

Application of the HINT code to Heliotron J Plasmas

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1. Introduction

Advantage of Helical Systems

1. nested magnetic surface
2. net-current free operation



In helical systems, many equilibria are obtained from VMEC equilibrium with fix-boundary constraint and net-current free.

For Heliotron J plasmas, **fixed boundary VMEC equilibrium** was mainly used for the analysis

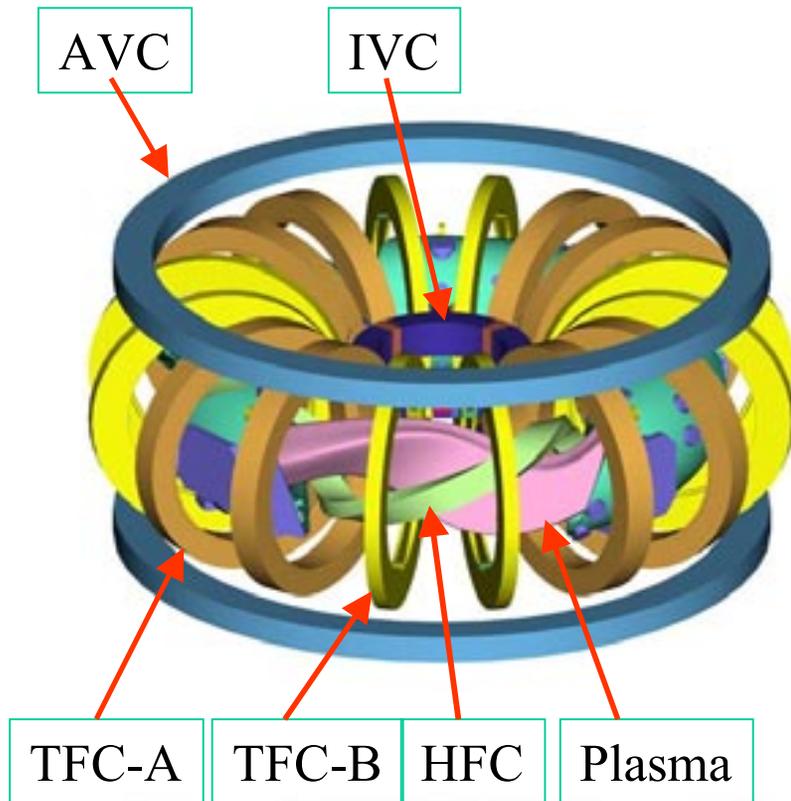
However, in equilibria at high beta operation or in experimental conditions...

1. **plasma boundary is changed by finite beta effect**
2. **net-currents can exist** e.g. bootstrap current, Ohkawa current

 It is necessary to do free boundary calculations

Heliotron J device

L=1/M=4 Helical Axis Heliotron Configuration



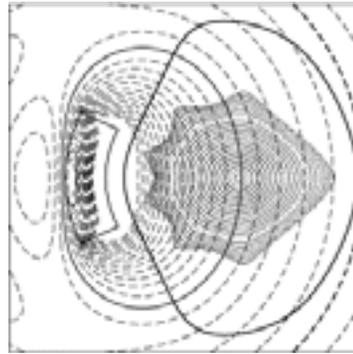
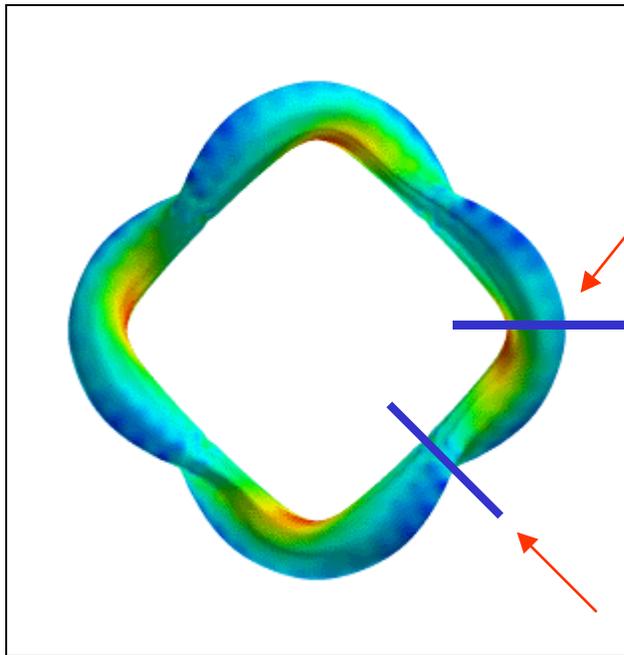
Device Parameters

Major Radius	1.2m
Helical coil Minor Radius	0.4m
Plasma Minor Radius	~0.15m
Magnetic Field Strength	1.5T
Helical Coil Current	0.96MA
Vertical Coil Current	0.84MA

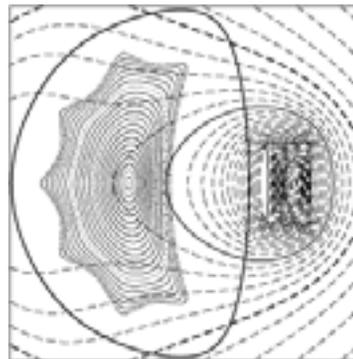
Bumpy component can be widely varied by two sets of TF coils (TA & TB)

Standard configuration (STD) of Heliotron J device

Top view of STD plasmas

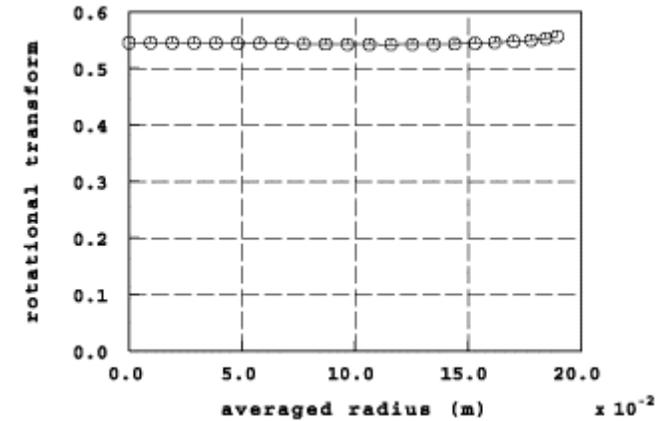


$$M\phi = 0$$

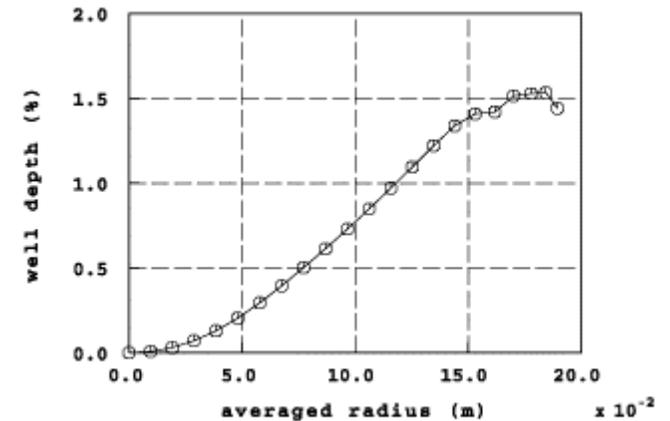


$$M\phi = 180$$

rotational transform



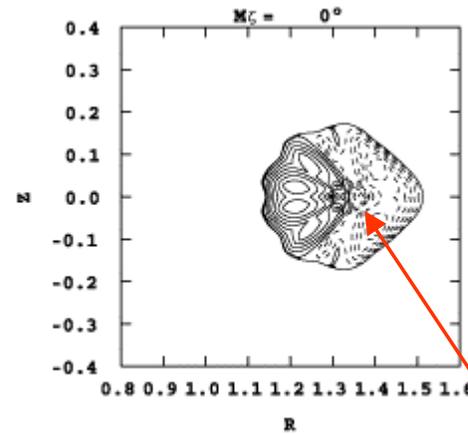
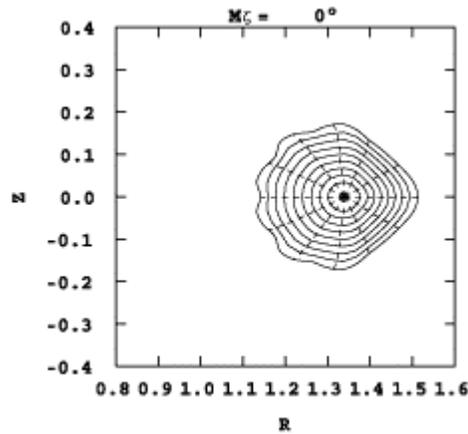
magnetic well



In standard configuration, rotational transform is very sensitive to finite β effect and magnetic well exist at vacuum.

Fixed boundary calculation using VMEC code

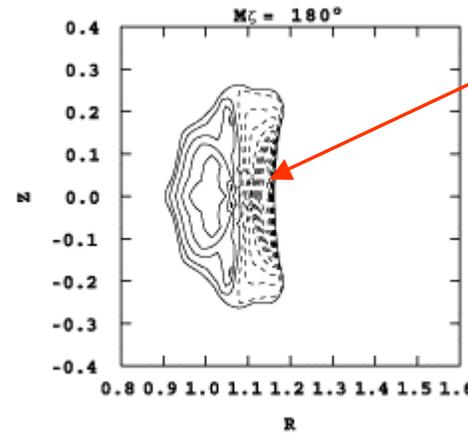
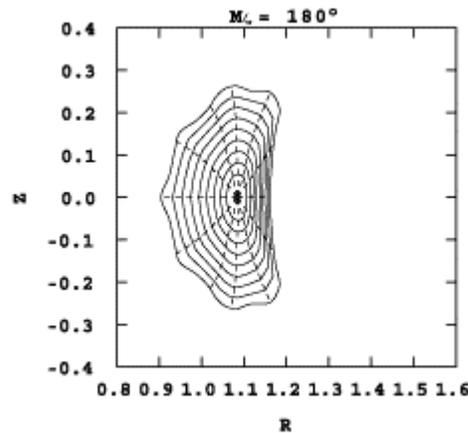
$\beta_{axis} \sim 1\%$ pressure profile $p = p_0(1-s)^2$



