Fluid Simulation on Pellet Ablation

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US/Japan Workshop on *Theoretical Considerations on Helical Plasmas*
Nassau Inn, Princeton, NJ
Nov. 18-20/2002

Fluid Simulation on Pellet Ablation

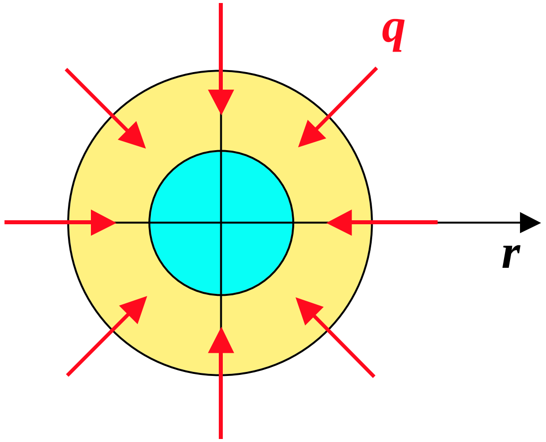
Pellet ablation problem

1. Toroidal curvature effect.
2. Elongated ablation cloud.
3. Sheath effect between ablation cloud and bulk plasma.

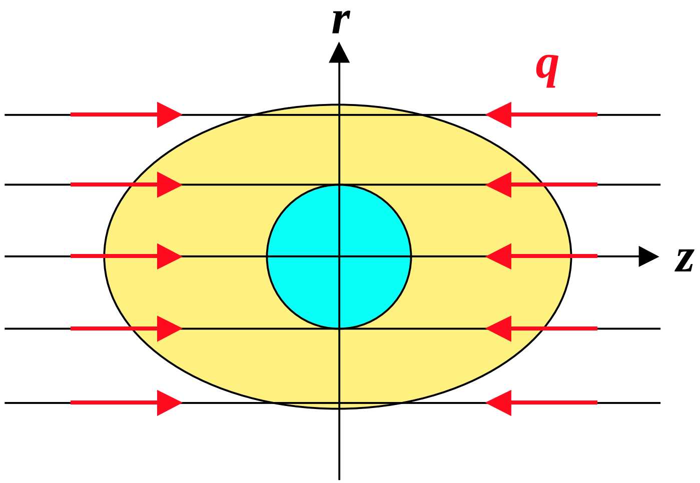
2D axisymmetric pellet ablation code is developed to study the interaction of the ablation cloud with B-field.

Present Talk

1. Atomic process with dissociation and ionization.
2. Dimensional Effect by heat flux along B-field.



1D spherical symmetry



2D cylindrical symmetry

Outline

1. Basic equations.
2. Electron heat flux.
3. Cubic-Interporated Pusudeparticle (CIP) method.
4. Spherical pellet in 1D spherical symmetry.
5. Spherical pellet in 2D cylindrical symmetry.
6. Conclusions.

Basic Equations

$$\mathrm{D_2solid} \xrightarrow{\text{Sublimation}} \mathrm{D_2gas} \xrightarrow{\text{Disociation}} \mathrm{D} \xrightarrow{\text{Ionization}} \mathrm{D^+ + e^-}$$

$$f_d = \frac{n_a + n_i}{2n_m + n_a + n_i} \qquad \frac{f_d^2}{1 - f_d} = 1.55\times10^{24}\,\frac{T^{0.327}[eV]}{n\left[cm^{-3}\right]}\exp\left(-\frac{\varepsilon_d}{T}\right)$$

$$f_i = \frac{n_i}{2n_m + n_a + n_i} \qquad \frac{f_i^2}{1 - f_i} = 3.0\times10^{27}\,\frac{T^{3/2}[eV]}{n\left[cm^{-3}\right]}\exp\left(-\frac{\varepsilon_i}{T}\right)$$

$$\varepsilon_\mathrm{s} = 0.01\,[\mathrm{eV}]$$

$$\varepsilon_\mathrm{d} = 4.48\,[\mathrm{eV}]$$

$$\varepsilon_\mathrm{i} = 13.6\,[\mathrm{eV}]$$

$$\frac{d\rho}{dt} = -\rho\nabla\cdot u$$

$$\rho\frac{du}{dt} = -\nabla p$$

$$\rho\frac{de}{dt} = -p\nabla\cdot u + H$$

$$p = p_\mathrm{s} + \left(\frac{1}{2} + \frac{1}{2}f_\mathrm{d} + f_\mathrm{i}\right)\frac{\rho T}{m}$$

$$e = (1 - f_\mathrm{s})e_\mathrm{s} + \left[\frac{1 - f_\mathrm{d}}{2(\gamma_\mathrm{m} - 1)} + \frac{f_\mathrm{d} + f_\mathrm{i}}{\gamma - 1}\right]\frac{kT}{m} + \frac{1}{2}f_\mathrm{s}\frac{k\varepsilon_\mathrm{s}}{m} + \frac{1}{2}f_\mathrm{d}\frac{k\varepsilon_\mathrm{d}}{m} + f_\mathrm{i}\frac{k\varepsilon_\mathrm{i}}{m}$$

$$\gamma_\mathrm{m} = 7/5 \qquad \gamma = 5/3$$

Electron Heat Flux.

1. Mono-energetic heat flux.

$$H = \frac{\partial q}{\partial z} = q\Lambda(E)$$

$$\frac{\partial E}{\partial z} = \frac{\rho}{m} L(E)$$

q : Electron heat flux

E : Average energy of electron

Λ : Effective energy flux cross section

L : Electron loss function

P.B.Parks, Phys. Fluid 21, 1735 (1978).

2. Maxwellian electron heat flux

$$\frac{q}{q_\infty} = \frac{1}{2}\left(\frac{w}{\mu_g}\right) K_2\left[\left(\frac{w}{\mu_\infty}\right)^{1/2}\right]$$

$$\mu_g = 0.64$$

$$w = 1.23\tau / \tau_\infty$$

$$\tau = \int_z^\infty n dz$$

$$\tau_\infty = \frac{k_B^2 T_{e\infty}^2}{4\pi e^2 \ln \Lambda}$$

$$\ln \Lambda = f_i \ln \Lambda_{ef} + (1 - f_i) \ln \Lambda_{eb}$$

$$\Lambda_{ef} = \frac{1.35 \times 10^{10} T_{e\infty}^2 [eV]}{n_e^{1/2} [cm^{-3}]}$$

$$\Lambda_{eb} = \frac{2.516 T_{e\infty} [eV]}{7.5}$$

K_2 : Modified Bessel

P.B.Parks, Phys. Plasmas 7, 1968 (2000).

