Ray tracing for ballooning modes in quasi-symmetric devices

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Overview

- Using COBRAVMEC to solve the ballooning equation
  - Range of parameters in ballooning space
- Ray equations
  - Integrating the ray equations
- Results for two configurations
  - QPS, NCSX, (still working on HSX)

Solving the ballooning eigenvalue equation

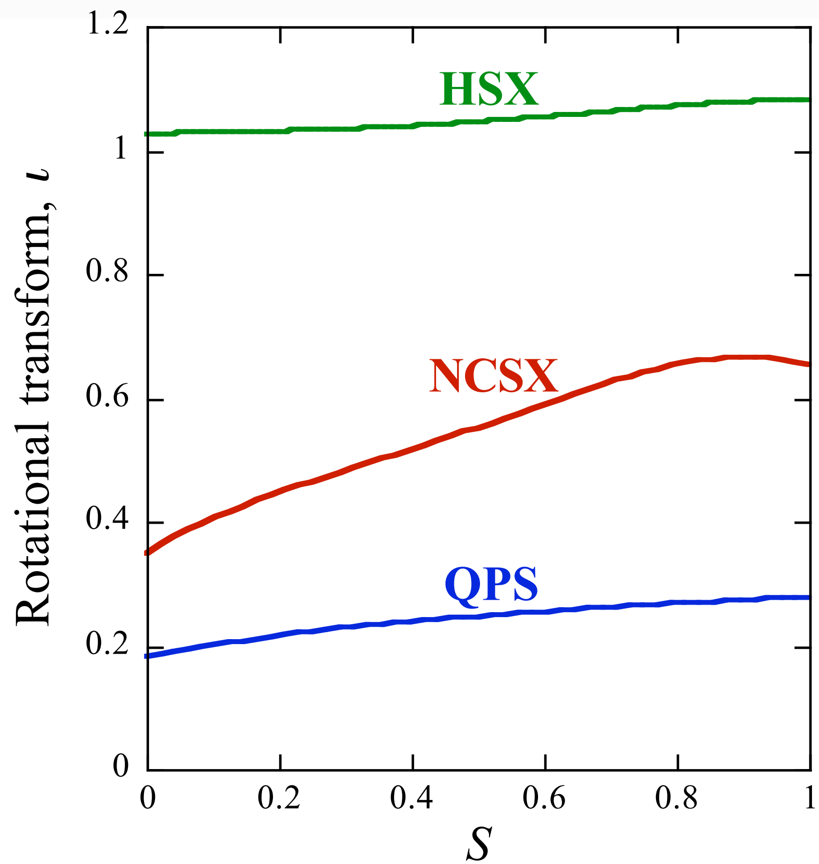
- We use COBRAVMEC to solve the ideal MHD ballooning equation
  - Given a VMEC equilibrium (wout file), COBRAVMEC obtains the MHD eigenvalue as a function of the normalized flux, S, the field line label, $\alpha = q\theta - \zeta$, and the ballooning parameter, $\theta_k = k_q/k_\alpha$.

$$\frac{\partial}{\partial\theta}\left[c_1(\theta \mid S,\theta_k,\alpha)\frac{\partial\hat{\xi}}{\partial\theta}\right] + c_2(\theta \mid S,\theta_k,\alpha)\hat{\xi} + \lambda c_3(\theta \mid S,\theta_k,\alpha)\hat{\xi} = 0$$

  - To obtain: $\lambda = \lambda(S,\alpha,\theta_k)$

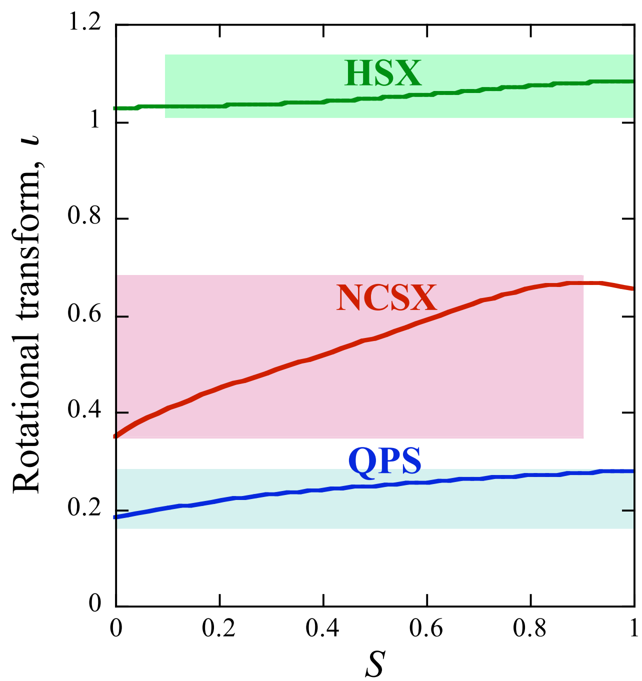
Issue concerning using the safety factor as the radial coordinate