Parallel current calculated from local equilibrium



Resonant rational surface exists



$$\iota / 2\pi = 4/7$$

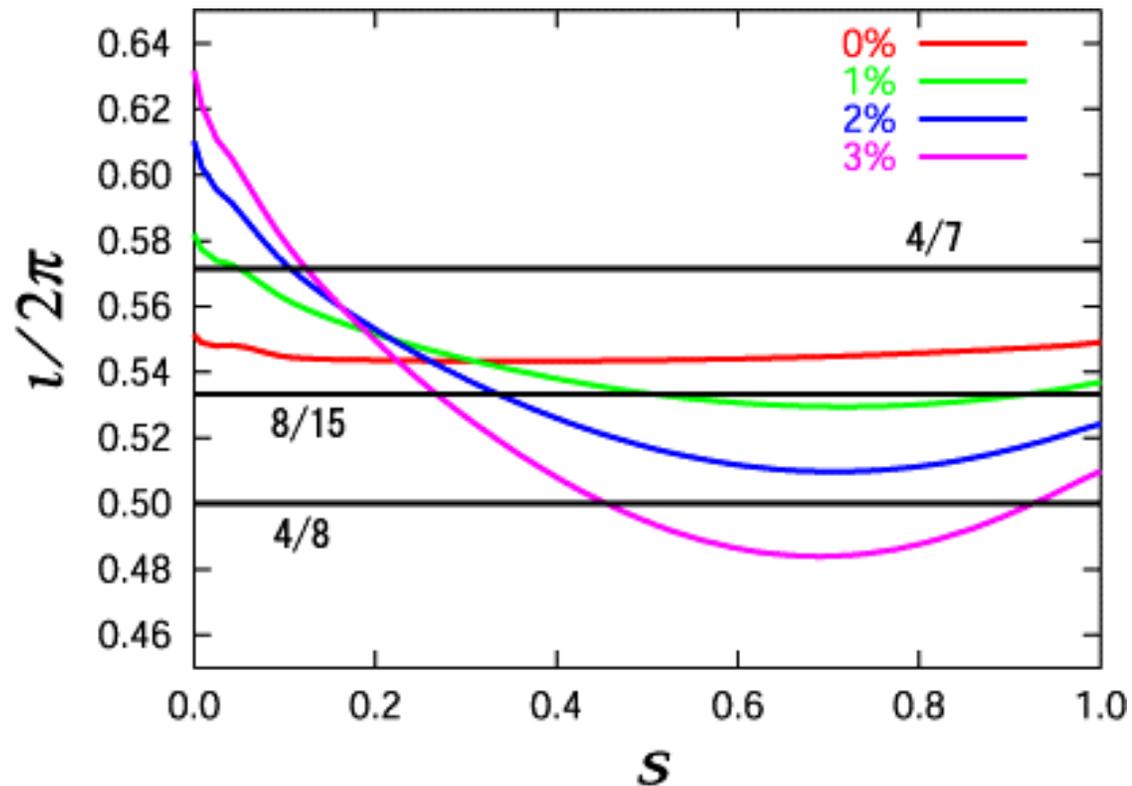
Flux surfaces

Parallel current

Rotational transform

pressure profile $p = p_0(1 - s)^2$

Rotational transform



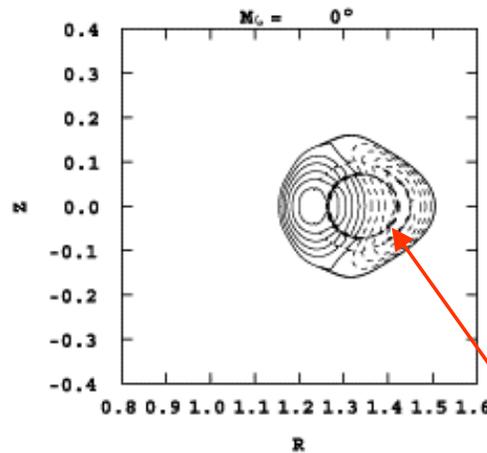
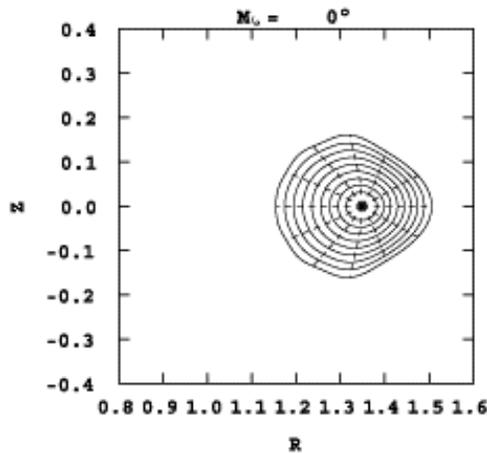
Rotational transform increased in inner region and decreased in outer region.

2. Free boundary calculation using VMEC code

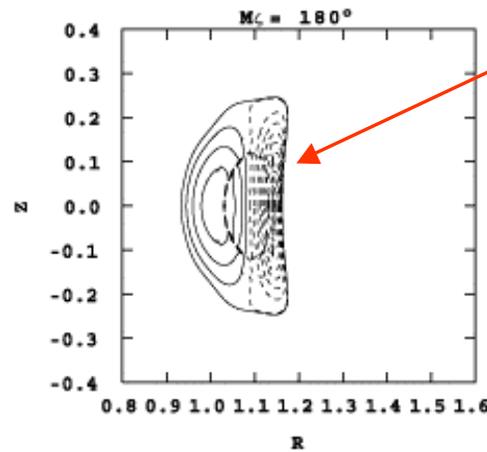
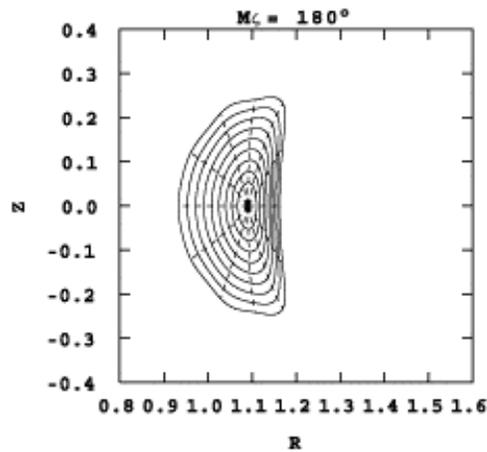
$\beta_{axis} \sim 1\%$

pressure profile $p = p_0(1-s)^2$

Total toroidal flux fixed



$NS = 61$
 $0 \leq m \leq 11$
 $-12 \leq n \leq 12$
 $N_u = 64$
 $N_v = 64$



$$\iota / 2\pi = 4/7$$

Position of rational surface changes because of larger Shafranov shift

It is difficult to reproduce wavy plasma boundary.

Flux surfaces

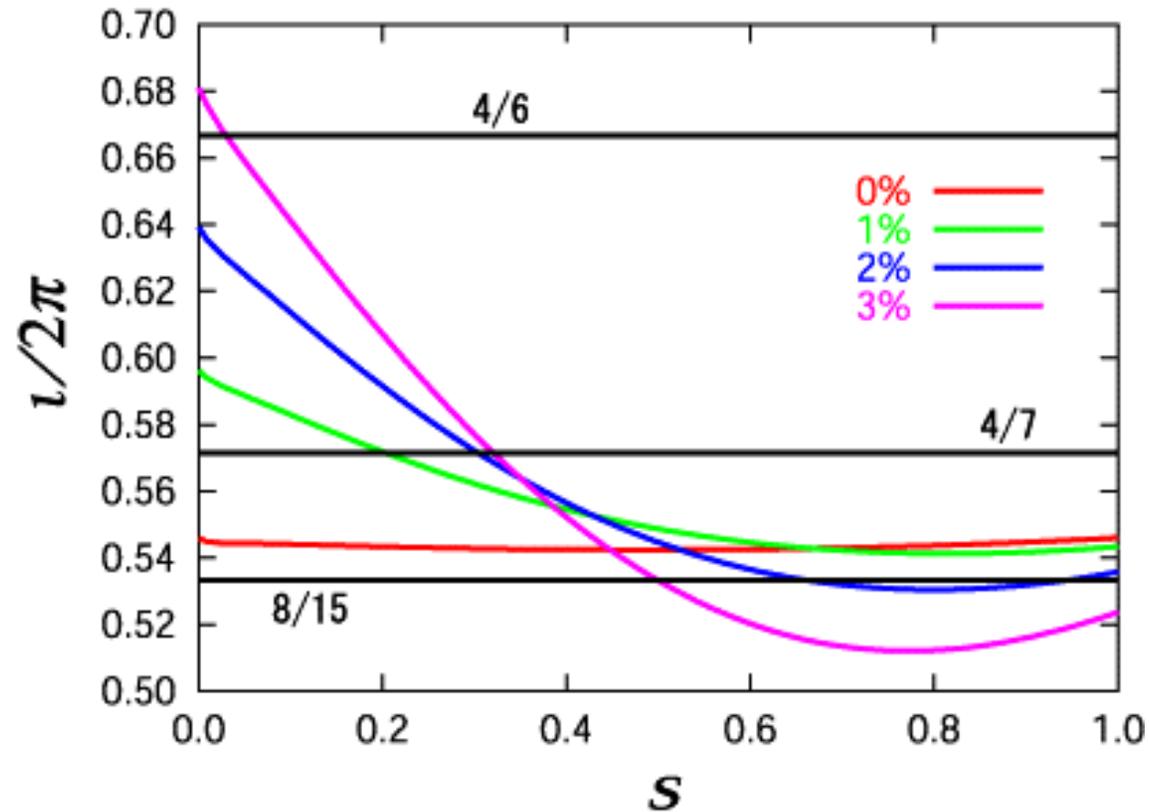
Parallel current

➡ Parameter survey!

Rotational transform

pressure profile $p = p_0(1 - s)^2$

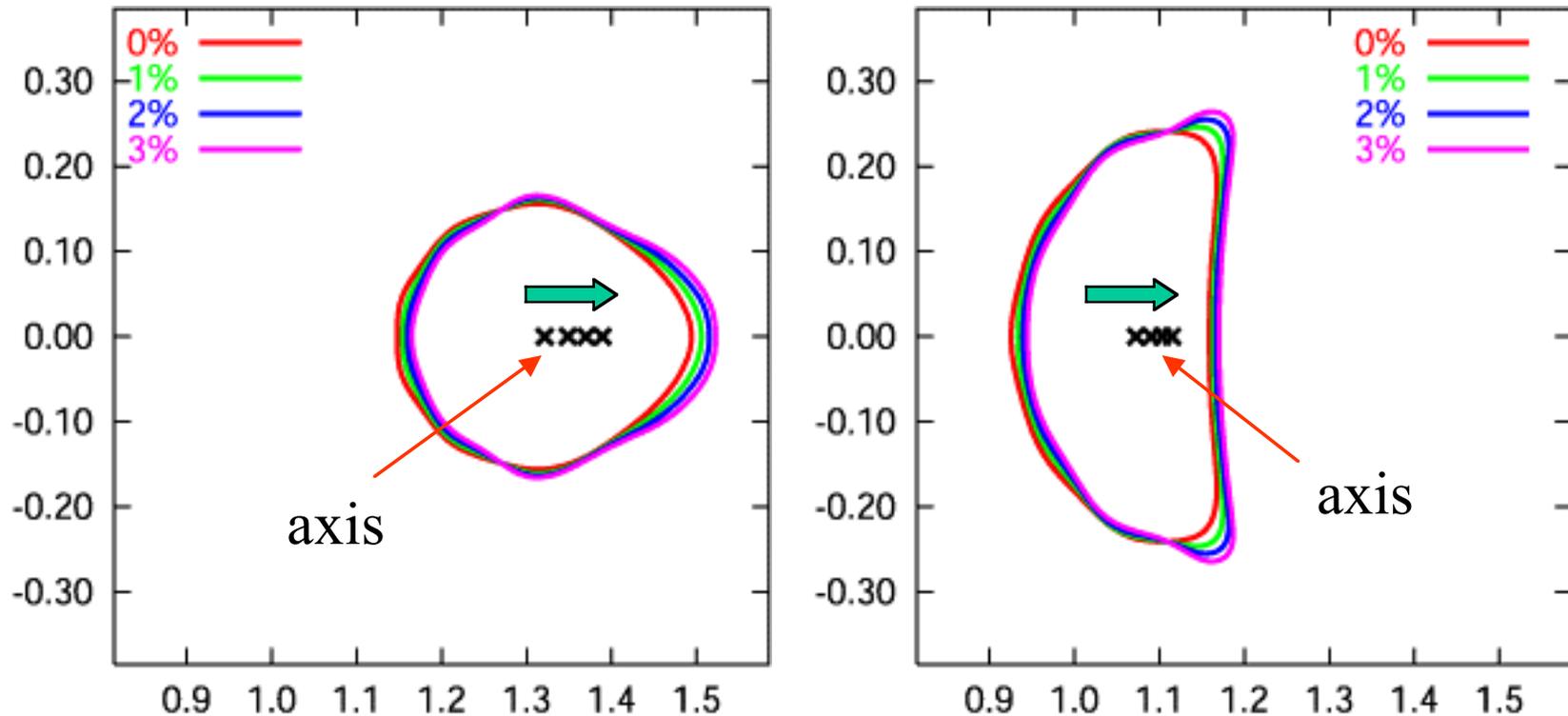
Rotational transform



Position of resonant surface changes because of Shafranov shift.