Compressible and incompressible fluids are simultaneously calculated by solving a Poisson equation on pressure.

$$\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot u$$
$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \nabla p$$
$$\frac{\partial e}{\partial t} = -\frac{p}{\rho} \nabla \cdot u$$

+ $p = \dfrac{\rho T}{m}$ replace ⇨

(1) (2) (3)

$$\nabla \cdot \left(\frac{1}{\rho^*} \nabla p^{n+1} \right) = \frac{p^{n+1} - p^*}{\Delta t^2 \left(\rho c_s^2 + \dfrac{P_{TH}}{\rho C_v T} \right)^*} + \frac{\nabla \cdot u^*}{\Delta t}$$

$$c_s^2 = \left.\frac{\partial p}{\partial \rho}\right|_T, \quad P_{TH} = T \left.\frac{\partial p}{\partial \rho}\right|_\rho, \quad C_v = \left.\frac{\partial e}{\partial T}\right|_\rho$$

$$\frac{p/\rho}{\Delta x^2} \sim \frac{p/\rho}{c_s^2 \Delta t^2} + \frac{u}{\Delta x \Delta t}$$

⇩

$$\left(\frac{\Delta t}{\tau_u} \right)^2 \sim \left(\frac{\tau_c}{\tau_u} \right)^2 + \frac{\Delta t}{\tau_u}$$

with $p/\rho \sim u^2$

τ_c : Sound

τ_u : Fluid motion

Incompressible $\quad u^2 << c_s^2 \Rightarrow \tau_u > \Delta t > \tau_c \quad (2) \to 0$

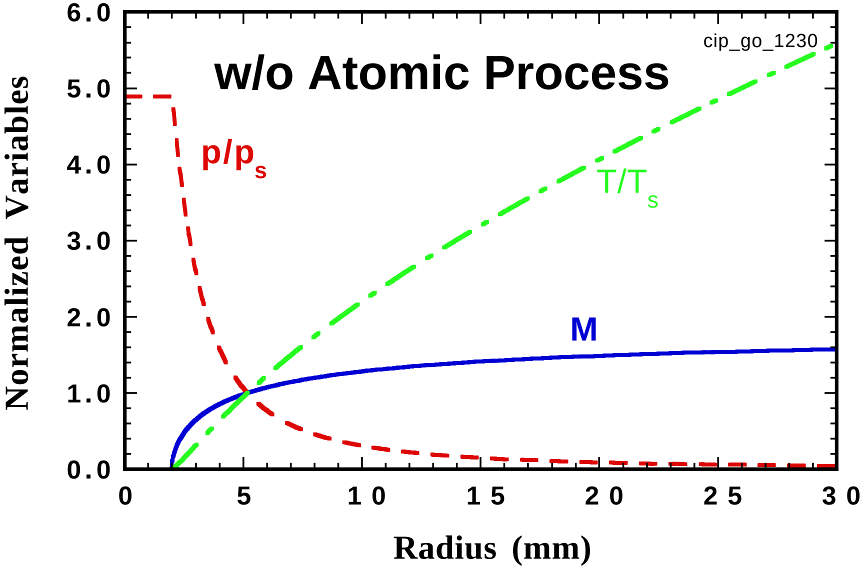
$$\frac{\nabla \cdot u^{n+1} - \nabla \cdot u^*}{\Delta t} = -\nabla \cdot \left(\frac{1}{\rho^*} \nabla p^{n+1} \right) \quad \Rightarrow \quad \nabla \cdot u^{n+1} = 0$$

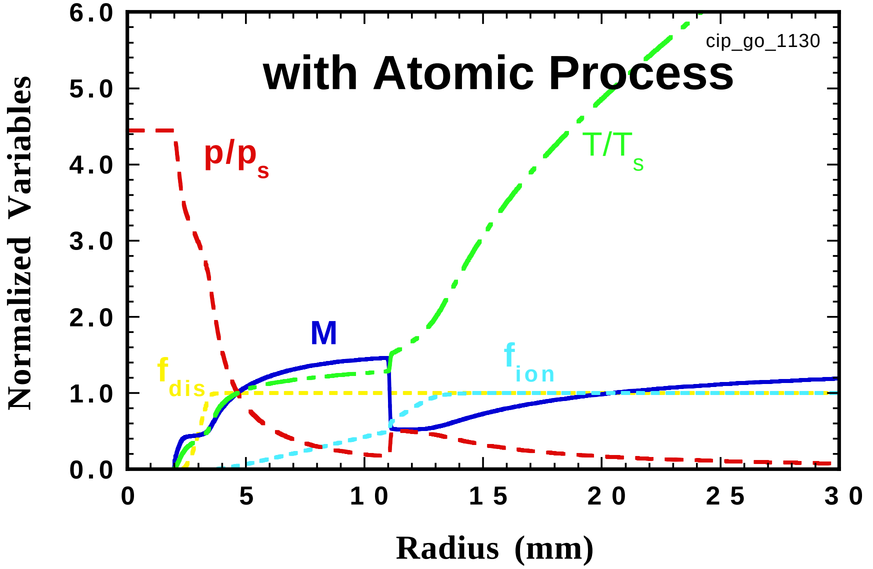
Compressible $\quad u^2 \geq c_s^2 \Rightarrow \tau_c \geq \tau_u > \Delta t \quad (1) \to 0$

$$0 = \frac{p^{n+1} - p^*}{\gamma p^* \Delta t^2} + \frac{\nabla \cdot u^*}{\Delta t} \quad \Rightarrow \quad \frac{p^{n+1} - p^*}{\Delta t} = -\gamma p^* \nabla \cdot u^*$$

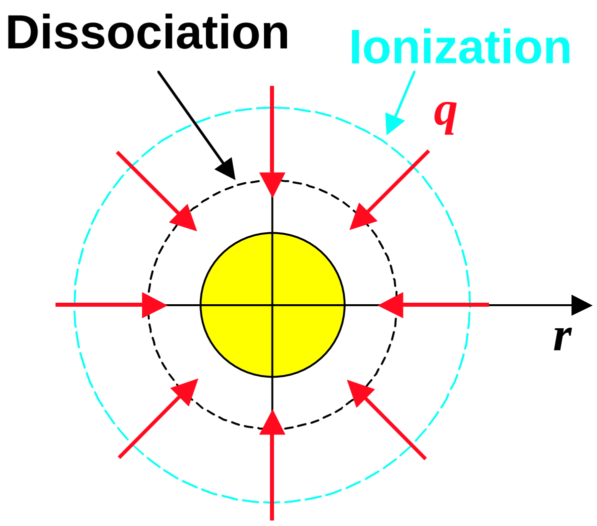
A shock is driven by ionization.