- For ray tracing, it is useful to use the safety factor, *q*, as the radial coordinate
  - ➢ For monotonic *q*-profiles this is not an issue (QPS)
  - ➢ For profiles with shear reversals (NCSX and HSX) this proves difficult for interpolation

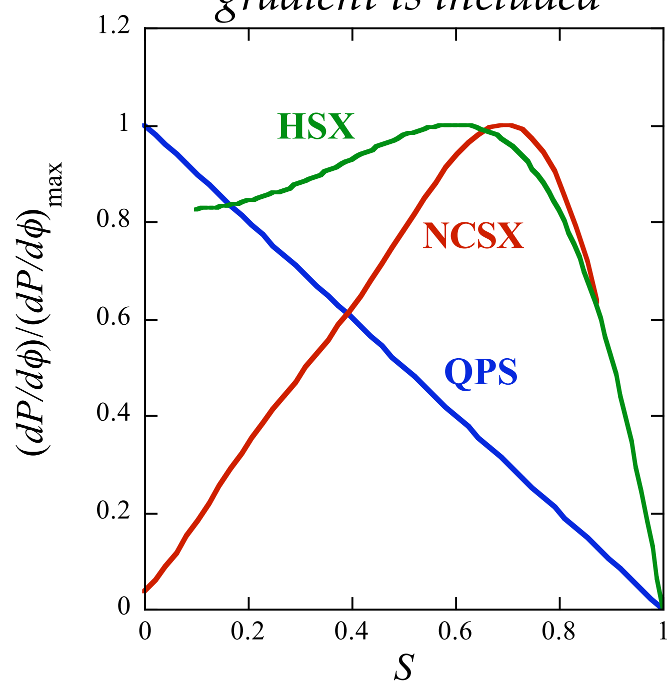


Issue concerning using the safety factor as the radial coordinate

- For now, we are using ray tracing only over the portion of flux surfaces with a monotonic *q*-profile



Region of max pressure gradient is included



The Ray Equations

- Dewar and Glassers ray tracing theory can be used to obtain information about the global eigenmodes

$$\lambda = \lambda(q, \alpha, \theta_k)$$

$$\dot{q} = \frac{\partial \lambda}{\partial k_q}, \quad \dot{\alpha} = \frac{\partial \lambda}{\partial k_\alpha}, \quad \dot{k}_q = -\frac{\partial \lambda}{\partial q}, \quad \dot{k}_\alpha = -\frac{\partial \lambda}{\partial \alpha}$$

- Using the values of λ on a 49x49x91 grid in (q, θ_k, α), we integrate the ray equations
  - Range for θ_k is 0 to π and for α is 0 to 2π

Results for three quasi-symmetric stellarators

- All of the unstable ballooning modes that we have examined for QPS, NCSX, and HSX, are localized in ballooning space
- We have also examined some rays for configurations for which the 3-dimensional shaping has been reduced

Transition from 2D to 3D:
The "equivalent" tokamak

- Let ν represent an artificial parameter used to transform a fixed boundary from axisymmetric to nonaxisymmetric

