STATUS OF THE 3-D EQUILIBRIUM RECONSTRUCTION PROJECT V3FIT

L.L. Lao for the V3FIT Team*

General Atomics, San Diego, CA, U.S.A.

National Stellarator Teleconference
April 10, 2003

***V3FIT Team:** *S.P. Hirshman and E.A. Lazarus Oak Ridge National Laboratory, L.L. Lao General Atomics, J.D. Hanson and S.F. Knowlton Auburn University*

OAK RIDGE NATIONAL LABORATORY
UT-BATTELLE

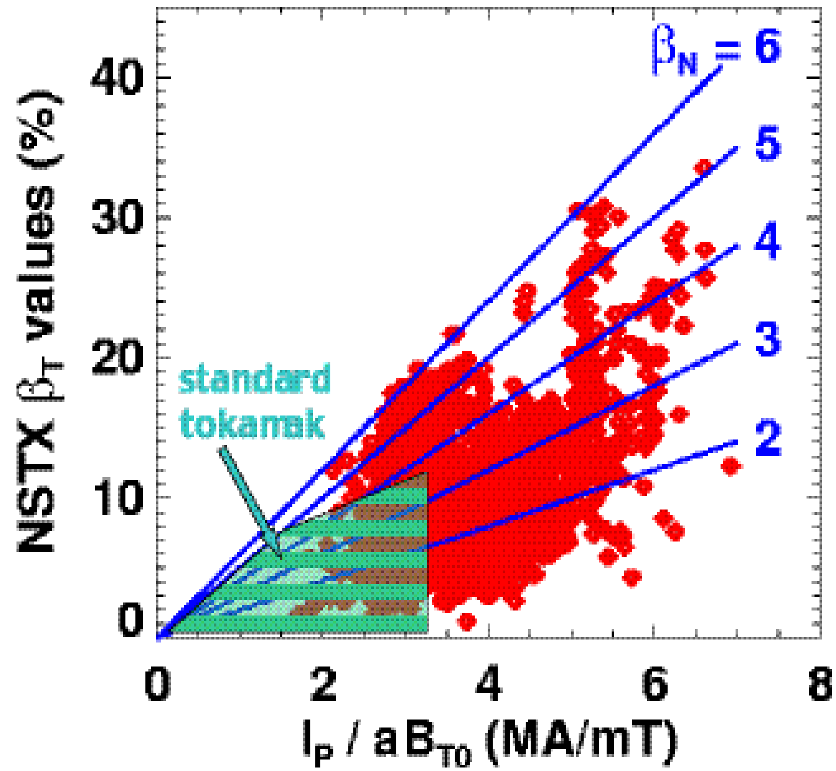
AUBURN UNIVERSITY

GENERAL ATOMICS

MOTIVATION

- Equilibrium reconstruction (ER) tools such as EFIT play a crucial role in the data analysis, control, and operation of tokamaks
  - — Routinely used to support experiments
- U.S. fusion program is expanding its efforts in 3-D toroidal systems (NCSX, CTH, QPS), but no similar ER capability exists
- Previous NCSX study shows that active control of the helical field is required to maintain attractive quasi-symmetry and stability properties as the plasma pressure and current depart from vacuum
  - — Need ER tools to support design of magnetic diagnostics
- These tools are also useful for analysis of departures from axisymmetry in tokamaks

NSTX β Reconstructed Magnetically Using EFIT



Menard, IAEA 2002
Sabbagh, Nuc Fusion 2001

THE GOAL OF V3FIT IS TO BUILD A 3-D ER CODE TO SUPPORT STELLARATORS SIMILAR TO EFIT IN TOKAMAKS

- **Collaboration project supported by DoE, presently includes GA, ORNL, and Auburn University**
- **Based on the efficient EFIT response function formalism**
  - **Lengthy calculations of inductance separated from rapid calculations of magnetic signals**
- **Presently efforts are concentrated on the VMEC equilibrium solver**
- **Two-phase approach**
  - **Phase 1: Develop 3-D response function formalism relating plasma and external current sources to measurements**
    - **Tools developed also useful to design of magnetic diagnostics for new stellarator devices**
  - **Phase 2: Integrate the 3-D response functions with VMEC and development of regression algorithms to build V3FIT**

V3FIT STATUS

- A 3-D response function approach has been formulated using magnetic reciprocity
- Two new codes V3RFUN / V3POST have been developed to provide an efficient tool for evaluation of magnetic diagnostics in 3-D toroidal system
  - Improved and numerically efficient Biot-Savart expression based on analytically simplified magnetic vector potential of current line-segment [1]
  - Have been benchmarked against EFIT DIII-D plasmas
  - Efforts underway to compare against the Wendelstein code DIAGNO for W7-AS cases. First comparisons show good agreement
  - Both are being applied to design NCSX and CTH magnetic diagnostics
  - A collaboration is underway to test the tools on the NIFS CHS stellarator
- Iteration schemes to integrate the response functions and VMEC and regression algorithms are being formulated to build V3FIT

[1] Hanson, Hirshman Phys. Plasmas 9, 4410 (2002)

TWO KEY EFIT FEATURES ARE RESPONSE FUNCTION APPROACH AND INTERLEAVE OF EQUILIBRIUM / FITTING

- **Measurements directly related to the current sources through the Green/response functions *G* and the Picard's iteration scheme**

$$C_i^{m+1}(\bar{r}) = \sum_j G_{C_i}(\bar{r},\bar{r}_{ej}) I_{ej} + \int_V dR' dZ' G_{C_i}(\bar{r},\bar{r}') J_\varphi \left[R', \psi^m(\bar{r}') \right]$$