$$r_p = 2\ \mathrm{mm} \quad T_e = 2\ \mathrm{keV} \quad n_e = 10^{20}\ \mathrm{m}^{-3} \quad t = 8\ \mu\mathrm{s}$$



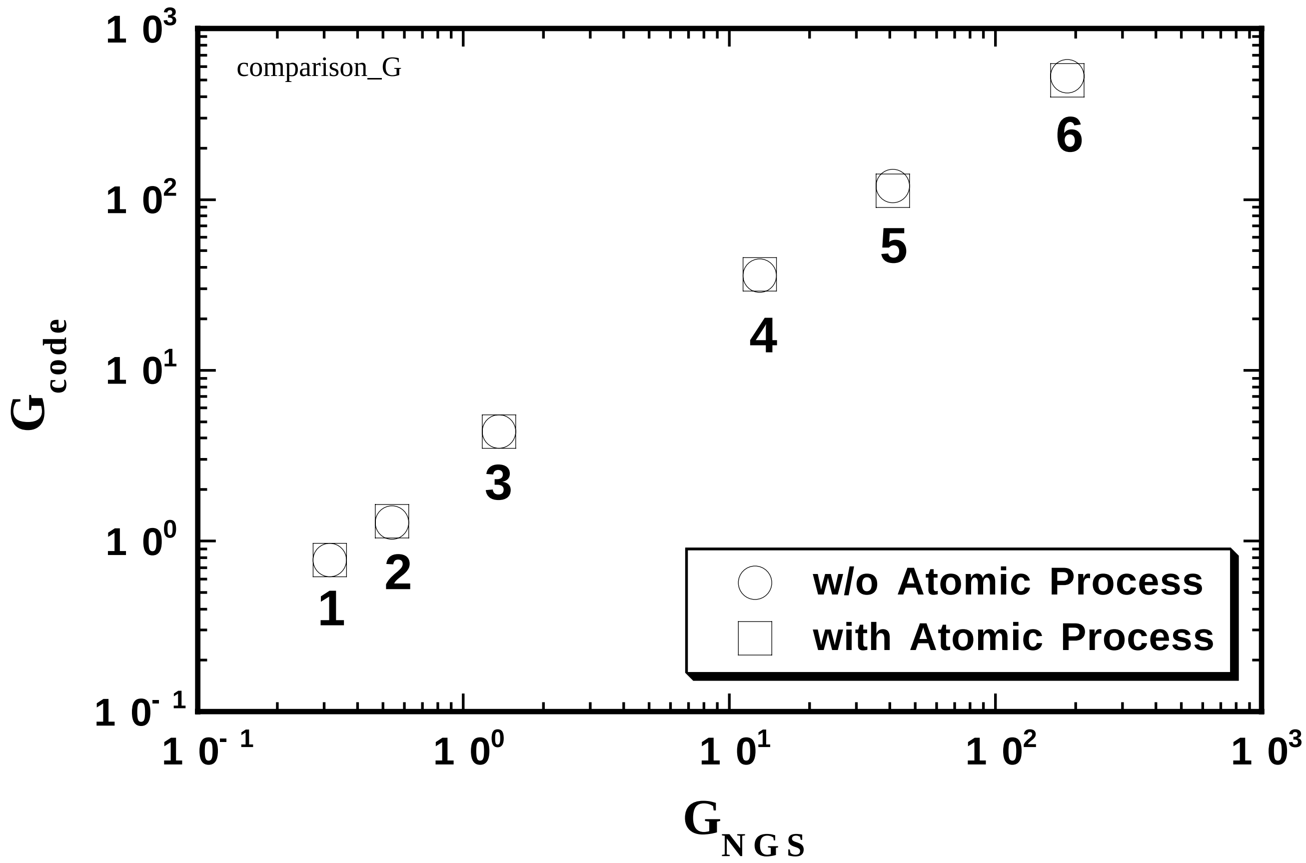


$$p_s = 2.8\ \mathrm{MPa}$$
$$T_s = 3.6\ \mathrm{eV}$$
$$r_s = 5.15\ \mathrm{mm}$$
$$G = 120\ \mathrm{g/s}$$



$$p_s = 2.4\ \mathrm{MPa}$$
$$T_s = 1.6\ \mathrm{eV}$$
$$r_s = 4.63\ \mathrm{mm}$$
$$G = 110\ \mathrm{g/s}$$

Difference of the ablation rate between with atomic process and without it is less than 10%.