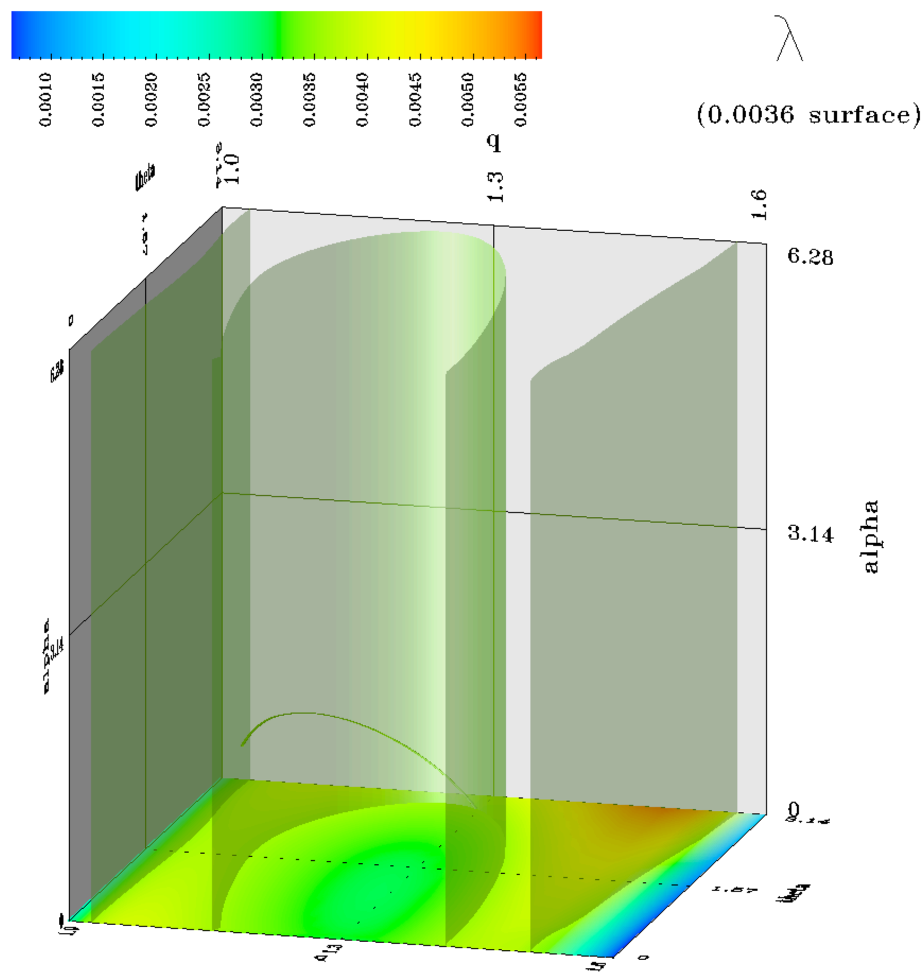
$$R(\theta,\zeta) = \sum_{m} R_{bc}(m,0)\cos(m\theta) + \nu \sum_{m,n\neq 0} R_{bc}(m,n)\cos(m\theta - n\zeta)$$

$$Z(\theta,\zeta) = \sum_{m} Z_{bs}(m,0)\sin(m\theta) + \nu \sum_{m,n\neq 0} Z_{bs}(m,n)\sin(m\theta - n\zeta)$$

  - $\nu = 0$ is the "equivalent" tokamak case
  - $\nu = 1$ is the QPS (or NCSX or HSX) case
- Use VMEC to calculate equilibria for $0 \le \nu \le 1$, keeping ι and $<|B|>$ fixed

