Current and Future Stellarator Theory Physics Topics

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Oak Ridge National Laboratory

Stellarator Theory Teleconference
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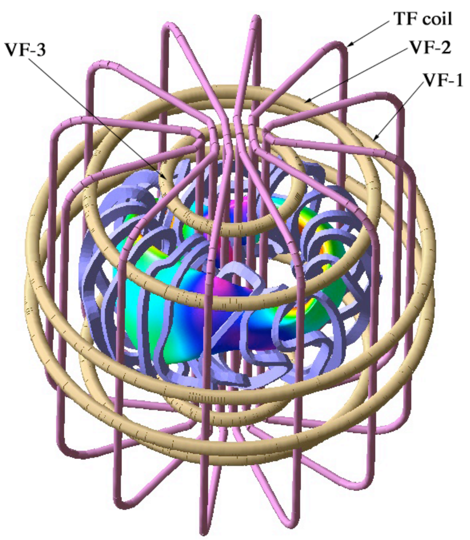
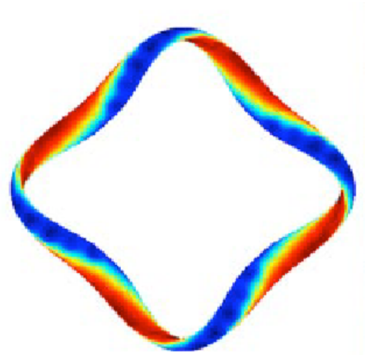
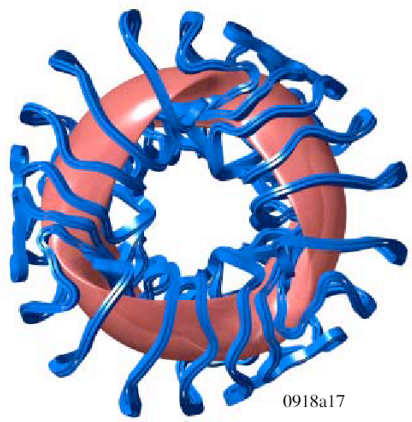






Stellarator physics has unique aspects that influence theory efforts:

- The configuration space of existing and planned stellarators is large
  - Torsatrons: LHD, CHS
  - Transport and bootstrap current optimized: W7-AS, W7-X
  - High flexibility heliacs: TJ-II, H-1A, Heliotron-J
  - Quasi-symmetric systems: HSX, QPS, NCSX, CHS-qa
  - Comparative studies of physics issues between configurations are of great value - international collaboration important
  - Design efforts dominantly involve theory and modeling
    - Optimization of coils and plasma, choice of physics themes
    - Although QPS/NCSX designs are now fixed, need to keep functioning optimization tools for future experimental flexibility analysis
- Compact stellarators require the development of new computational tools
  - Standard expansion methods are not adequate, strong poloidal/toroidal coupling
  - We have learned to effectively utilize massively parallel computers
    - Optimization, Monte Carlo transport, moments method transport, magnetic island studies, RF heating, finite-n ballooning
  - This is coupled with development of efficient algorithms: VMEC2000, COBRA, STELLOPT, COILOPT
  - Visualization is important

Important current and future stellarator theory topics:

Equilibrium reconstruction

Flexibility studies, island suppression

Neoclassical transport and flows using moments methods

Stability of Alfvénic modes

Finite-n ballooning

Resistive MHD

Microturbulence

Non-diffusive transport models

Monte Carlo modeling of energetic particle confinement

Reactor studies

Stellarator research topics

- **Equilibrium**
  - Improvements to VMEC code
  - Flux surface reconstruction for 3D systems
- Transport
- Stability
- Optimization, configuration development
- RF Heating

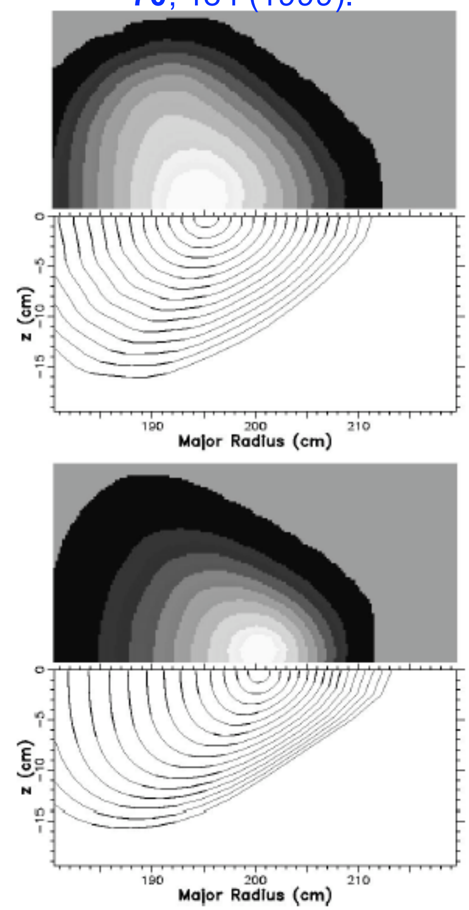
VMEC has been essential in the optimization of new stellarators and the analysis of existing experiments.

- Minimizes the plasma potential energy:

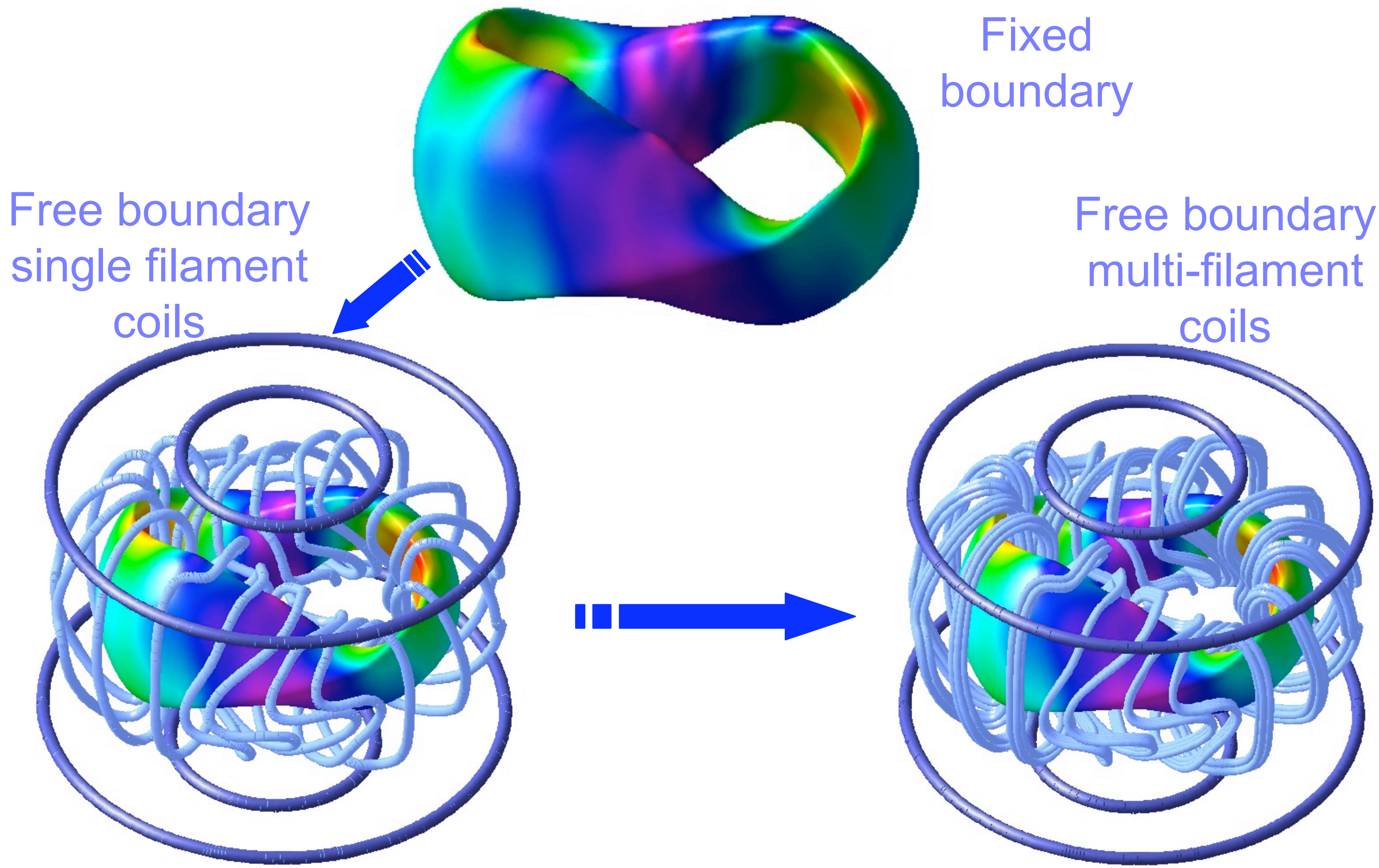
$$\int \left(B^2/2 + p\right) dV$$

- Assumes nested flux surfaces
- Optimization would be unfeasible without rapidly calculated accurate 3D equilibria
- Used in stellarator labs worldwide
- Significant improvements recently in speed and accuracy
  - new preconditioner
  - MHD residuals brought down to machine precision levels
  - short runtimes maintained
  - Extended to non-stellarator symmetry

Reconstructed W7-AS X-ray emissivity contours and VMEC surfaces at $\beta(0)$ = 1% and 4.4%
(A. Weller, et al., Rev. Sci. Inst. **70**, 484 (1999).



Various boundary options are available in the VMEC equilibrium code



Stellarator/tokamak equilibrium reconstruction

"Magnetic diagnostic responses for compact stellarators," Steven P. Hirshman, Edward A. Lazarus, James D. Hanson, Stephen F. Knowlton, and Lang L. Lao, Phys. Plasmas 2, 595 (2004)

- Goal: reconstruction of 3D equilibria consistent with external flux signal measurements (V3FIT)
  - Uses similar methods as those developed for stellarator optimization
  - 3D systems are more complex than tokamaks, but offer less degeneracy (I.e., more information) for this process than axisymmetric devices
  - Divergence of Biot-Savart methods near line segment currents resolved
    - J. Hanson, S. P. Hirshman, Phys. of Plasmas 9, 4410 (2002)
- Improvements in Motional Stark Effect (MSE) diagnostic support
  - New MSE module written for full 3D systems
    - also allows up-down asymmetric tokamaks to be analyzed
  - Robust/fast mapping code developed for (X, Y, Z) to (ψ, θ, ζ) transformation
    - crucial for interpretation of MSE measurements (also used with AORSA - RF heating)

Rapid, accurate equilibrium reconstruction will be essential for the next generation of compact stellarator hybrids (NCSX, QPS, CTH)

- Time-varying plasma pressure/currents will modify boundary shape
- Need dynamic coil current control of 3D shaping to achieve physics goals

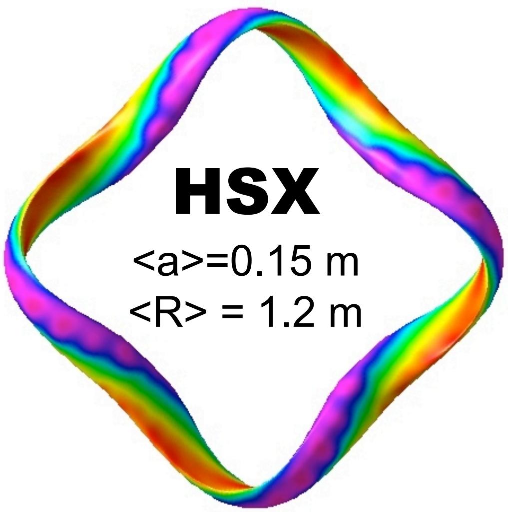
Stellarator research topics

- Equilibrium
- **Transport**
  - Moments method
    - Based on DKES transport coefficients
    - Self-consistent prediction of viscosities, flows, electric field, bootstrap current
  - Monte Carlo method
    - Non-local transport
    - Direct calculation of viscosities for moments method
- Stability
- Optimization, configuration development
- RF Heating

Moments Method for Stellarator Transport

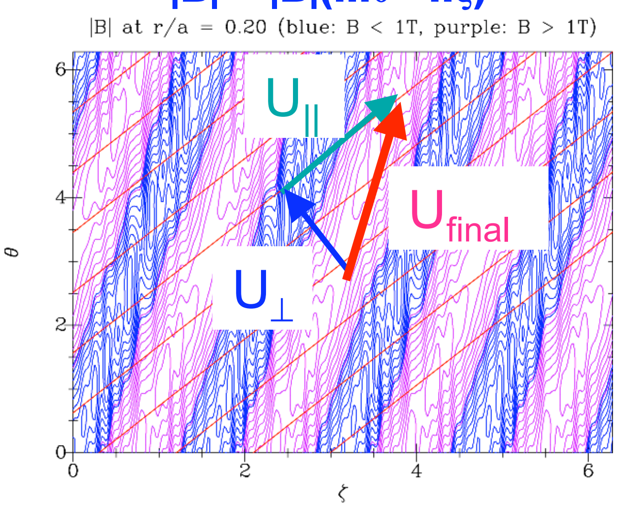
- QPS/NCSX/HSX have been optimized:
  - so that neoclassical losses << anomalous losses
  - remaining transport-related differences are in the parallel momentum transport properties
- Recently, a theoretical framework has been developed that allows quantitative, self-consistent assessment of the parallel and perpendicular transport in 3D systems:
  - H. Sugama and S. Nishimura, Physics of Plasmas, **9 (**November, 2002) 4637.
  - Viscosities (derived in terms of DKES coefficients) incorporate all needed kinetic information
  - Allow multiple species to be decoupled at the macroscopic level
  - Provides particle/energy fluxes, viscosity tensor, flows, and bootstrap current
- This motivates development of an analog of the NCLASS code for stellarators
- Calculation of flow velocity profiles for stellarators is motivated by:
  - Relevance to turbulence suppression/enhanced confinement regimes
  - Impact on magnetic island formation and growth
  - Impurity accumulation/shielding studies
- More accurate collisional bootstrap current prediction, and ambipolar electric field estimation