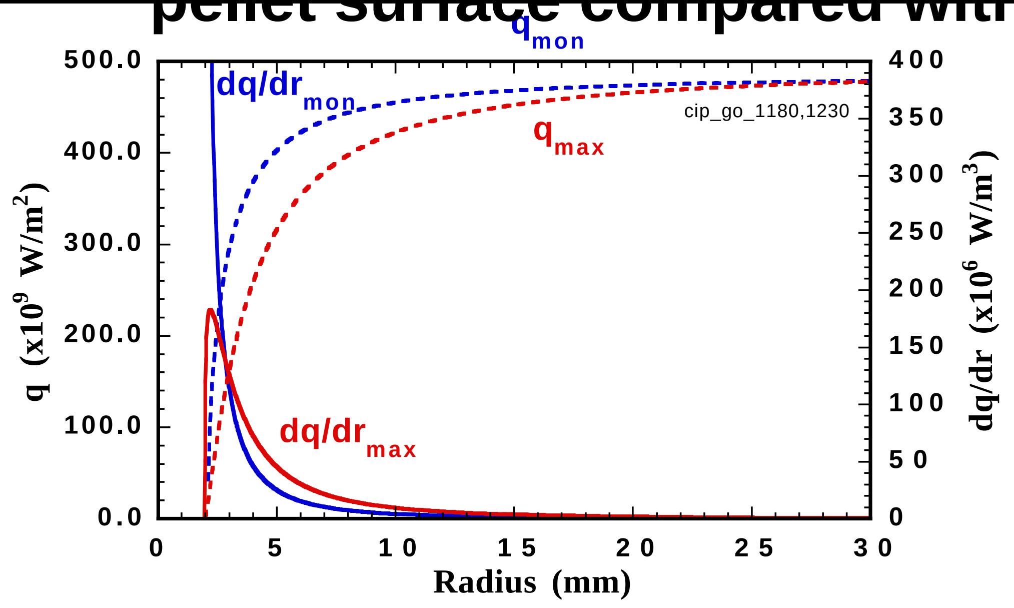


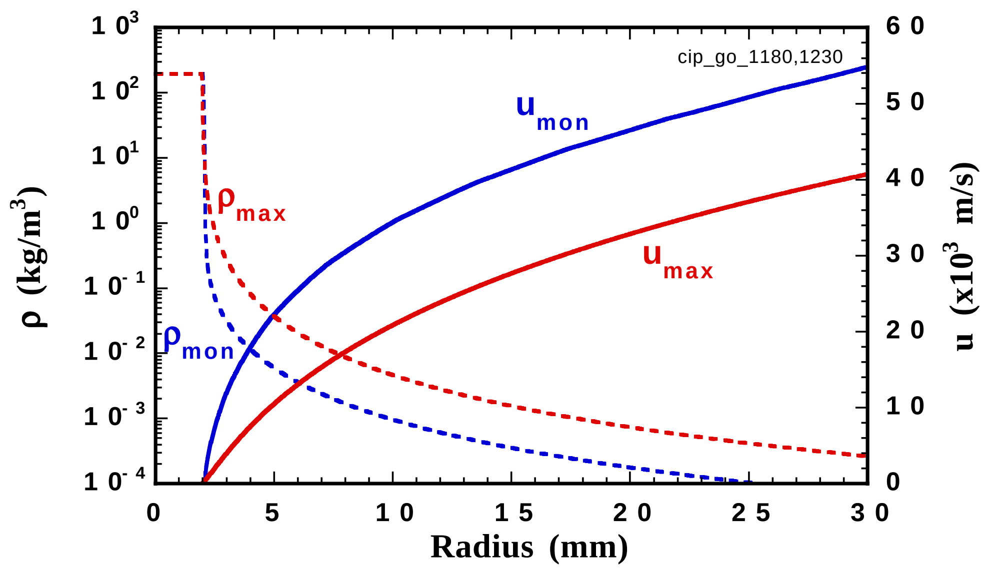
The use of a Maxwellian heat flux enhances the ablation rate by about a factor of three in comparison with NGS model.

Parameters used in the code.

Case	Radius (mm)	Tempereture (keV)	Density (m^{-3})
1	0.5	0.5	0.1×10^{20}
2	0.5	0.5	0.5×10^{20}
3	1.0	0.5	0.5×10^{20}
4	1.0	2.0	0.5×10^{20}
5	2.0	2.0	1.0×10^{20}
6	2.0	5.0	1.0×10^{20}

A Mono-energetic heat flux is deposited around the pellet surface compared with a Maxwellian one.





Ablation rate

$$G = 4\pi r^2 \rho u$$

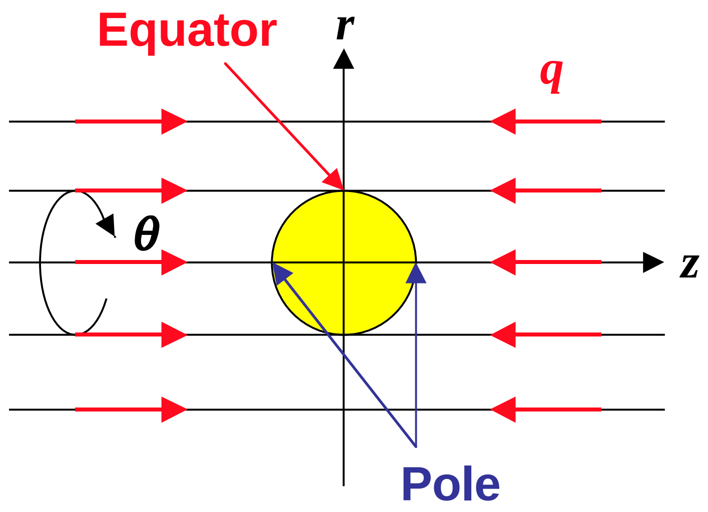
$$\rho = \rho_0 \exp(-\nabla \cdot u \Delta t)$$

$$\frac{G_{\mathrm{mon}}}{G_{\mathrm{max}}} = \frac{u_{\mathrm{mon}}}{u_{\mathrm{max}}} \exp\left(-\frac{u_{\mathrm{mon}} - u_{\mathrm{max}}}{L} \Delta t\right)$$

Geometry for simulation

$\mathrm{D_2}$ solid : $r_p = 2$ mm

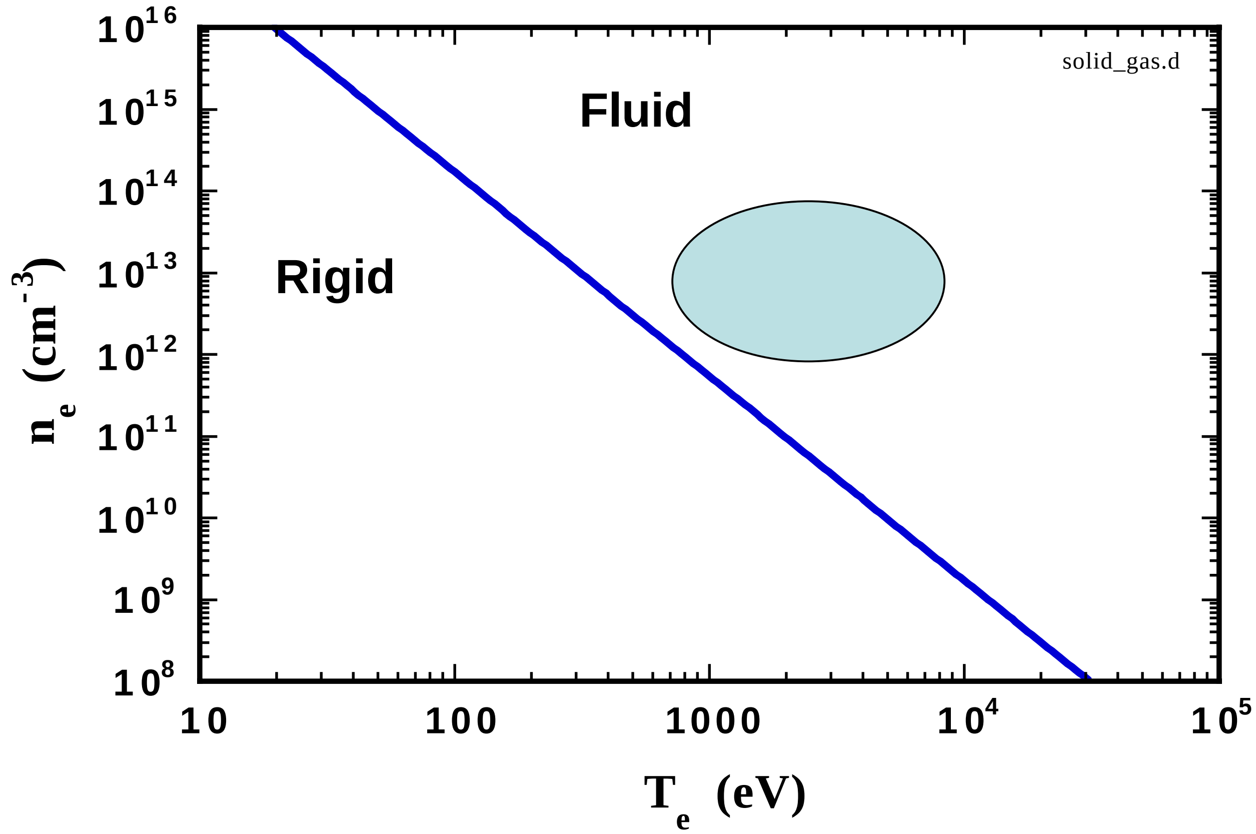
Bulk Plasma : $T_e = 2$ keV $n_e = 10^{20}$ m^{-3}



