Exploration of Transport-Optimized Stellarator Configuration Space

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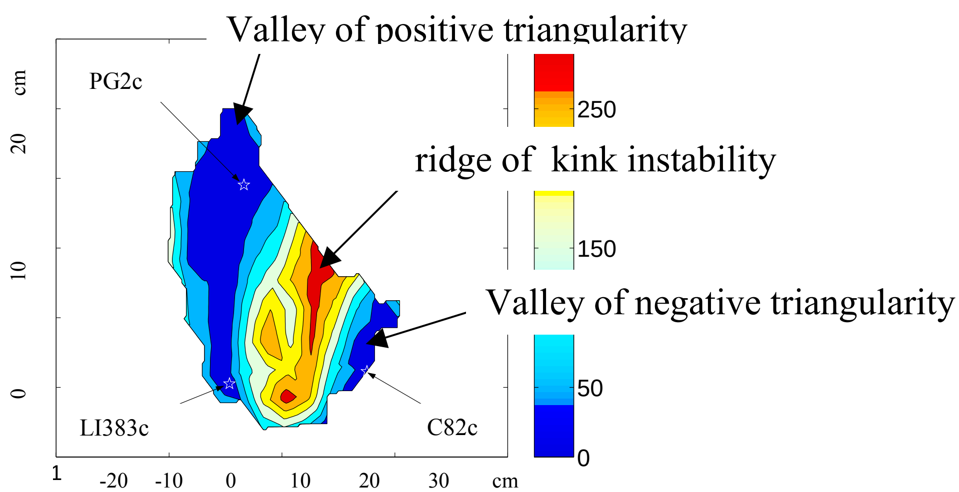
-Motivation:

In earlier work[1], we have found that a wide range of previously-discovered quasi-axisymmetric (QA) stellarator configurations could be put into a small number of classes, each corresponding to a basin in configuration space **z** of a cost function $\chi^2(P_i)$. (Here, $P_i(\mathbf{z})$ = physics performance measures, eg, measures of neoclassical confinement, stability to kink and ballooning modes, etc.)

[1]H.E. Mynick, N. Pomphrey, S. Ethier, Phys. Plasmas **9**, 869 (2002).



Here, we begin to extend this exploration to encompass other transport-optimized (XO) approaches, including quasi-omnigenous (QO), quasi-poloidal (QP), and quasi-helical (QH) designs, to ascertain the topography of the P_i and χ^2 over **z**, and to gain a better understanding of this topography.

-Formulation:

-Optimization occurs in configuration space $\mathbf{z}\equiv\{z_{j=1,..Nz}\}$. Here, $\mathbf{z}$ is specialized to a **fixed-boundary** set $\mathbf{z}\rightarrow\mathbf{X} \equiv\{X_{j=1,..Nx}\}=\{R_{\mathbf{m}1},Z_{\mathbf{m}1},R_{\mathbf{m}2},\ldots Z_{\mathbf{m}N(x/2)}\}$ needed for a description of the boundary, plus a small number of other parameters describing the equilibrium, eg, central pressure p_0, toroidal flux Φ_a, toroidal current I_p. [Here, $\mathbf{m}\equiv(m,\tilde{n}\equiv n/N_f)$, N_f=num field periods]

-Optimizer minimizes cost fn $\chi^2(P_i)$. Typically $\chi^2=\Sigma_i\chi_i^2$, $\chi_i^2 = w_i^2\ P_i$, where w_i^2=weights, and the P_i are M_p physics figures of merit, eg:

$P_1\equiv\ P_{Bmn} =\Sigma_{\psi k}\ w_{1k}^2\Sigma_{m,n\neq 0}B_{\mathbf{m}}^2()/B_0^2$ = QA-ness measure,

$P_2\equiv\ P_{Ripp}^2 =\Sigma_{\psi k}\ w_{2k}^2\varepsilon_{ef}^3$, with ε_{ef}=effective ripple (from NEO code[2]),

$P_3\equiv\ P_B = \Sigma_{\psi k,\theta 0}\ w_{3k}^2\lambda_B^2$, with $\lambda_B=-\omega_B^2$=ballooning eigenvalue (from COBRA),

$P_4\equiv\ P_K = w_{4k}^2\lambda_K^2$, with $\lambda_K= \omega_K^2$=kink eigenvalue (from TERPSICHORE).

