Nonlinear MHD Analysis for LHD Plasmas

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   - Bursting Activity
   - Stable Path to High Beta Regime
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Introduction

- Discrepancy between Experiments and Linear Stability Theory in LHD

  Experiments : Achievement of $\langle\beta\rangle = 3.2\%$ in inward-shifted config. ($R_{ax} = 3.6$m)

  Linear Theory : Destabilization of ideal interchange mode at much lower β for smooth pressure profiles

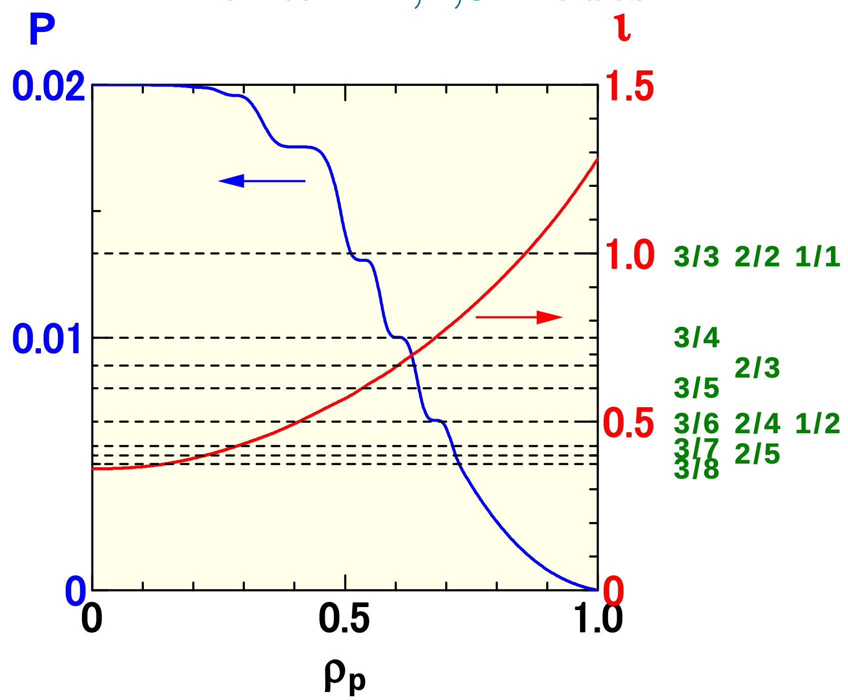
- Stabilizing Mechanism ?

  Prediction by linear stability analysis
  3D equilibrium calculation
  (K.Ichiguchi, et al., NF(2001)181)

  – Effective stabilization due to Local pressure flattening at low order resonant surfaces

  (Interchange mode $\Leftarrow$ pressure gradient)
  $\Downarrow$
  Such profile is generated automatically ?
  Mode Overlapping becomes crucial ?

Pressure profile stable for $n = 1,2,3$ modes



- Development of Nonlinear Calculation Code

Basic Equations

- **Reduced MHD Equations (based on improved stellarator ordering) for Ψ(Poloidal flux), Φ(Stream function), P(Pressure)**

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{R}{R_0}\right)^2 \boldsymbol{B}\cdot\nabla\Phi + \frac{1}{S}J_\zeta, \qquad \ldots \qquad \textbf{Ohm's Law}$$

$$\frac{dU}{dt} = \left(\frac{R}{R_0}\right)^2 \left(-\boldsymbol{B}\cdot\nabla J_\zeta + \frac{\beta_0}{2\epsilon^2}\nabla\Omega\times\nabla P\cdot\nabla\zeta\right) + \nu\hat{\nabla}_\perp^2 U, \quad \ldots \ \textbf{Vorticity Equation}$$

$$\frac{dP}{dt} = \kappa_\perp \Delta_* P + \epsilon^2 \kappa_\parallel \left(\frac{R}{R_0}\right)^2 \boldsymbol{B}\cdot\nabla(\boldsymbol{B}\cdot\nabla P). \qquad \ldots \qquad \textbf{Equation of State}$$

$$\Omega = \frac{1}{2\pi}\int_0^{2\pi} d\zeta \left(\frac{R}{R_0}\right)^2 \left(1 + \frac{|\boldsymbol{B}_{eq}(R,\zeta,Z) - \overline{\boldsymbol{B}_{eq}}(R,Z)|^2}{B_0^2}\right), \qquad \boldsymbol{B}\cdot\nabla = \frac{R_0 B_0}{R^2}\frac{\partial}{\partial\zeta} - \nabla\Psi\times\nabla\zeta\cdot\nabla$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}_\perp\cdot\nabla, \quad \boldsymbol{v}_\perp = \left(\frac{R}{R_0}\right)^2 \nabla\Phi\times\nabla\zeta, \quad U = \hat{\nabla}_\perp^2\Phi = \left(\frac{R}{R_0}\right)^2 \nabla\cdot\nabla_\perp\Phi, \quad J_\zeta = \Delta_*\Psi = \left(\frac{R}{R_0}\right)^2 \nabla\cdot\left(\frac{R_0}{R}\right)^2 \nabla_\perp\Psi$$

- **Toroidal Effects $\Longleftarrow$ Toroidally averaged 3D equilibrium (VMEC code)**
- **Perturbation : Expanded in Fourier series with Multi-Helicity**
- **No Average Flow**

Configuration and Linear Stability

- **Vacuum configuration : Inward-shifted LHD with R_{ax}=3.6m**
- **Equilibrium conditions**
  - ◇ Free boundary and no net-current conditions
  - ◇ $P = P_0(1-\rho^2)(1-\rho^8)$ ...close to experimental result for $\langle\beta\rangle < 1\%$, $(\langle\beta\rangle \sim 0.43\beta_0)$
- **Focus on equilibria at $\beta_0 = 0.5\%$ and 1.0% (linearly unstable)**