2D cylindrical symmetry

Solid deuterium behaves like incompressible fluid in the pressure > 0.5 MPa

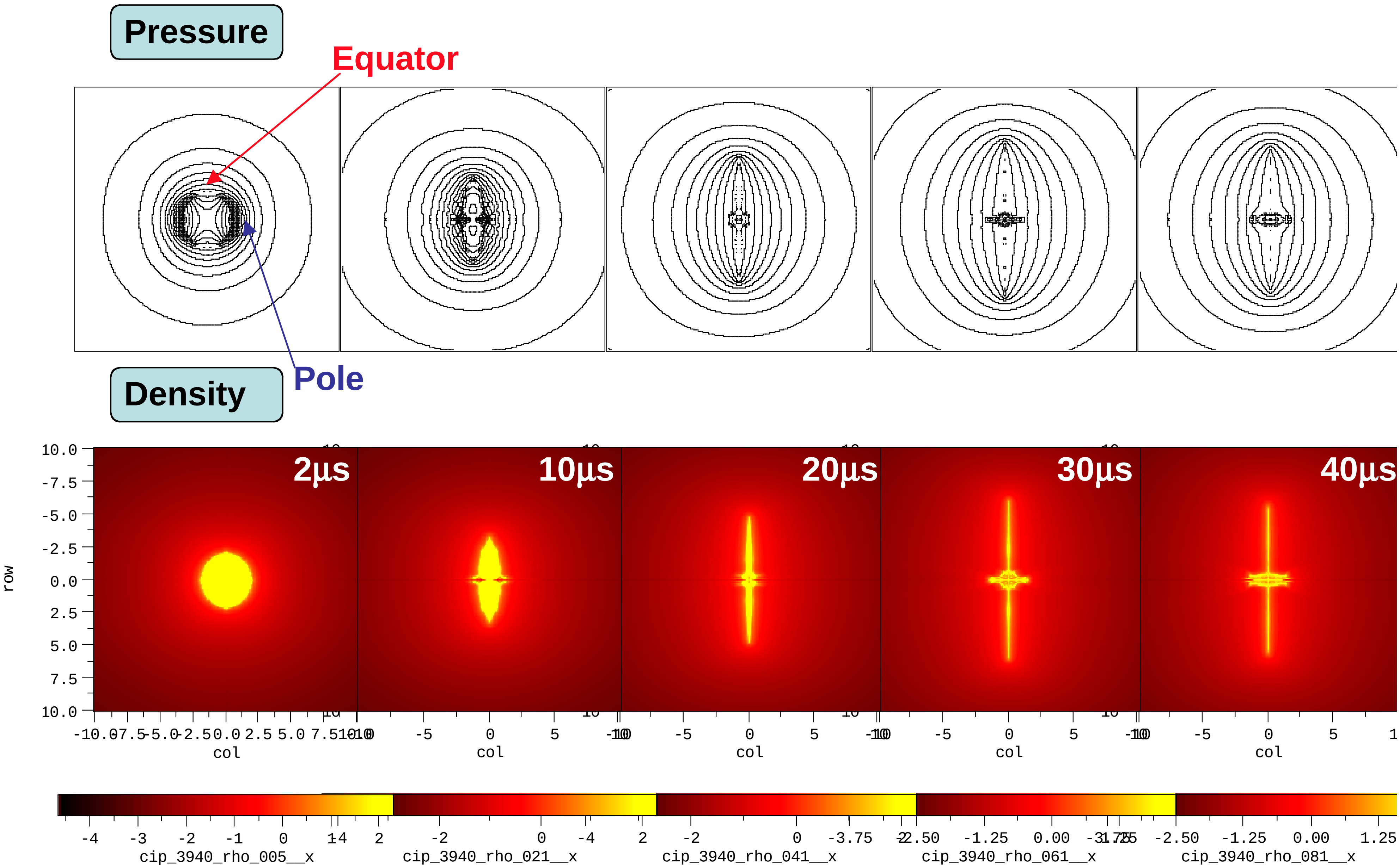
D_2 solid r_p = 2mm



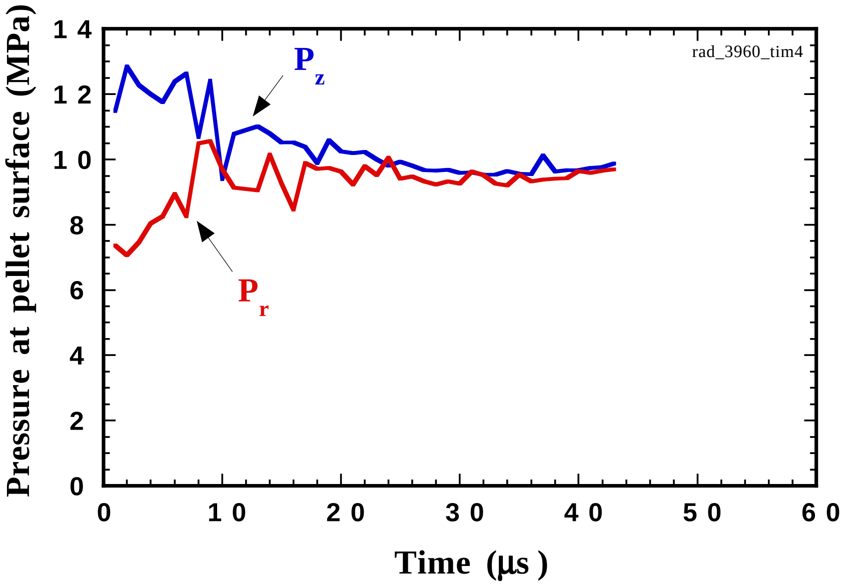
Solid deuterium behaves like incompressible fluid in the pressure > 0.5 MPa