- **Equilibrium and fitting iterations are interleaved**
  - **Inverting Δ^* approximately conserving magnetic fluxes and fields**

$$\psi = \psi_{ext} + \psi_P$$

$$\Delta^* \psi_P^{m\theta} = -\mu_0 R J_\varphi(R, \psi^m, \alpha), \psi^{m+1} = (1 - \vartheta)\psi^m + \vartheta \psi^{m\vartheta}$$

$$\psi_{ext}(\bar{r}) = \sum_j G_\psi(\bar{r},\bar{r}_{ej}) I_{ej} \qquad \bar{\bar{R}}\bar{\alpha} = \bar{M}$$

- **Non-linear optimization efficiently solved by transforming into a sequence of linear optimization problems**

Lao, *et sl* Nuc. Fusion 25, 1611 (1985)

V3RFUN / V3POST ARE DEVELOPED TO PROVIDE AN EFFICIENT TOOL TO COMPUTE 3-D MAGNETIC RESPONSES

- **Both codes have been written and tested**
- **V3RFUN computes and tabulates magnetic response functions for a given magnetic diagnostic set *(lengthy calculations)***
  - **Magnetic probe and flux loop signal vectors $\bar{S}$ are related to the current vectors through the response matrixes $\bar{\bar{R}}^{S}_{EXT}$ and $\bar{\bar{R}}_{S}^{PLA}$ by magnetic reciprocity**

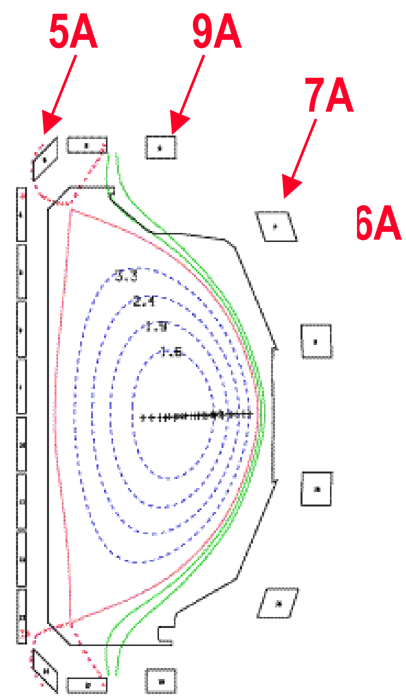
$$\bar{S} = \bar{\bar{R}}^{S}_{EXT}\,\bar{I}_{EXT} + \bar{\bar{R}}^{S}_{PLA}\,\bar{I}_{PLA} = \bar{\bar{R}}^{S}_{EXT}\,\bar{I}_{EXT} + \bar{\bar{R}}_{S}^{PLA}\,\bar{I}_{PLA}$$

  - **Response matrixes represented in cylindrical (R, ϕ, Z) coordinates**
- **V3POST evaluates magnetic responses for a given magnetic diagnostic set and for various plasma equilibria *(rapid calculations)***
  - **VMEC equilibrium**
  - **Plasma current distribution represented in VMEC (s, u, v) coordinates**
  - **Bilinear interpolation to map plasma response to VMEC coordinates**
- **This formulation allows V3FIT to be built by extending the efficient EFIT response function approach to 3-D**

V3RFUN / V3POST HAVE BEEN BENCHMARKED AGAINST EFIT DIII-D MAGNETIC RESPONSES

- DIII-D external coil 6A. Flux loops PSI6A and PSF7FA

	PSI6A (Web/rad)	*PSF7FA (Web/rad)*
Analytic	-0.3384	-0.1386
V3POST	-0.3384	-0.1386
EFIT	-0.3362	-0.1375
Measured	-0.3373	-0.1401



DIII-D

- DIII-D AT equilibrium, plasma contribution

PLASMA	*PSF5A (Wb)*	*PSI6A (Wb)*	*PSF9A (Wb)*	*MPI6FA322 (Tesla)*
EFIT	0.62708	3.1785	1.3081	0.12635
V3POST	0.62868	3.1786	1.3106	0.12439
Difference (%)	0.25449	0.00259	0.19151	1.5578

- Comparison against DIAGNO 3-D W7-AS configuration underway. Initial results show good agreement

ON-GOING V3FIT TASKS AND PLAN

- Testing a surface formulation (virtual casing principle) for plasma response in V3POST
- Benchmark V3RFUN / V3POST against Wendelstein codes DIAGNO and MFBE
- Applications of V3RFUN / V3POST for design of magnetic diagnostics in NCSX and CTH
- Applications of V3RFUN / V3POST to study DIII-D error magnetic fields
- CTH (NIFS) collaboration to test V3RFUN / V3POST
- Develop numerical schemes to integrate the 3-D response functions into the VMEC equilibrium iteration procedure and explore regression algorithms
- Build a 3-D filament reconstruction code to test and explore 3-D equilibrium reconstruction issues (Lao / Zarnstoff)
  - Plasma current distributions represented using filaments

CHALLENGES

- **VMEC is efficient but assumes nested flux surface. Effects of magnetic islands ? Magnetic stochasticity ?**
- **Direct equilibrium solver ?**
- **Need an efficient method to find magnetic surfaces if non-flux coordinates are used**