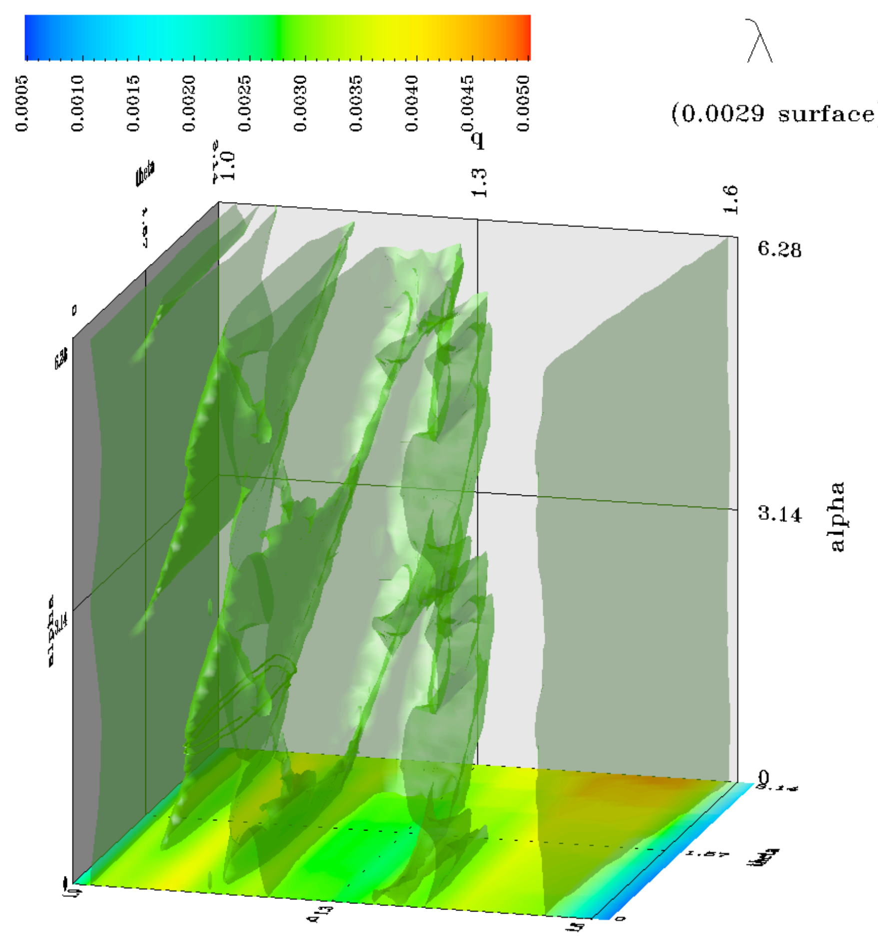
QPS at $\beta = 2.5\%$

- The equivalent tokamak case (i.e., ν=0)
  - All surfaces are stable
  - Surfaces of constant λ are independent of α
  - A constant λ surface and a ray on that surface are shown



QPS at $\beta = 2.5\%$

- Moderate shaping (ν=0.25)
  - All surfaces are still stable
  - Surfaces of constant λ are extended along (but not independent of) α
  - A constant λ surface and a ray on that surface are shown