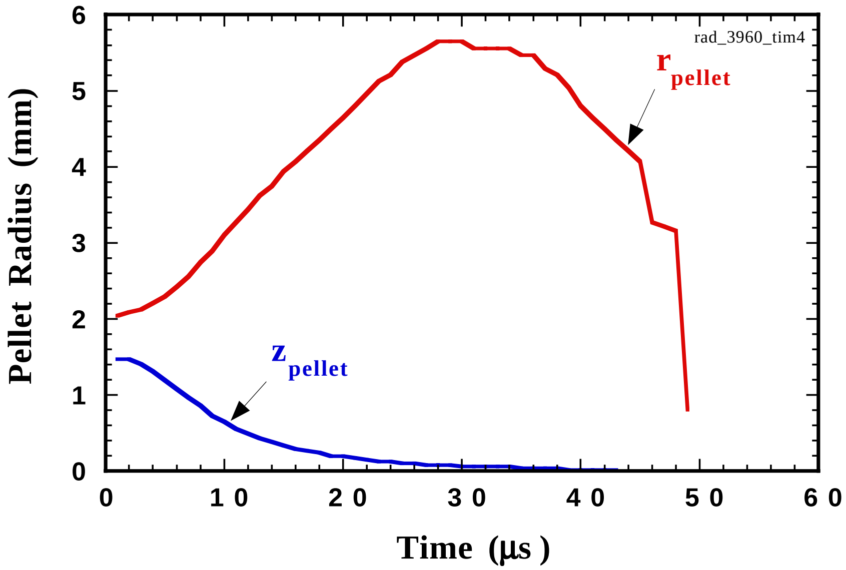
P.C.Souers, Hydrogen Properties for Fusion Energy (1986) p.85

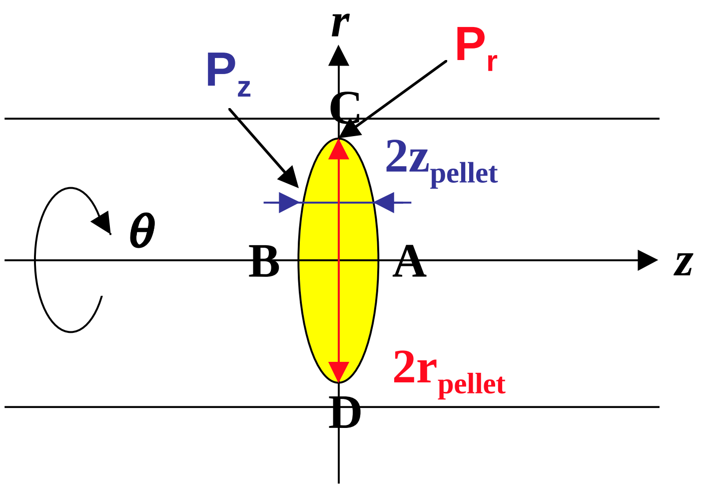
A pellet is deformed by nonuniform heat flux on the pellet surface.



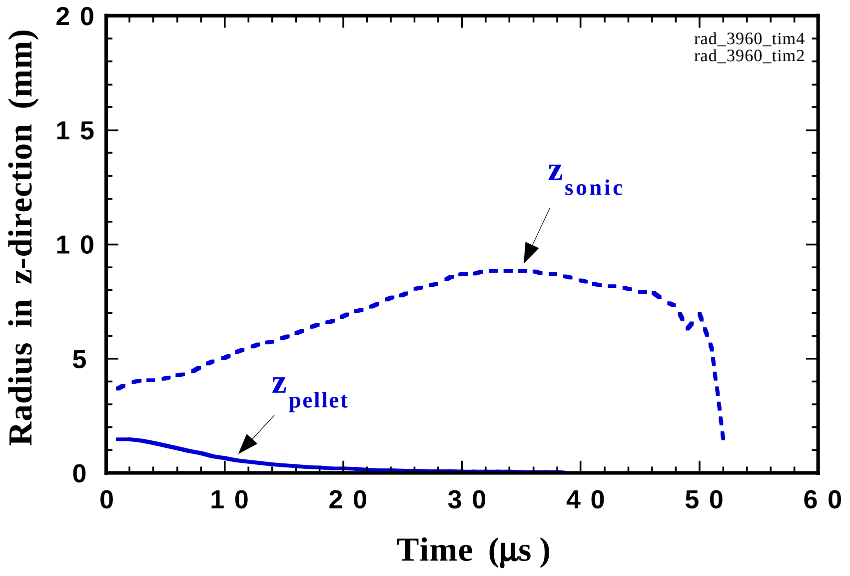
A pellet is deformed by nonuniform pressure on the pellet surface.