◇ Ideal β limit for low-n mode :
$\beta_0 \simeq 0.1\%$

◇ $0.37 \leq \iota \leq 1.8$

◇ Parameters for Nonlinear Calculation

$S = 10^6$

$\nu = 10^{-4}$

$\kappa_\perp = 10^{-6}$ ($\sim 0.1 m^2/sec$)

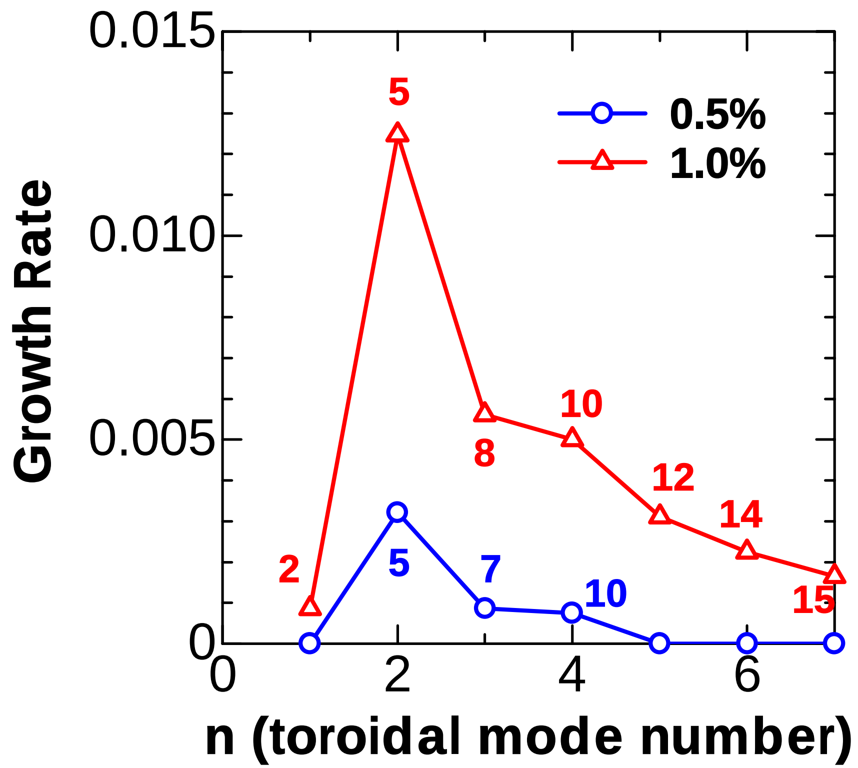
$\epsilon^2\kappa_\parallel = 10^{-2}$ ($\epsilon = 0.16$)

