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FLEXIBILITY AND ROBUSTNESS CALCULATIONS FOR NCSX

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Abstract

In order to achieve the scientific goals of the NCSX mission, the NCSX device must be capable of supporting a wide range of variations in plasma configuration about a reference baseline equilibrium. We will demonstrate the flexibility of NCSX coils to support such configuration variations, and demonstrate the robustness of performance of NCSX plasmas about reference design values of the plasma current (I_p), beta (β), and profile shapes. The robustness and flexibility calculations make use of free-boundary plasma equilibrium reconstructions using a combination of non-axisymmetric modular coils, and axisymmetric toroidal and poloidal field coils. The primary computational tool for the studies is STELLOPT, a free-boundary optimization code which varies coil currents to target configurations with specific physics properties, such as good quasi-axisymmetry (QA) and stability to kink and ballooning modes. Among various numerical experiments that will be reported are (1) An investigation of plasma performance as β and I_p are varied with fixed profile shape – stable configurations with low effective helical ripple, ε_h, are found over a wide region of the $I_p - \beta$ plane, with a stable path from a vacuum state to a full current high–β configuration with $\beta > 6.5\%$; (2) An examination of plasma performance when plasma profiles are varied about reference forms at fixed I_p and β – a wide range of configurations is found whose β-limits exceed 3.0% and which have good QA, including configurations with finite current at the plasma edge; (3) An examination of the ability to control the external transform, $\iota(s)$, by varying coil currents while constraining the toroidal field and plasma current to be constant – substantial changes in $\iota(s)$ can be achieved, demonstrating an important control knob for transport and stability experiments on NCSX; (4) A demonstration of the ability to control the degree of QA-ness while maintaining plasma stability – providing a means to systematically explore the role of QA in improving the transport properties of stellarator plasmas; 5) A demonstration of the use of STELLOPT in defining experiments which elucidate the role of 3-D shape (stabilization in setting the β-limits separately for kink and ballooning modes.

Introduction

To achieve the scientific goals of the NCSX mission, the NCSX device must be capable of supporting a wide range of variations in plasma configuration about the reference baseline equilibrium. We demonstrate the flexibility of NCSX coils to support such configuration variations and demonstrate the robustness of performance of NCSX plasmas for substantial variations about reference design values of the plasma current, β, and profile shapes.

The robustness and flexibility calculations make use of free-boundary equilibrium reconstructions of NCSX plasmas. Most of the calculations reported here use a 21-modular coil design named M1017 which contain 7 coils per field period including a coil on the v=0 symmetry plane (the bean-shaped cross section). The 21 modular coils are grouped into four circuits, one circuit for each distinct coil shape. Like-shaped coils are connected in series and circuit currents are controlled independently. Figure 1 shows a rendition of the coils. A supplementary toroidal field coil set provides a weak (± 0.3 T at R = 1.4 m) 1/R field that can be in either direction relative to the

modular coils. It provides the capability to vary the external rotational transform at fixed toroidal field. A poloidal field coil system provides inductive current drive and low-order axisymmetric multipole fields for additional shape flexibility. Several design variants have been obtained subsequent to M1017, including an attractive 18-modular coil (3 independent coil shapes)/5-poloidal field coil design. We have verified that this 18 coil design has comparable flexibility to that of M1017.

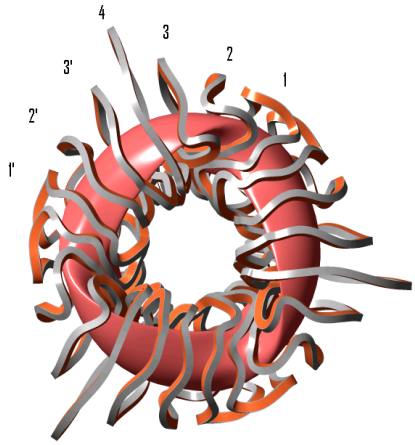

Figure 1: M1017 modular coils used for flexibility studies. Independent coil types are labelled 1 – 4 (primed numbers are stellarator-symmetric coils)

The primary computational tool for the flexibility studies is STELLOPT, a VMEC-based free-boundary optimizer which varies coil currents to target configurations with specific physics properties, such as stability to kink and ballooning modes and good quasi-axisymmetry(QA). Essential code modules within STELLOPT include an equilibrium solver (VMEC[1]), stability analysis codes (TERPSICHORE[2] for kink modes, COBRA[3] for ballooning modes), and a QA analyser (NEO[4] which evaluates QA-ness by calculating the effective helical ripple, ε_h).