Change of Last Closed Flux Surface

pressure profile $p = p_0(1 - s)^2$



LCFS changed and plasmas shifted to outward by finite beta effect



It is necessary to do “free boundary calculation”

3. Free boundary calculation by HINT code

Outline of HINT

- relaxation method (initial value problem)
- Eulerian coordinate(rotating helical coordinate)

Rotating helical coordinate

Rotating helical coordinate (u^1, u^2, u^3) consists of rectangular grid (u^1, u^2) called “box” on $u^3 = \text{const.}$ plane. “box” rotates along toroidal direction.

Scheme of relaxation process (2-steps)

A-step

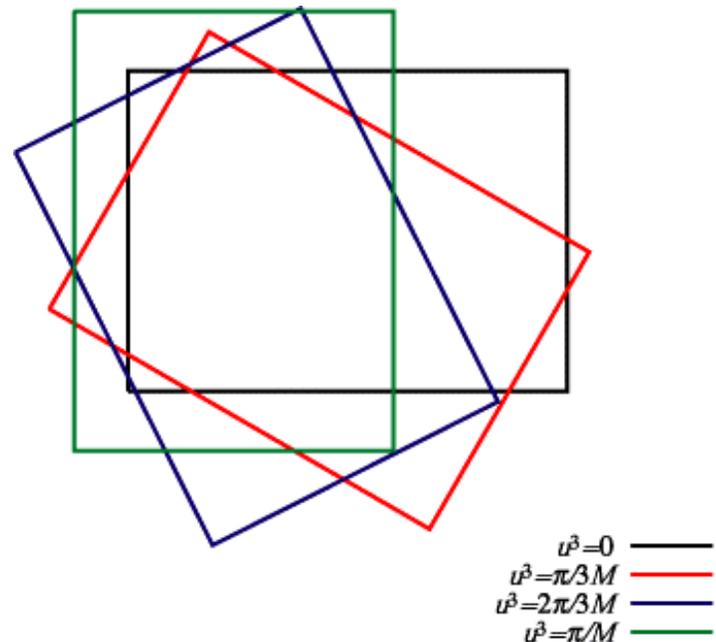
Relaxation of plasma pressure

B-step

Relaxation of magnetic field



Main loop



A-step : Relaxation of plasma pressure

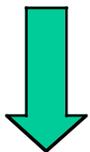
$$\mathbf{B} \cdot \nabla p = 0 \quad \longleftrightarrow \quad \bar{p} = \int p \frac{dl}{B} / \int \frac{dl}{B}$$

This process calculates constant pressure along field line.

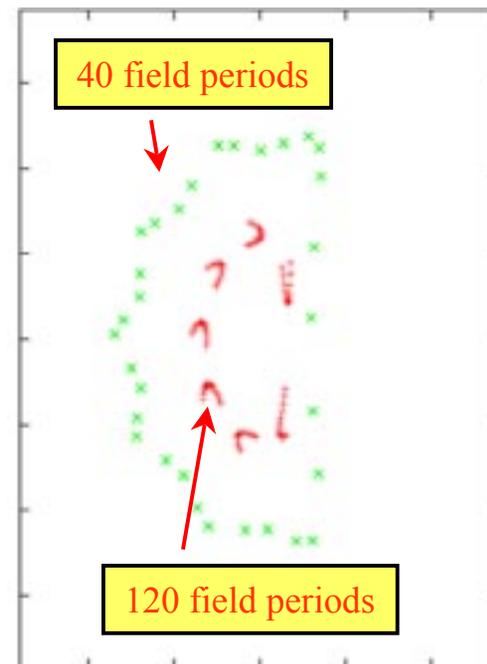
Averaged pressure is calculated on all grid points

Technical issue

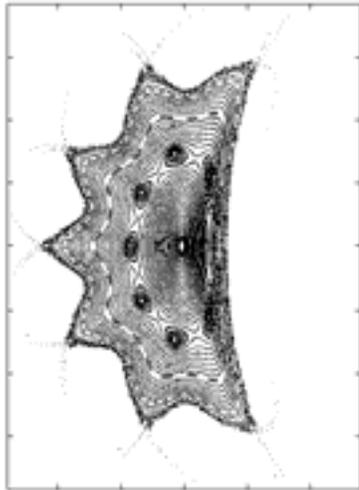
In a magnetic island or stochastic region,
field lines need to be traced very long!
In particular, **for low shear stellarators**,
this problem is very severe.



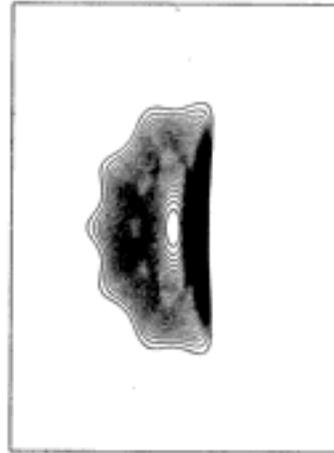
Computational time consuming !



pressure distribution in the magnetic islands



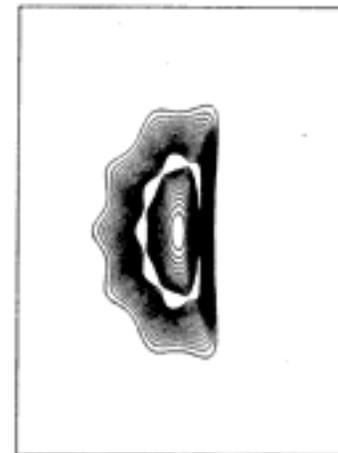
Flux surfaces



40 field periods



120 field periods



240 field periods

For traced around 240 field period to all grid points, computational time is about 3 hours. (SX-6, single CPU)

Grid number: $(u^1, u^2, u^3) = (137, 137, 55)$



Improved scheme of pressure relaxation

Outline

- Relaxed pressure should be the same all along field line.
- From single field line, we can know the relaxed pressure not just for the computational grid point but also many points over a flux surface.

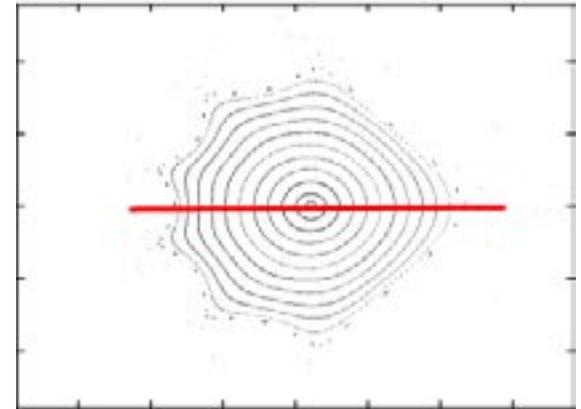


We used this information to help find the pressure at a grid near the flux surface.

Scheme

1. Some field lines are traced from start points.
2. Averaged pressure is calculated on field line obtained from step-1.
3. Relaxed pressure is calculated from the pressure of step-2 using interpolation.

Start points



Number of start points are 500 on $u^2=u^3=0$ plane. Field lines are traced for 1000 field period.



About 30 minutes!

Improved scheme of pressure relaxation (detail)

From step-1 and Step-2,

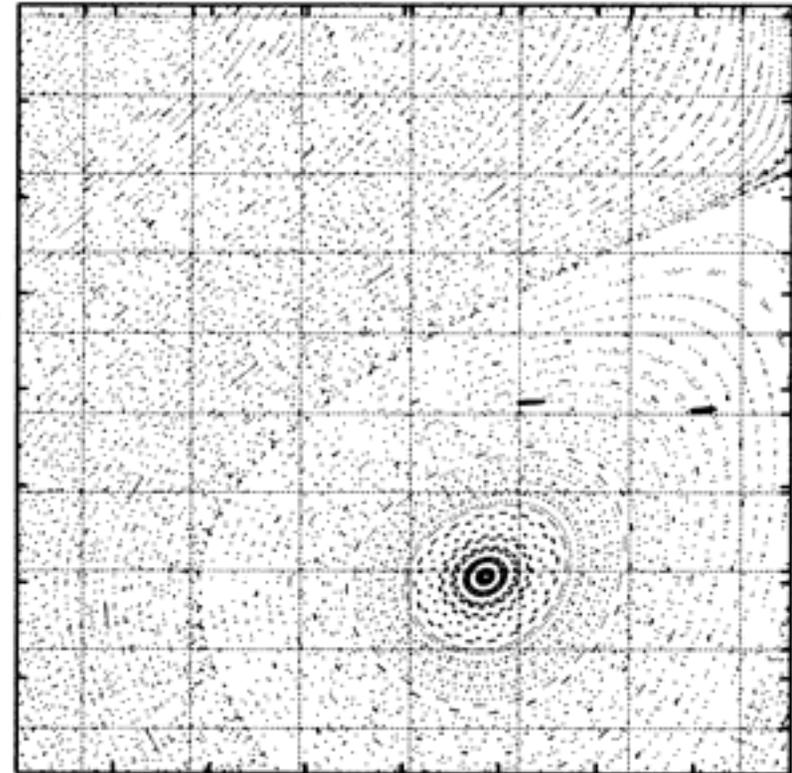
position and pressure of field lines are obtained.