▷ $\beta_0 = 0.5\%$: $2 \leq n \leq 4$ modes unstable

▷ $\beta_0 = 1.0\%$: $1 \leq n$ modes unstable

▷ m=5,n=2 mode dominant for both cases

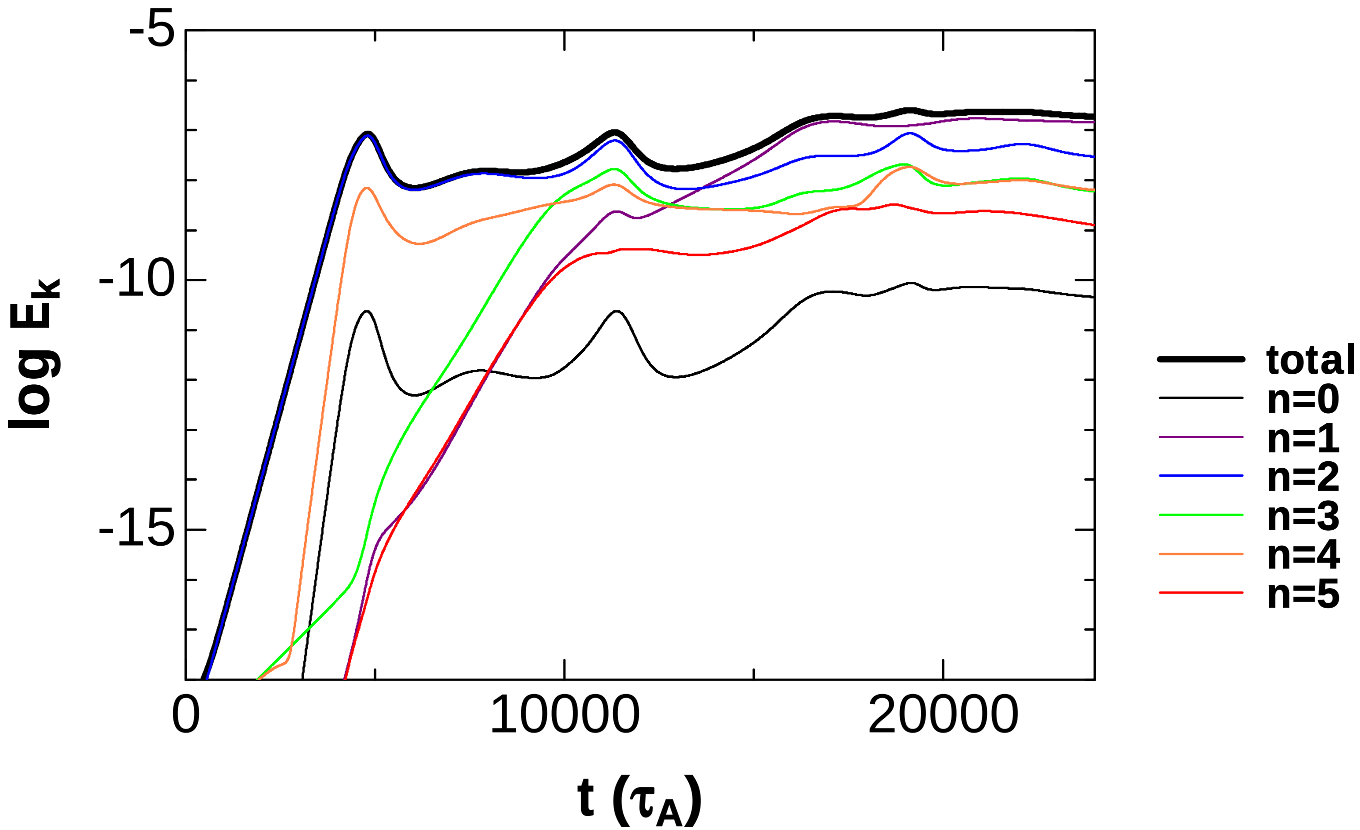
Linear growth rate with the parameters



Mild Saturation at $\beta_0 = 0.5\%$

- **Nonlinear evolution at $\beta_0 = 0.5\%$ for $P = P_0(1-\rho^2)(1-\rho^8)$** $(0 \leq n \leq 5,\ 0 \leq m \leq 15)$

Time Evolution of Kinetic Energy

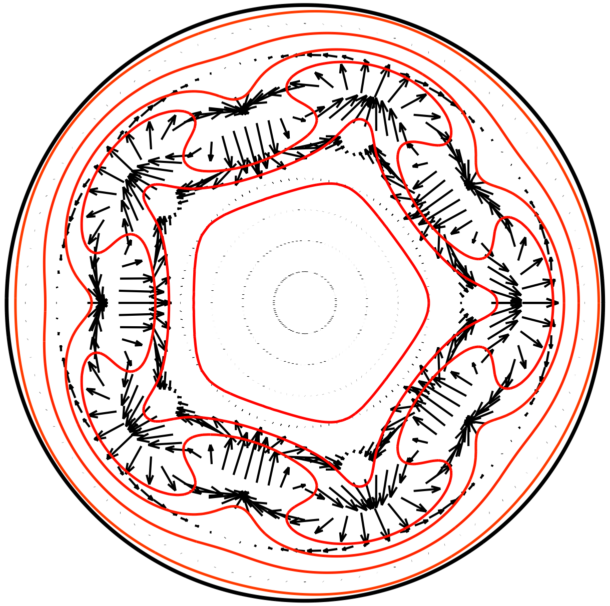


- ▷ **Linear phase** : **Dominant linear mode : m=5,n=2**
- ▷ **Nonlinear phase** : **Slow and small variation of E_k in low level ($\lesssim 10^{-7}$)**
  Small humps at $t = 4800, 11325, 16000\tau_A$ $\Rightarrow$ What happens ?

Mild Saturation at $\beta_0 = 0.5\%$ (cont.)

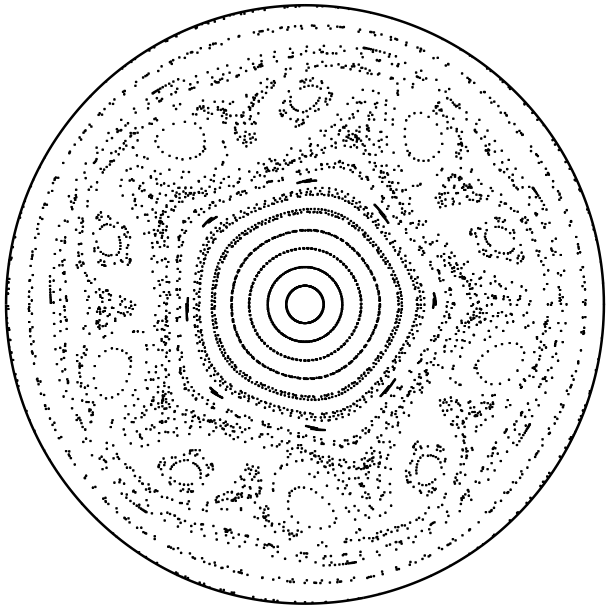
Saturation of m=5,n=2 mode

Pressure Contour and Flow Pattern
($t = 4800\tau_A$, $\zeta = 0, \rho \leq 0.4$)



- ▷ 2m (10) vortices around $\iota = 2/5$ surface interchange low and high pressure regions.
- ▷ Mushroom-like structure of pressure

Poincaré Plot of Field Line
($t = 4800\tau_A$, $\zeta = 0, \rho \leq 0.4$)

