EQUILIBRIUM AND STABILITY IN STELLARATORS: THOUGHTS FROM A TOKAMAK PERSPECTIVE

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But the outrageous and just plain wrong statements are entirely my own!

MHD STABILITY CRITERIA APPEAR TO BE VIOLATED IN STELLARATOR EXPERIMENTS

- **Stellarator experiments have substantially exceeded the stability limits predicted from local Mercier and ballooning code calculations:**
  - **LHD and W7-AS have significantly exceeded the predicted β limits**
- **Global calculations are closer to experimental stability limits:**
  - **But even these appear to be violated in recent W7-AS experiments**
- **This superficially appears to be a quite different situation from the standard paradigm in tokamaks:**
  - **Ideal MHD appears to predict not just tokamak stability limits but also growth rates and mode structures in many situations**
- **How can the two situations be resolved?**

> ⇒ **Stellarators and tokamaks do have the same underlying physics based on Maxwell's Equations and Newtonian mechanics!**

AT SECOND GLANCE THE TWO SITUATIONS ARE NOT ALL THAT DIFFERENT

- **Tokamaks also routinely violate some MHD stability limits:**
  - **MHD limits are open to interpretation and cannot be applied blindly as absolute hard limits**
  - **MHD limits can be sensitive to details in the equilibrium**
- **There are also some important distinctions between tokamaks and stellarators that may produce superficially different behavior**
  - **MHD theory, as applied to both, assumes the existence of nested flux surfaces:**
    - ⇒ **In tokamaks this is sometimes not the case but normally it is an accurate assumption**
    - ⇒ **In stellarators this is not always the case:**

Surfaces may not exist !

They may exist but be non-nested !

  - **We already know this to be partly true! But:**
    - ⇒ **Given the sensitivity of the stability to the equilibrium the assumption of nested flux surfaces might be a poor approximation for stability even if islands are small**

TOKAMAKS ALSO ROUTINELY VIOLATE SOME MHD STABILITY LIMITS

- **The most well known example is the internal kink instability:**
  - **Tokamaks routinely operate with $q < 1$**
  - **The sawtooth instability is a consequence of the internal kink but is not at all well described by it**
    - ⇒ **Non-ideal effects are important for low growth rate modes**
    - ⇒ **Nonlinear consequences are usually benign**

- **Tokamaks also routinely violate Mercier interchange stability limits:**
  - **The Mercier limit is normally close to the internal kink limit but appears to be largely irrelevant in tokamaks**

- **Ballooning modes can have consequences in tokamaks near 'the β limit':**
  - **Interchange modes are in principle a special case of ballooning**
    - ⇒ **But the consequences of reaching the ballooning limit are not always devastating**

- **In H-mode Tokamaks also routinely reach intermediate n external mode stability limits:**
  - **ELMs appear to be the result of these instabilities**
    - ⇒ **Nonlinear consequences are generally benign**

IT IS NOT EVEN CLEAR THAT LOCAL MHD STABILITY CRITERIA SHOULD BE RELEVANT FOR STELLARATORS

- **Localized modes predicted to be unstable for β well below the global MHD limits should be stabilized by kinetic effects:**
  - **Finite n corrections are needed for physically meaningful predictions**
  - **In tokamaks, finite toroidal mode number n corrections to ballooning and Mercier stability are generally small**
    - ⇒ **The infinite n calculation accurately reflects the real limit**
  - **In stellarators, the global stability codes in principle incorporate the high n localized modes with low and intermediate n**
    - ⇒ **In practice the high n modes are numerically excluded**
- **In tokamaks high and low n are uncoupled and evaluated separately:**
  - **In Stellarators, they are coupled in principle and this is not accounted for in the localized criteria**
- **It is more realistic to ignore localized Mercier and ballooning limits in Stellarators and just use low and intermediate n global calculations:**
  - **By excluding the high n modes that in practice are stabilized by finite orbit effects the global codes are more closely reflecting the physics**
    - ⇒ **In the global calculations the range of n needs to be terminated at the limit where finite orbit effects become important**

TOKAMAK STABILITY LIMITS DEPEND SENSITIVELY ON THE EQUILIBRIUM

- **It is not normally sufficient to fit the equilibrium to just the global characteristics of the discharge:**
  - **Stability depends quite sensitively on the details of both the current density (or safety factor) and pressure profiles**
    - ⇒ **One can obtain widely varying results depending on the form assumed for the profiles for similar global parameters**
    - ⇒ **Profiles need to be measured accurately and used in reconstructing the equilibrium for the stability calculations**

- **In Stellarators the equilibrium is believed to be known largely from the external coils: But**
  - **The ι profile is often taken from the vacuum profile**
    - ⇒ **It is not normally measured in the discharge and may be different at finite β**
  - **The pressure profile is not known as a function of flux**
    - ⇒ **At most it is measured as a function of space and the mapping to flux space needed for the equilibrium depends on the ι profile**