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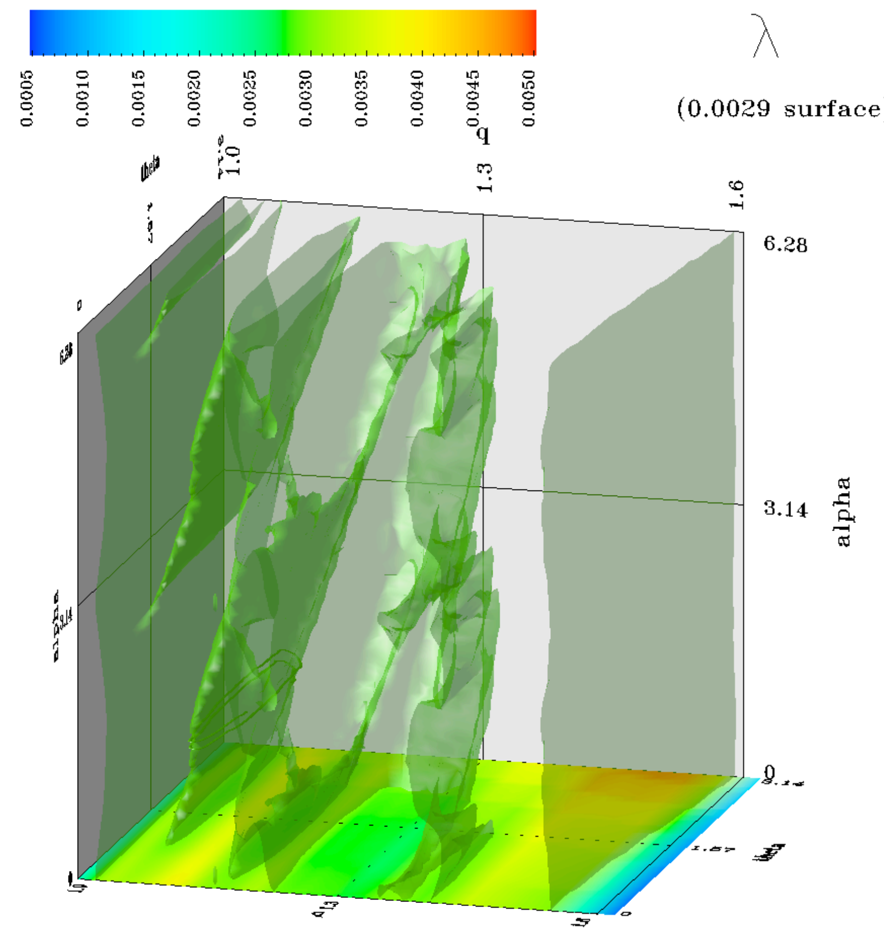
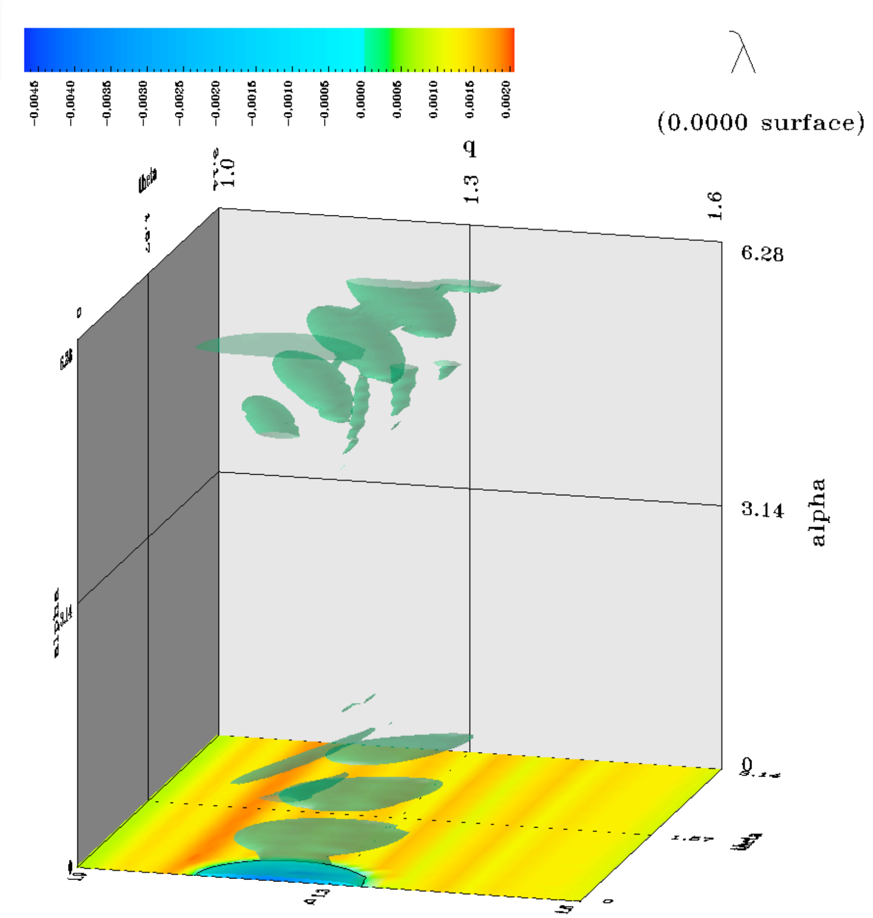