Relaxed pressure on $u^3=const.$ is calculated from

$$p(u^1, u^2) = p_0 + M_1 u^1 + M_2 u^2$$

Sampling points around grid points



Reference: S.S.Lloyd, H.J.Gardner *et al* Conf. Computational Physics 2000, Mon AM-2-4
Y.Suzuki, *et al* (to be submitted)

B-step: relaxation of magnetic field

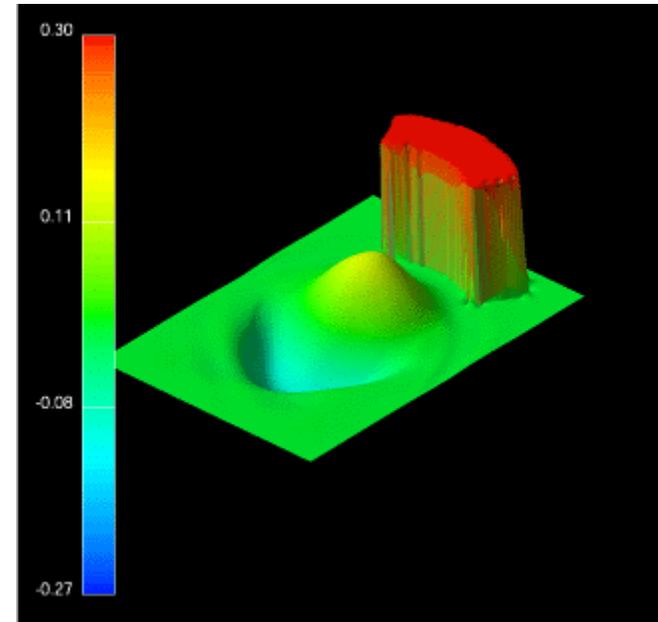
relaxation process using time evolution of the dissipative MHD equations

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) = -f_C (\nabla p - (\mathbf{j} - \mathbf{j}_0) \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta (\mathbf{j} - \mathbf{j}_0))$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

$$f_C = \begin{cases} 1 & B \leq B_C \\ (B_C / B)^2 & B \geq B_C \end{cases}$$



coil current in "box"

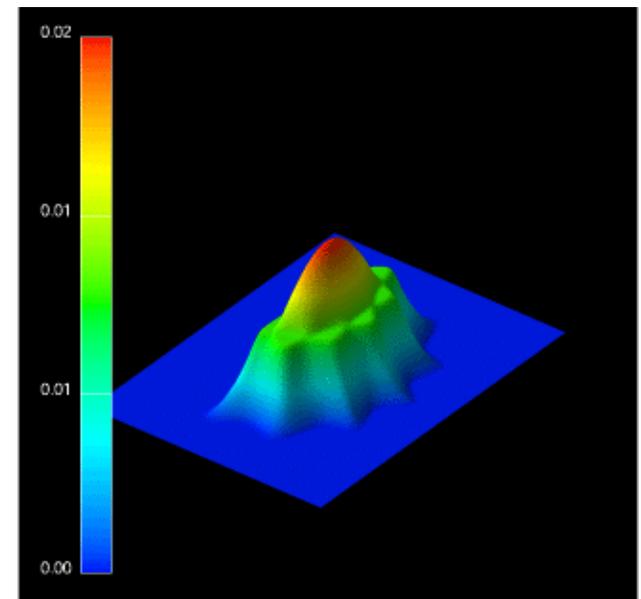
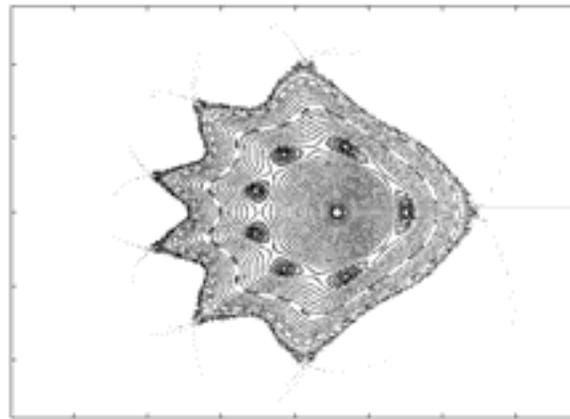
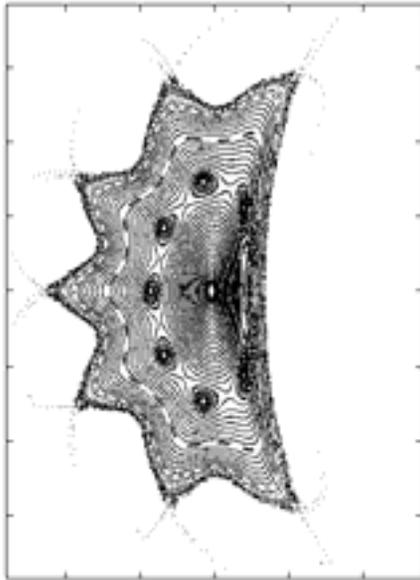
For Heliotron J, strong coil current exist in "box", so the CFL (Courant-Friedrichs-Levy) condition is very severe because of strong field.

Introducing the factor f_C \longrightarrow The CFL condition is satisfied!

Equilibrium of Standard configuration

Initial pressure distribution $p = p_0(1 - s)^2$

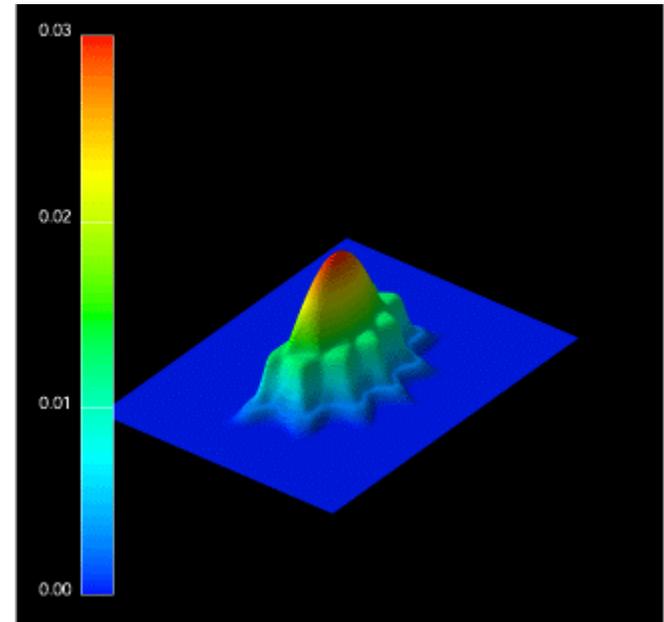
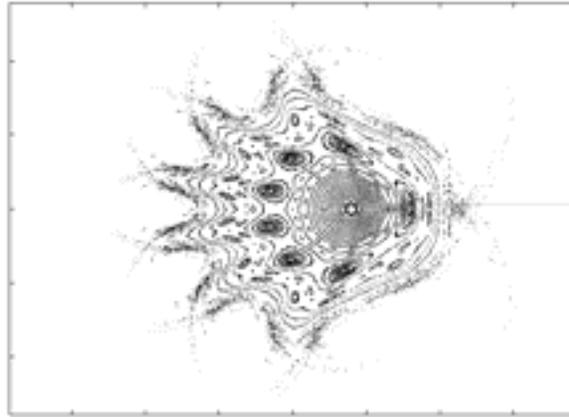
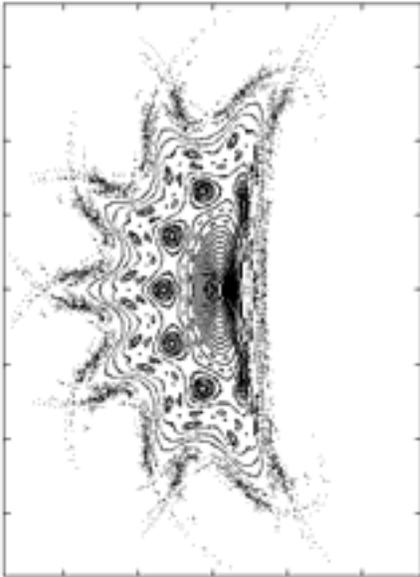
$\beta_{axis} \sim 1.5\%$



Equilibrium of Standard configuration

Initial pressure distribution $p = p_0(1 - s)^2$

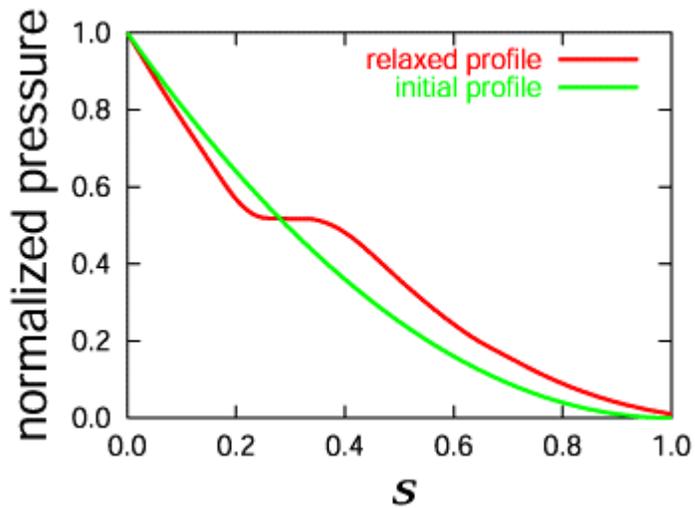
$\beta_{axis} \sim 3\%$



Profiles of plasma pressure

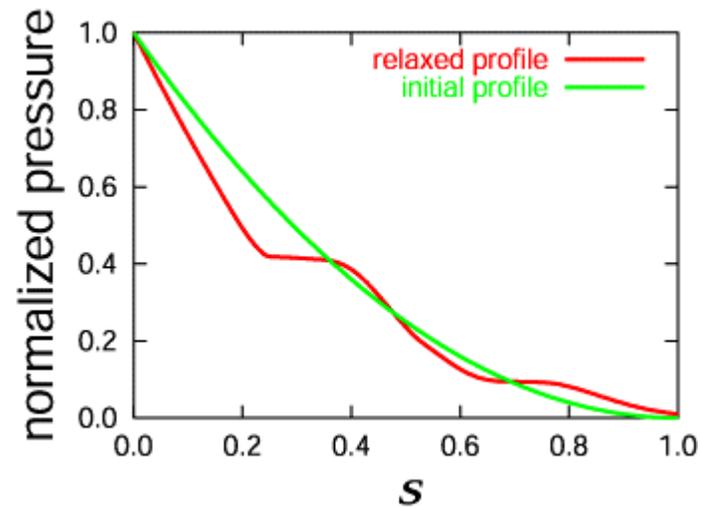
Initial pressure distribution $p = p_0(1 - s)^2$

$\beta_{axis} \sim 1.5\%$



Pressure profile is flat at 4/7

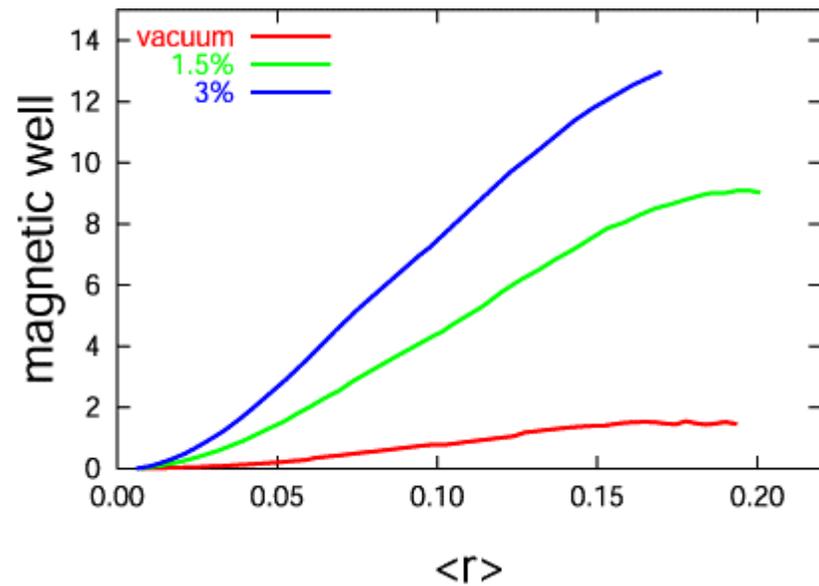
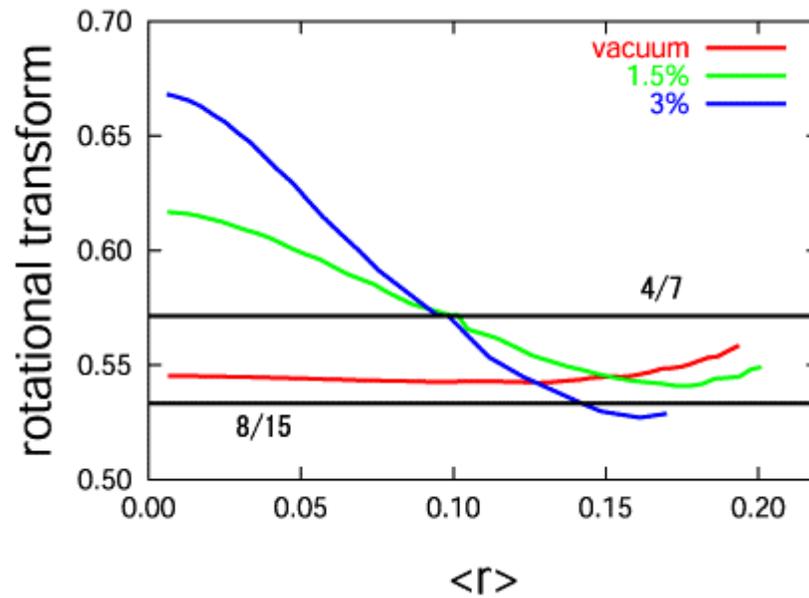
$\beta_{axis} \sim 3\%$