Plasma Performance as β and I_p are varied

First, we verify that coil currents can be found which produce stable plasmas with good QA as I_p and β are varied from their reference values. It is important, for example, to demonstrate that there are stable paths which connect low-β states (e.g., the vacuum "s1" state with I_p = 0 kA, β = 0%) to high-β states (e.g., the reference full current "s3" state with I_p = 174 kA, β = 4.2%). For these calculations, bootstrap-consistent reference profiles of pressure and current (see curves labelled $\alpha = 0.0$ and $\gamma = 0.0$ in Figures 2a and 3) were used, and the free-boundary optimizer was asked to seek stable configurations with low ε_h over a matrix of I_p, β values. Table 1 shows that 14 of 18 optimized configurations obtained with different I_p, β values were stable with respect to kink and ballooning modes. The remaining 4 configurations were stable to ballooning modes and had small kink eigenvalues. Our experience shows that re-optimization with adjusted weights will probably stabilize these cases. Each configuration is, by construction, marginally stable at the given I_p, β (since the optimizer was run in a mode which provides a cost function penalty for instability but no reward for stability margin), showing that configurations with low β-limits are easily dialled. In almost all cases, good QA-ness was obtained, measured by effective ripple amplitudes $\varepsilon_h < 1.5\%$ at the magnetic surface with normalized toroidal flux

value s=0.5. A stable configuration with β = 5.0% was found with I_p = 174kA and B_T = 1.7T. The modular coil current variation was < ± 15% over the $I_p - \beta$ plane. The auxiliary TF field variation was less than ± 0.10 T.

β[%]→ Ip[kA]	*0.0*	*1.0*	*2.0*	*3.0*	*4.0*	*5.0*
43.5		ε_h = 2.11%	ε_h = 1.30%	ε_h =1.30%	ε_h = 0.86%	
87		ε_h = 1.56%	ε_h = 1.60%	ε_h =1.28%	ε_h = 0.86%	
130.5		ε_h = 0.77%	λ^K_0 = -6.0e-5 λ^K_1 = -2.7e-5 ε_h = 0.84%	λ^K_0 = -8.8e-6 λ^K_1 = -2.2e-5 ε_h =1.14%	ε_h = 0.67%	
174	ε_h = 0.79%	λ^K_0 = -1.9e-5 λ^K_1 = -2.0e-5 ε_h = 0.81%	λ^K_0 = -2.3e-5 λ^K_1 = -2.3e-5 ε_h =1.43%	ε_h = 0.79%	ε_h =1.29%	ε_h =1.66%

Table 1: I_p - β scan results (B_T = 1.7 T)

- *Blue boxes => Kink and Ballooning modes are stable at this I_p,β.*
- *Red boxes => Kink is unstable with small λ^K, but ballooning modes are stable.*

NOTE: Config with I_p = 174 kA, β = 5.0% is stable! (Stable plasmas with β up to 6.5% have been found for a related coilset).

Plasma Performance as profiles are varied

We now examine plasma performance (β-limits, QA measure ε_h) when plasma profiles are varied about reference forms at fixed I_p, B_T. Here we wish to verify that the reference design configuration is not sitting atop a pinhead optimum so that as the profiles are varied the performance drops precipitously. A 1-parameter sequence of J.B profiles, labelled by parameter α, describing the effect of varying the current in the core of the plasma is shown in Fig. 2a. As α is increased from $0 \rightarrow 1$, the J.B profile varies from the reference (hollow) bootstrap profile to a peaked current profile with a toroidal flux dependence of $1\text{-}s^2$. Using the reference p(s) and I_p = 174 kA, B_T = 1.7 T, STELLOPT finds stable configurations with $\beta \geq 3.0\%$ for $0 \leq \alpha \leq 0.5$, with $\varepsilon_h \leq 1.3\%$ at s=0.5.