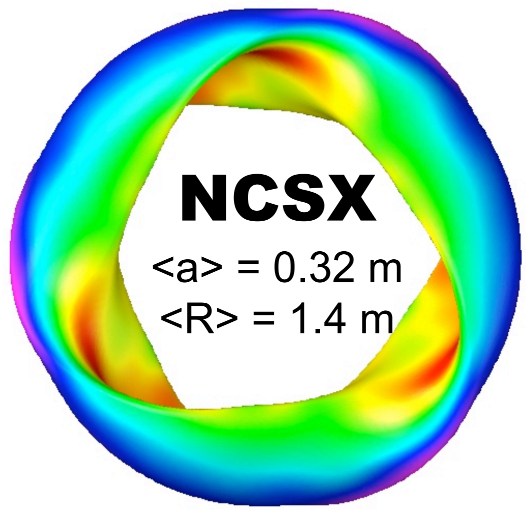
Advances in stellarator optimization have allowed the design of 3D configurations with magnetic structures that approximate: straight helix/tokamak/connected mirrors:



Quasi-helical symmetry

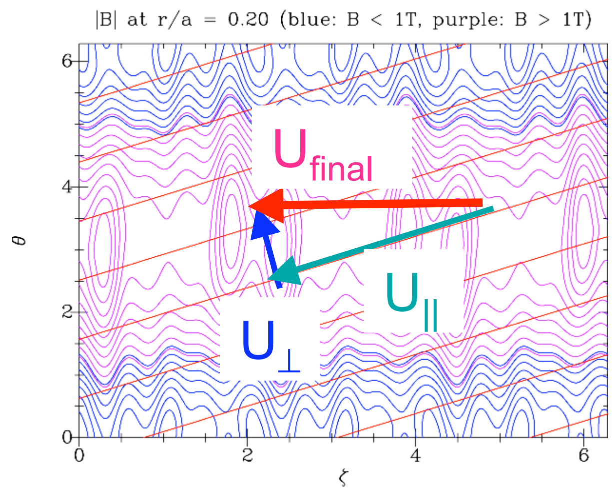
$|B| \sim |B|(m\theta - n\zeta)$

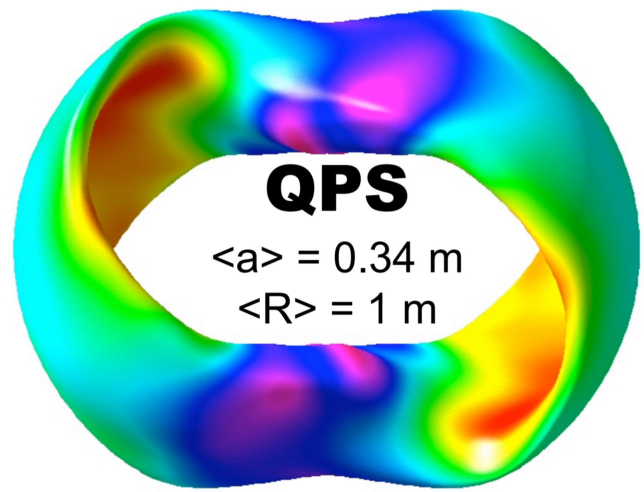




Quasi-toroidal symmetry

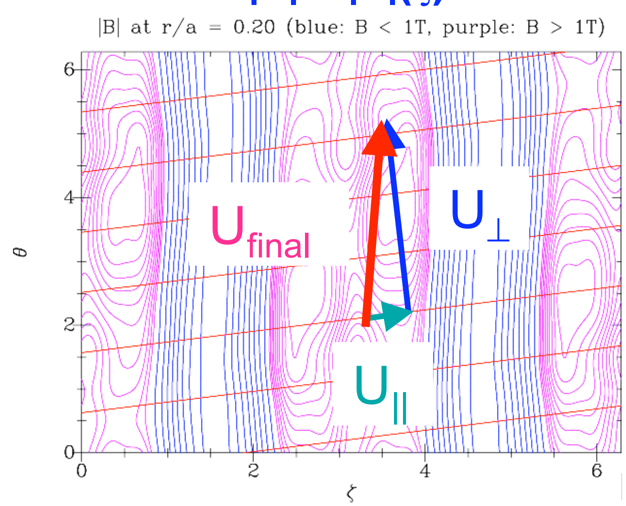
$|B| \sim |B|(\theta)$





Quasi-poloidal symmetry

$|B| \sim |B|(\zeta)$



Moments Method Closures for Stellarators

The parallel viscous stresses, particle and heat flows are treated as fluxes conjugate to the forces of parallel momentum, parallel heat flow, and gradients of density, temperature and potential:

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a/T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \left\langle u_{\|a}B\right\rangle / \left\langle B^2\right\rangle \\ \dfrac{2}{5p_a}\left\langle q_{\|a}B\right\rangle / \left\langle B^2\right\rangle \\ -\dfrac{1}{n_a}\dfrac{\partial p_a}{\partial s} - e_a\dfrac{\partial\Phi}{\partial s} \\ -\dfrac{\partial T_a}{\partial s} \end{bmatrix}$$

- Analysis of Sugama and Nishimura related monoenergetic forms of the M, N, L viscosity coefficients to DKES transport coefficients
- Combining the above relation with the parallel momentum balances and friction-flow relations

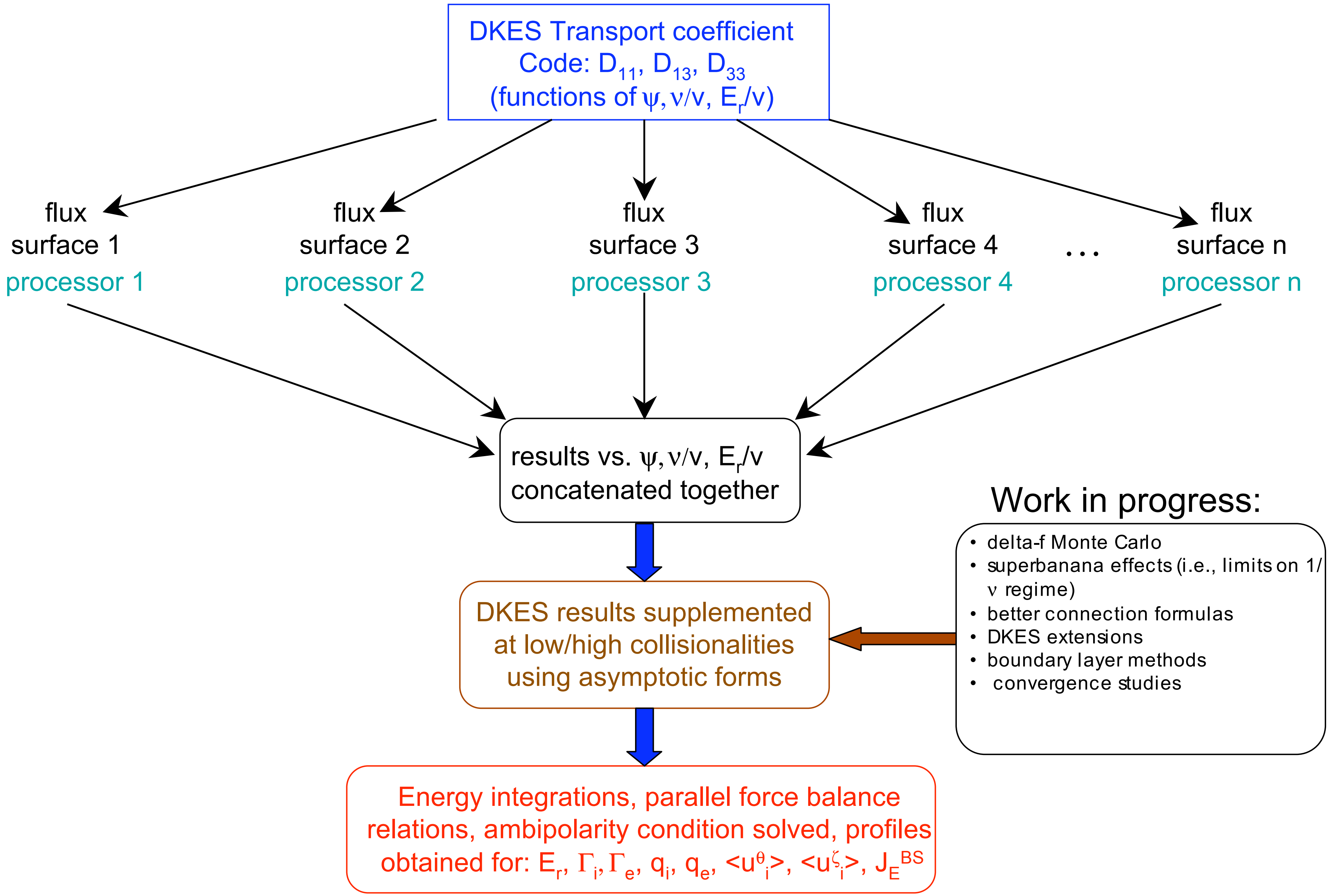
$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle - n_a e_a\left\langle BE_\|\right\rangle = \left\langle BF_{\|a1}\right\rangle$$

$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle = \left\langle BF_{\|a2}\right\rangle$$

$$\begin{bmatrix} \left\langle BF_{\|a1}\right\rangle \\ \left\langle BF_{\|a2}\right\rangle \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{12}^{ab} & l_{22}^{ab} \end{bmatrix} \begin{bmatrix} \left\langle Bu_{\|b}\right\rangle \\ \dfrac{2}{5p_b}\left\langle Bq_{\|b}\right\rangle \end{bmatrix}$$

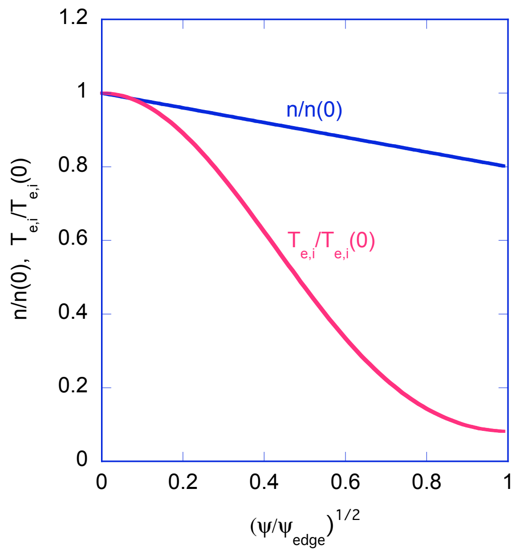
Leads to coupled equations that can be solved for <$u_{\|a}$B>, <$q_{\|a}$B>, Γ_a, Q_a

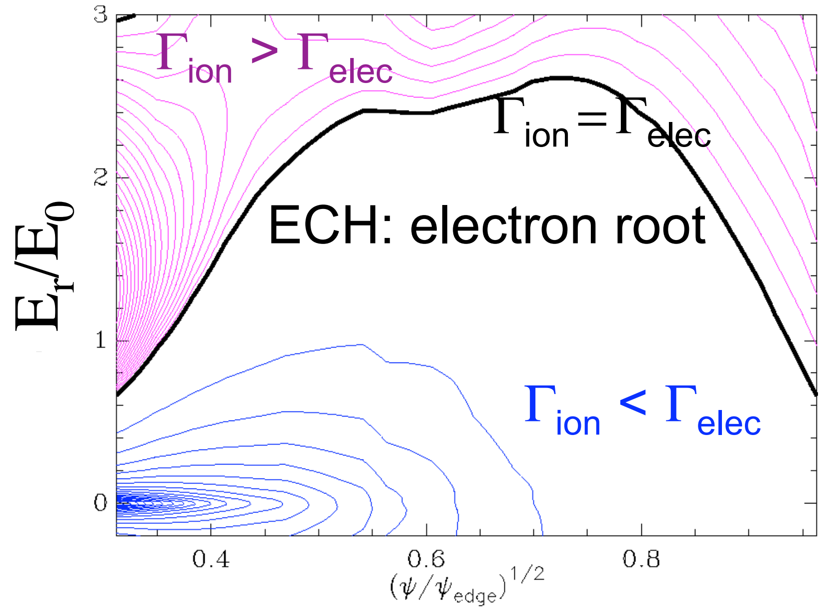
Parallel Environment for Neoclassical Transport Analysis (PENTA)



The flow model will be applied to two parameter ranges with radially continuous/stable electric field roots

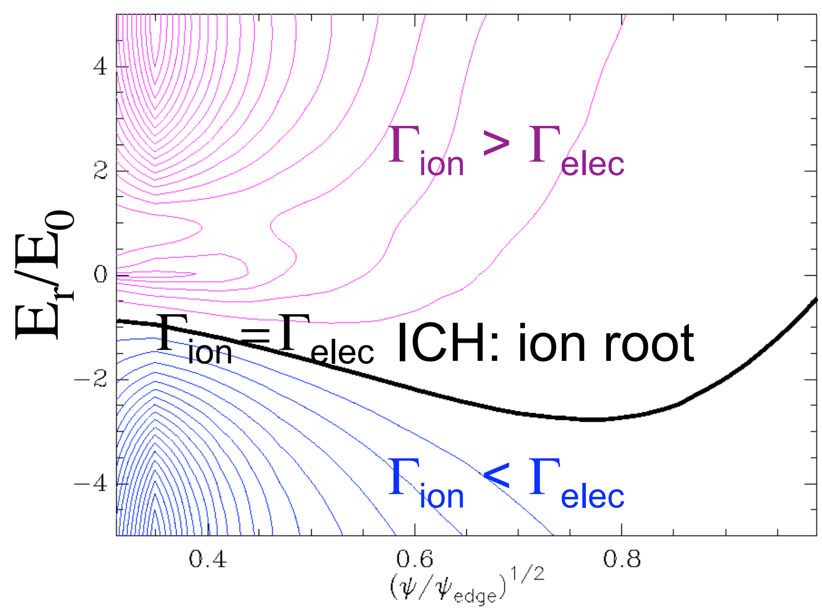
- ECH regime:
  - $n(0) = 2 \times 10^{19}\ m^{-3}$, $T_e(0) = 1.8$ keV, $T_i(0) = 0.2$ keV
- ICH regime
  - $n(0) = 8 \times 10^{19}$ m-3, Te(0) = 0.5 keV, $T_i(0) = 0.4$ keV