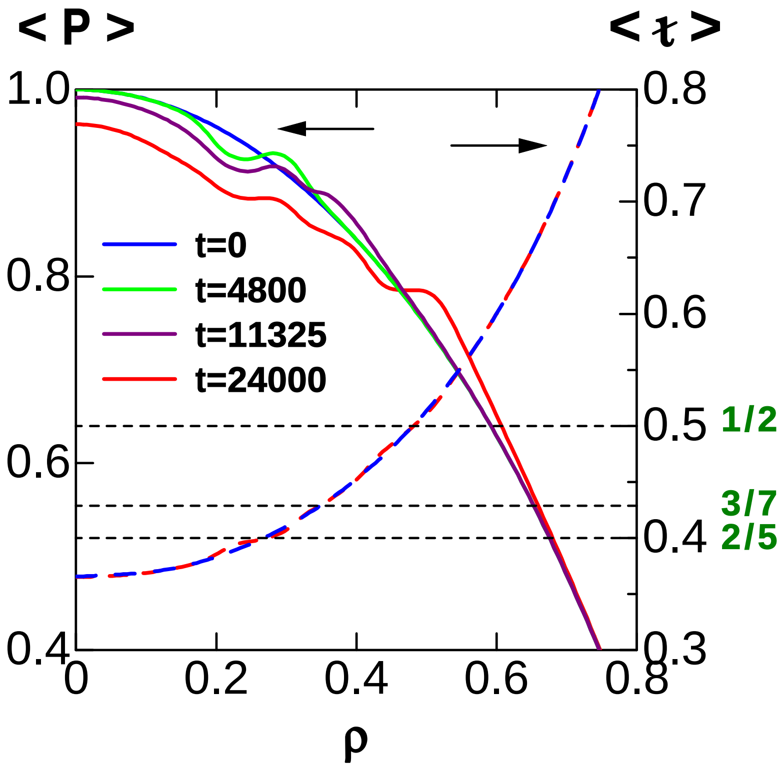


- ▷ Driven reconnection of field lines due to vortices
- ▷ 2m (10) magnetic islands

Mild Saturation at $\beta_0 = 0.5\%$ (cont.)

Average Pressure and Rotational Transform

$(\langle P \rangle = P_{eq} + \tilde{P}_{00})$



Single mode saturation :

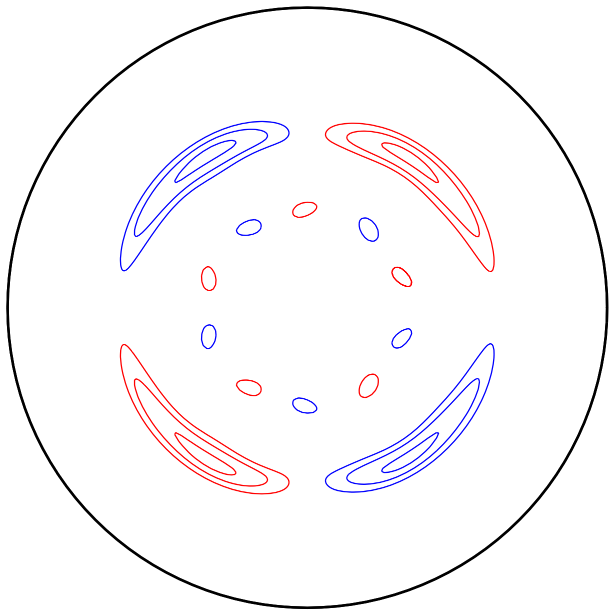
- ▷ Local pressure flattening
- ▷ Steepening pressure gradient in both sides
- ▷ Excitation of other single mode

Sequence : m=5,n=2 ⇒ m=7,n=3 ⇒ m=2,n=1

- ▷ Indirect interaction through local $\langle P \rangle$ variation
- ▷ Locally flat $\langle P \rangle$ profile (as predicted in linear analysis)