A second sequence of current profiles is displayed in Fig. 2b. Here, we vary the current in the edge region of the plasma by adding a term $\delta\, s^8$ to the reference J.B(s) with different values of δ. Using the reference p(s) and I_p = 174 kA, B_T = 1.7 T we know there are coil currents for which a configuration with β = 5.0% is stable (see Table 1). Remarkably, keeping β = 5.0% and the coil currents fixed as δ is increased, stability is maintained as $J.B^{edge}/J.B^{max}$ is raised to 50%! One of the fundamental issues confronting Advanced Tokamak physics is how to deal with the tendency for tokamak β-limits to degrade as current builds in the edge region (e.g., as bootstrap current develops due to an edge temperature pedestal). This stability of stellarators to edge currents is in contrast with tokamak behavior (as pointed out by M.I. Mikhailov and V.D. Shafranov[5]) and leads to the intriguing possibility that H-mode profiles may be beneficial to NCSX.

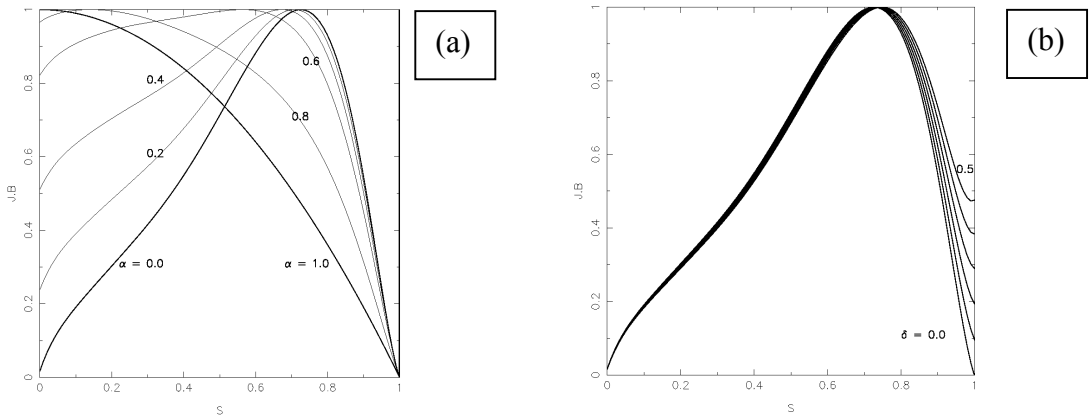

Figure 2: 1-parameter profile variations of current:
(a) α controls J.B variation in core region (α = 0.0 corresponds to reference J.B profile)
(b) δ controls J.B variation in edge region (δ = 0.0 is reference profile).

Finally, a 1-parameter sequence of pressure profiles is shown in Fig. 3. The profiles are parameterized by $\gamma \in (0,1)$ with increasing γ corresponding to increasing peakedness of the pressure profile. Using STELLOPT, we find the stable range of profiles is $0 \leq \gamma \leq 0.8$ (with $\gamma = 1.0$ stable at $\beta = 2.5\%$). Low ε_h values are obtained in almost all cases, with maximum coil current variations ~ 50 kA.

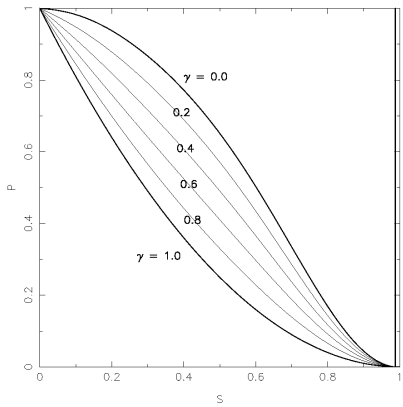

Figure 3: 1-parameter profile variations of pressure: γ controls peakedness of pressure profile (γ = 0.0 is reference profile).