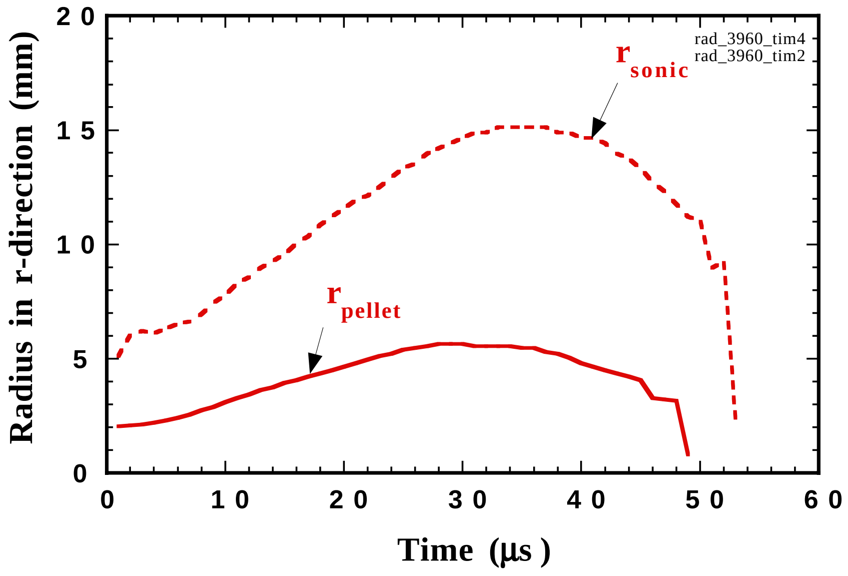


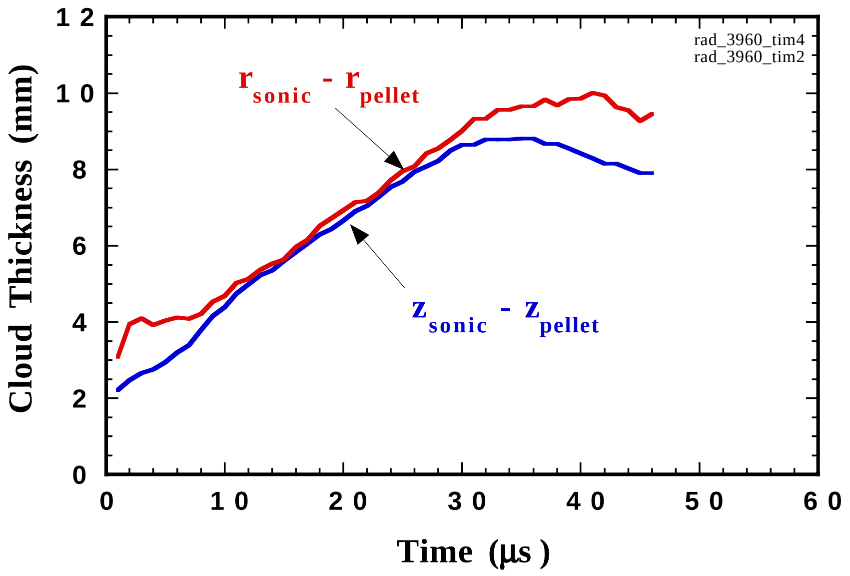


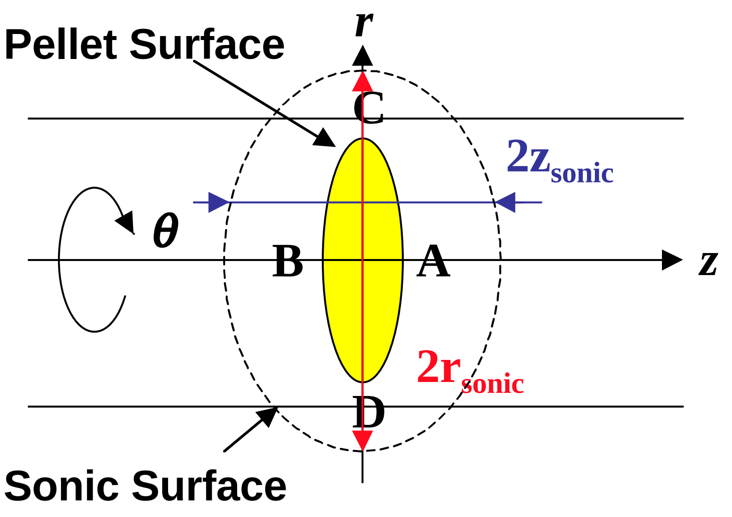


A cloud thickness in the z-direction is fairly equal to one in the r-direction.