[2] V.V. Nemov, S.V. Kasilov, W. Kernbichler, M.F. Heyn, Phys.Plasmas **6**, 4622 (1999).

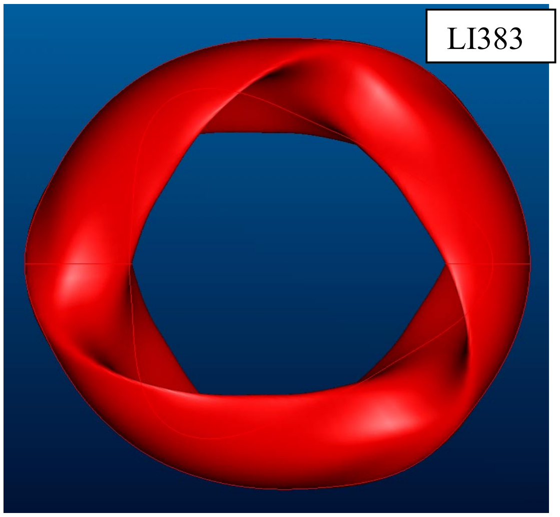
- -Transport-optimized ‘seed’ configurations:

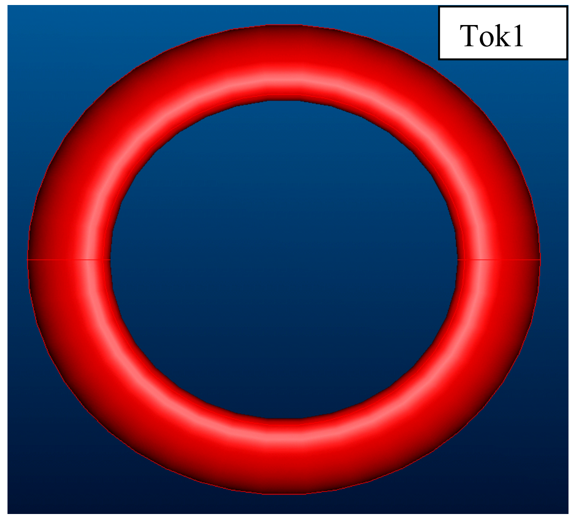
- We begin with a collection of `seed’ configurations representing the various approaches to transport-optimization, and study the relations among these.

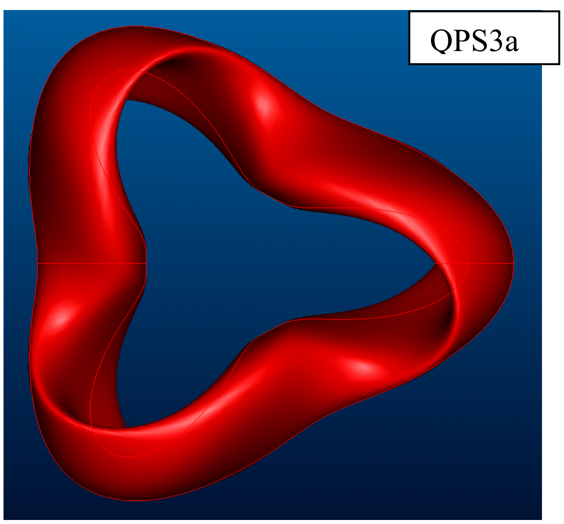
- All normalized to R_{00} = 1.68 m, N_f = 3, for continuous transitions between configs, more apples-to-apples comparisons. All in a space with N_x=70.
  - LI383 =QA, optimized basis of NCSX design
  - Tok1 =tokamak, derived from LI383 by setting all R_{mn},Z_{mn} =0 for n≠0.
  - QPS3a = From optimized ORNL/PPPL QPS design, but changing N_f = 2 → 3, so not optimized.
  - QHS3DS = QO design with dominant (n,m)=(1,-1),optimized at N_f = 3 (D.Spong).
  - QPS3DS = QO design with dominant (n,m)=(1,0), optimized at N_f = 3 (D.Spong).
  - HSX3a = From HSX design (so QH), but changing N_f = 4 → 3, so not optimized.