Flexibility to Control External Transform

We now demonstrate the important capability of NCSX coils to effect substantial changes in the external field contribution to $\iota(s)$. This will be an important control knob in the experiment, allowing tests of the importance of avoiding low-order rational surfaces in the plasma region and will provide a significant means for controlling discharge evolution. The iota flexibility experiments shown here assume fixed B_T and I_p, and induce $\Delta\iota$ by causing appropriate changes in plasma shape . (Increasing/decreasing $\iota(s)$ by changing B_T and I_p is trivial). We seek to determine the maximum change in $\iota_{ext.}$ that can be induced by the coils without regard for the actual plasma performance produced by the variations. Of course in the actual experiment the iota flexibility will be exploited in specific ways, most of which will require stable plasmas to be generated (unless stability boundaries are being explored).

Figure 4 shows results from a series of numerical experiments where STELLOPT was used to program changes in iota while keeping the shear essentially constant. As a baseline plasma we choose an "s2" state with I_p = 174 kA, β = 0.0%. Axis and edge iota values are $\iota(0) = 0.44$, $\iota(1) = 0.65$. The figure shows plasma boundaries and calculated iota profiles for cases where $\iota(0)$ and $\iota(1)$ were programmed to change by ±0.1 and ±0.2 from the baseline s2 values. As expected, a substantial change in boundary shape is required to induce the large changes in $\Delta\iota$ (up to 50% of $\iota^{ref}(0)$ and 30% of $\iota^{ref}(1)$). For $\Delta\iota = +0.2$ coil currents vary by ~120 – 150 kA-t and the maximum supplied $B_T^{Aux} \approx -0.1$ T.

Figure 5 shows results from a similar calculation where global shear is programmed to change by constraining $\iota(0)$ to remain fixed at the reference s2 value $\iota(0) = 0.44$, while asking for the edge value to increase/decrease from the reference $\iota(1) = 0.65$ by ±0.1 and ±0.2. For this sequence we find maximum coil current variations of ≈ 80kA-t, with $\Delta B_T^{Aux} \approx +0.4$ T for $\Delta\iota = -0.2$.

Clearly the NCSX coilset has a substantial capability to scale the iota profile or change the shear, a capability that will be exploited in transport and stability experiments. We have found similar flexibility to change the $\iota(s)$ profile for s1 states with I_p = 0 kA, a flexibility that can be used to control $\iota(s)$ in high-iota startup scenarios.