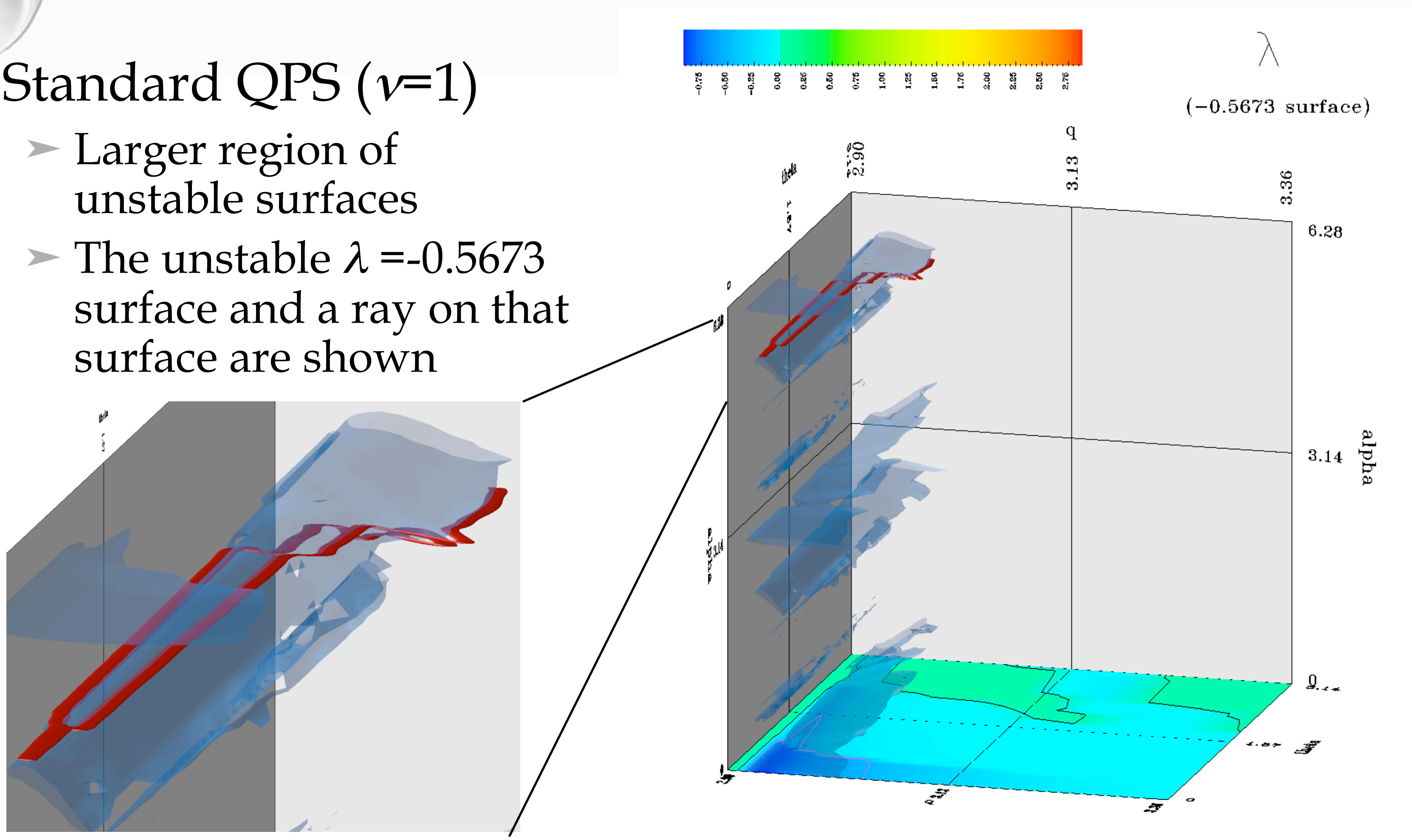
QPS at $\beta = 2.04\%$

- Standard QPS (ν=1)
  - Localized region of weakly unstable surfaces
  - Surfaces of constant λ are localized to a small range in α
  - The marginal stable λ surface is shown