THE ASSUMPTION OF NESTED FLUX SURFACES MAY NOT BE REASONABLE FOR ESTIMATING LINEAR STABILITY

- **The assumption of nested flux surfaces may be invalid:**
  - **At least it may be an insufficiently good approximation to yield the observed stability**
  - **Finite β can deteriorate the nested vacuum surfaces and given the sensitivity of the stability to the equilibrium configuration**
    - ⇒ **Stability predictions using nested surfaces could be meaningless at finite β**

- **The islands and stochastic regions may be small but they may be ubiquitous throughout significant regions of the cross section:**
  - **Local flattening of the profiles and non-nested topology may yield very different stability from the 'nearby' nested configuration**
    - ⇒ **The nested configuration may be linearly unstable but evolve nonlinearly to a configuration with 'braided' surfaces or thin islands, with flattened profiles in these regions**
  - **The new configuration will be linearly stable**
    - ⇒ **The linear stability calculation using the approximate nearby nested flux surface equilibrium will yield the wrong result!**

EQUILIBRIUM STABILITY AND TRANSPORT ARE NOT SEPARABLE IN STELLARATORS

- Existence of a nested flux surface equilibrium can be considered as either an equilibrium or a stability problem:
  - Unstable equilibria with nested surfaces will evolve to a nearby state with non-nested surfaces with lower energy if it is physically possible
  - Transport is strongly dependent on underlying equilibrium magnetic topology and in turn determines the possible equilibrium profiles

- Equilibrium codes can be considered stability codes:
  - An equilibrium computed under certain constraints must be stable unless those constraints can be avoided by a physically valid motion:
  - Otherwise any iterations for force balance in which an iterative error mimics an allowed perturbation will evolve away from the equilibrium unless constrained to not do so
  - A variational code will find the energy minimizing state unless constrained to not do so

VMEC, PIES, AND HINST EQUILIBRIUM CODES CAN GUARANTEE VARYING DEGREES OF STABILITY

- **VMEC imposes simply nested flux surfaces:**
  - **Profiles assumed for p(ψ) and a function specifying current density j**
    - ⇒ **Equilibria should be stable to all topology preserving and profile preserving (i.e. fixed p(ψ) and j) MHD instabilities**

- **PIES and HINST have few constraints on the equilibrium:**
  - **Profiles assumed for p(ψ) and a function specifying the current density j (an integration constant on each flux contour for PIES)**
    - ⇒ **Equilibria should be stable to all profile preserving (i.e. fixed p(ψ) and j) MHD instabilities**

GUARANTEE OF STABILITY IS SUBJECT TO IMPORTANT CAVEATS

- **Claim is that convergence to physically unstable equilibria is not possible unless constraints are imposed on the numerical procedure that prevent either:**
  - **Symmetry breaking perturbations away from force balance or**
  - **Equilibrium states without specific symmetries**

$\Rightarrow$ Lack of convergence does not imply lack of stable equilibrium

- **PIES and HINST assume p = constant for flux surfaces inside islands:**
  - **Pressure is a different function of flux in separate simply connected regions**
    - **$\Rightarrow$ p is not a *single valued* function of ψ**
  - **States with different prescriptions for the multiple values for p and j in different simply connected regions (islands etc.) are possible and may be physically accessible**
    - **$\Rightarrow$ The actual profiles will be determined by transport and the topology of the region**

MAJOR QUESTION: SHOULD WE IGNORE MHD STABILITY PREDICTIONS BASED ON NESTED FLUX SURFACE EQUILIBRIA

- **Local stability criteria probably should be ignored:**
  - **There is little reason that infinite n should provide a physical limit**
  - **Finite n corrections appear to be large given the difference between the global code limits and the infinite n localized limits**
- **Global MHD stability is probably valid but must be applied to the right equilibrium:**
  - **Need to use the measured equilibrium profiles**
  - **May need to construct a non-nested flux surface equilibrium (with islands)**
  - **Flux surfaces might not even exist**
- **The nonlinear consequences are crucial in interpreting the results of a stability calculation:**
  - **Generally it might be expected that internal modes surrounded by a fairly robust and stable outer shell might be benign**
  - **Is there a way to quantify this without doing the full nonlinear calculation?**

FINAL QUESTION: WHAT SHOULD WE DO? HOW SHOULD WE PROCEED?

- **Is there a role for equilibrium and stability codes based on nested flux surfaces?**
  - **Under what conditions is nested surfaces a valid approximation for stability calculations?**
  - **Does linear instability of a nested flux surface equilibrium simply result in benign nonlinear evolution to a 'nearby' non-nested state?**

- **Can we formulate the stability problem in terms of finding nonlinearly stable equilibria:**
  - **Is it possible to develop a general equilibrium code with few imposed constraints that can guarantee stability**
  - **How can one distinguish a failure of the numerical scheme to converge from nonexistence of a stable nearby equilibrium?**