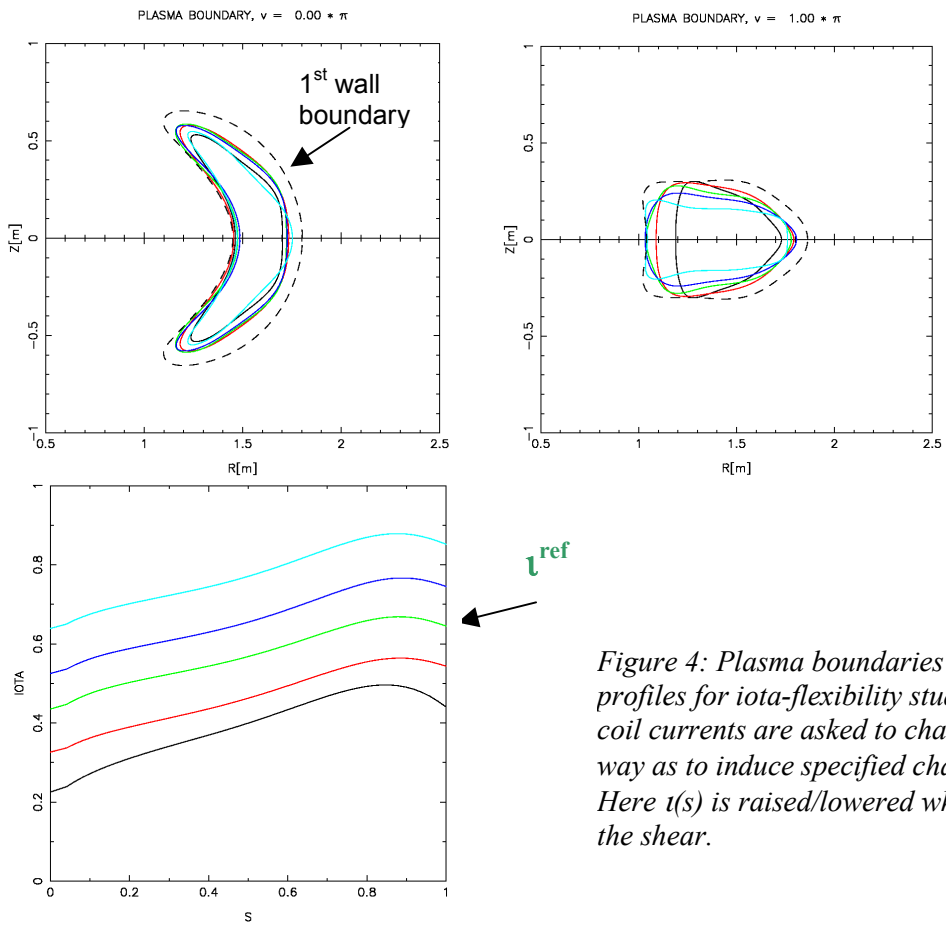

Figure 4: Plasma boundaries and iota profiles for iota-flexibility studies where coil currents are asked to change in such a way as to induce specified changes in ι(s). Here ι(s) is raised/lowered while preserving the shear.

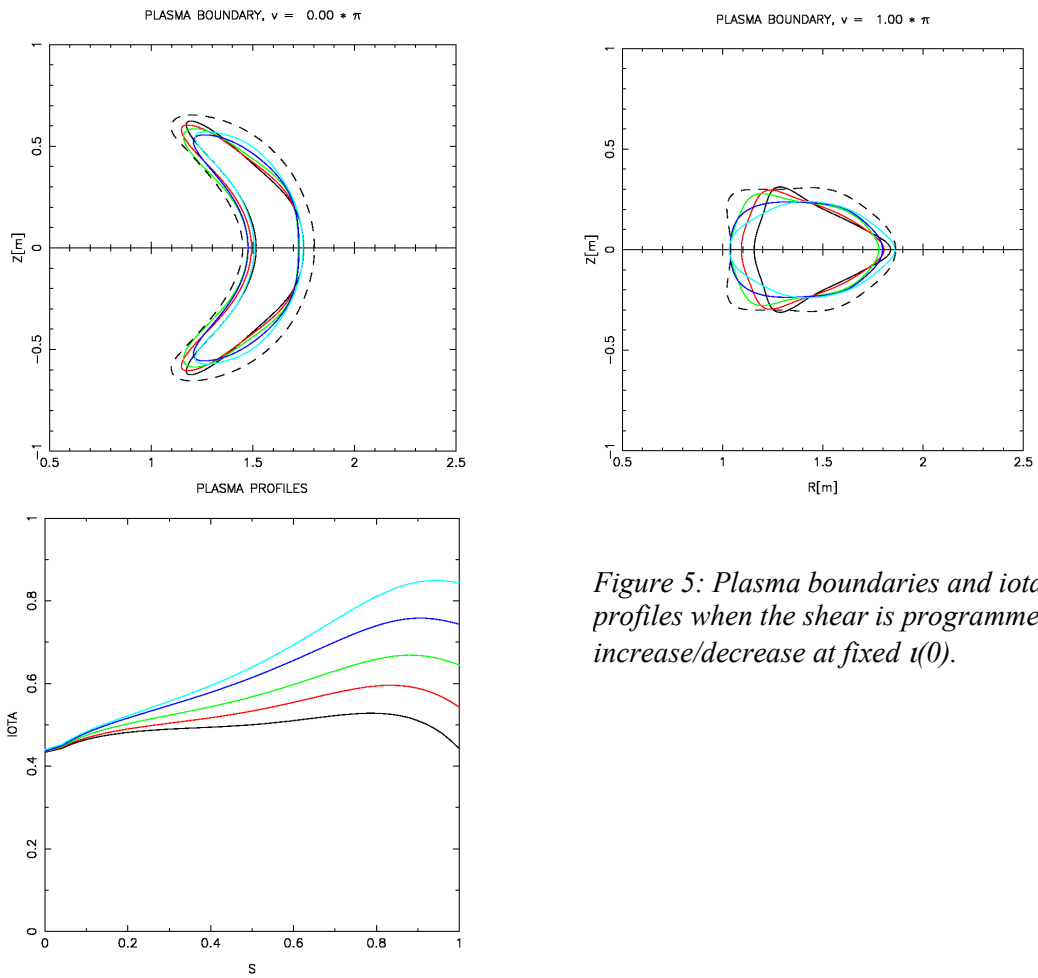

Figure 5: Plasma boundaries and iota profiles when the shear is programmed to increase/decrease at fixed ι(0).