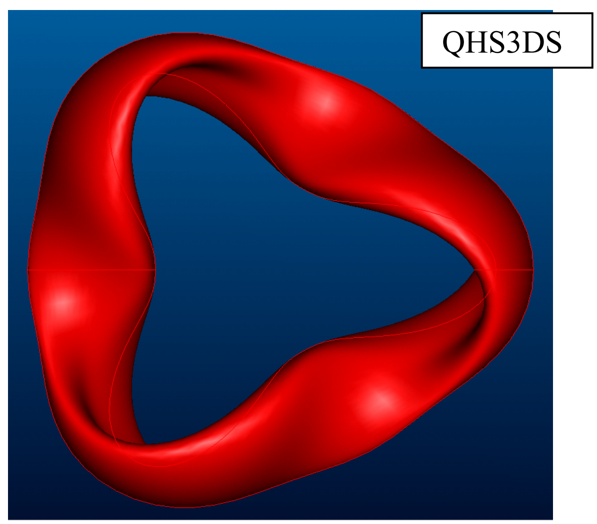
Configuration	LI383	Tok1	QPS3a	QHS3DS	QPS3DS	HSX3a
A=R/a	4.365	3.989	2.734	3.772	4.950	9.992
<B> (T)	1.13	0.90	1.13	1.13	1.13	1.13
Ip (kA)	-150	-460	47.3	39.4	34.8	0.0
ι_0	.39	.02	.36	.59	.39	.33
ι_a	.65	.68	.38	.76	.48	.75

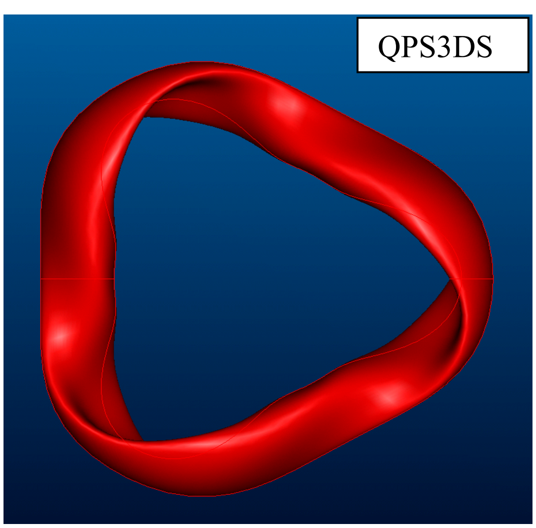
-Seed Configurations, cont.:

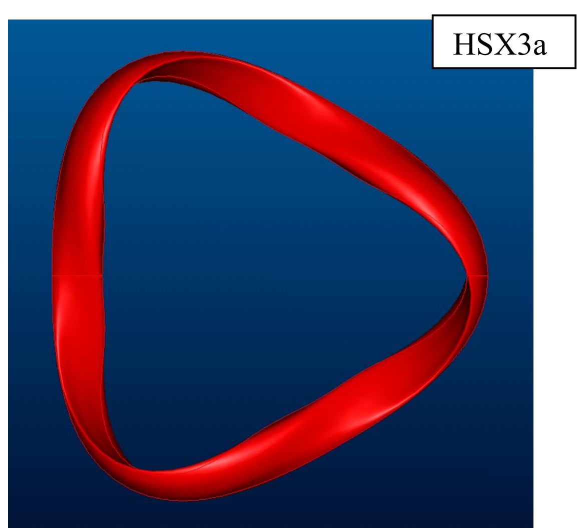




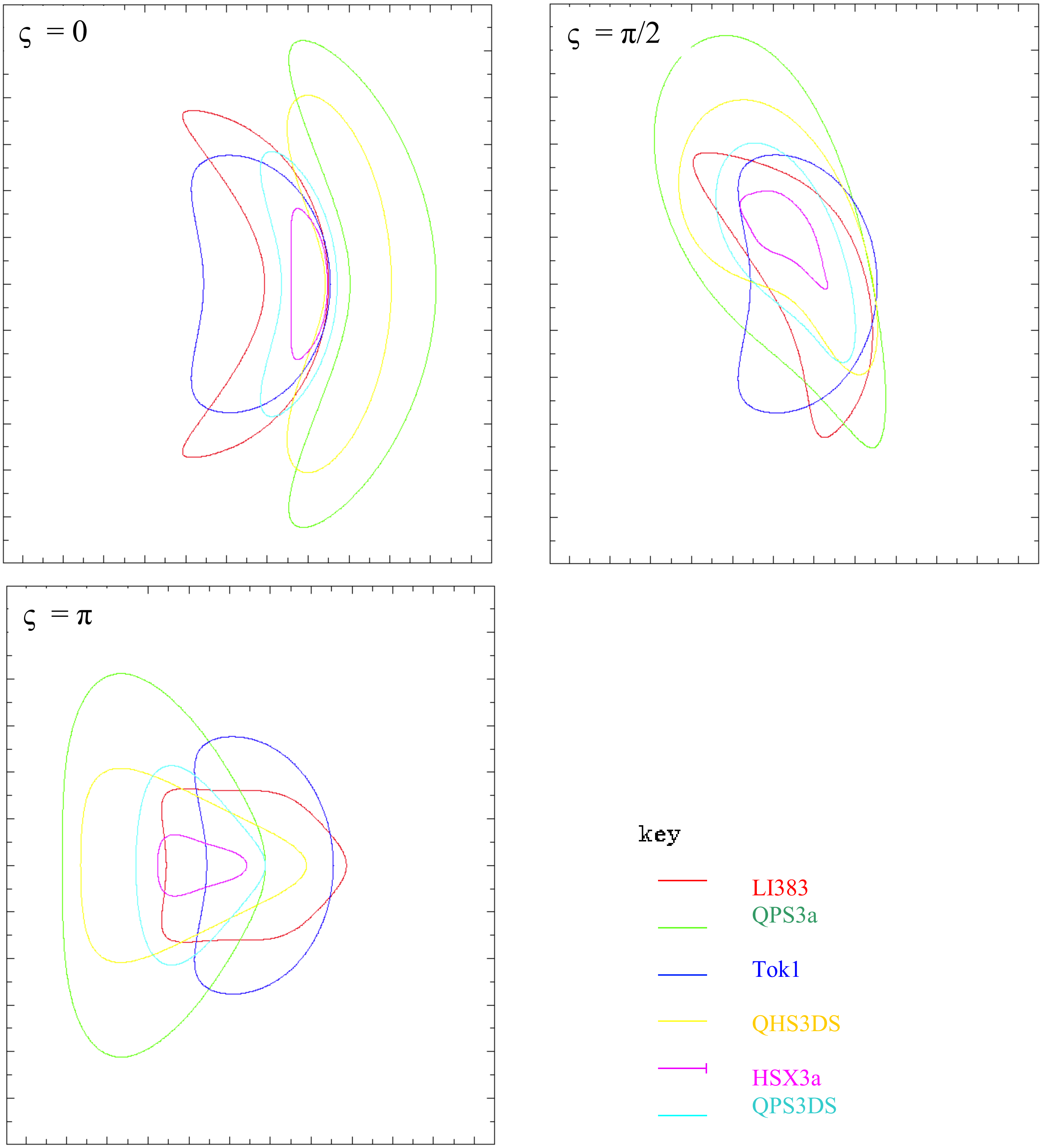








o Poloidal Cross-Sections:



-To compare these, introduce metric d[,] on **z**-space:

$$d[\mathbf{z}_1,\mathbf{z}_2] = |\mathbf{z}_1-\mathbf{z}_2|, \quad |\mathbf{z}|=(\Sigma'_j z_j^2)^{1/2} \quad .$$

conf1	conf2	d[conf1,conf2](cm)
-QA subspace:		
LI383c	PG2c	19.1
LI383c	C82c	22.3
-extended XO space:		
LI383	QPS3a	61.0
LI383	Tok1	29.1
LI383	QPS3DS	34.4
LI383	QHS3DS	45.7
LI383	HSX3a	57.9
QPS3a	QPS3DS	52.5
QPS3a	QHS3DS	37.2
QPS3DS	QHS3DS	25.6

- In terms of this metric, the distances between configurations in this extended subspace is several times what it was the the QA subspace.