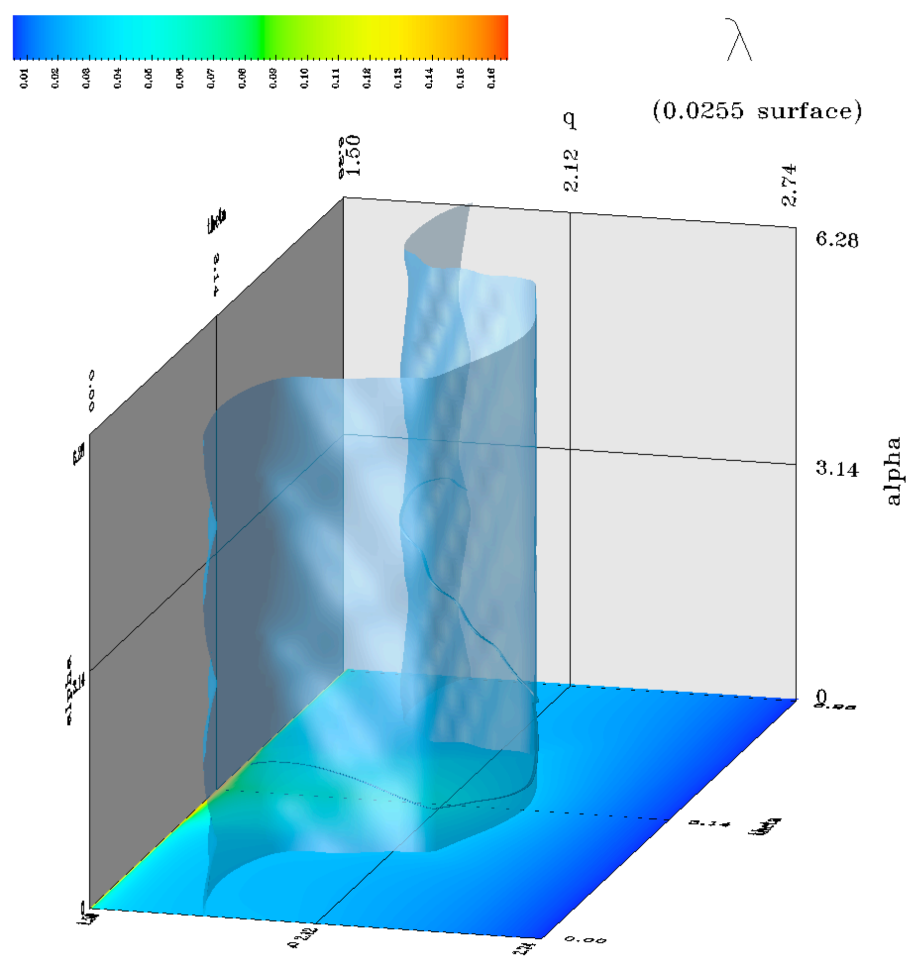
QPS at $\beta = 4.0\%$

- Standard QPS (ν=1)
  - Larger region of unstable surfaces
  - The unstable λ =-0.5673 surface and a ray on that surface are shown



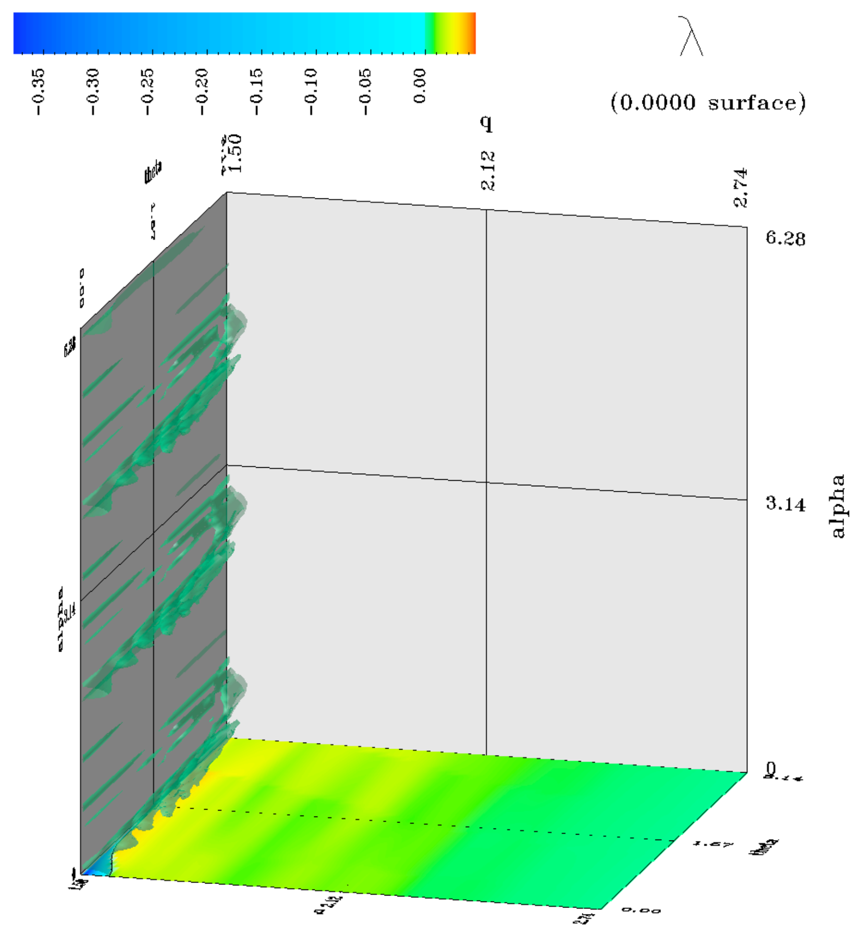
NCSX at $\beta = 4.88\%$

- Weakly shaped (ν=0.05)
  - All surfaces are stable
  - Surfaces of constant λ are still bumpy and extended in α
  - Range in θ_k is now 0 to 2π
  - *A* stable λ shown along with a ray on that surface