Pressure profile is flat at 4/7 and 8/15

Rotational transform and magnetic well

Initial pressure distribution $p = p_0(1 - s)^2$



For $\beta_{axis} \sim 3\%$, magnetic shear is strong at 4/7



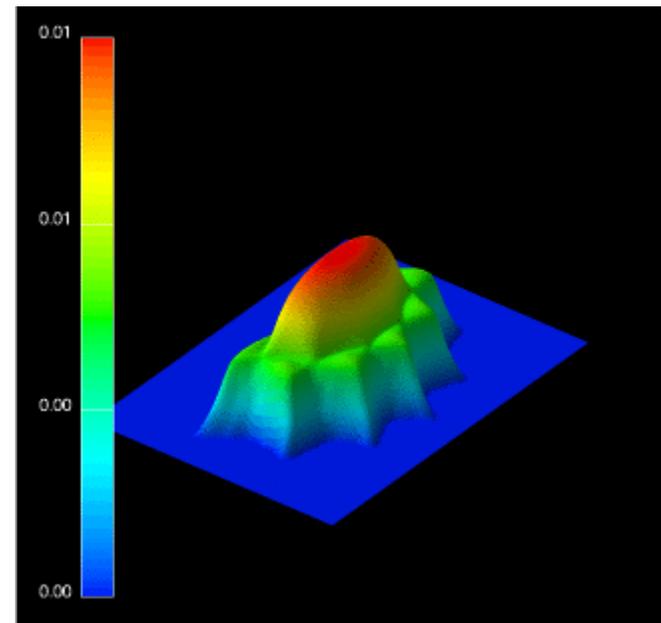
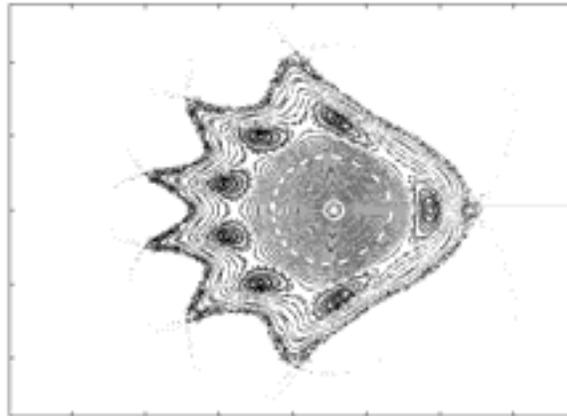
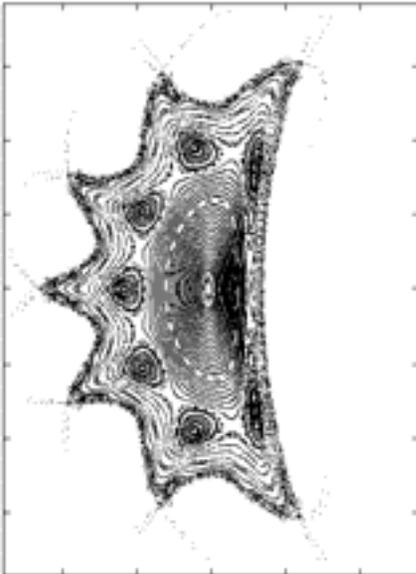
island width is small

Magnetic well is deeply due to beta

Effect of pressure distribution

Initial pressure distribution $p = p_0(1 - s^2)^2$

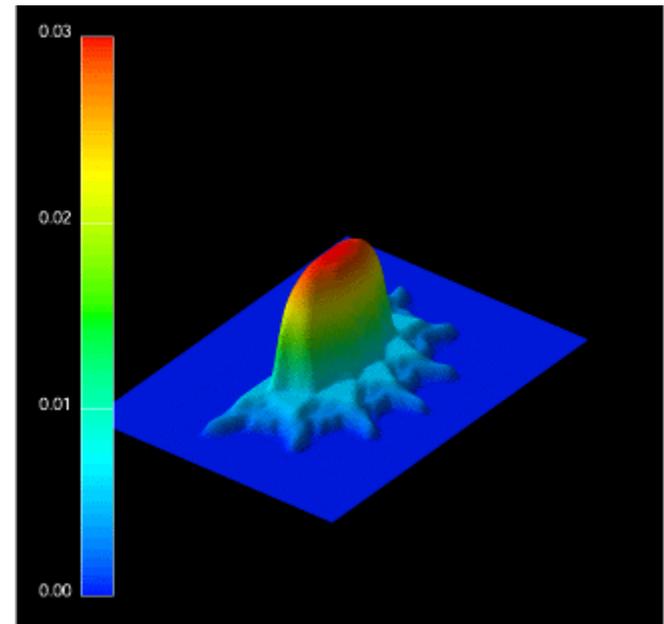
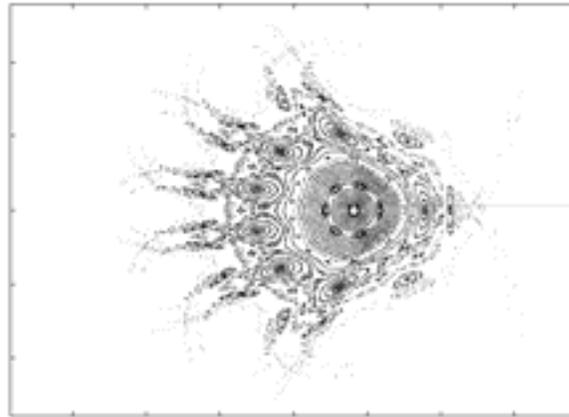
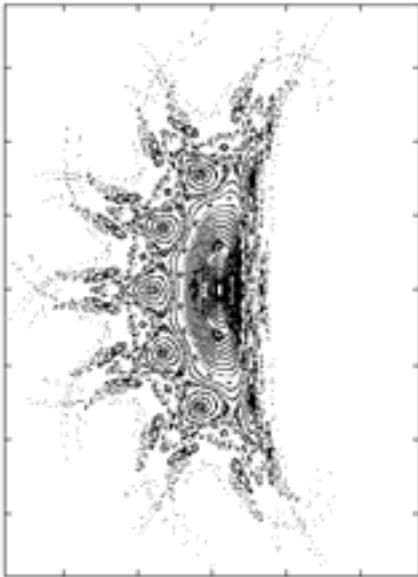
$\beta_{axis} \sim 1.5\%$



Effect of pressure distribution

Initial pressure distribution $p = p_0(1 - s^2)^2$

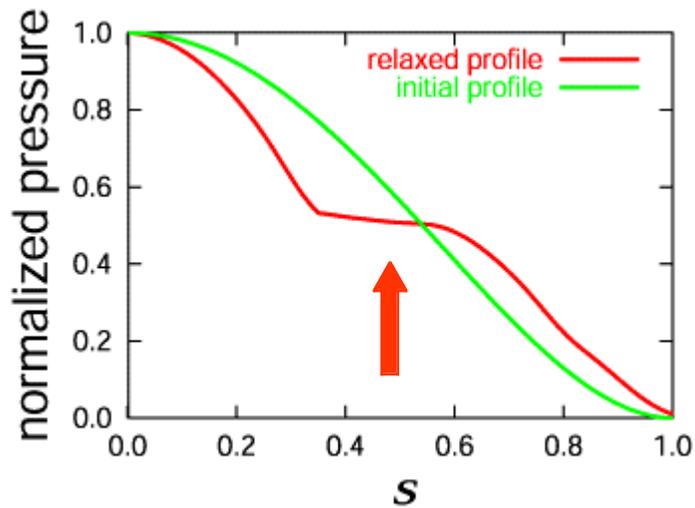
β axis~3%



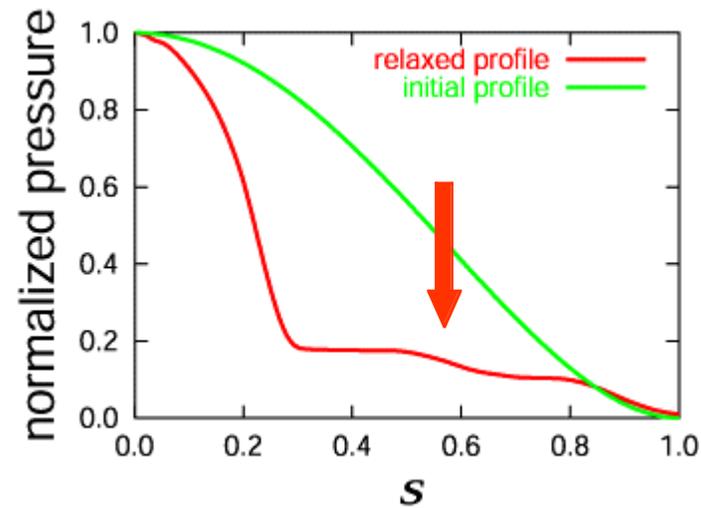
Profiles of plasma pressure

Initial pressure distribution $p = p_0(1 - s^2)^2$

$\beta_{axis} \sim 1.5\%$



$\beta_{axis} \sim 3\%$



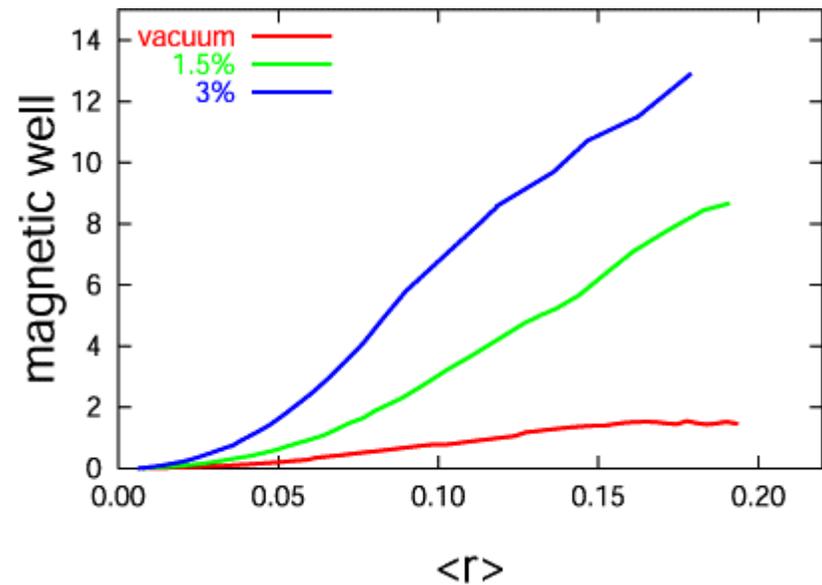
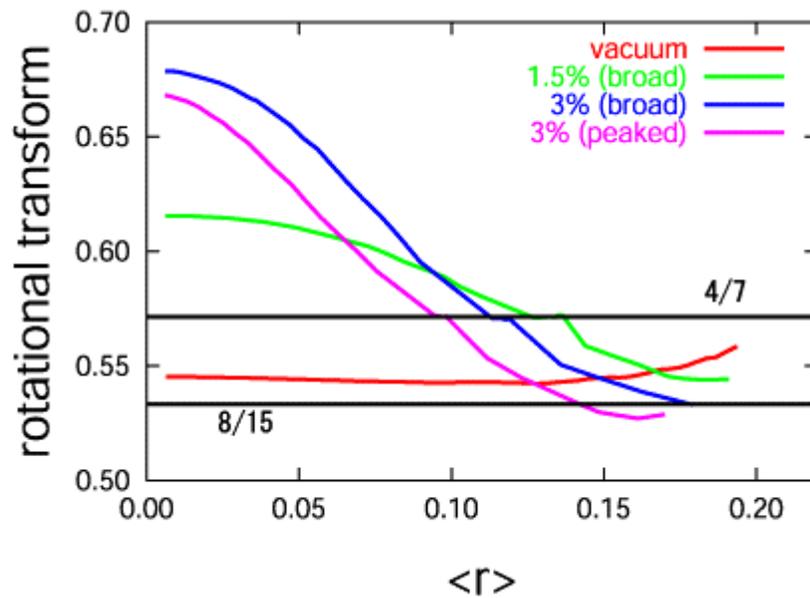
Flattening of pressure is wide in edge