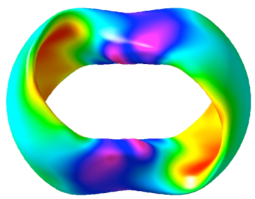


Roots chosen to give stable restoring force for electric field perturbations through:

$$\frac{\varepsilon_\perp}{q}\frac{\partial E_r}{\partial t} = \Gamma_e - \Gamma_i$$



For QPS the ambipolar electric field is compared both using the usual method and the newer approach with viscous couplings



Conventional $\Gamma_i = \Gamma_e$:

$$D_{11}^{ion}\left(\frac{n_i'}{n_i} - \frac{eE_r}{T_i}\right) + D_{12}^{ion}\frac{T_{ion}'}{T_{ion}}$$

$$= D_{11}^{elec}\left(\frac{n_{elec}'}{n_{elec}} + \frac{eE_r}{T_{elec}}\right) + D_{12}^{elec}\frac{T_{elec}'}{T_{elec}}$$

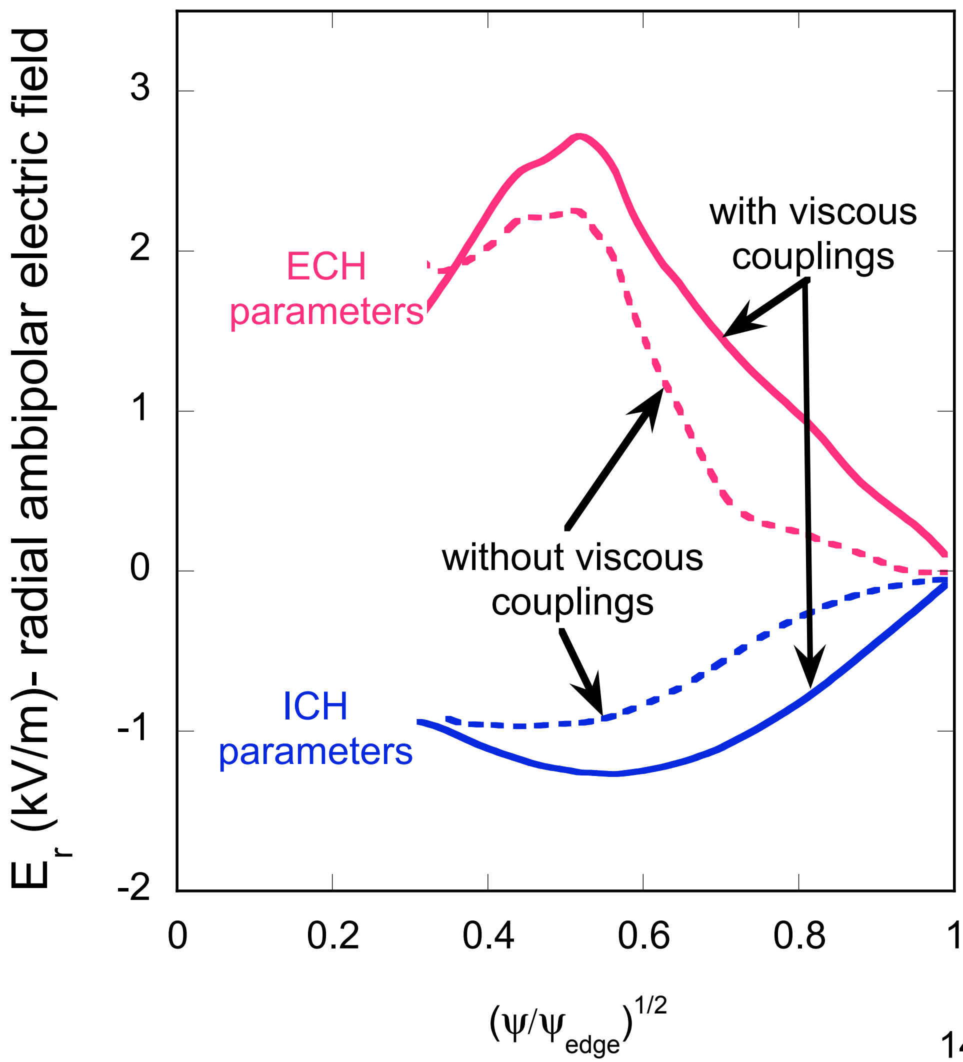
Viscous-coupled $\Gamma_i = \Gamma_e$:

$$L_{11}^{ee}X_{e1} + L_{12}^{ee}X_{e2} + L_{11}^{ei}X_{i1} + L_{12}^{ei}X_{i2}$$

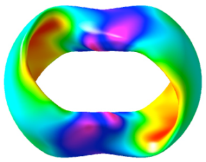
$$= L_{21}^{ie}X_{e1} + L_{22}^{ie}X_{e2} + L_{21}^{ei}X_{i1} + L_{22}^{ei}X_{i2}$$

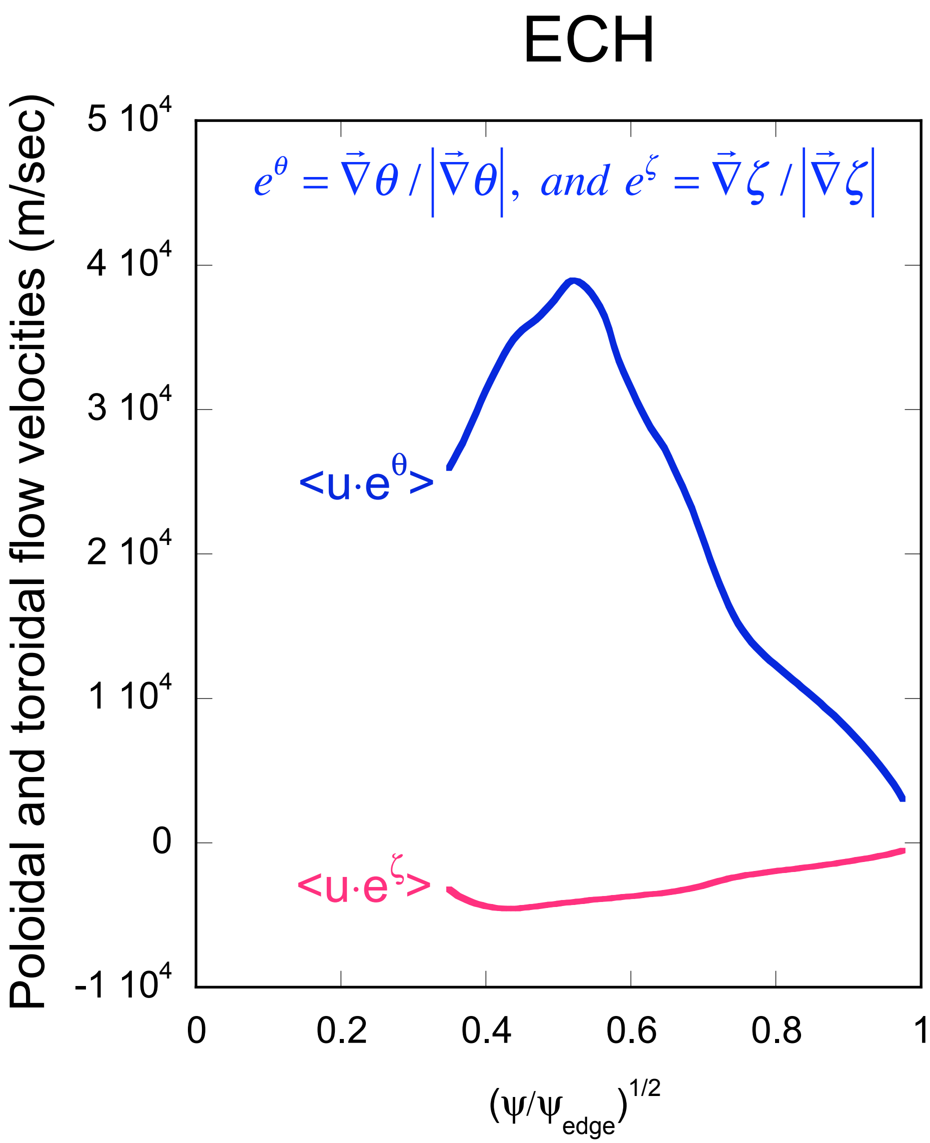
$$where \;\; X_{a1} = -\frac{1}{n_a}\frac{\partial p_a}{\partial r} - e_a\frac{\partial \phi}{\partial r}$$

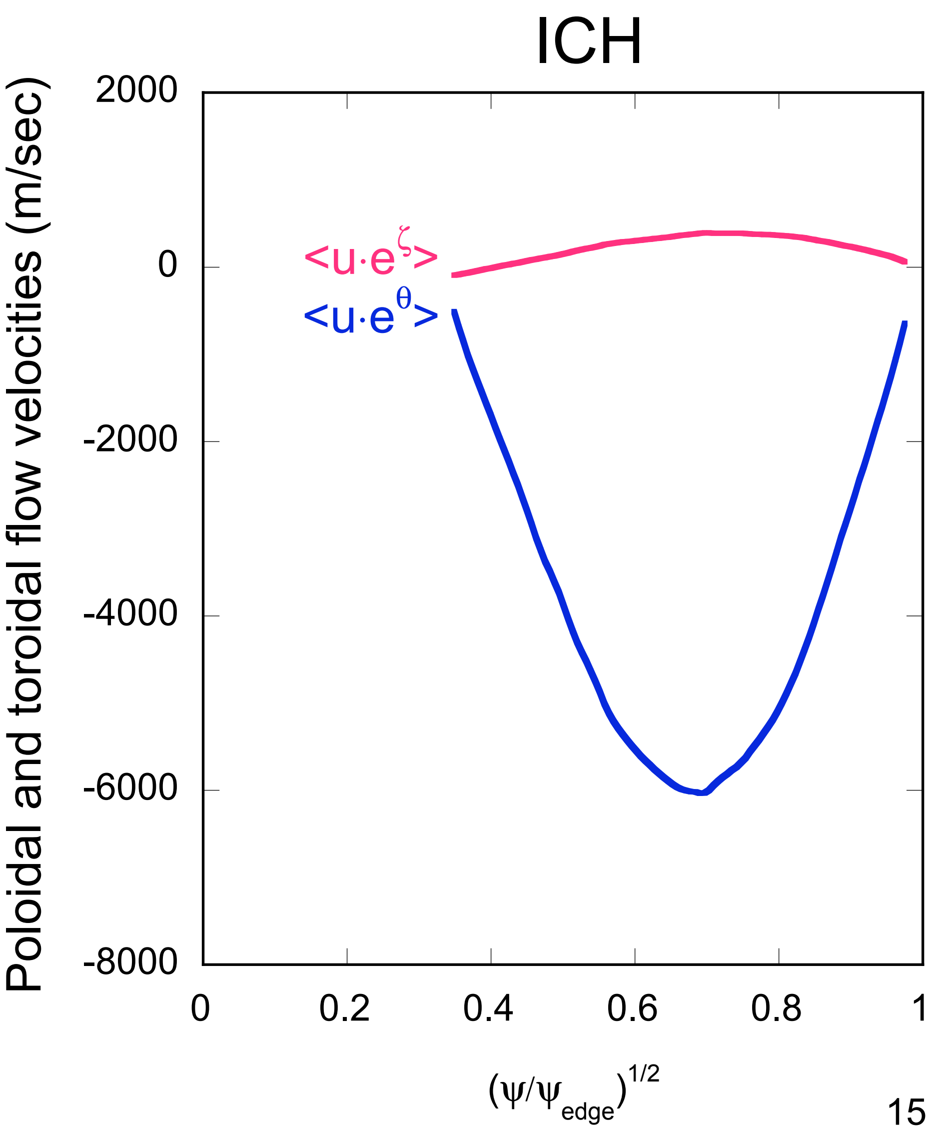
$$X_{a2} = -\frac{\partial T_a}{\partial r}$$



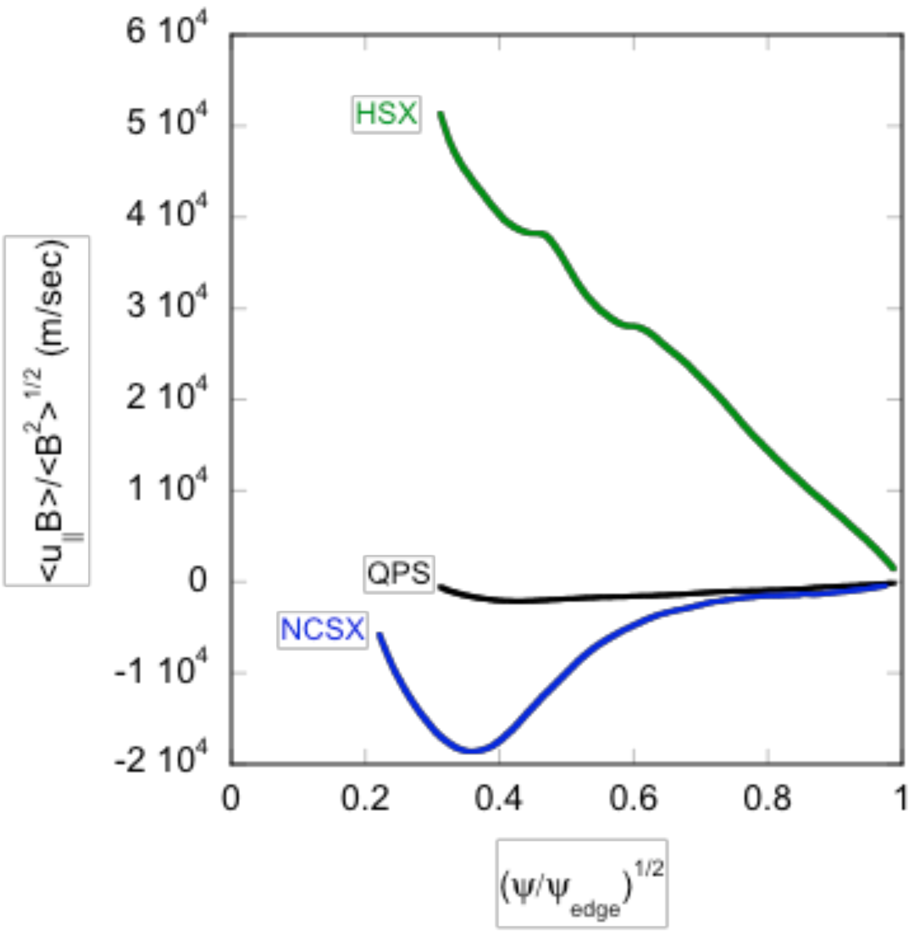
QPS flow velocities are dominated by poloidal components.