Stream Lines at final state

$(t = 24000\tau_A,\ \zeta = 0, \rho \leq 0.8)$

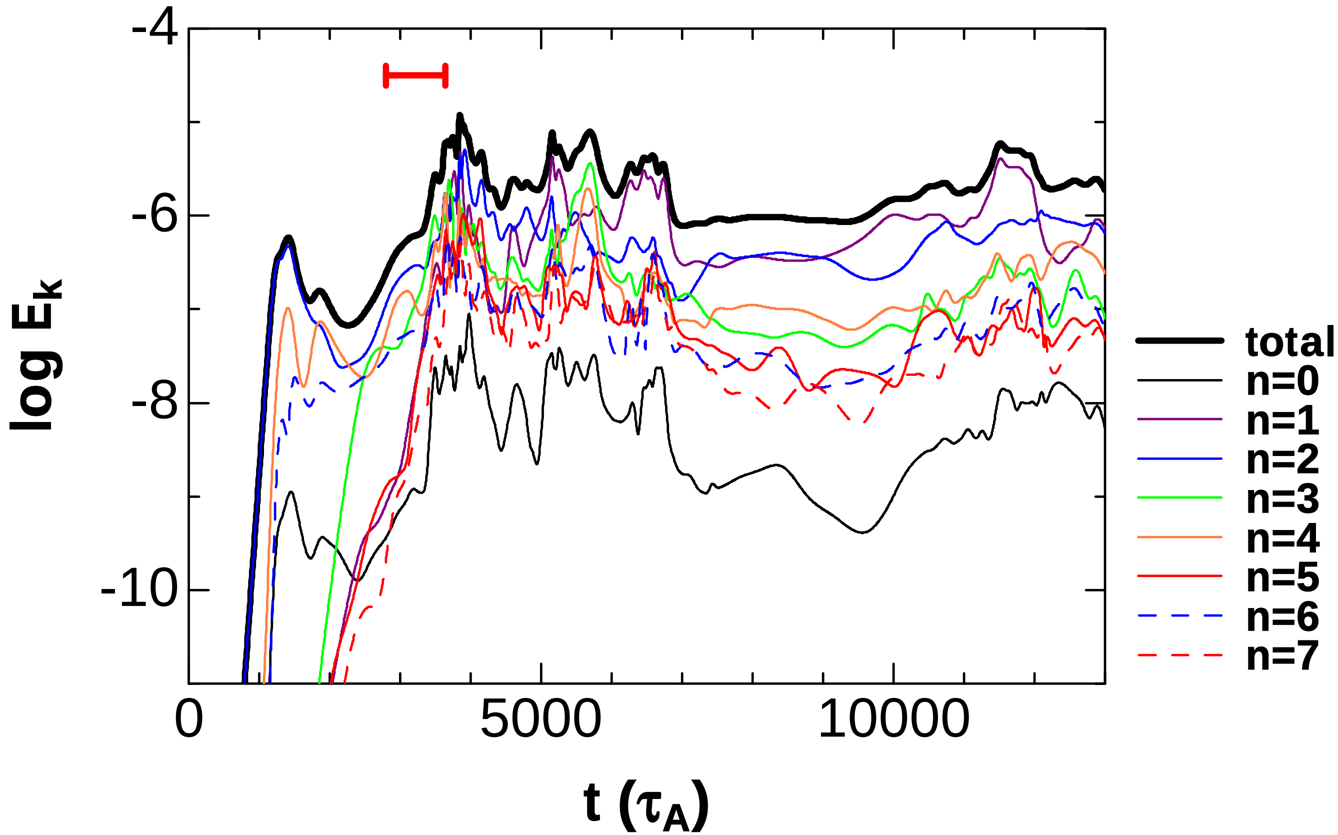


- ▷ Separated mode structure in ρ due to weak driving force

Bursting Activity at $\beta_0 = 1.0\%$

- Nonlinear evolution at $\beta_0 = 1.0\%$ for $P = P_0(1-\rho^2)(1-\rho^8)$ ($0 \le n \le 7$, $0 \le m \le 22$)

Time Evolution of Kinetic Energy



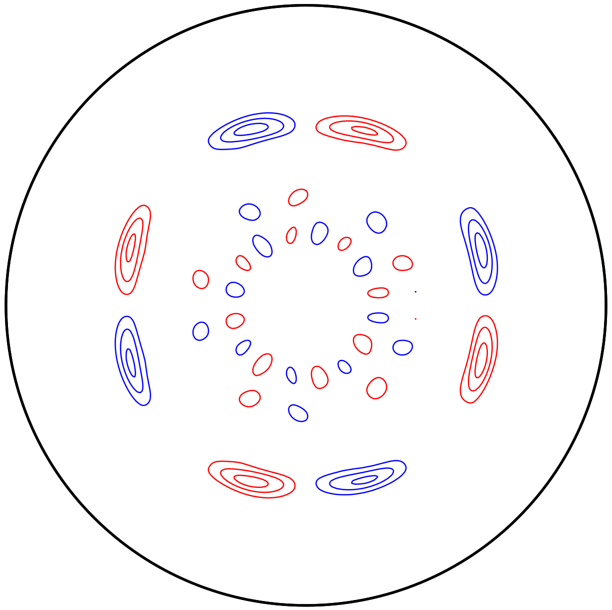
- ▷ Linear phase : Dominant linear mode : m=5,n=2
- ▷ Nonlinear phase : Bursting behavior of E_k in high level ($\lesssim 10^{-5}$) $\Rightarrow$ Mechanism?

  Simultaneous excitation of many modes
  due to enhancement of driving force

Bursting Activity at $\beta_0 = 1.0\%$ (cont.)

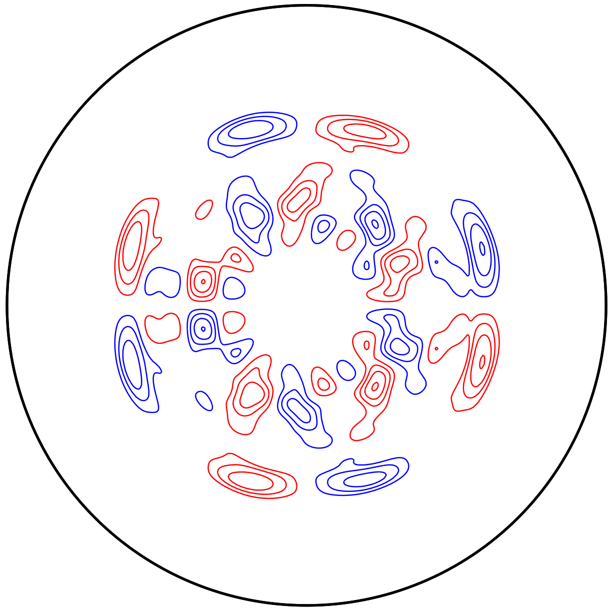
Stream Lines $(\zeta = 0, \rho \leq 0.8)$

- $t = 2800\tau_A$
  (before burst)



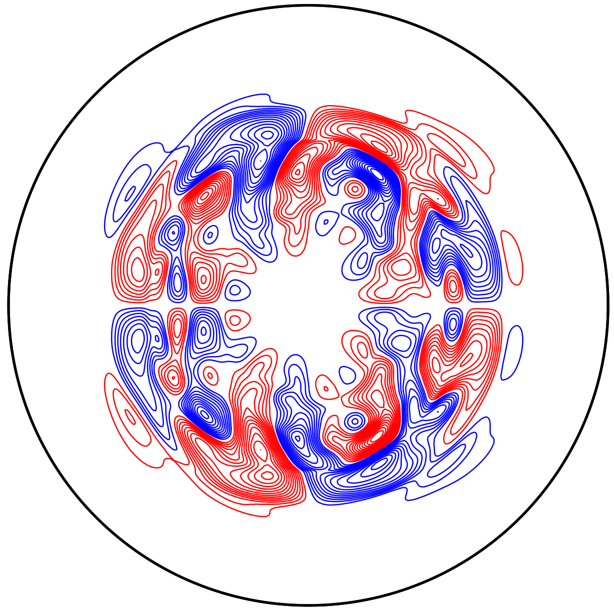
▷ Localization around each resonant surface