Averaged beta value is decreased

Rotational transform and magnetic well

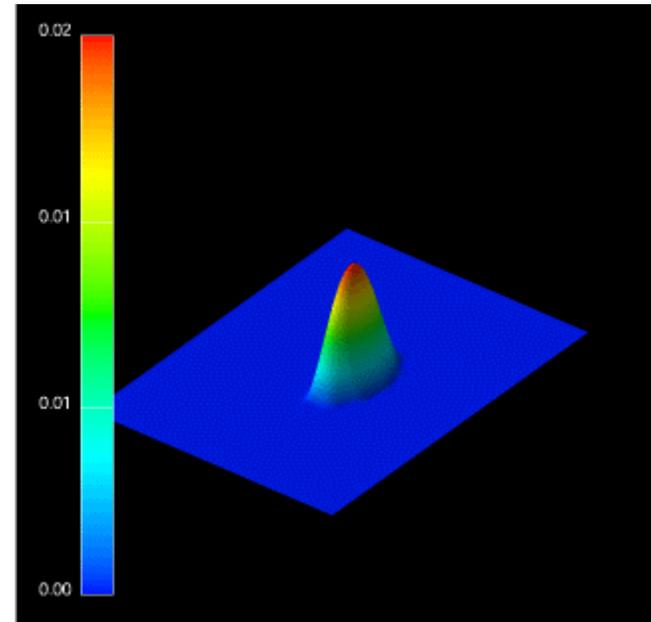
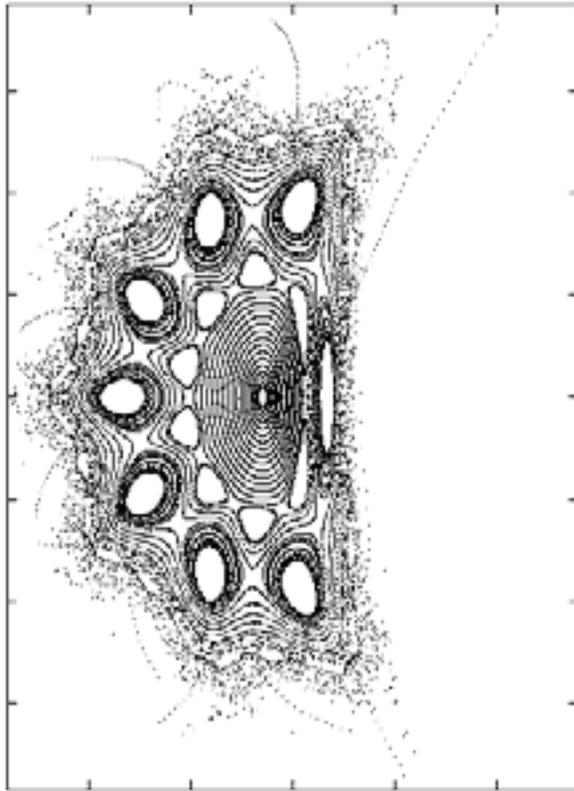
Initial pressure distribution $p = p_0(1 - s^2)^2$



Effect of vertical field

Initial pressure distribution $p = p_0(1 - s)^2$
 $\beta_{axis} \sim 1.5\%$

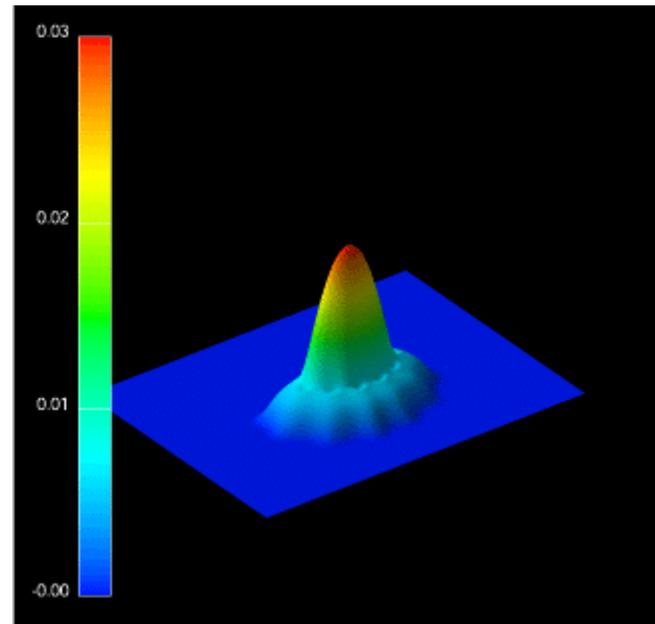
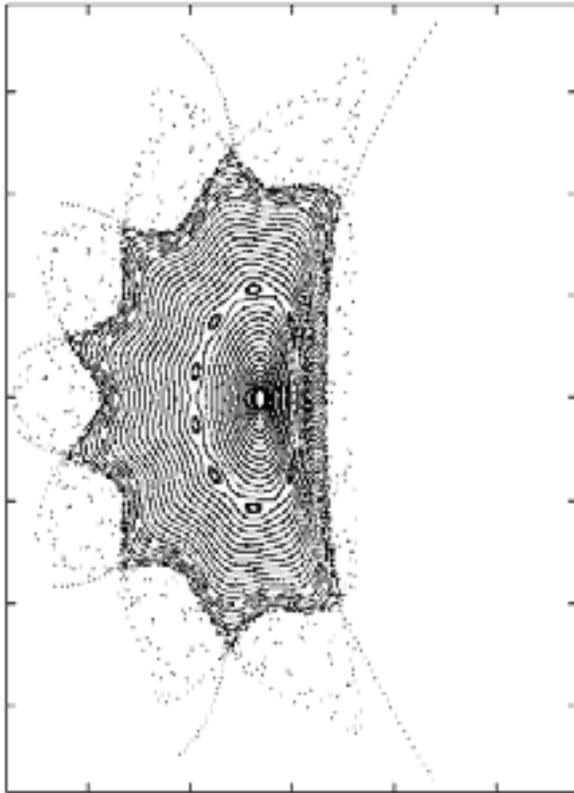
Shafranov shift is reduced by
vertical field control



Most of plasma pressure diffused because of 4/8 island chain

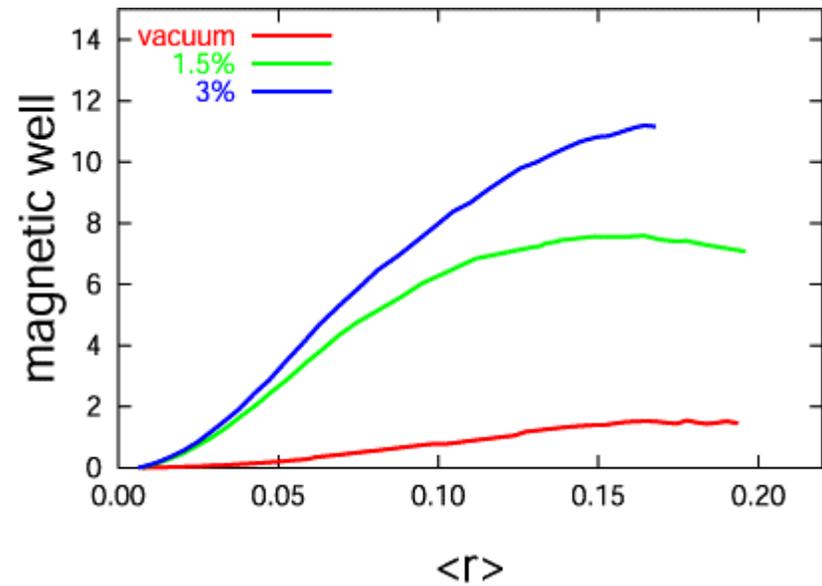
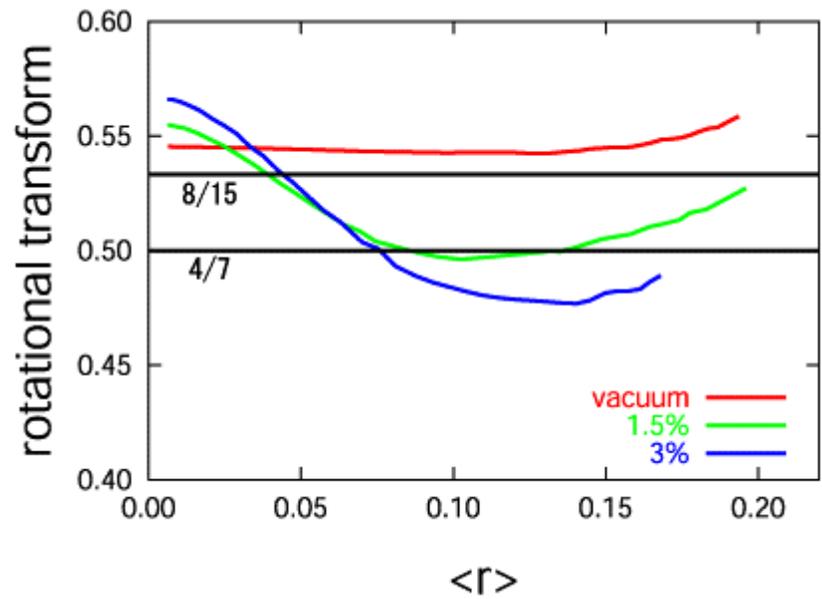
Effect of vertical field