Flexibility to vary QA-ness

The ability to generate configurations with good QA is an essential requirement for NCSX. For a systematic exploration of the role of QA in improving the transport properties of stellarator plasmas it is necessary to have the ability to control the degree of QA-ness while maintaining plasma stability.

Figure 6 shows a plot of helical ripple amplitude, ε_h, as a function of normalized toroidal flux, s, for three NCSX configurations with I_p = 43.5 kA, β = 3.0%. Among the configurations, ε_h varies by a factor of 9 at s = 0.25 (r/a ≈0.5), and by a factor of 4 at s = 0.5. Each configuration is stable to kink and ballooning modes. Also shown is the variation in each of the stellarator coil current. At the time of these calculations the operating version of STELLOPT did not target a specific value of ε_h. Instead, ε_h was minimized. In order to obtain the highest ε_h case, we used intuition gained through many optimization runs and artificially pinned the coil current of modular coil # 4 at –281 kA-t. STELLOPT was allowed to vary the three other modular coil currents. In spite of this restriction, the kink and ballooning modes were able to be stabilized.

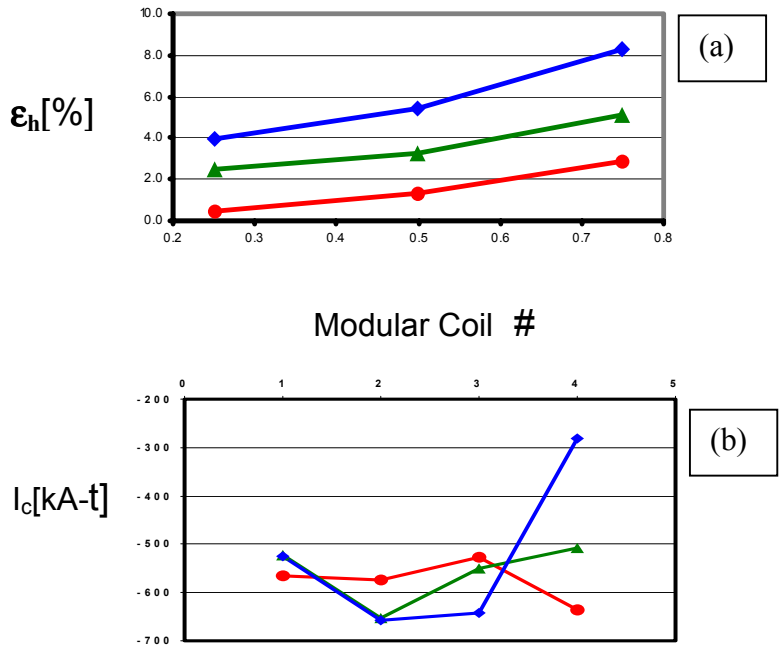

Figure 6:
(a) Effective helical ripple, $\varepsilon_h(s)$, for three different configurations.
(b) Currents in each of the four modular coils for the three configurations with different degrees of quasi-axisymmetry.

Flexibility to Explore 3D Shape Stabilization

The final set of flexibility experiments we describe illustrate the type of experiment that can be run on NCSX aimed at understanding the physics that determines stability boundaries and the role of 3D shape stabilization. First, we re-state that each stable free-boundary configuration in the $I_p - \beta$ scan (Table 1) lies at a point of marginal stability with respect to kink and ballooning mode stability for the given profiles. Two such configurations are:

C1) $I_p = 43.5$ kA, $\beta = 1.0\%$, $\iota(0) = 0.44$, $\iota(1) = 0.50$ and
C2) $I_p = 43.5$ kA, $\beta = 3.0\%$, $\iota(0) = 0.40$, $\iota(1) = 0.44$.