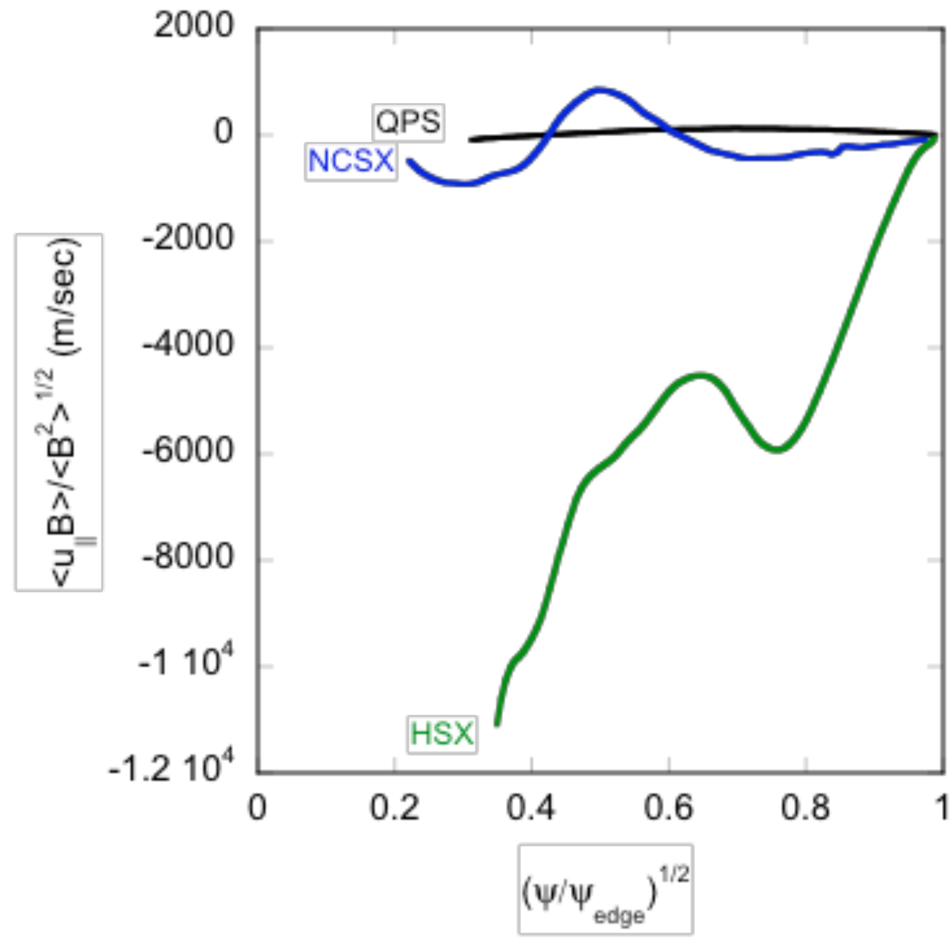




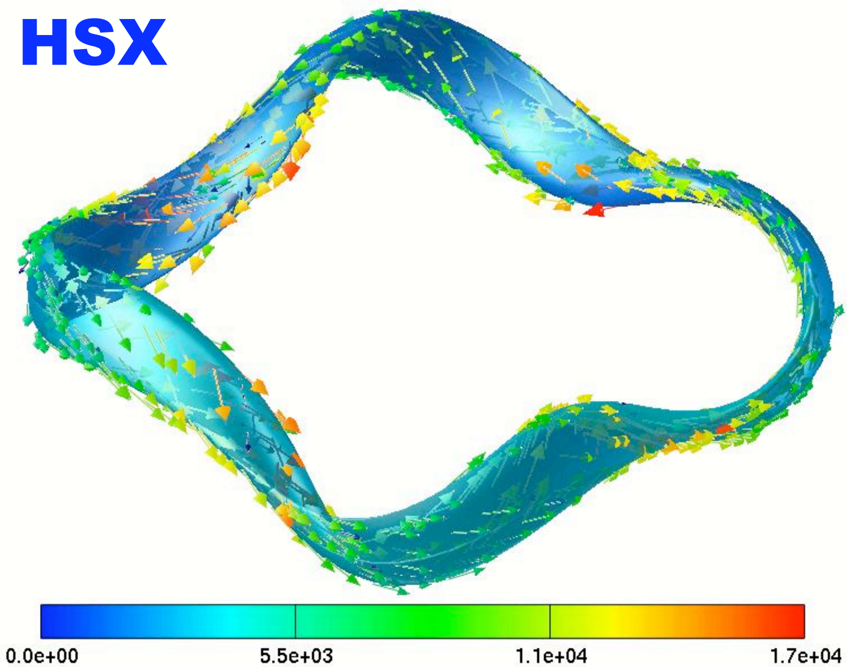


The neoclassical flow response occurs through the parallel velocity component

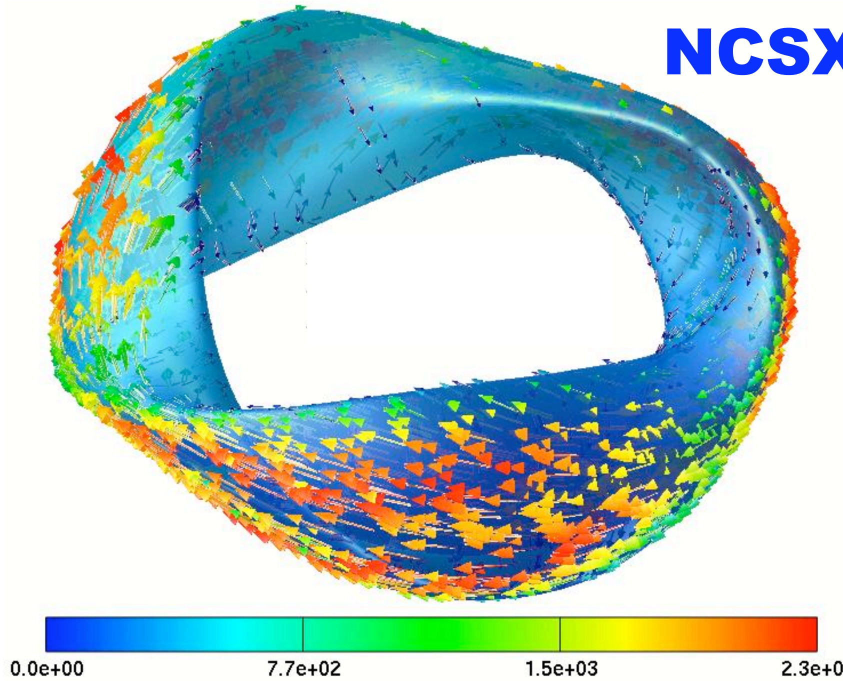




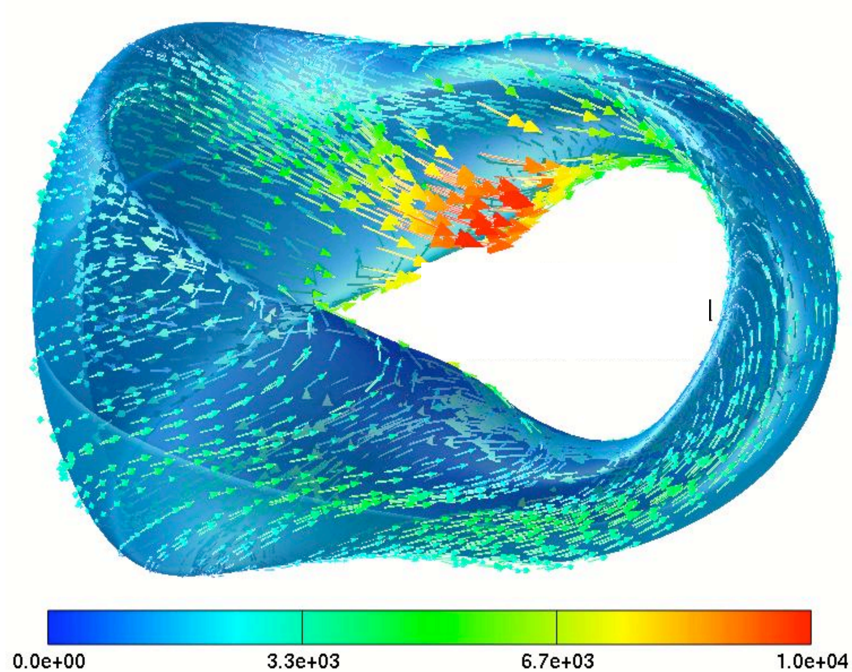
HSX


0.0e+00
5.5e+03
1.1e+04
1.7e+04

NCSX


0.0e+00
7.7e+02
1.5e+03
2.3e+03

QPS


0.0e+00
3.3e+03
6.7e+03
1.0e+04

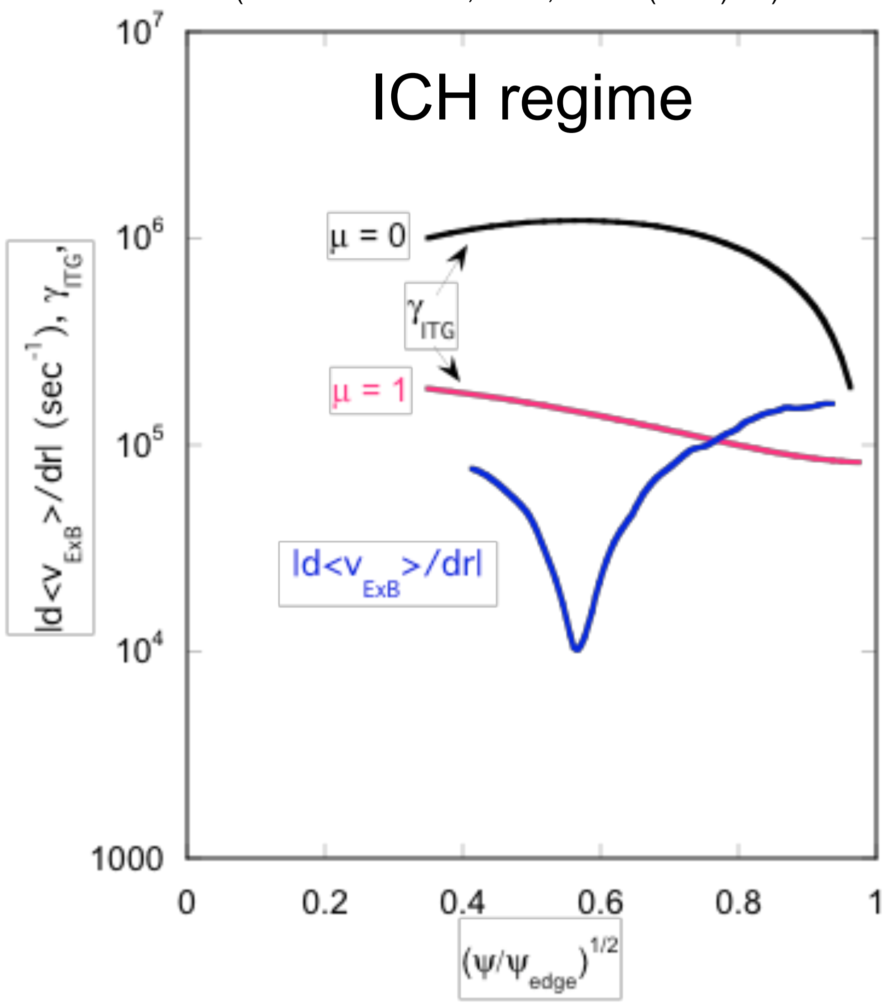
Even without external torques, flux-surface-average velocity shearing rates can be comparable to estimates (tokamak-based) of ITG growth rates. Also velocity shearing within flux surfaces can be ~0.1 τ_{A0}

Transport barrier formation condition:

E × B shearing rate > $\gamma_{ITG} = (C_S / L_T)(L_T/R)^{\mu}$

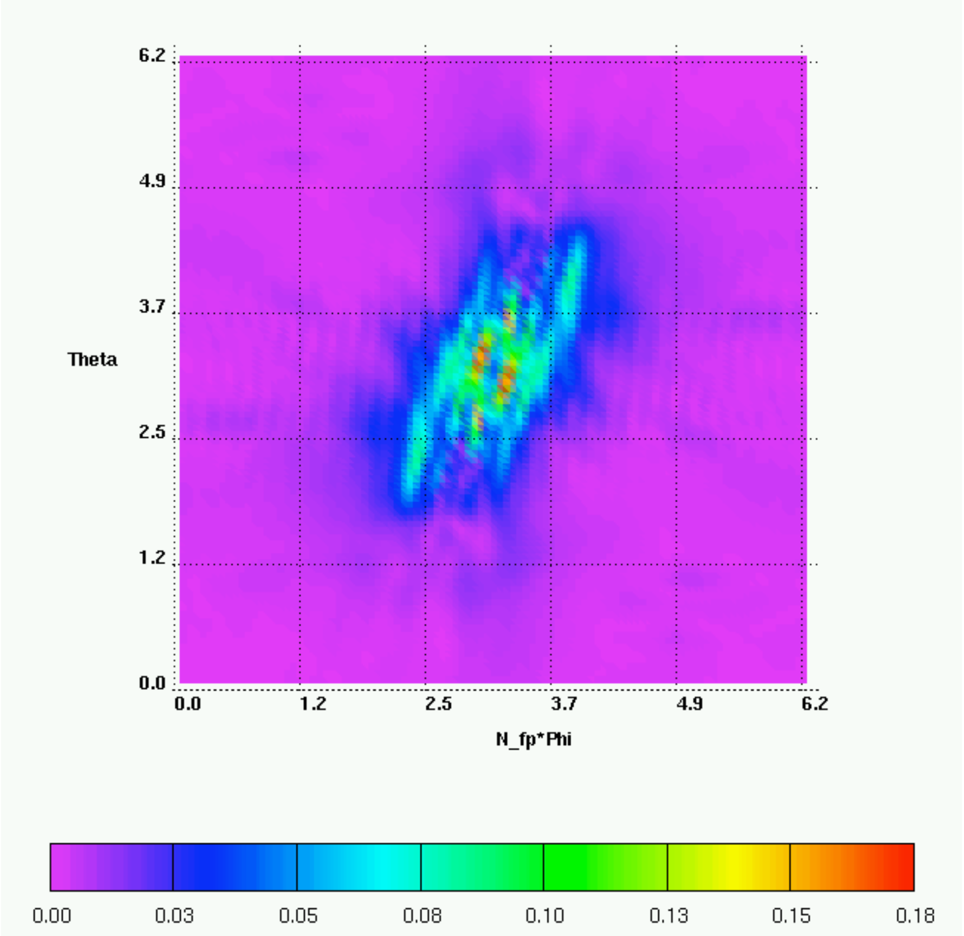
where $0 < \mu < 1$, C_S = sound speed

(from J. W. Connor, et al., NF **44** (2004) R1)

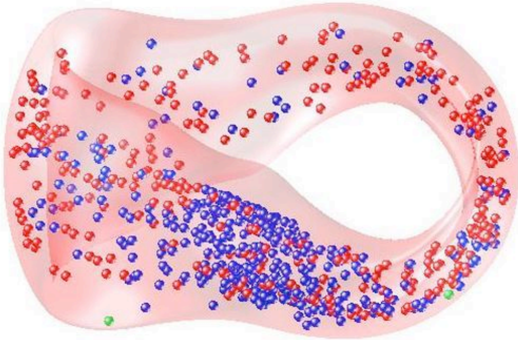
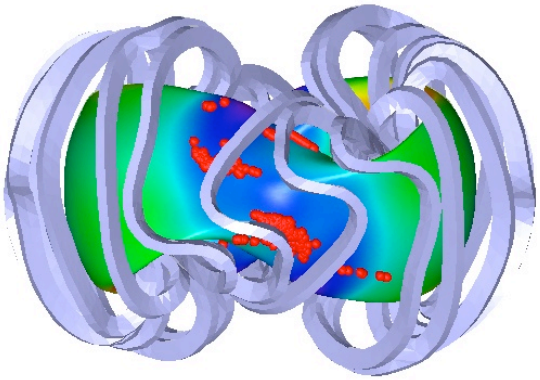
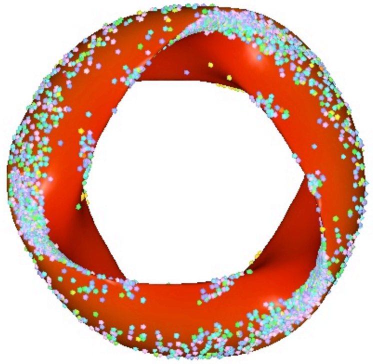


Flow shearing along **B**

$$\tau_{A0}\left|\left(\hat{b}\bullet\vec{\nabla}\right)\vec{\mathrm{v}}\right| \quad \text{where} \quad \tau_{A0} = \langle R \rangle / \mathrm{v}_{A0}$$



Monte Carlo particle simulations are used for a variety of stellarator transport phenomena

- DELTA5D code developed at ORNL as an outgrowth of the earlier MCMP code [R. Fowler, J. Rome, J. F. Lyon, Phys. Fluids 28, 338 (1995)]
  - Computational structure:
    - Particles are followed (using LSODE) in parallel on 100 - 2000 processors
    - Diagnostics accumulated locally and merged together at the end
  - Global energy confinement times, local diffusion coefficient
  - Neutral beam slowing down and heating efficiency
  - Bootstrap current
  - RF tail confinement and production
  - Alpha particle confinement
  - Electron tail and runaway losses

DELTA5D will be extended to run efficiently on ORNL's Cray X1 vector computer and to calculate viscosities in general 3D systems

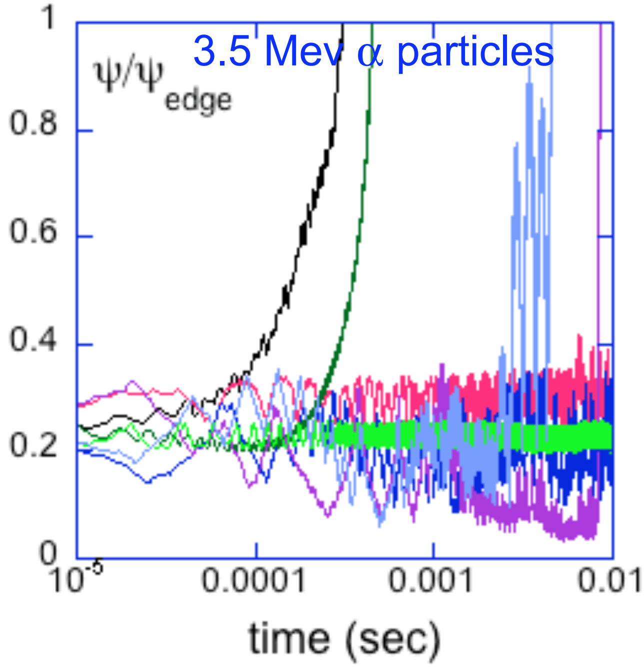
- Multiscale MHD LDRD project (ORNL/PPPL)
  - Optimize DELTA5D and M3D for Cray vector architecture
  - Develop electron viscosity-bases closures for M3D code
  - Application: neoclassical tearing instabilities in tokamaks and stellarator hybrids
- Moments method for transport
  - Direct calculation of viscosities to supplement DKES coefficients
- Non-diffusive transport
- Transport in the presence of magnetic islands
  - Orbit following code developed based on direct calculation of B from coils (D. Strickler)

Fractional derivative diffusion equations allow a natural generalization of Fick's law for nonlocal transport:

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} \quad \Longrightarrow \quad \frac{\partial^\beta n}{\partial t^\beta} = D\frac{\partial^\alpha n}{\partial x^\alpha}$$

Riemann-Liouville definition of fractional derivative:

$$\frac{\partial^\alpha n}{\partial x^\alpha} = \frac{1}{\Gamma(2-\alpha)}\int_0^x (x-x')^{1-\alpha}\, n(x')\,dx'$$



- Developed by Diego del-Castillo-Negrete and Ben Carreras to characterize plasma turbulence transport
- Fractional diffusion models can incorporate a variety of new effects:
  - Waiting time distributions
  - Anomalous (super/sub diffusive) transport
  - Asymmetrical transport
  - Intermittency
- Stellarator regimes
  - Low collisionality transport
  - Transport in the presence of islands
  - Transport in the presence of turbulence
  - Energetic particle transport

Stellarator research topics

- Equilibrium
- Transport
- **Stability**
  - **Ballooning, second stability**
  - **Alfvén gap structures in 3D systems**
- Optimization, configuration development
- RF Heating

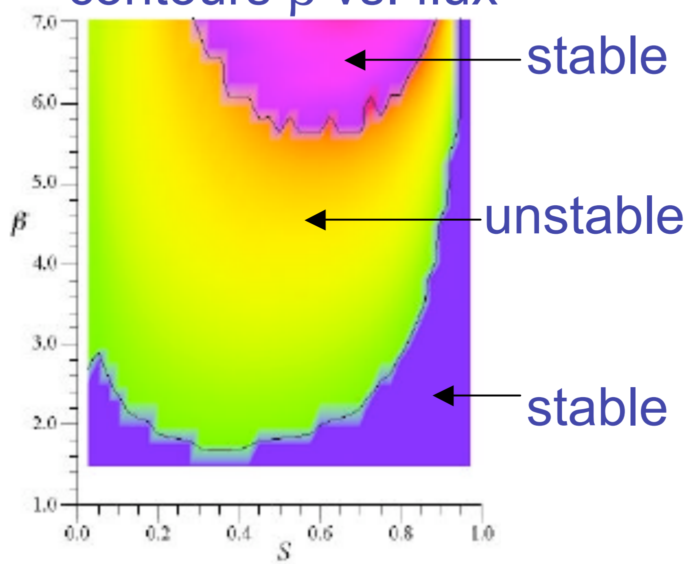
The COBRA code has allowed rapid high-n ballooning calculations for 3D systems

"COBRA: An Optimized Code for Fast Analysis of Ideal Ballooning Stability of Three-Dimensional Magnetic Equilibria," R. Sanchez, et al., J. Comp. Physics **161**, 576 (2000).; "High-β Equilibria of Drift-Optimized Compact Stellarators," A. S. Ware, et al., Phys. Rev. Lett. **89**, 125003-1 (2002).; "Second ballooning stability in high-, compact stellarators," A. S. Ware, et al., Phys. Plasmas **11**, 2453 (2004)

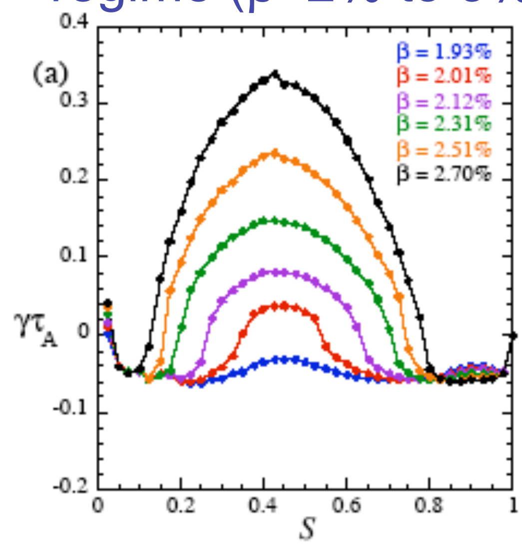
$$\rho\gamma^2\left(\frac{k_\perp^2}{B^2}\right)\Phi - \vec{B}\cdot\vec{\nabla}\left(\frac{k_\perp^2}{B^2}\right)\vec{B}\cdot\vec{\nabla}\Phi - \frac{p'}{B^2}\left(\vec{k}_\perp\times\vec{B}\right)\cdot\vec{\kappa}\Phi = 0 \qquad \text{where} \quad \vec{k}_\perp = \vec{\nabla}\theta - \iota(\psi)\vec{\nabla}\zeta - \frac{d\iota}{d\psi}(\zeta-\zeta_k)$$

- COBRA code is ~100 times faster than other options for 3D ballooning
- Used directly in optimizer
- Facilitated identification of stellarator second stable regimes (QPS/W7-AS)

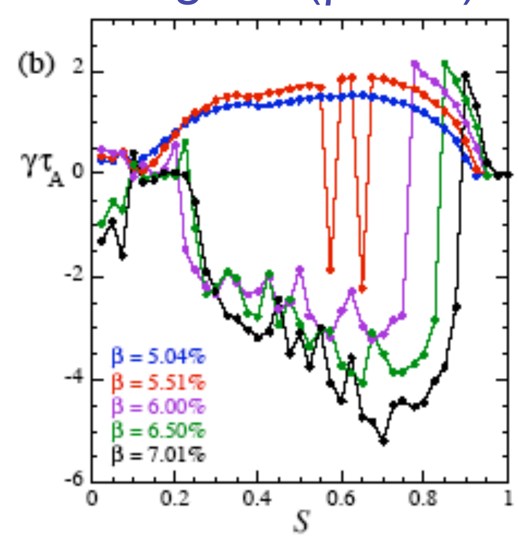
Growth rate contours β vs. flux



first unstable regime (β=2% to 3%)



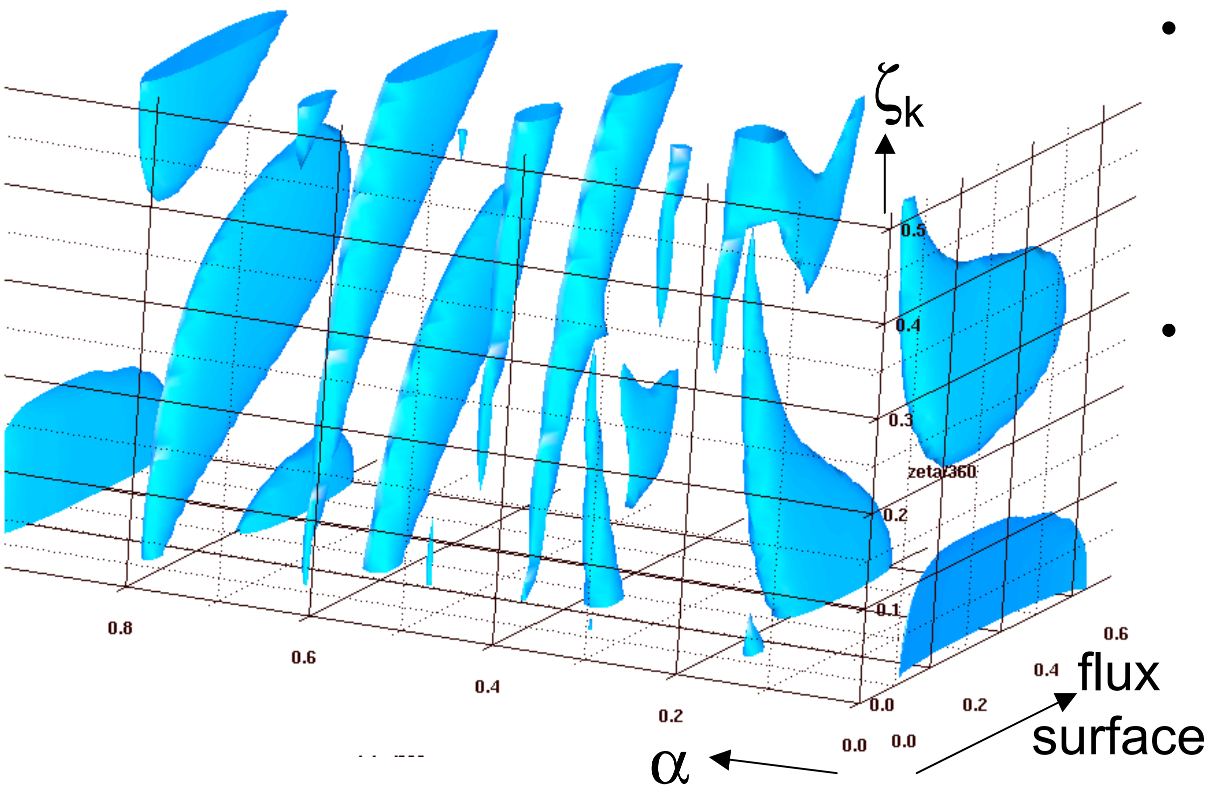
second stable regime (β=6%)