-Some initial results:

- Choose weights w_i so that $\chi_i^2 \approx 1$ for marginally acceptable P_i.

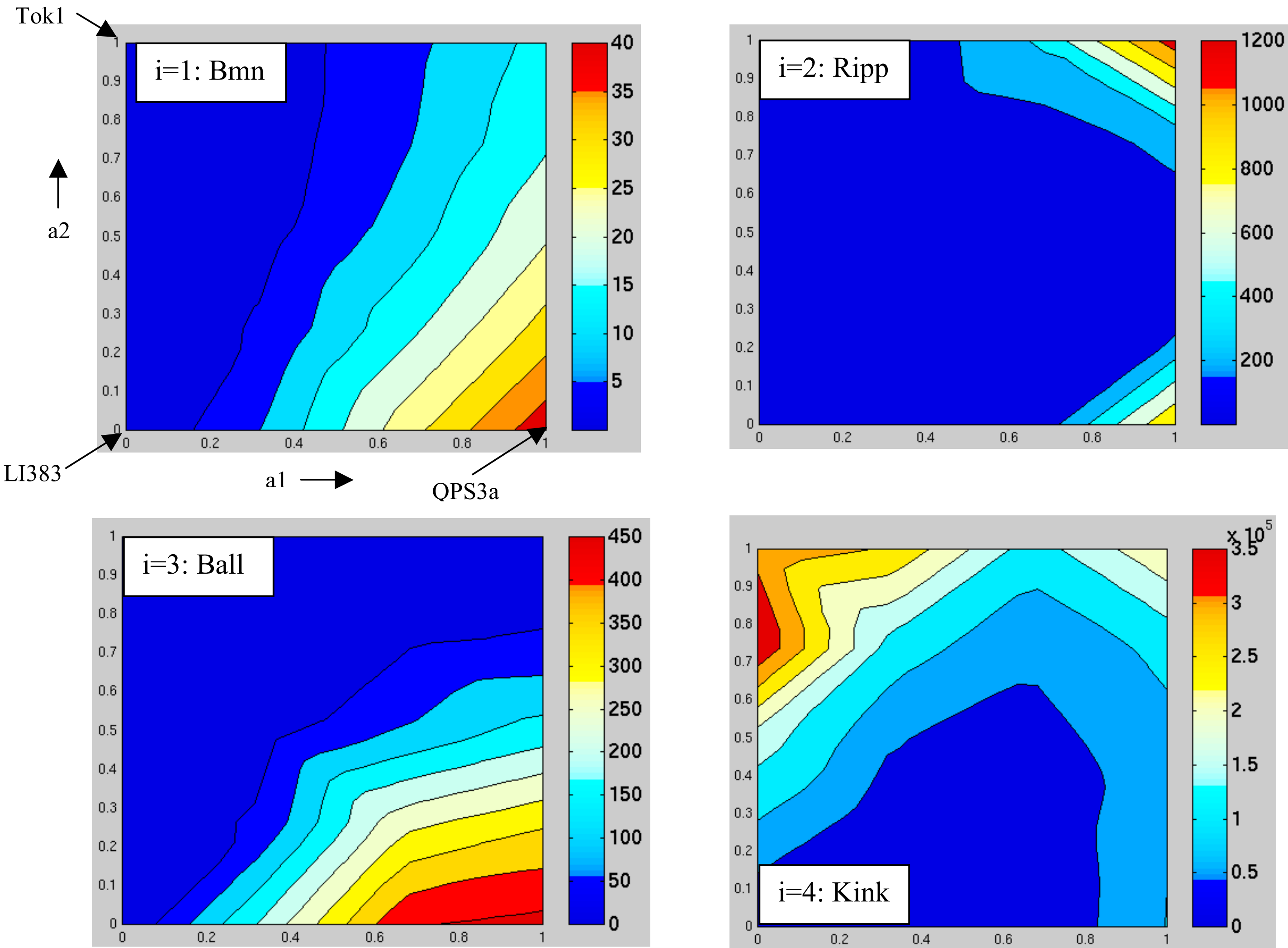
Configuration	LI383	Tok1	QPS3a (*)	QHS3DS	QPS3DS	HSX3a (*)
χ^2 contribs χ_i^2:						
Bmn (*)	.0381	1.325e-06	43.530	11.740	31.530	3.806
Ripp	.03097	7.292e-12	998.00	.43860	.52270	2.441E+04
Ball	0.662	0.00	465.9	637.4	629.7	2398.
Kink	0.00	4.065e+05	1.016e5	.7410	0.00	??