- $t = 3200\tau_A$
  (triggering burst)



▷ Overlapping of modes with different helicity

- $t = 3640\tau_A$
  (high-level burst)



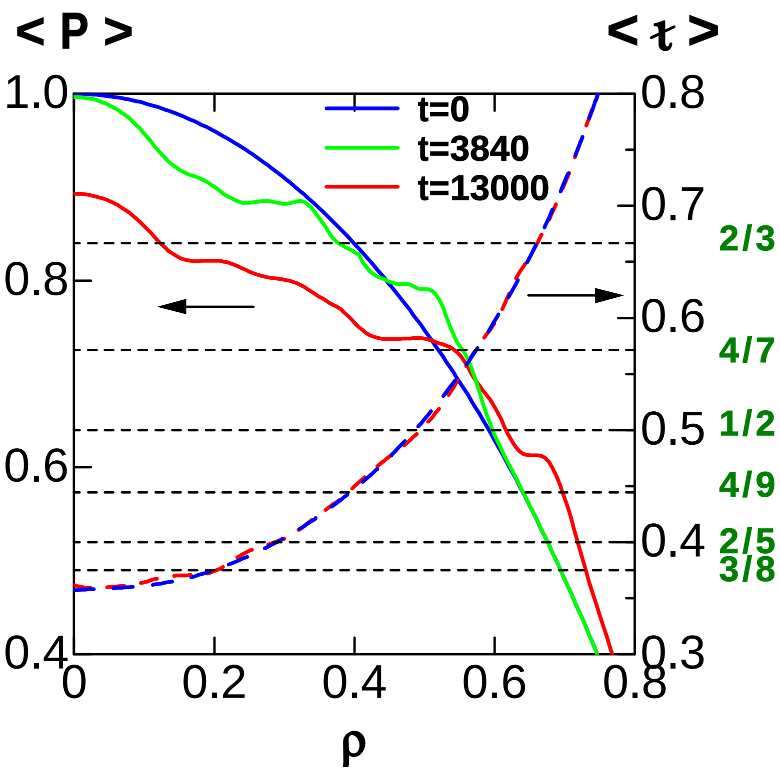
▷ Generation of large scale vortices in ρ

Mechanism of Bursting Activity

Mode Overlapping (direct interaction) $\Longrightarrow$ Release of wide range Free Energy $\Longrightarrow$ Strong Cooperative Flow

Bursting Activity at $\beta_0 = 1.0\%$ (cont.)

Average Pressure and Rotational Transform



▷ During Burst ($t = 3840\tau_A$) :
Wide flat-pressure regions
due to large scale vortices

▷ Final state ($t = 13000\tau_A$) :
Substantial reduction
of pressure
in whole core region ($\rho \lesssim 0.5$)

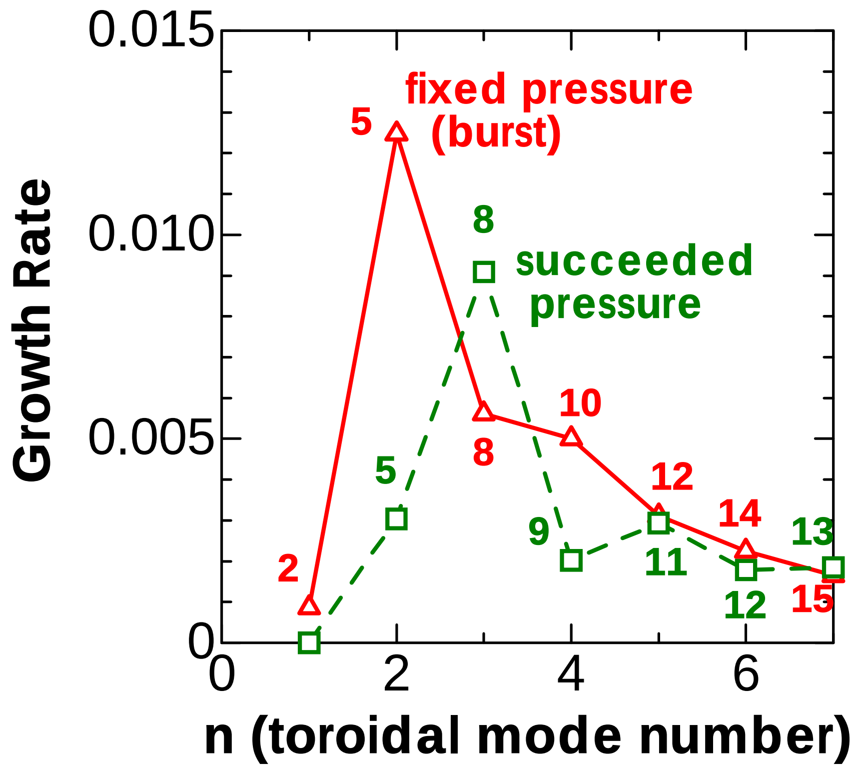
Stable Path to High Beta Regime

- Bursting activity for fixed pressure profile may limit achievable β . However, pressure profile must be succeeded as β increases.
- Nonlinear analysis at $\beta_0 = 1.0\%$ with pressure profile saturated at $\beta_0 = 0.5\%$

Procedure for initial equilibrium

- Employment of $\langle P \rangle$ profile saturated at $\beta_0 = 0.5\%$
- Increase of beta to $\beta_0 = 1.0\%$
- Free boundary equilibrium