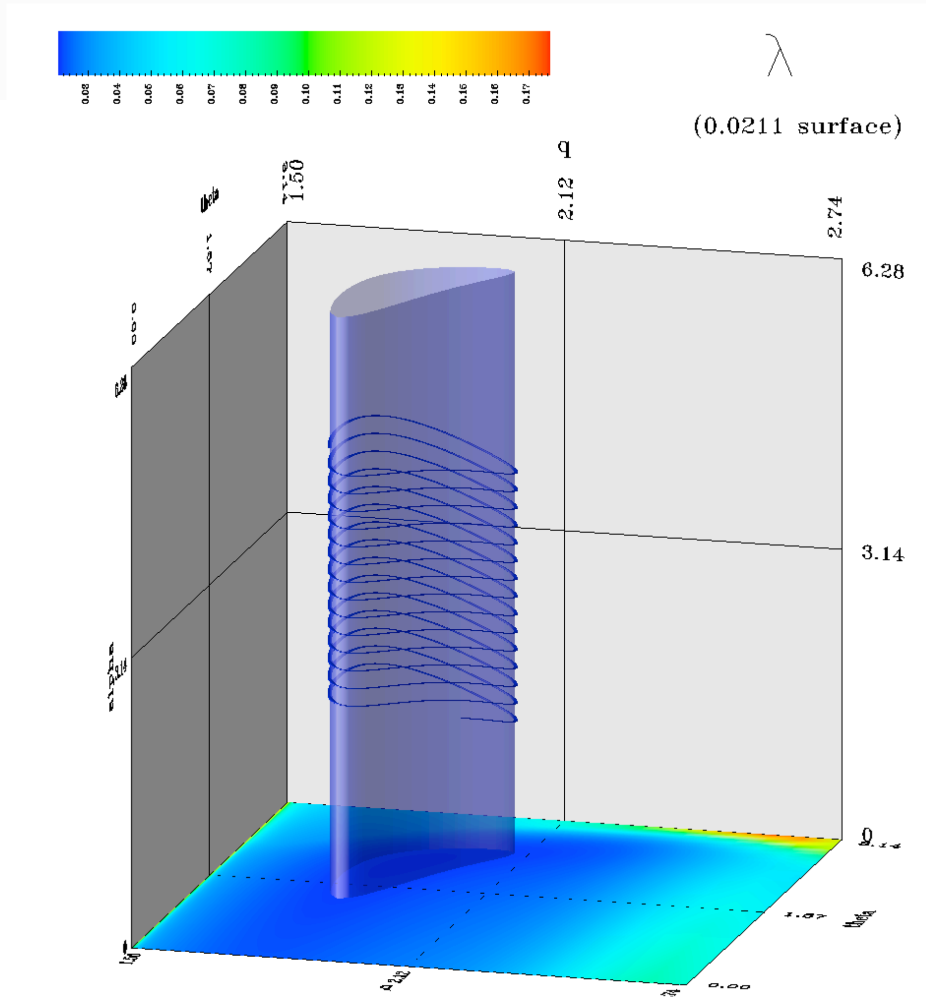
NCSX at $\beta = 4.88\%$

- Standard NCSX (ν=1)
  - Localized regions of instability
  - Surfaces of constant λ are localized in α
  - Range in θ_k is now 0 to 2π
  - The marginal stability surface is shown
  - Region of instability localized to the edge (lowest q values)



NCSX at $\beta = 5\%$ with a $p = p_0(1 - S)^2$ profile

- "Tokamak" case (ν=0)
  - No unstable regions
  - Standard cylindrical type rays



NCSX at $\beta = 5\%$ with a $p = p_0(1 - S)^2$ profile

- Standard NCSX (ν=1)
  - Larger unstable regions
  - Regions of instability localized in α