(*) $\chi_1^2 = \chi_{Bmn}^2$ =QA-ness measure relevant only for LI383 and Tok1.)
(*) Transport-optimized approach, but configuration non-optimized.

- We study the relations among these seeds, by Plotting the χ_i^2 over 2D cuts in **z**-space, with planes defined by 3 seed configurations at points $\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_2$, and a general point $\mathbf{z}$ given by coordinates (a_1, a_2), where

$$\mathbf{z} = \mathbf{z}_0 + a_1(\mathbf{z}_1 - \mathbf{z}_0) + a_2(\mathbf{z}_2 - \mathbf{z}_0) \quad :$$

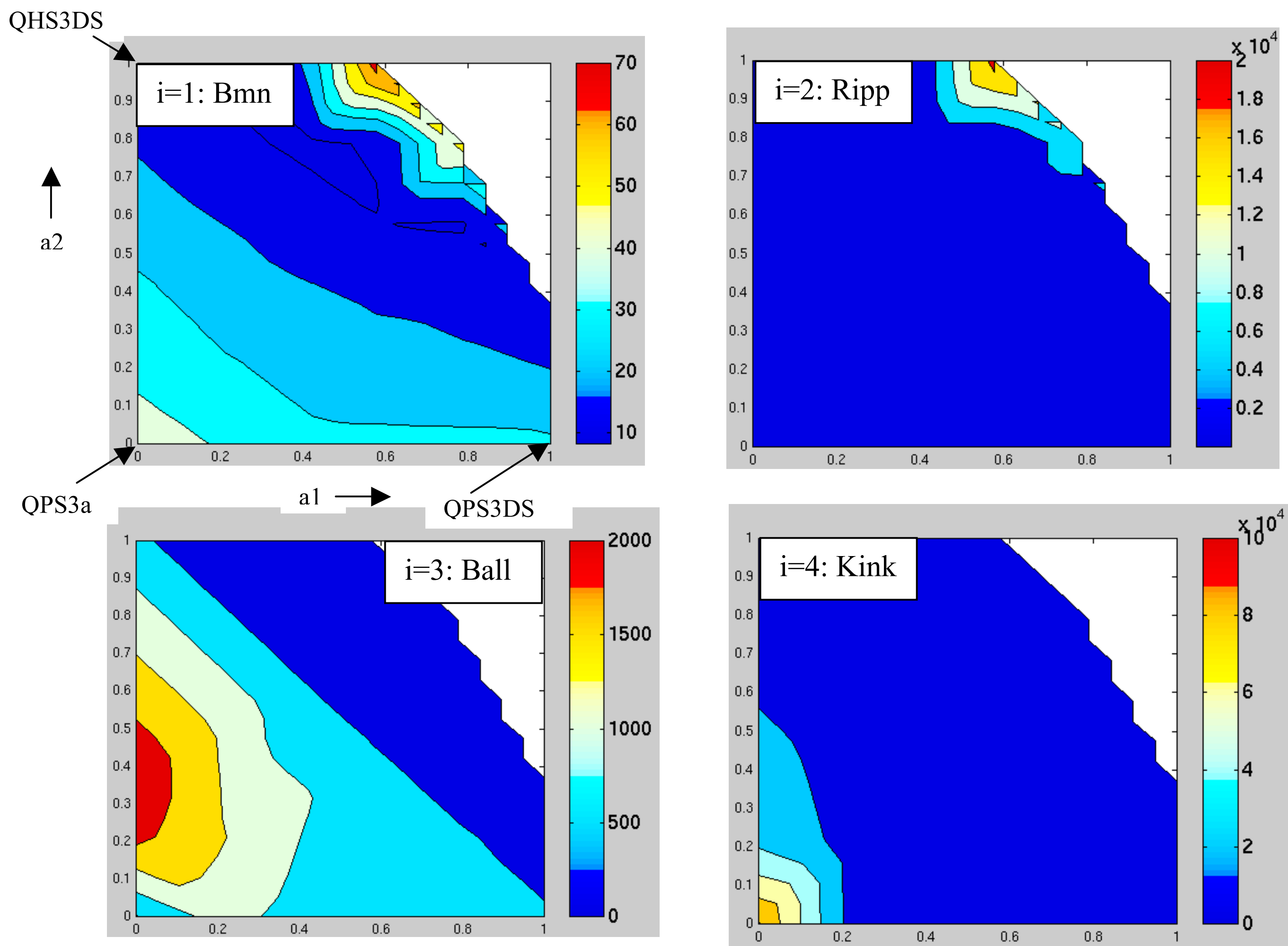
-Map-1: Compare χ_i^2 of 3 quite different configurations, viz LI383,QPS3a,Tok1.