Initial pressure distribution $p = p_0(1 - s)^2$
 $\beta_{axis} \sim 3\%$



Rotational transform and magnetic well

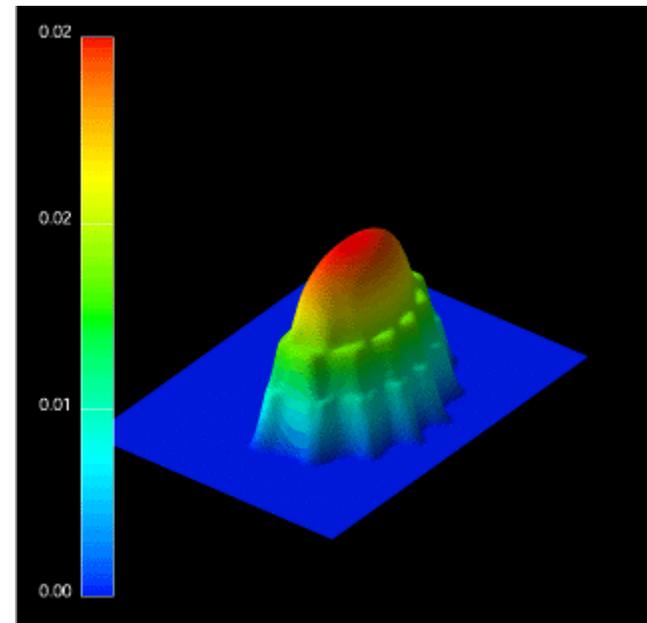
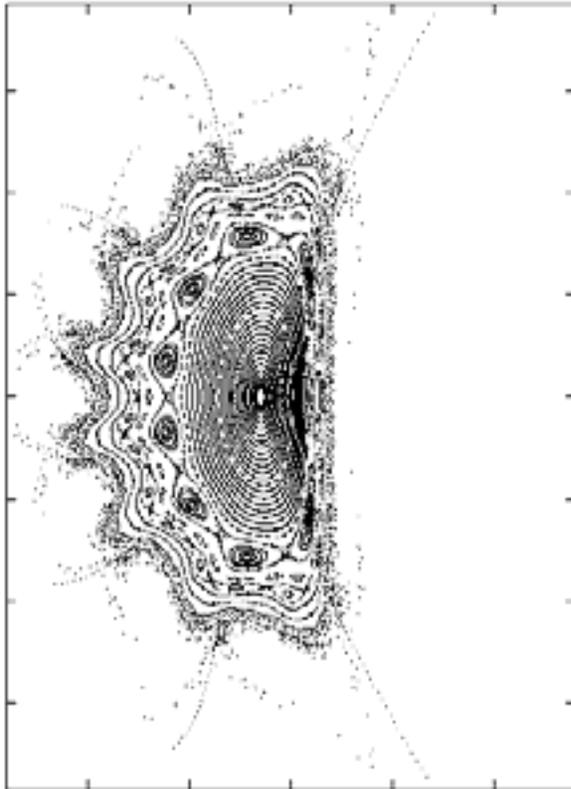
Initial pressure distribution $p = p_0(1 - s)^2$



Effect of vertical field and broad pressure distribution

Initial pressure distribution $p = p_0(1 - s^2)^2$

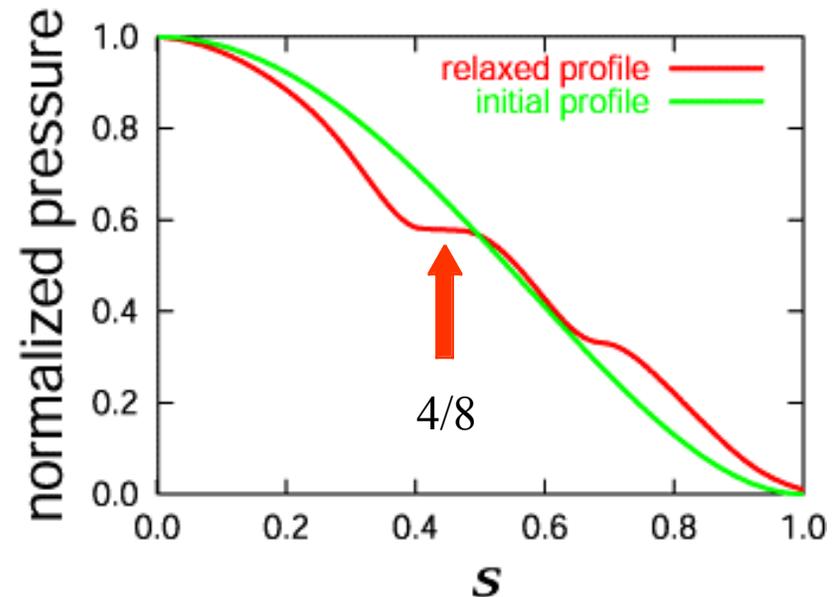
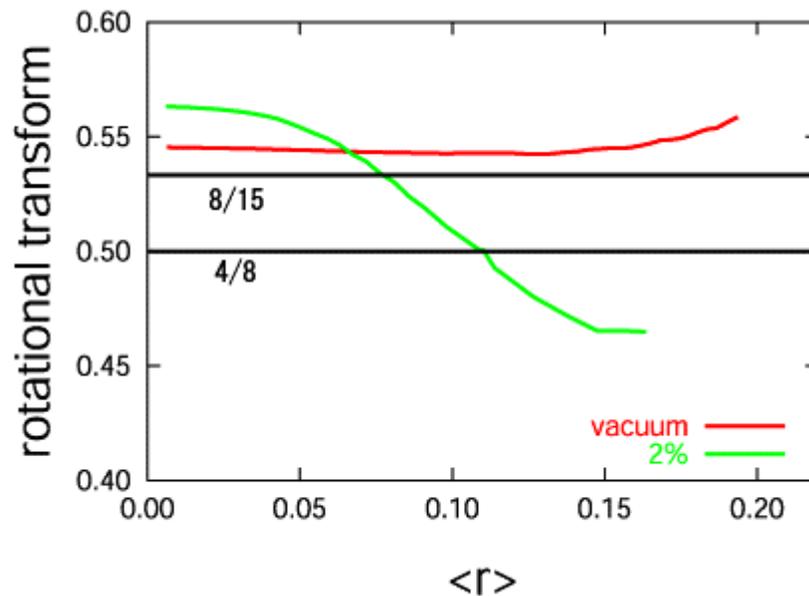
$\beta_{\text{axis}} \sim 2\%$



Rotational transform and pressure profile

Initial pressure distribution $p = p_0(1 - s^2)^2$

$\beta_{axis} \sim 2\%$



Magnetic shear is strong at 4/8 because of broad pressure profile and vertical field



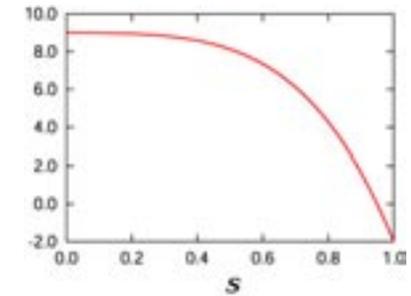
Island width of 4/8 is reduced

Equilibrium with negative net-current

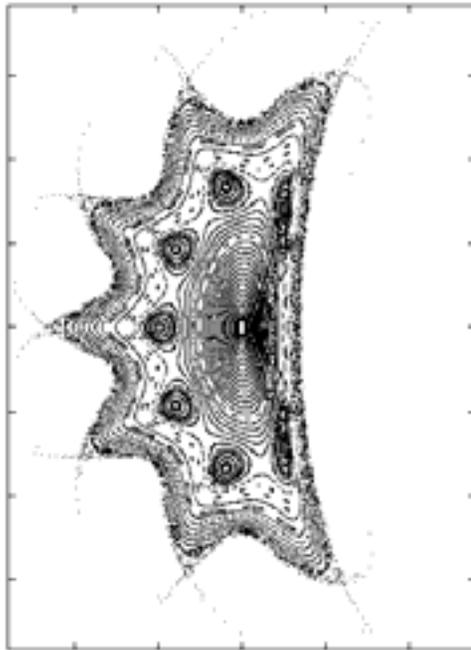
Initial pressure distribution $p = p_0(1 - s)^2$

β axis ~ 2%

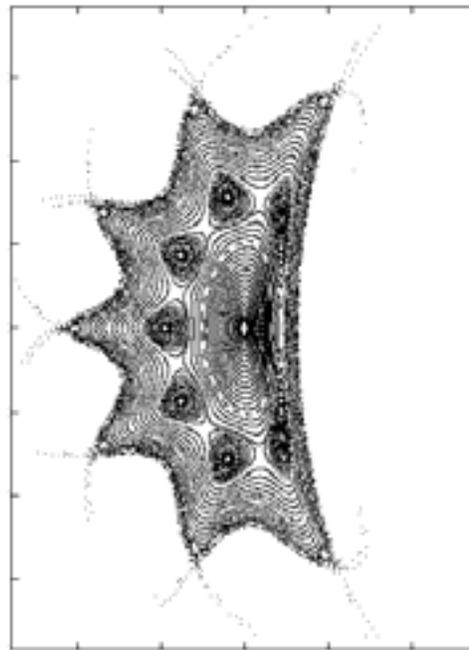
Fixed current profile



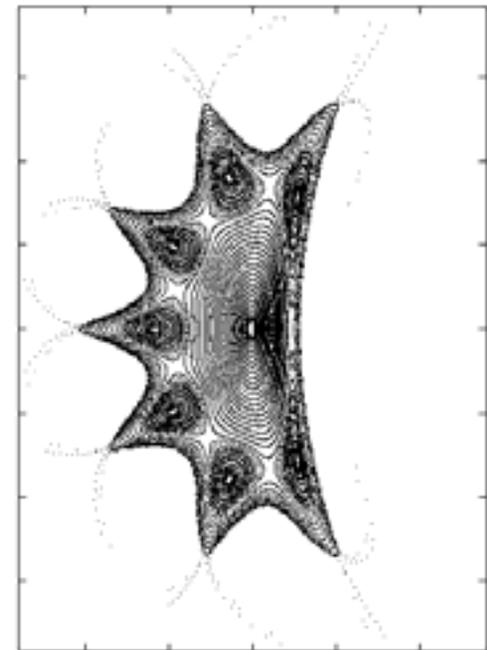
No net-current



-1kA



-2kA



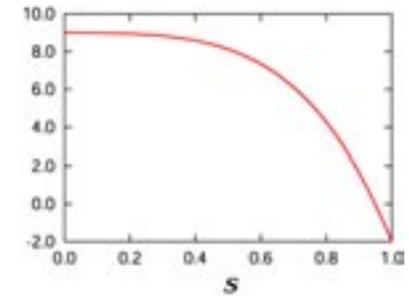
Shafranov shift is large because of negative net-current

Equilibrium with positive net-current

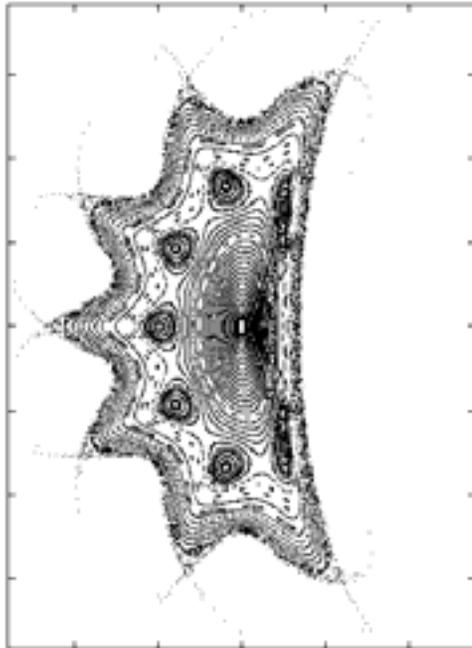
Initial pressure distribution $p = p_0(1 - s)^2$

