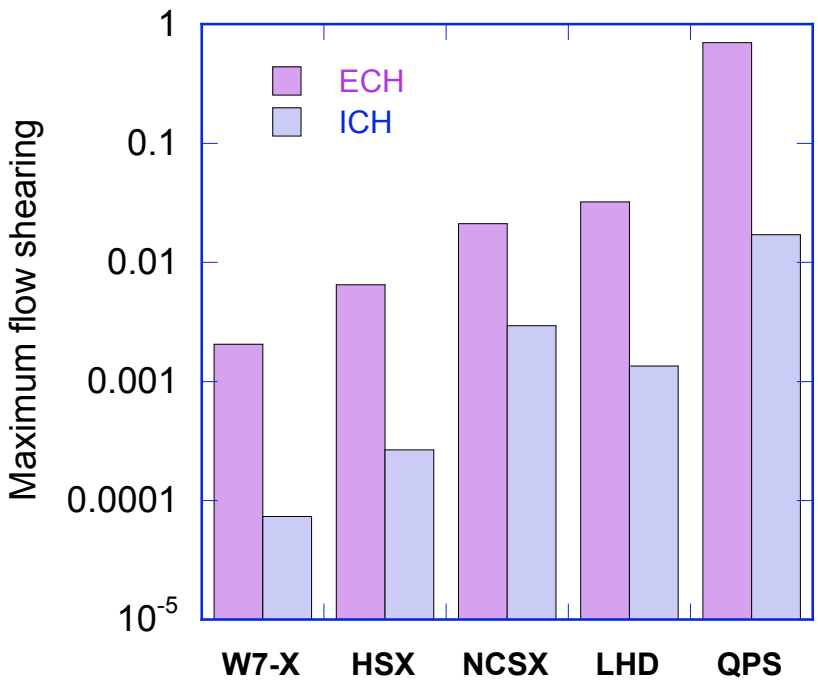
QPS, NCSX, W7-X, LHD

$\gamma = 0.2$ to 1.6×10^5 sec^{-1}

Even without external torque, QPS, NCSX, LHD, W7-X shearing rates can exceed $\gamma_{DTEM\text{-}ITG}$ at edge region

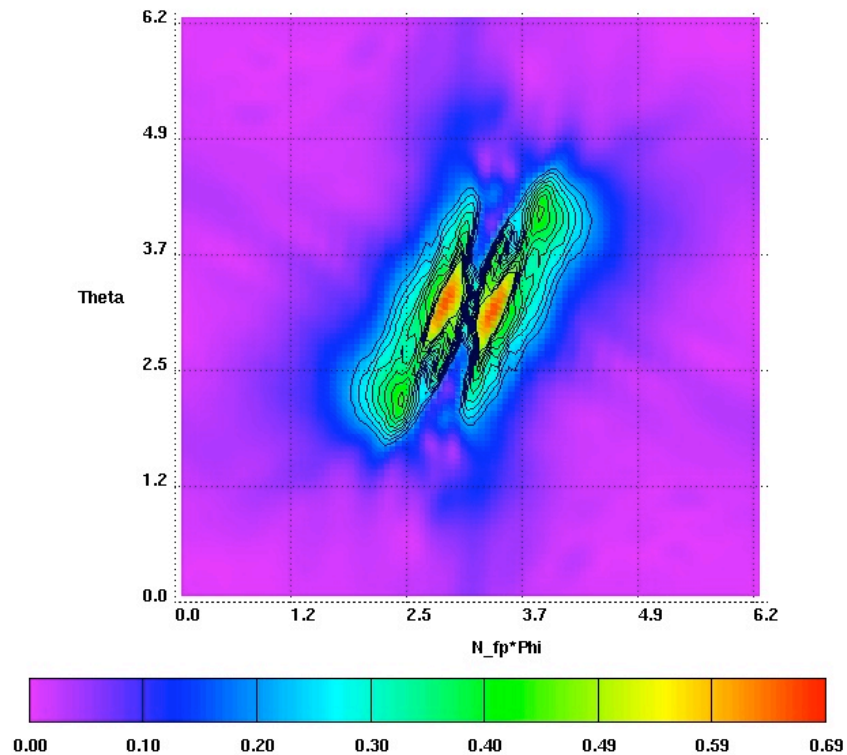
Flow variations within flux surfaces can also impact MHD ballooning/interchange thresholds:

- Maximum flow shearing rates are ~0.5 of Alfvén time
- Could influence MHD stability thresholds



QPS flow shearing along **B**

$$\tau_{A0}\left|\left(\hat{b}\bullet\vec{\nabla}\right)\vec{u}\right| \quad \text{where } \tau_{A0}=\langle R\rangle / \mathrm{v}_{A0}$$



outboard

inboard

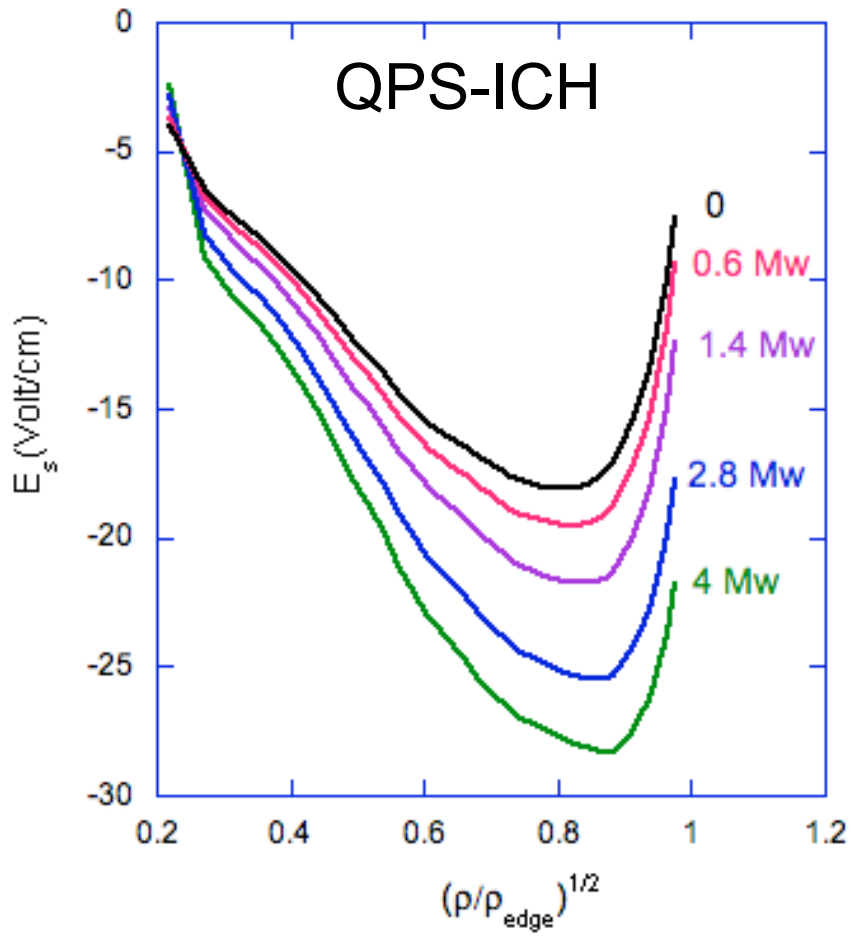
outboard

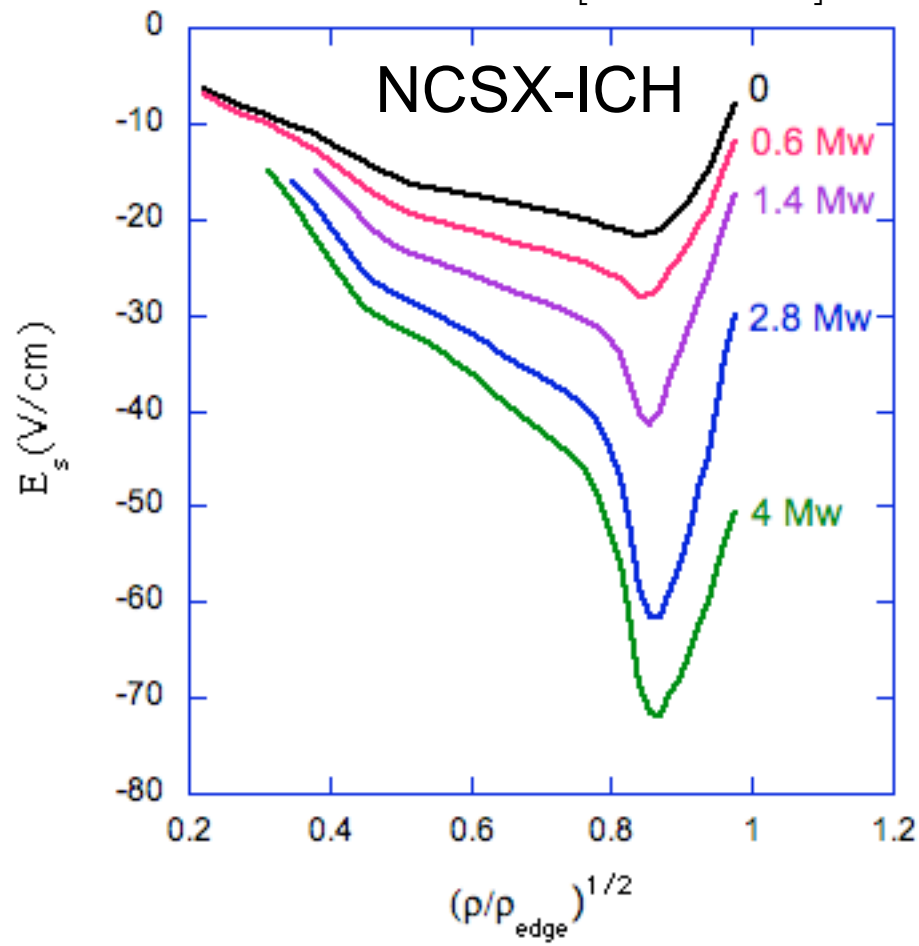
E_r is modified by parallel momentum source

(40 keV H^0 beam, $F_{||}^{(i,e)}$ = 0.1, 0.2, 0.5, 0.7 Nt/m^3)

Same viscosity coefficients that relate gradients in n, T, ϕ to $u_{||}$ also imply that changes in $u_{||}$ modify Γ.

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a/T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \langle u_{||a}B\rangle/\langle B^2\rangle \\ \frac{2}{5p_a}\langle q_{||a}B\rangle/\langle B^2\rangle \\ -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a\frac{\partial\Phi}{\partial s} \\ -\frac{\partial T_a}{\partial s} \end{bmatrix}$$





Conclusions

- **A self-consistent ambipolar model has been developed for the calculation of flow profiles in stellarators**
  - Ambipolar electric field solution with viscous effects
  - Predicts profiles of E_r, $\langle u_{||} \rangle$, $\langle u^{\theta} \rangle$, and $\langle u^{\zeta} \rangle$; 2D flow variations
- **Magnetic field structure influences flows in quasi–symmetric stellarators**
  - QPS: poloidal flows dominate over toroidal flows
  - W7-X: poloidal flows dominate, but flows reduced from other systems
  - NCSX: toroidal flows dominate except near the edge
  - HSX: flows are dominantly helical/toroidal
  - LHD: toroidal flows dominate for ICH case, poloidal flows for ECH case

Conclusions (contd.)

- **Ambient flow shearing rates approach levels that could suppress turbulence**
  - ITG, ballooning
  - Further stellarator-specific turbulence work needed, but with 2D neoclassical flow fields
  - Collaborations initiated with LHD, HSX to further apply model and look for correlations with confinement data
- **The sensitivity of flow properties to magnetic structure is an opportunity for stellarators**
  - Experimental tests of flow damping in different directions
  - Possible new hidden variable in confinement scaling data
  - Stellarator optimizations/flexibility studies -> should use $E \times B$ shear (turbulence suppression to complement neoclassical transport reduction