Overlays of the plasma boundaries for configurations C1 and C2, and of the corresponding iota profiles is shown in Fig. 7. Beta limits are $\beta_{lim} = 1.0\%$ for C1, and $\beta_{lim} = 3.0\%$ for C2. Marginal stability is verified by raising β at fixed plasma boundary shape and noting the appearance of an unstable mode.

The $\iota(s)$ profile for C1 is $\iota(1) = 0.50$. The question naturally arises whether the reduced β-limit of this configuration, relative to C2, is due to the de-stabilizing influence of the $\iota(1) = ½$ rational surface. The previously demonstrated capability of the NCSX coilset to change the iota profile can be used to test such a question. STELLOPT was re-run for the case $I_p = 43.5$ kA, $\beta = 1.0\%$ with the additional constraint that the plasma shape be consistent with coil currents such that $\iota(1) = 0.44$,

the same edge value as C2. A successful solution, C1-mod, was found and is shown in Fig. 8. Stabilization of C1-mod relative to C2 is clearly due to 3D shaping.

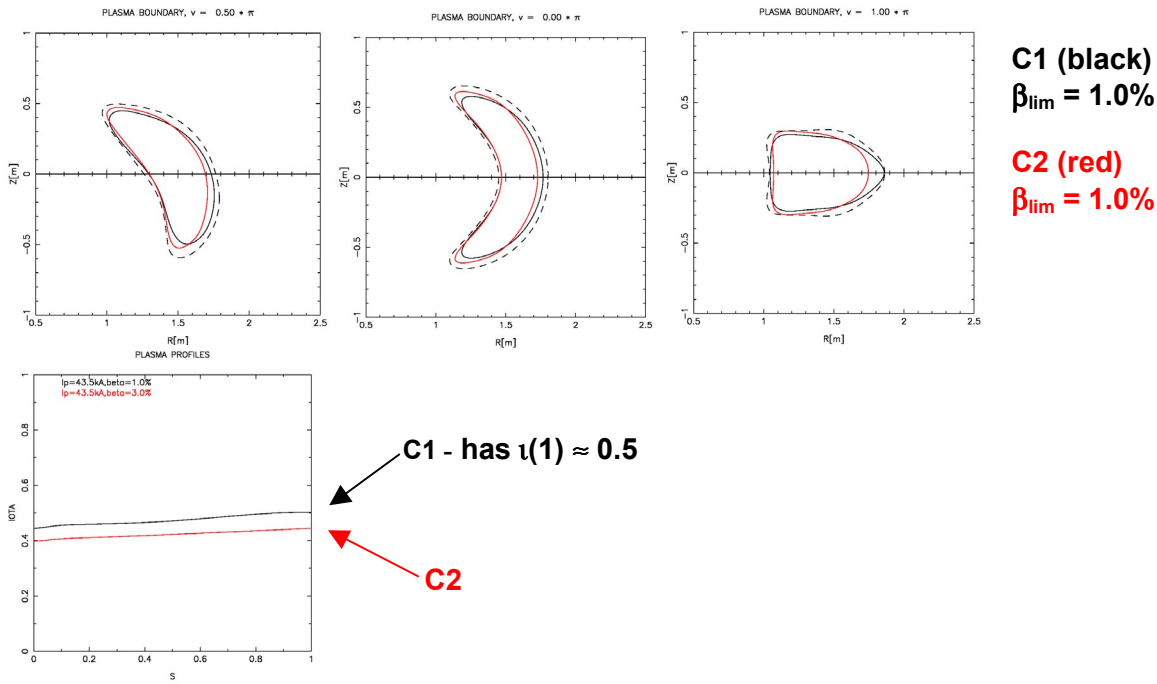

Figure 7: Overlay of plasma boundaries and iota profiles for the cases I_p = 43.5 kA, β = 1.0% and I_p = 43.5 kA, β = 3.0% used to illustrate MHD stabilization by 3D shaping.