-Ballooning plot reflects the good ballooning stability of the ARIES-1 like shape of Tok1, as well as of LI383, whose n=0 harmonics came from such a tokamak.

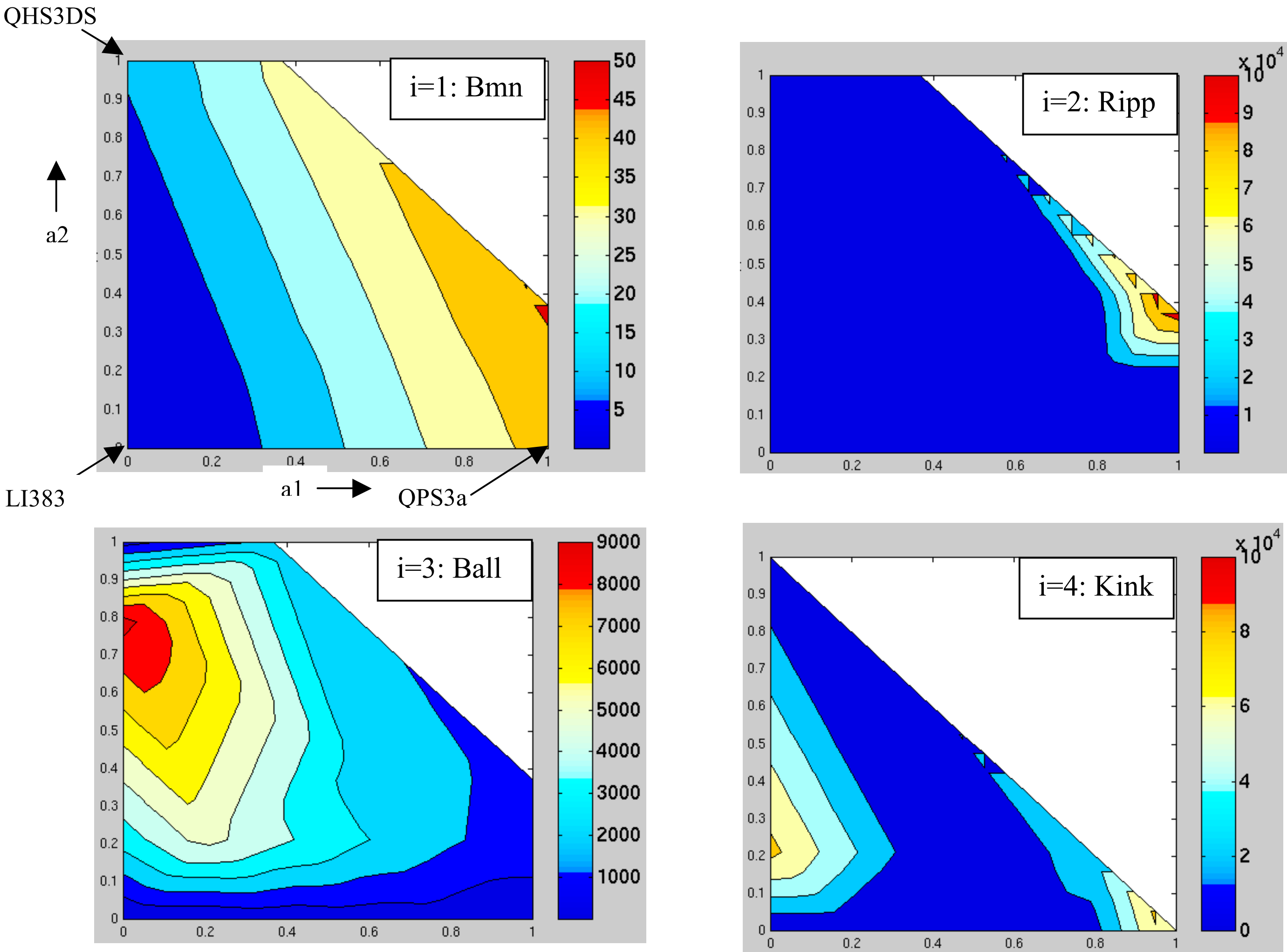
-Kink plot reflects that stellarators, having external transform, are more stable to kinks than tokamaks without close conducting walls.