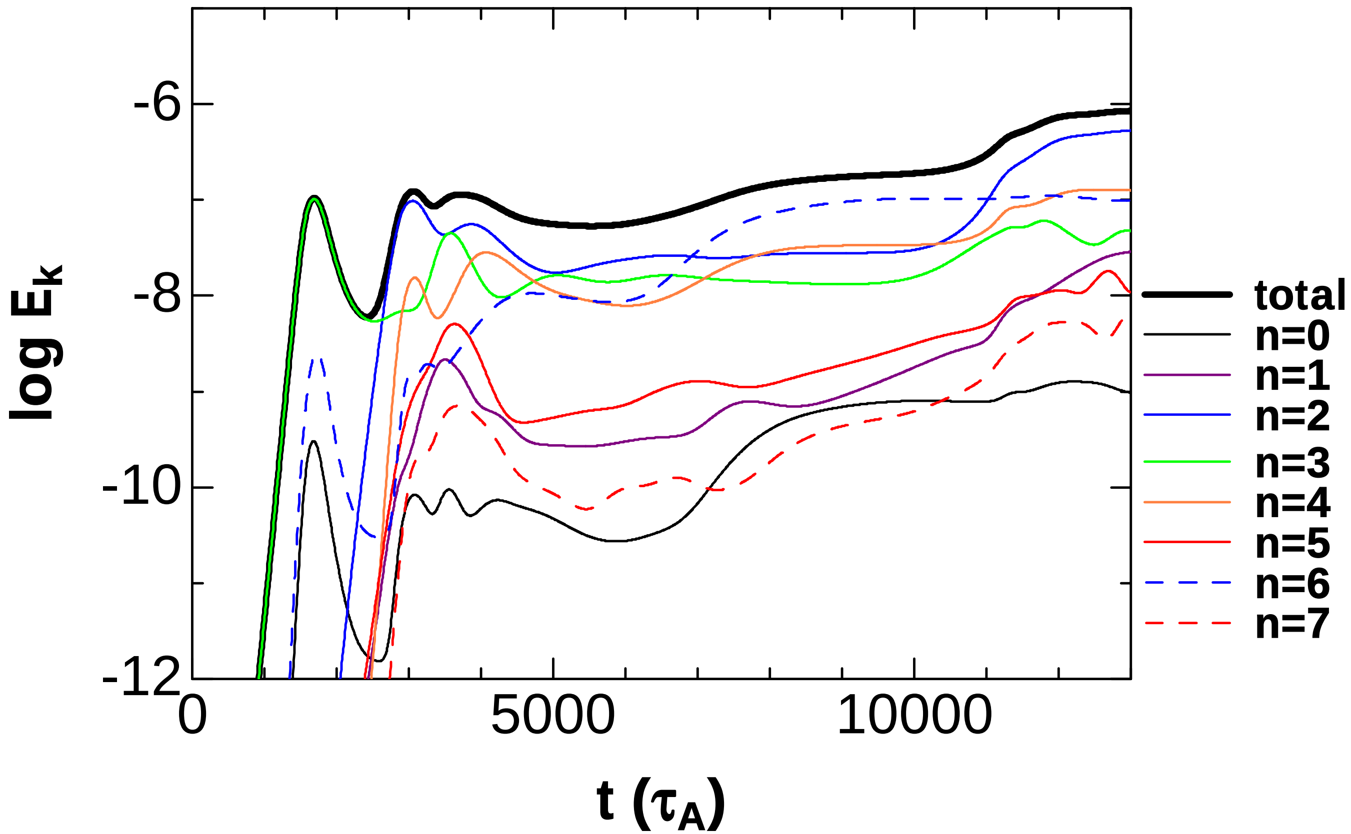
Linear Growth Rate



▷ Reduction in n=1,2,4 modes

Stable Path to High Beta Regime (cont.)

Time Evolution of Kinetic Energy



▷ Nonlinear phase : Mild saturation as in $\beta_0 = 0.5\%$ case

Small and Slow variation in E_k in low level ($\lesssim 10^{-6}$)