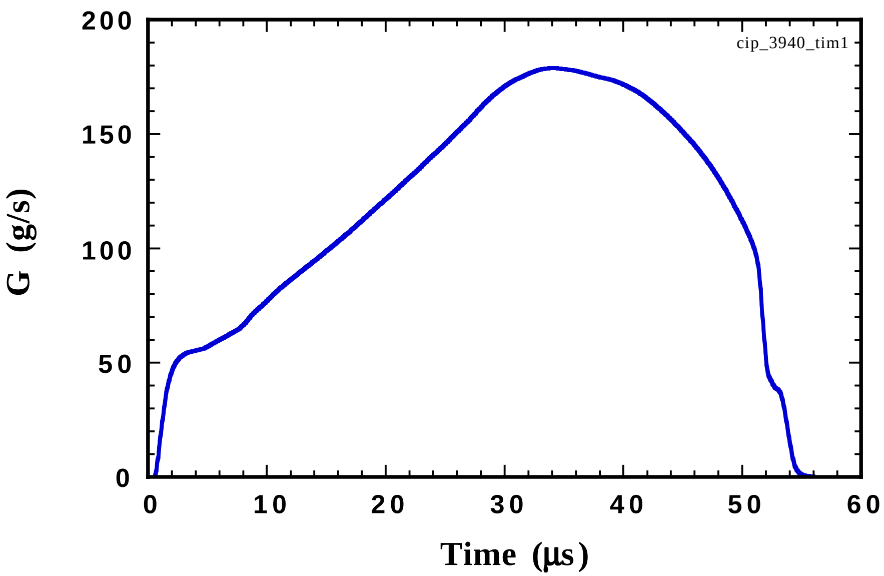


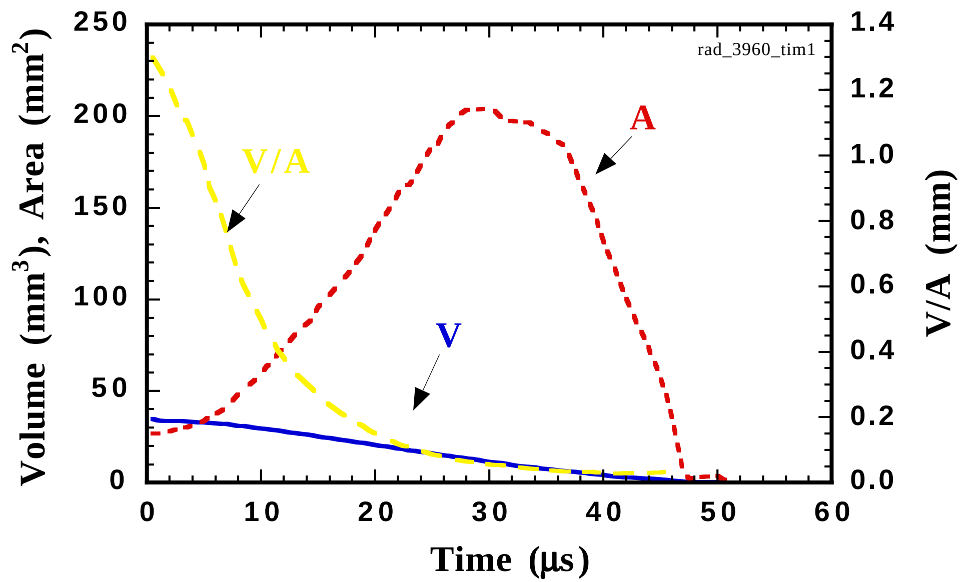


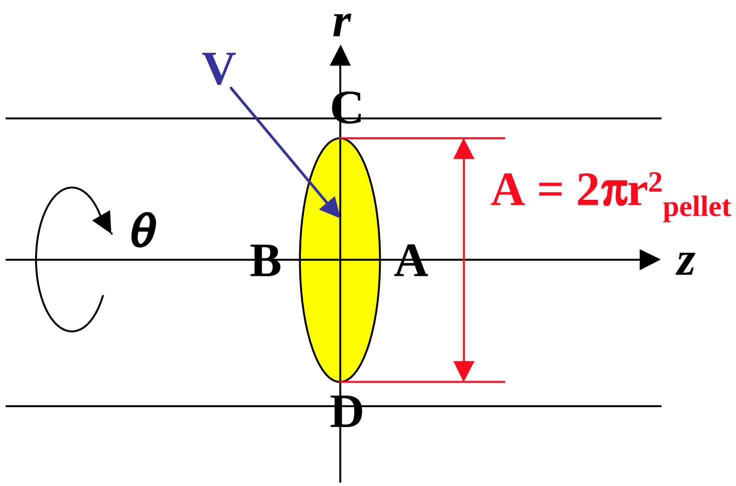




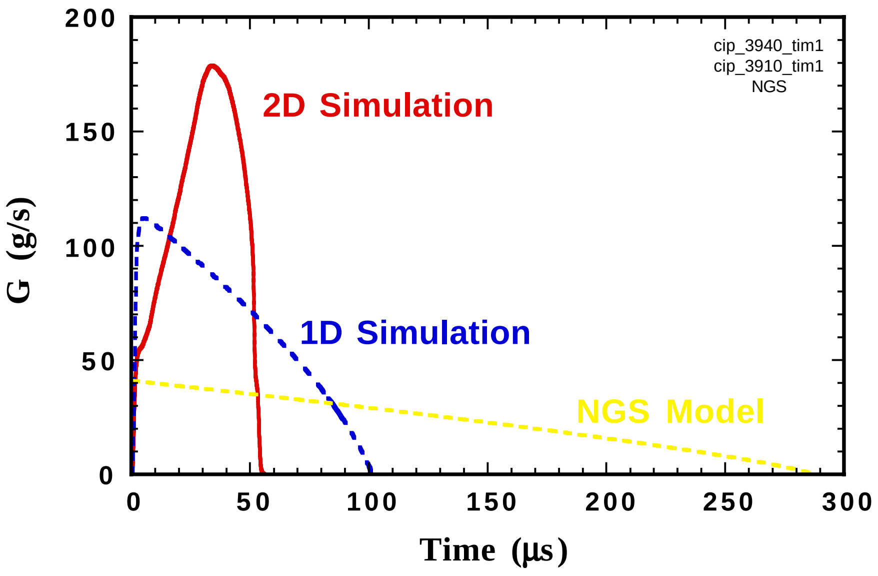
Volume & Effective Area.

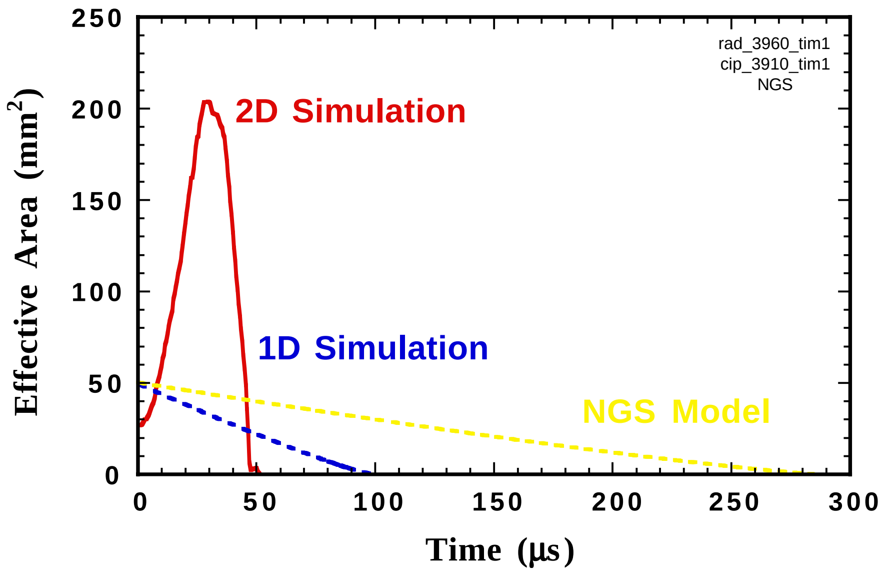






Comparison between 1D and 2D cases on ablation rate.





1. Difference between 2D and 1D : Pellet deformation
2. Difference between 1D and NGS : Heat flux treatment

NGS Model

$$G = Cr_p^{4/3} = -4\pi r_p^2 \dot{r}_p \rho_{solid}$$

$$\frac{G}{G_0} = \left(1 - \frac{5G_0}{9M_0}t\right)^{4/5}$$

$$\frac{A}{A_0} = \left(1 - \frac{5G_0}{9M_0}t\right)^{6/5}$$

G_0:Initial ablation rate

M_0:Initial mass

Conclusions.

1. A stationary shock wave is driven because expanding ablation cloud looses the ionization energy.

2. A shock is not driven by dissociation because dissociation is dominant in the subsonic region.

3. Difference of the ablation rate between with atomic process and without it is less than 10%.

4. In the 2D cylindrical geometry with heat flux parallel to B-field, the pellet is deformed by nonunifrom ablation pressure.

5. Pellet life time becomes shorter than one of the ablation model due to the pellet deformation.

Future work

1. Flow due to JxB force will be clarified with 2D code, and ablation channel will be also shown because the ablation flow across the B-field is impeded.