-Map-2: Compare χ_i^2 of the QO configs, viz QPS3a,QPS3DS,QHS3DS:



-A continuous valley appears to connect the 2 optimized configurations QPS3DS, QHS3DS. A balloooning stability ridge separates QPS3a from QHS3DS. The less optimized character of QPS3a is apparent.

-Map-3: a1 axis as Map-1, rotate a2 axis to include QHS3DS, hence LI383, QPS3a, QHS3DS.



-As for the QA subspace, one finds the variation of the P_i over $\mathbf{z}$ is smooth and not very rapid, so that there is again not a very large number of basins of χ^2.

-Summary & Discussion:

-We have initiated a study of the topography of the physics performance measures P_i and cost function $\chi^2(P_i)$ over the space **z** of transport-optimized (XO) toroidal configurations, extending earlier explorations of the QA-subspace of that space.

-As found in the earlier QA-subspace studies, the variation of the P_i over the XO-space appears smooth and not very rapid, suggesting that there will again be only a modest number of basins of χ^2, and therefore distinct classes of XO configurations.

-We plan to obtain greater insight into why the P_i vary in this way, with complementary (semi)analytic studies analyzing numerical results such as those presented here. We thereby should be better able to:

- predict how the $P_i(\mathbf{z})$ will look for changed parameters, eg, for $N_f \neq 3$.

- understand how the $P_i(\mathbf{z})$ will vary in directions in which these numerical maps have not been made.
- make guesses at fruitful regions of this space to use automated search to find optimal configurations.

- The cost function χ^2 used thus far needs improvement, to provide a more apples-to-apples comparison for this broader range of configurations, and to be more reactor relevant. For example,
  - The current transport measures $P_{Bmn,Ripp}$, adequate for the QA-optimizations leading to LI383, need refinement to a more uniform transport measure P_{xport} not dependent on QA-ness (as is P_{Bmn}), and capturing ripple, symmetric, and turbulent contributions.
  - χ^2 needs to include a transport measure for alpha confinement.
  - The P_i need refinement to be valid for comparisons across the much broader range of parameters considered here, eg, for R/a. (Eg, the current use of ε_{ef} alone in P_2 is incorrect for wide variations in R/a.)