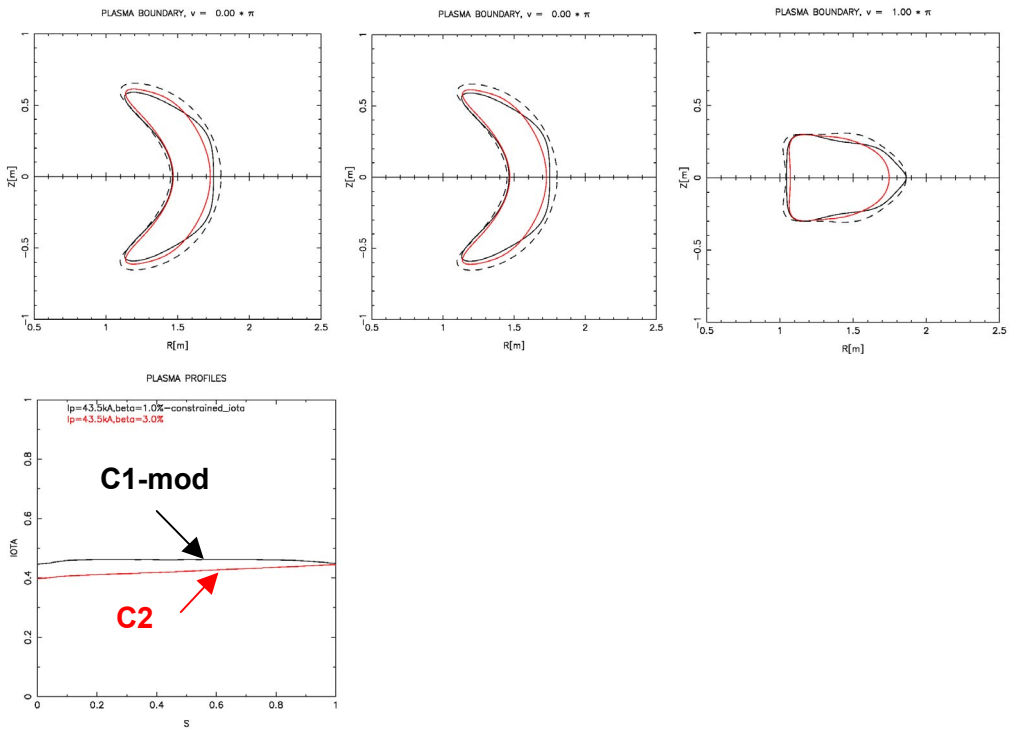

Figure 8: Overlay of plasma boundaries and iota profiles for the ι(1) = 0.44 constrained I_p = 43.5 kA, β = 1.0% configuration and the I_p = 43.5 kA, β = 3.0% configuration. With ambiguity of ι(s) = ½ surface removed, stabilization is seen to be due to 3D shaping.

Conclusions

The performance (β-limits, QA-ness measure ε_h) of NCSX plasmas is robust with respect to substantial variations of Ip and profile shapes about reference design values. The NCSX coil system has considerable flexibility to support the wide variety of experiments required by the NCSX mission.

A more complete description of flexibility and robustness calculations for NCSX can be found in Chapter 9 of the "NCSX Physics Validation Review", which can be accessed at **http://www.pppl.gov/ncsx/pvr/pvr.html**

Acknowledgements

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References

[1] S.P. Hirshman, W.I. Van Rij and P. Merkel, Comput. Phys. Communications **43**, 143 (1986)
[2] D.V. Anderson, A. Cooper, U. Schwenn and R. Gruber, *Linear MHD Stability Analysis of Toroidal 3D Equilibria with TERPSICHORE* in "Theory of Fusion Plasmas", J. Vaclavik, F. Troyon and E. Sindoni eds (Soc. Ital. Fisica – Editrice Compositori, Bologna, 1988)
[3] R. Sanchez, S.P. Hirshman, J.C. Whitson and A.S. Ware, J. Comput. Physics **161**, 189 (2000)
[4] V.V. Nemov, S.V. Kasilov, W. Kernbichler and M.F. Heyn Physics of Plasmas **6**, 4622 (1999)
[5] M.I. Mikhailov and V.D. Shafranov, Nucl. Fusion **30**, 413 (1990)