3D global ballooning reconstruction and ion FLR

(collaborators: R. Sanchez, A. Ware, C. Hegna, Diego del-Castillo Negrete, S. Hudson)

Constant ballooning growth rate isosurfaces vs. flux surface, $\alpha=\theta-\iota\zeta$, and ζ_k



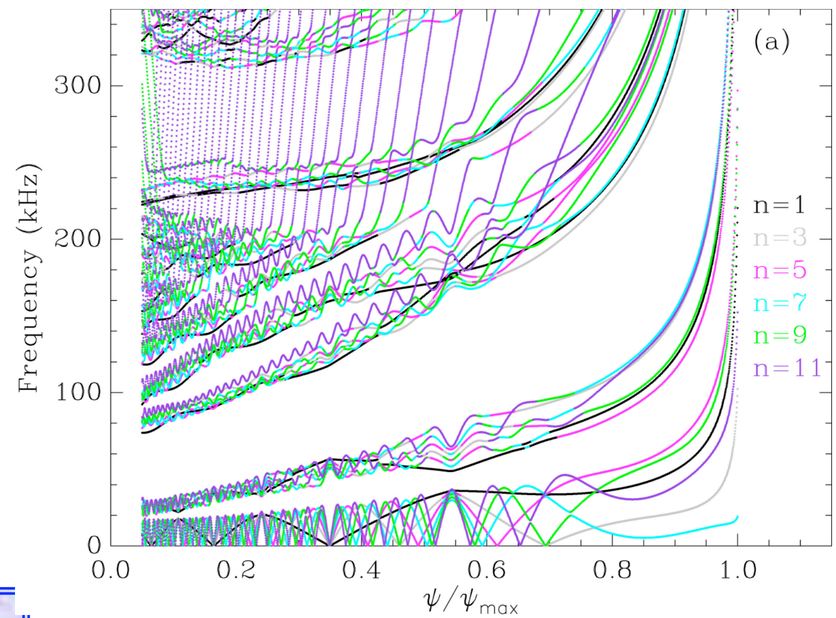
- Global ballooning reconstruction involves WKB ray-tracing on constant γ surfaces with physical quantization rules
- Ion FLR effects
  - ballooning boundaries that are more relevant to experiments
  - access to second stability
- High-n eikonal approach converts the PDE's of MHD to a set of trivially parallel problems
  - COBRA code for local eigenvalues
  - Raytracing: ODE integrations

Alfvén Gap structure in 3D systems - STELLGAP code

"Shear Alfvén Continua in Stellarators," D. A. Spong, R. Sanchez, A. Weller, Phys. Plasmas **10**, 3217 (2003).

- MHD spectroscopy (iota profile, . . .)
- AE's limit operational range, enhance fast ion losses
  - quote from Fritz Wagner regarding Alfvén instabilities: "If you don't find them, they'll find you."
- In larger devices ➡ wall damage concerns

$$\mu_0 \rho \omega^2 \frac{|\nabla \psi|^2}{B^2} E_\psi + \vec{B} \cdot \vec{\nabla} \left\{ \frac{|\nabla \psi|^2}{B^2} (\vec{B} \cdot \vec{\nabla}) E_\psi \right\} = 0$$



Stellarator Alfvén Couplings

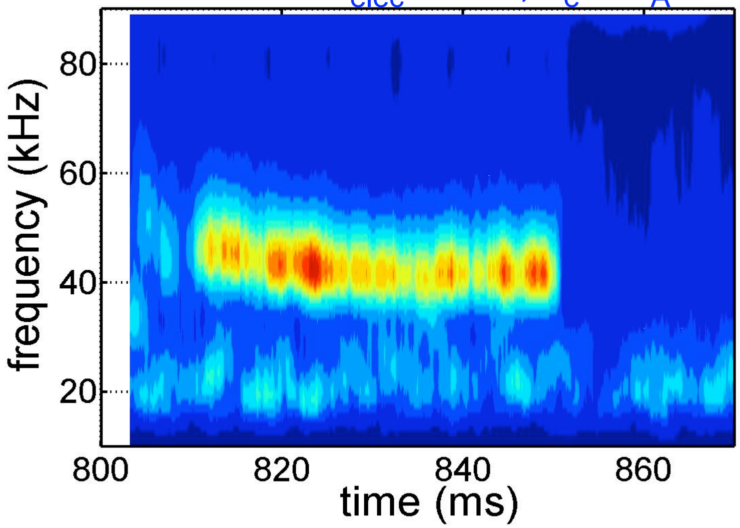
$$k_{\|,m,n} = -k_{\|,(m+\Delta),(n+\alpha N_{fp})} \qquad \Delta,\ \alpha = \text{integers}$$

$$\iota = \frac{2n + \alpha N_{fp}}{2m + \Delta} \qquad \omega = \frac{v_A}{R} \frac{\Delta n - \alpha m N_{fp}}{2m + \Delta}$$

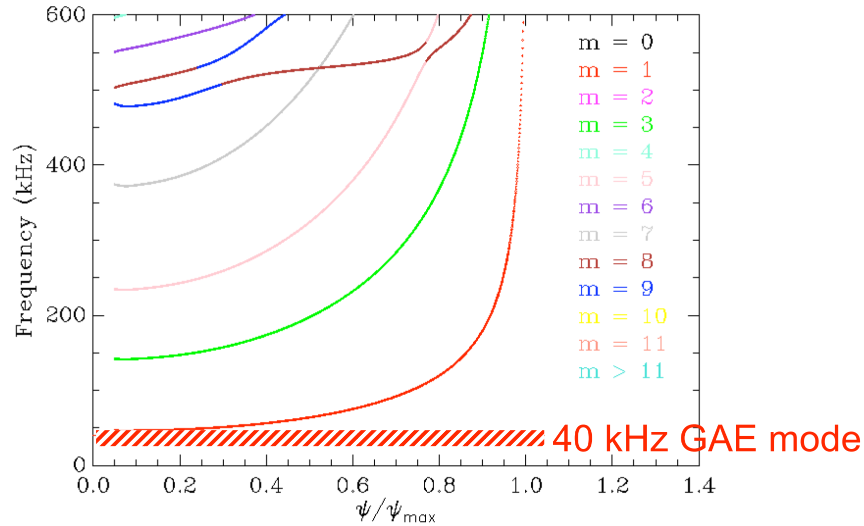
- GAE (global Alfvén mode): $\alpha=0, \Delta=0$
- TAE (toroidal Alfvén mode): $\alpha=0, \Delta=\pm 1$
- EAE (elliptical Alfvén mode): $\alpha=0, \Delta=\pm 2$
- NAE (noncircular Alfvén mode): $\alpha=0, |\Delta|>2$
- MAE (mirror Alfvén mode): $\alpha=1, \Delta=0$
- HAE (helical Alfvén mode): $\alpha=1, \Delta\neq 0$

Two recent observations of Alfvén instabilities: HSX and TJ-II

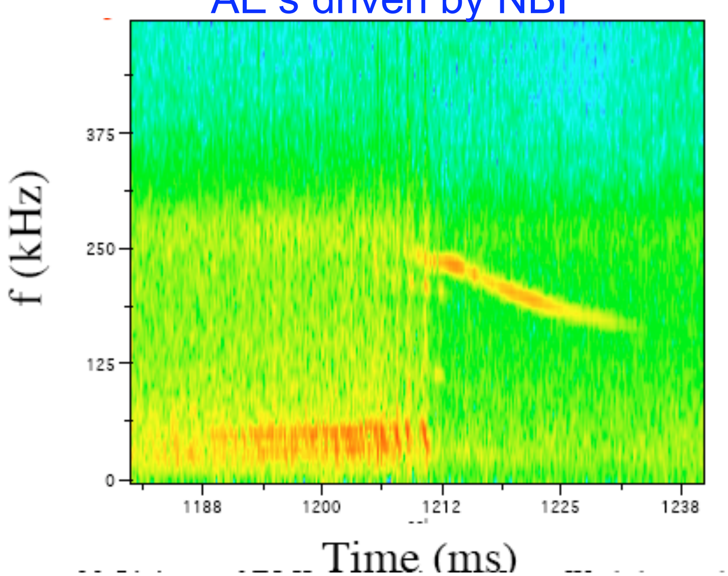
C. Deng, D. Brower, HSX:
AE's are driven by ECH electron tail with $\omega_{*elec}/\omega > 1$, $v_e \sim v_A$

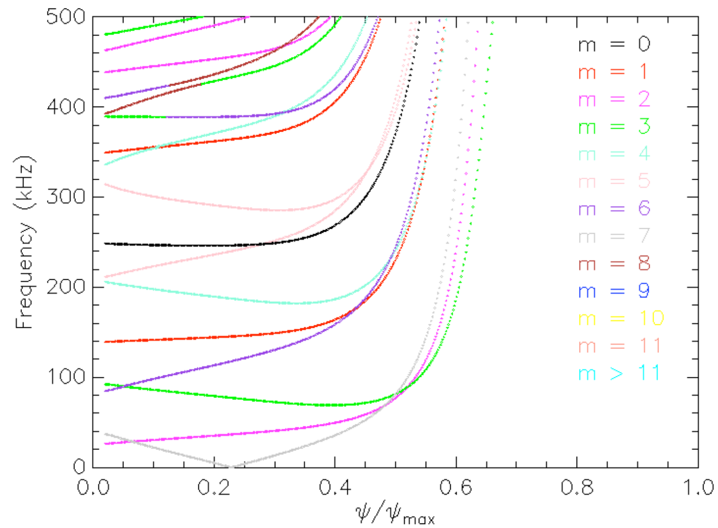


STELLGAP continua



I. Garcia-Cortes, T. Estrada TJ-II:
AE's driven by NBI

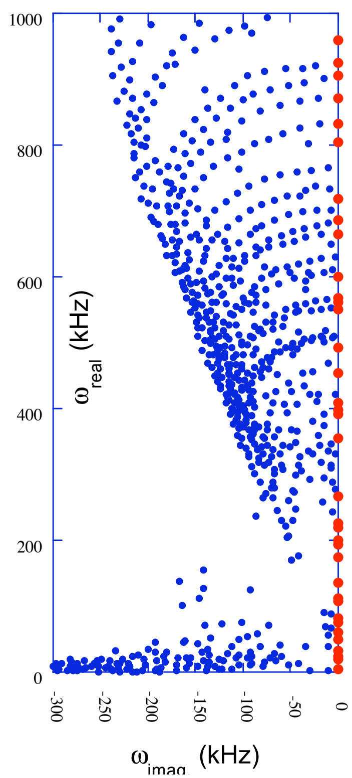




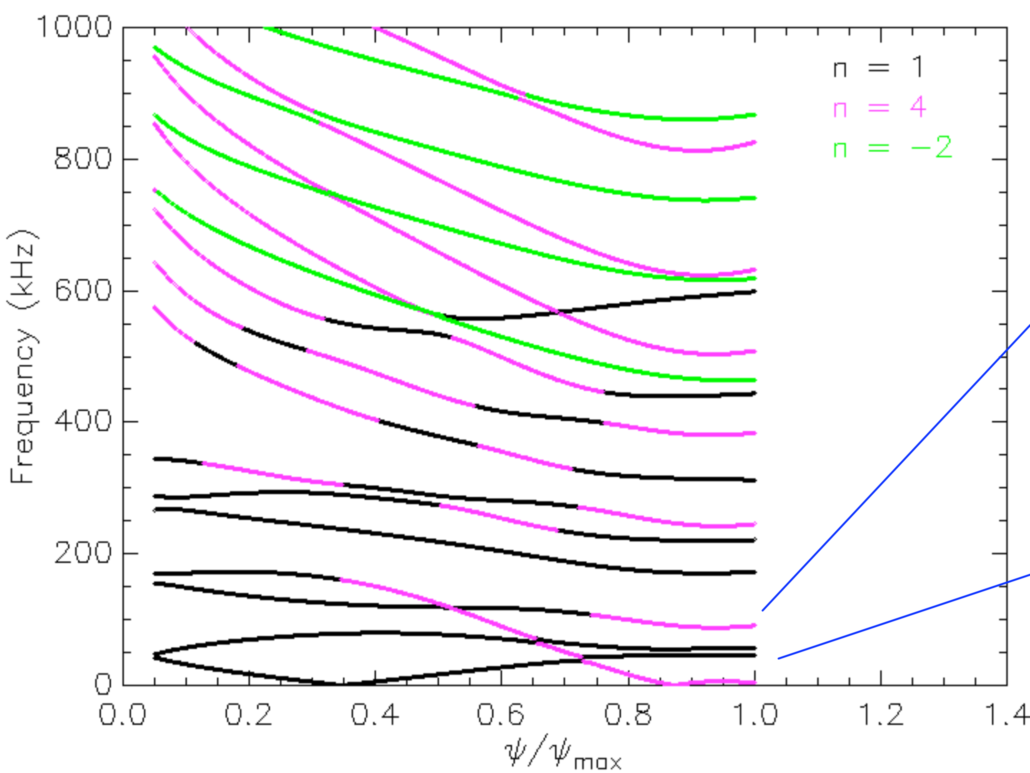
Alfvén Instabilities in 3D configurations