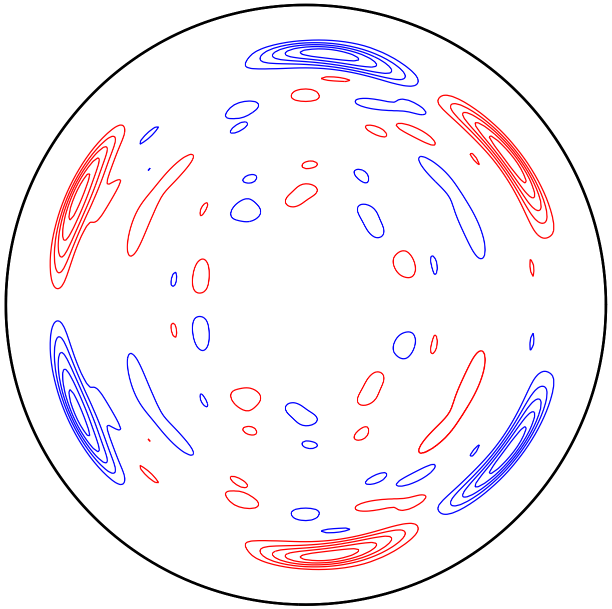
No bursting activity

$\Uparrow$

Reduction of driving force through deformation of initial pressure profile

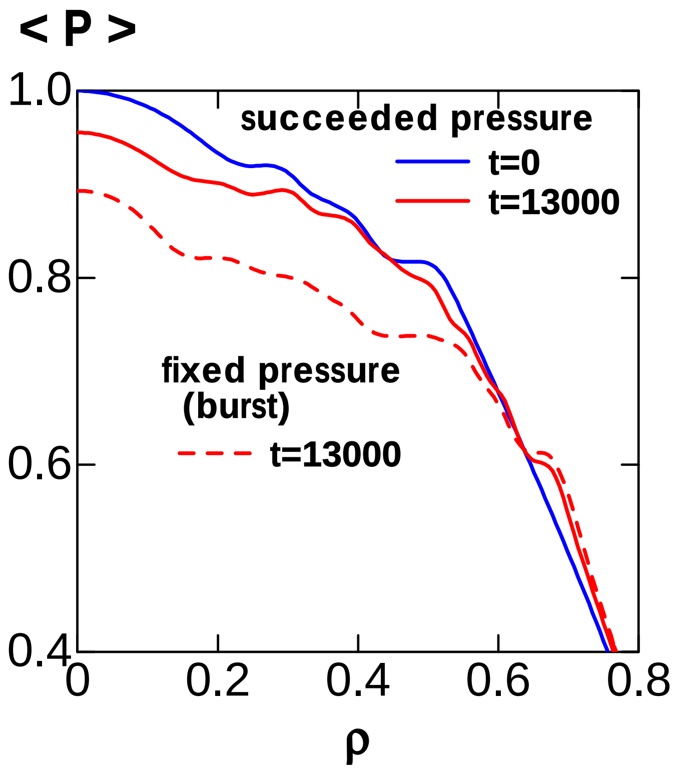
Stable Path to High Beta Regime (cont.)

Stream Lines at final state ($t = 13000\tau_A$, $\zeta = 0, \rho \leq 0.8$)



- ▷ Localization around resonant surfaces due to reduction of driving force
- ▷ Weak interaction between modes

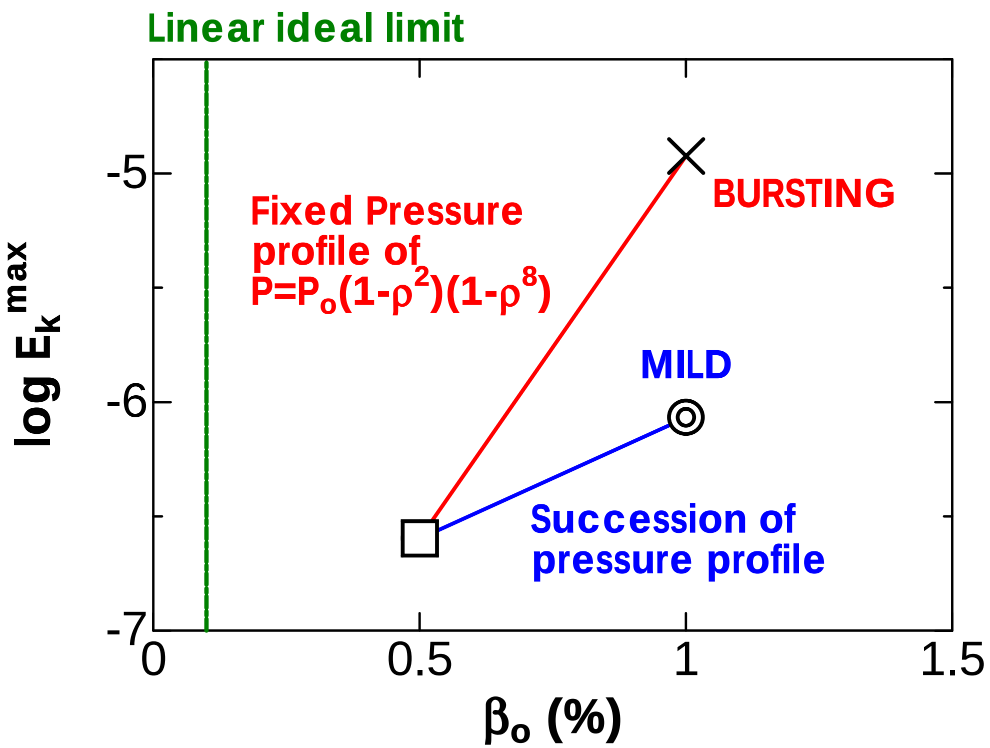
Average Pressure Profile



- ▷ Reduction of pressure is much smaller than in bursting case.

Stable Path to High Beta Regime (cont.)

Diagram of stable path to high beta regime in LHD



- ▷ Fixed Profile : High fluctuation level ⇒ Bursting : Unstable Path
- ▷ Succeeded Profile : Low fluctuation level ⇒ Mild : Stable Path

Succession of pressure profile : Stabilizing mechanism
Self-organization of pressure

LHD plasma traces the stable path to high β regime.

Conclusions

- **The nonlinear evolution of the interchange mode with multi-helicity is examined in the inward-shifted low-beta LHD equilibria with almost parabolic pressure profile.**

- **In the sufficiently low β case, the modes with different helicity interact indirectly through the local pressure variation.**
  Saturated pressure profile is locally flattened, as is predicted in the linear analysis.

- **Increasing β with the pressure profile fixed enhances the driving force.**
  Direct interaction of modes through the mode overlapping results in a bursting activity in the kinetic energy.
  The bursting activity may limit the achievable β, because the pressure is globally reduced in the core region.

- **The bursting behavior is suppressed if the saturated pressure profile is succeeded in the increase of β.**
  This results indicate that the pressure profile can be self-organized so that the LHD plasma should achieve high beta regime along a stable path.