- MHD spectroscopy/antenna excitation
  - mode structure calculation
- Linear stability model (damping effects)
- Nonlinear models

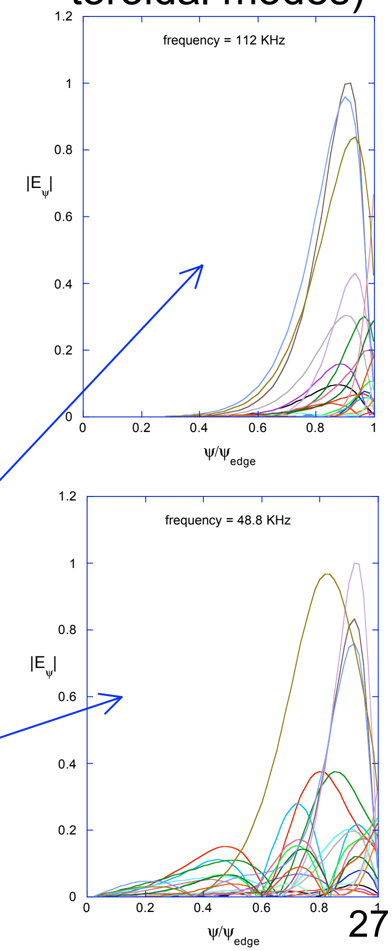
Discrete 3D Alfvén frequency spectrum (blue = damped red = neutrally stable)



Stellarator Alfvén continuum (color indicates dominant toroidal mode number)



AE radial mode Structure (multiple toroidal modes)

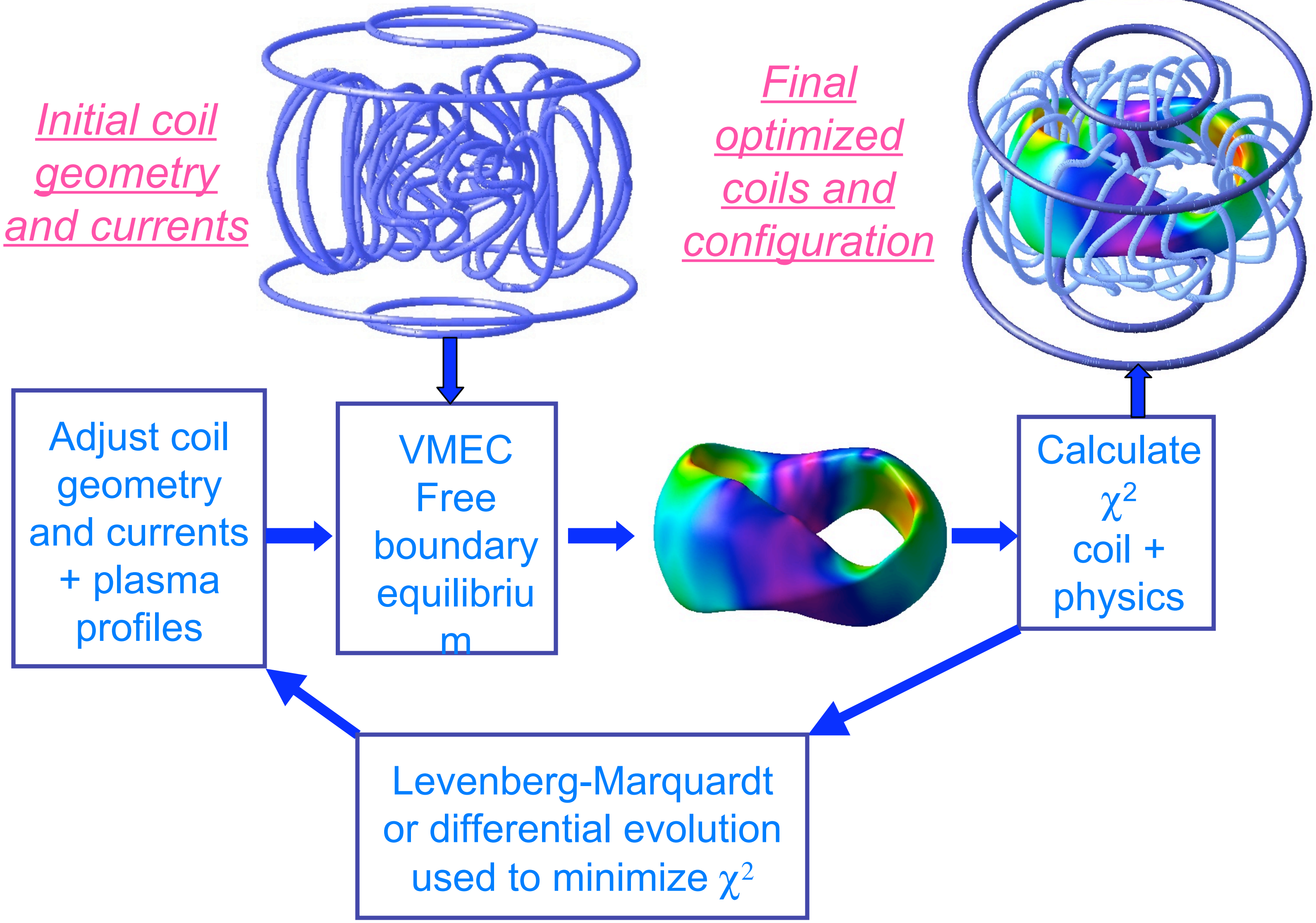


Stellarator research topics

- Equilibrium
- Transport
- Stability
- **Optimization, configuration development**
  - **Merged coil-plasma STELLOPT code**
  - **Flexibility studies**
  - **Island suppression**
- RF Heating

Merged coil/plasma stellarator optimization

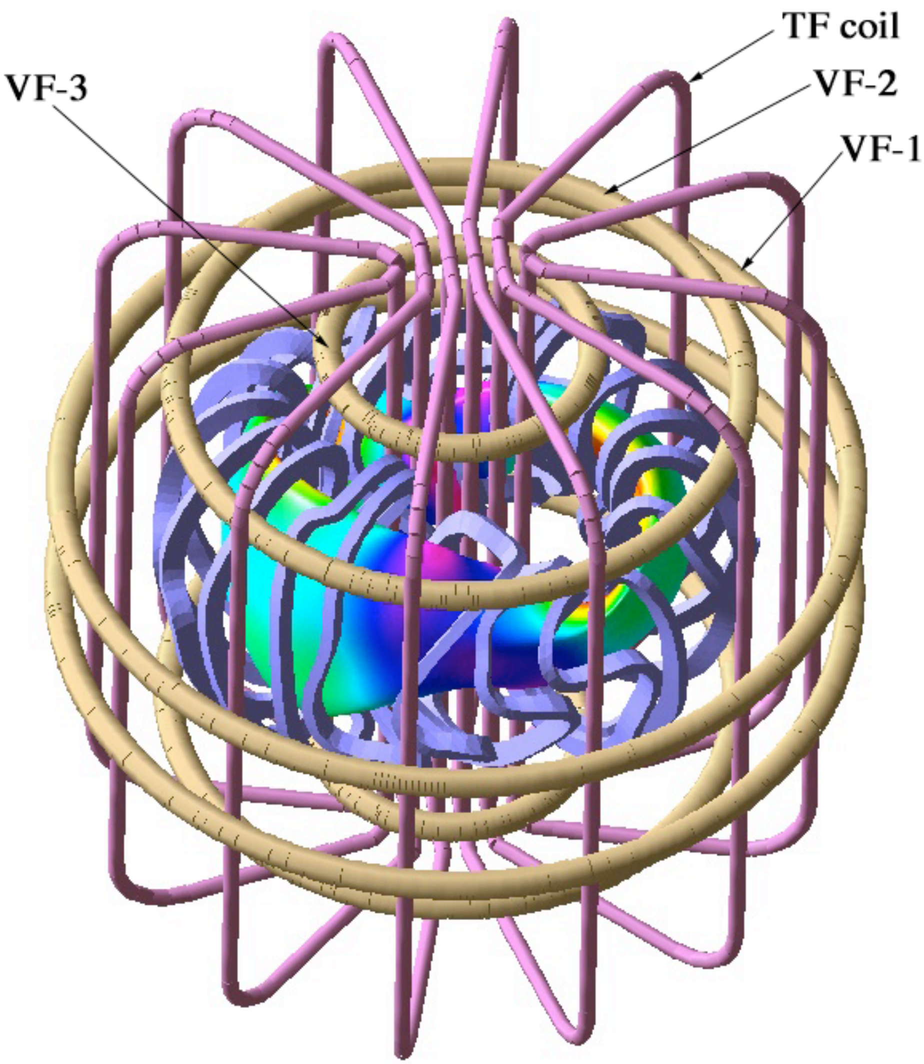
"Development of a Robust Quasi-Poloidal Compact Stellarator," D. J. Strickler, S. P. Hirshman, D. A. Spong, M. J. Cole, J. F. Lyon, B. E. Nelson, D. E. Williamson, A. S. Ware, Fusion Science & Technology, 45(1), 15 (2004); "Designing Coils for Compact Stellarators," Fusion Science & Technology 41(2), 107 (2002).



Coil geometry is typically characterized by several hundred parameters

QPS offers substantial flexibility through 9 independently variable coil currents

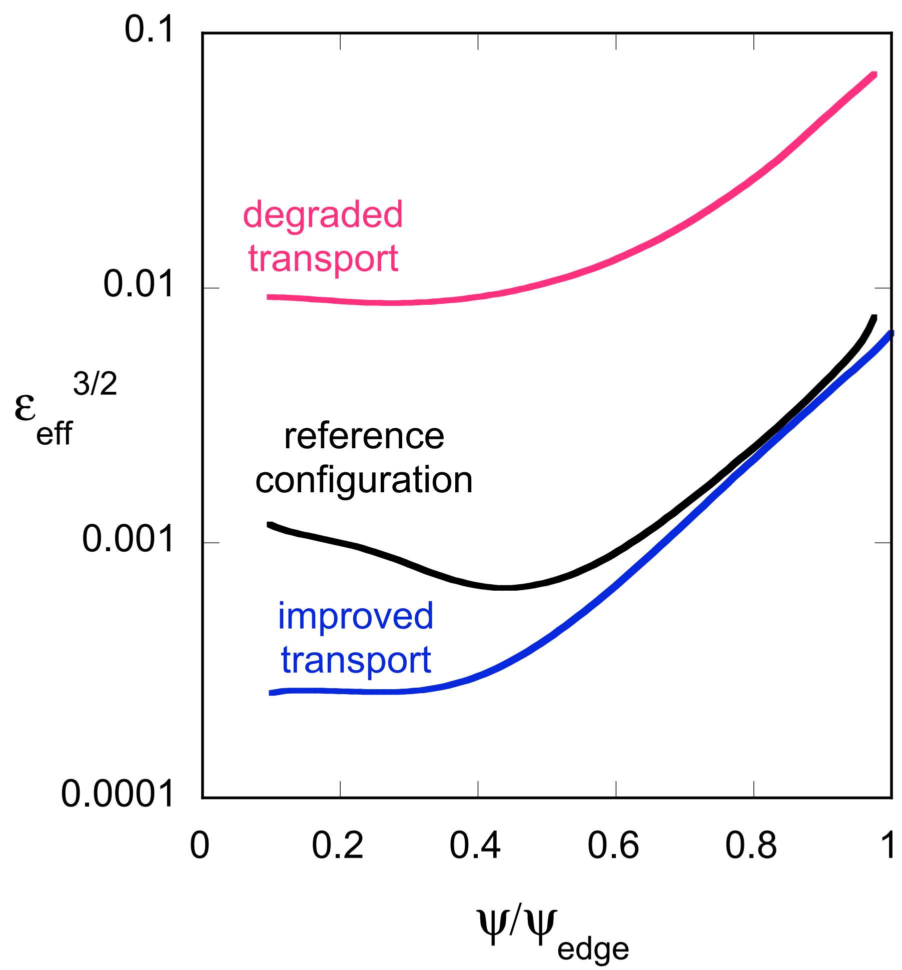
"QPS Transport Physics Flexibility using Variable Coil Currents," D. A. Spong, D.J. Strickler, S.P. Hirshman, J.F. Lyon, L.A. Berry, D. Mikkelsen[1], D. Monticello[1], A. S. Ware, accepted for publication in Fusion Science and Technology (2004).



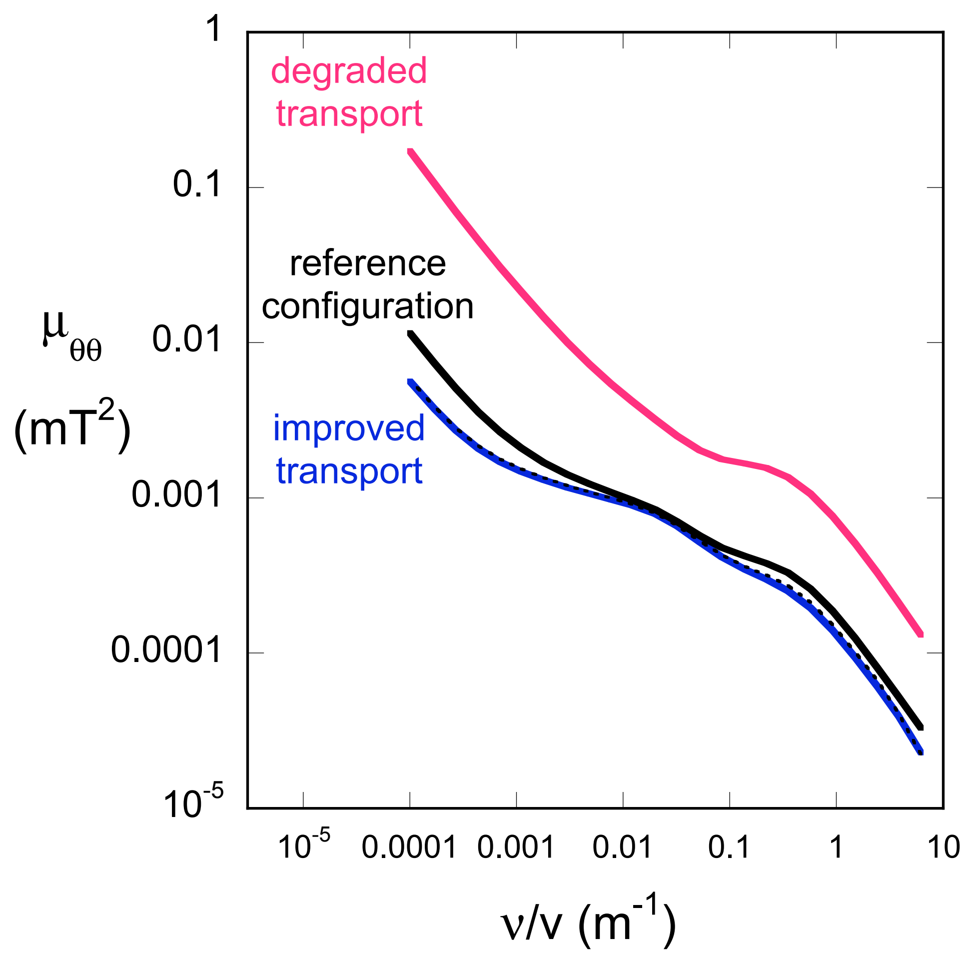
- Flexibility is a significant advantage offered by stellarator experiments
- Flexibility will aid scientific understanding in:
  - Flux surface fragility/island avoidance
  - Neoclassical vs. anomalous transport
  - Transport barrier formation
  - Plasma flow dynamics
  - MHD stability
- QPS offers flexibility through:
  - 5 individually powered modular coil groups
  - 3 vertical field coil
  - toroidal field coil set
  - Ohmic solenoid
    - Variable ratios of Ohmic/bootstrap current
- The STELLOPT optimization code is an effective tool for identifying physics flexibility opportunities

QPS can vary low collisionality transport levels by a factor of ~25 and poloidal viscosity by a factor of 5-10

Low collsionality transport



Viscosity

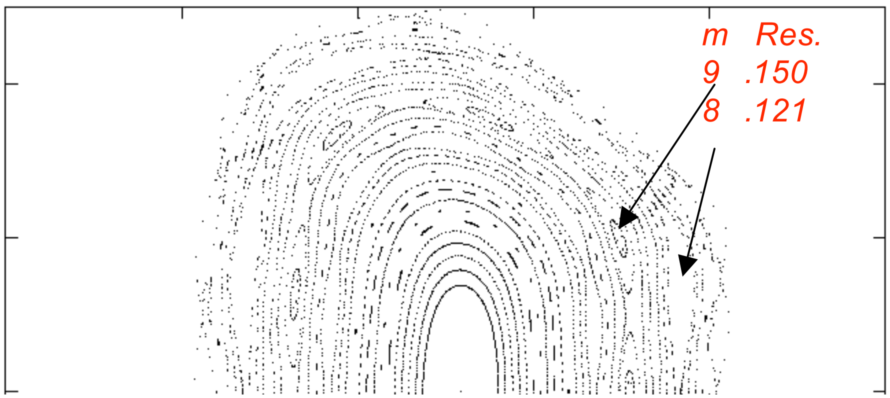


Islands have been suppressed at low A using two strategies:

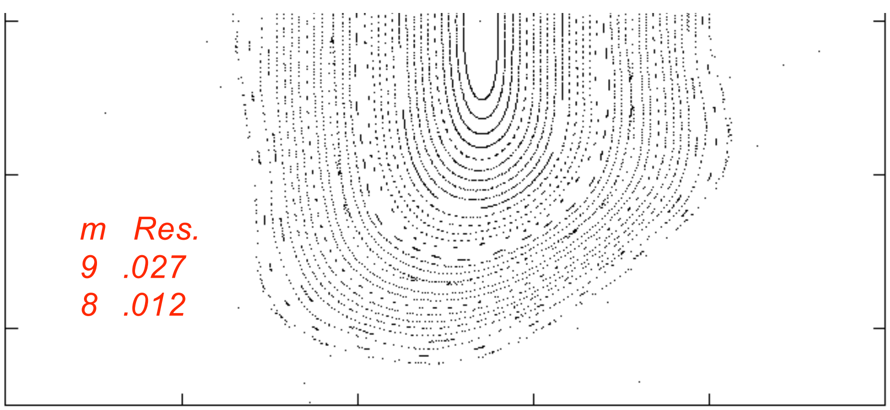
Direct targeting of residues

method of Cary and Hanson
(Phys. Fluids **29**, 1986)

QPS vacuum field - reference coil currents

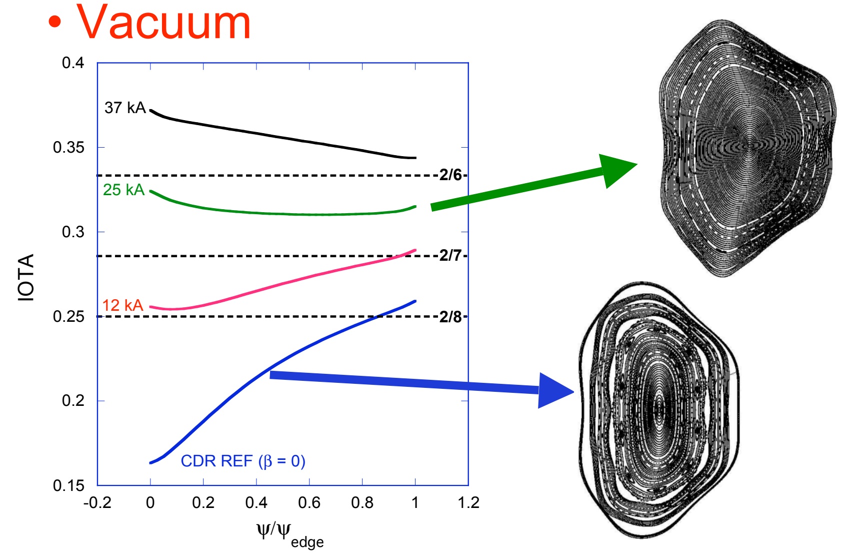


Final state - minimum residues

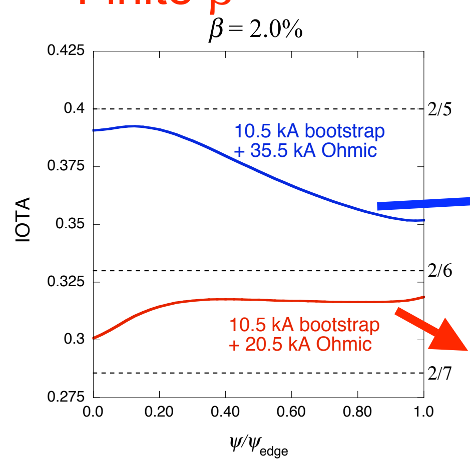


Targeting of iota profile to avoid major resonances

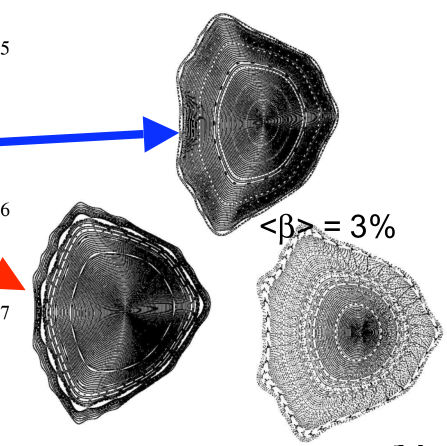
• Vacuum



• Finite β



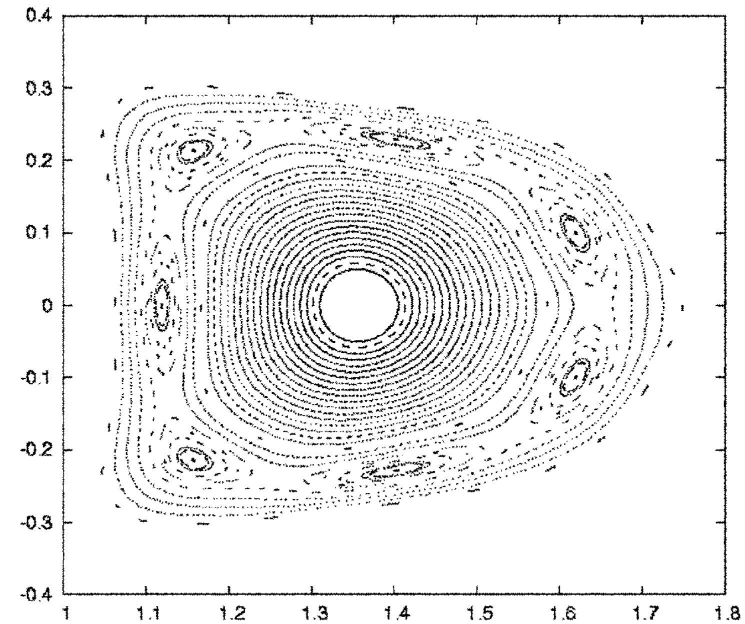
results from PIES code - D. Monticello



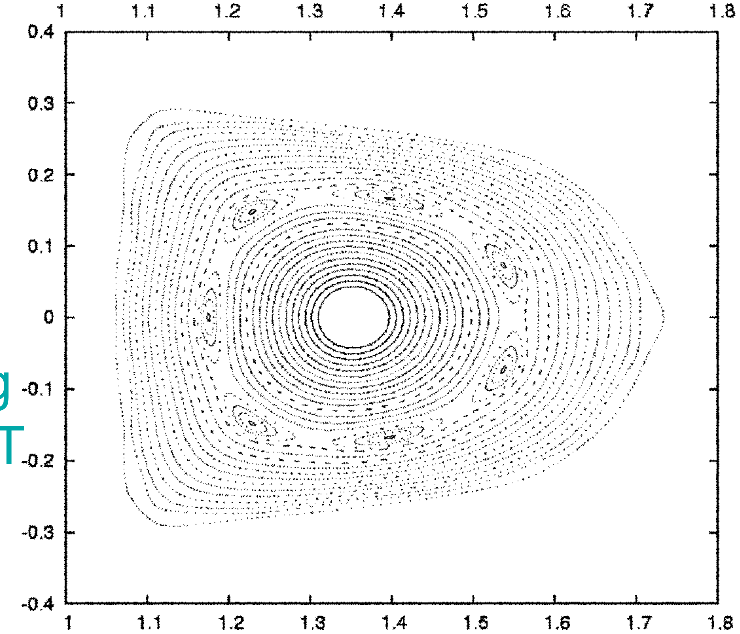
A vacuum field optimization module (VACOPT) is being applied to NCSX error field correction

- Magnetic islands can be produced by:
  - Coil fabrication errors
  - Coil deformations during operation (thermal expansion, magnet forces, etc.)
- Using residue targeting and performing changes in coil rotation and translation, the fabrication errors can be corrected.
- Using coil current variation, the second type of error can also be corrected.

Surfaces with coil fabrication errors

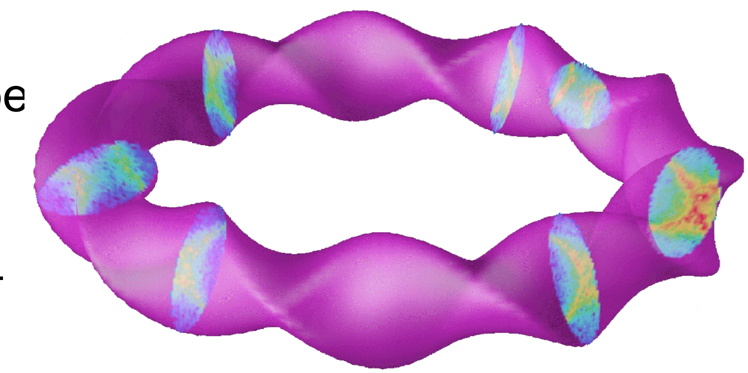


Surfaces after coil repositioning with VACOPT

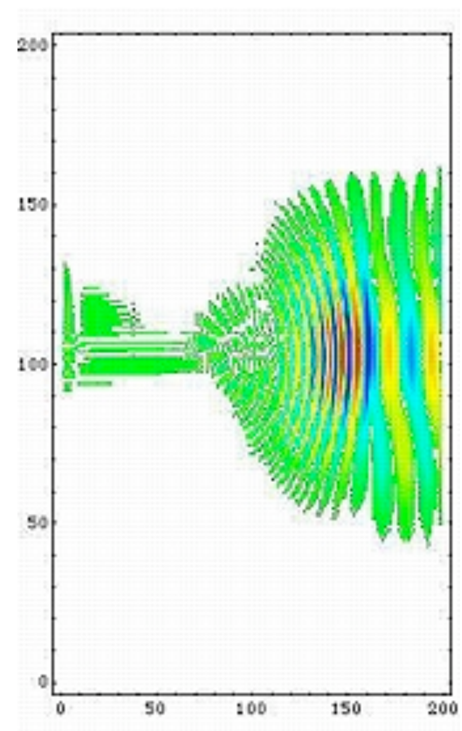


RF Heating Theory for 3D systems

E. F. Jaeger, L. A. Berry, D. Batchelor - RF SciDAC)



- Wave Propagation and RF
  - – 3D full wave RF calculations have been develope using the AORSA code for stellarator geometry
  - – Computationally intensive for 3D solutions
  - – Physics of non-local wave-particle interactions – conductivity operator



- RF Heating - future topics
  - –More comprehensive wave-plasma interaction calculations
  - –geometric optics and full wave RF heating calculations for QPS and NCSX
  - –Develop strategies for controlling current drive and flow shear in addition to heating
  - –Develop better models of plasma response to RF absorption (with radial drifts/3D effects)