$\beta_{axis} \sim 2\%$

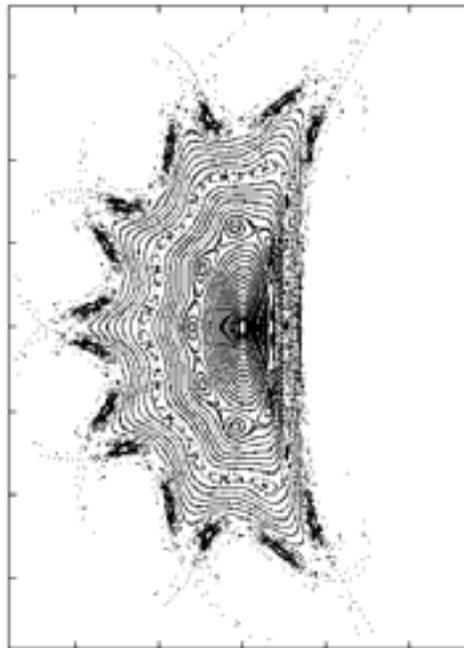
Fixed current profile



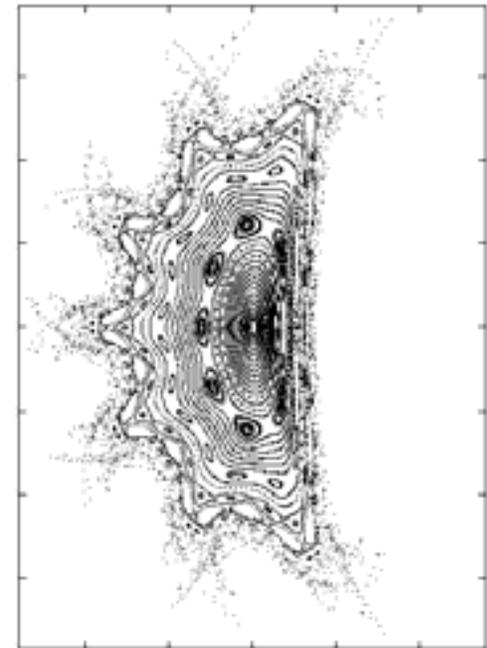
No net-current



1kA



2kA

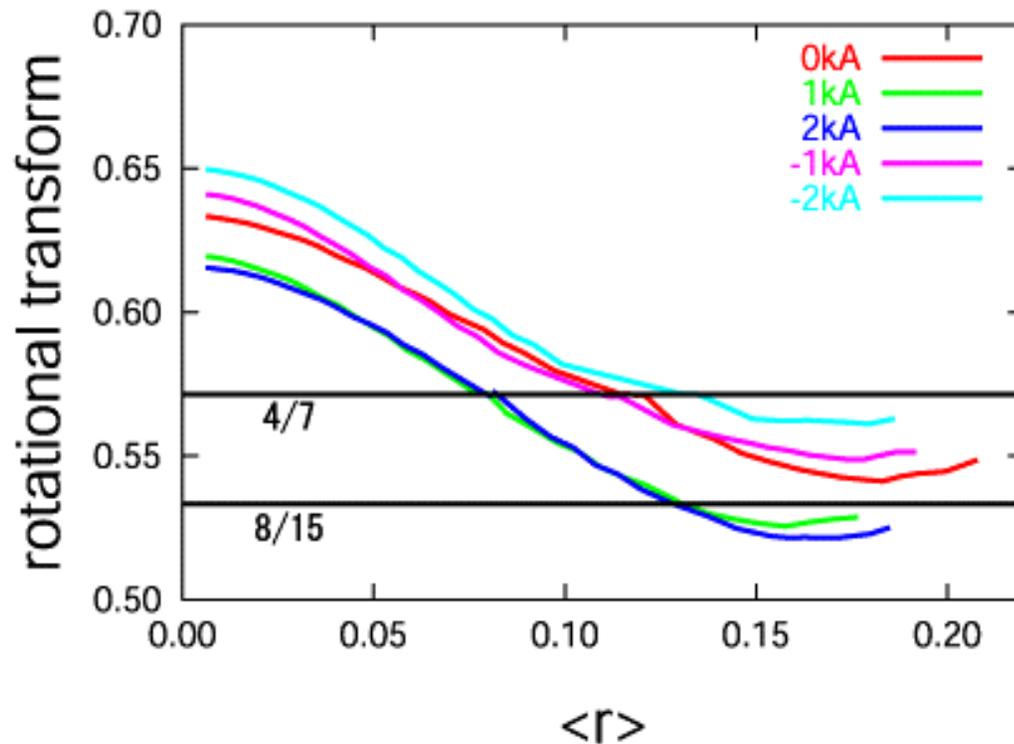


Shafranov shift is small because of positive net-current

Rotational transform

Initial pressure distribution $p = p_0(1-s)^2$

$\beta_{axis} \sim 2\%$



Positive current

➡ Rotational transform **decreased**

Negative current

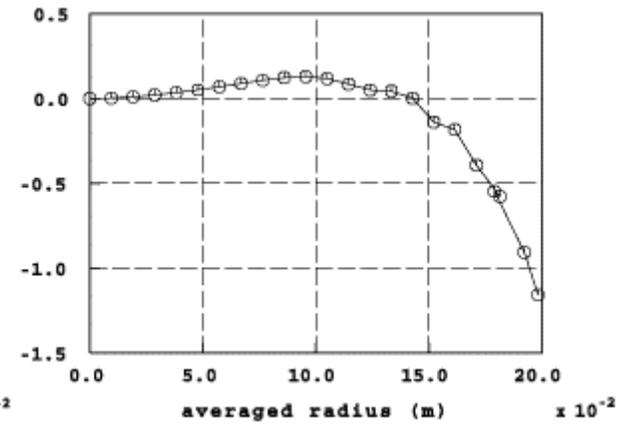
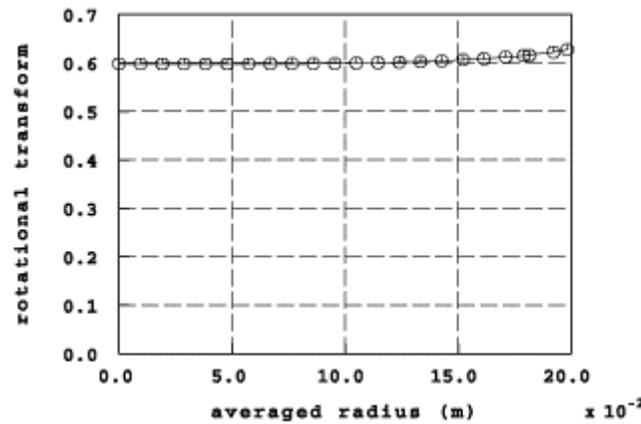
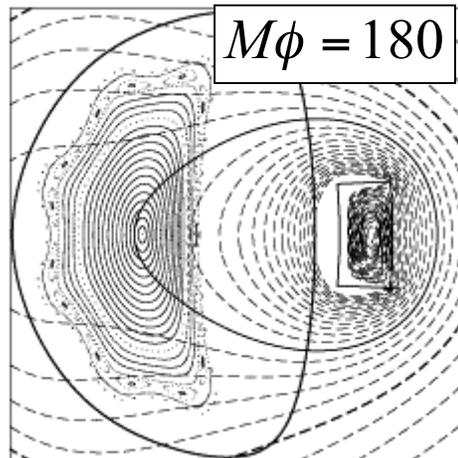
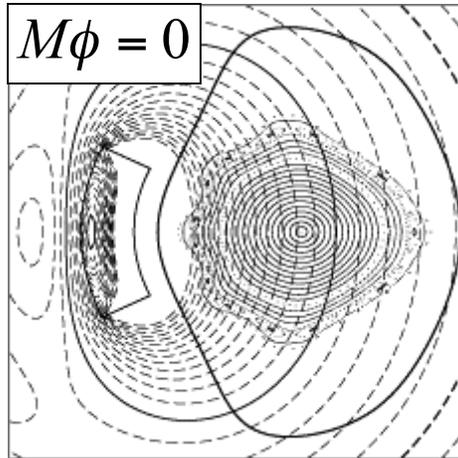
➡ Rotational transform **increased**

The Shafranov shift changes due to net-current.

The rotational transform also changes by net-current.

Results of other configuration

New configuration avoid low order resonance (HV120)



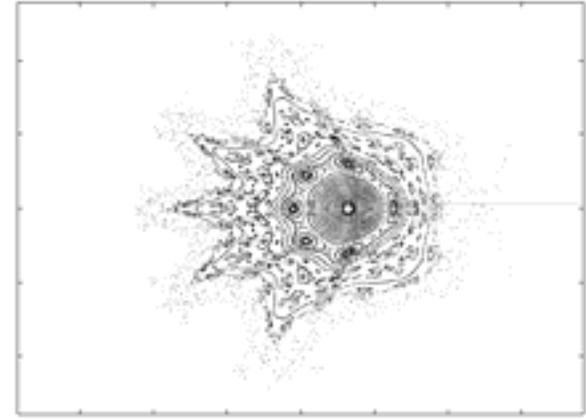
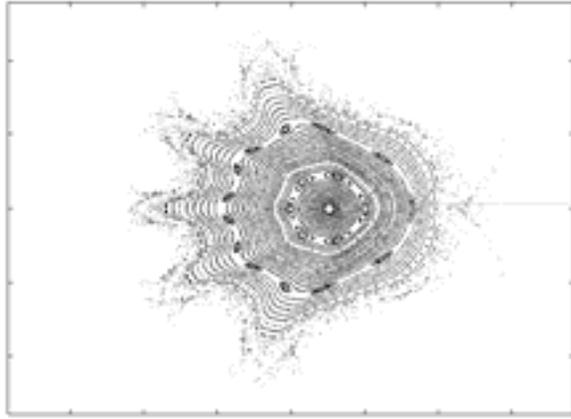
Profile is flat (~ 0.6)



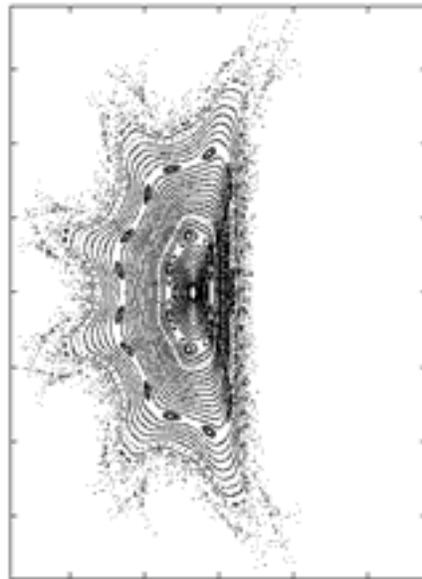
Magnetic hill in edge

MHD Equilibrium of HV120 configuration

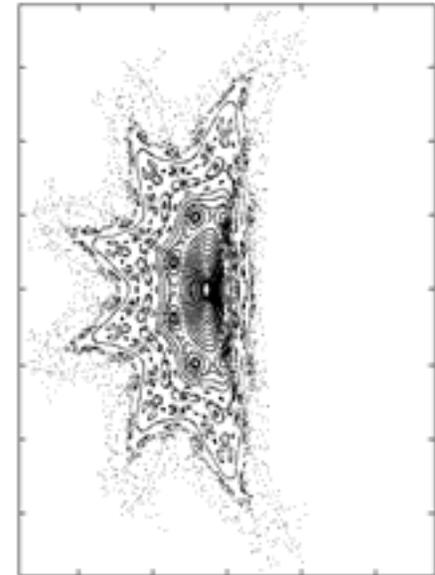
Initial pressure distribution $p = p_0(1 - s)^2$



$\beta_{\text{axis}} \sim 2\%$



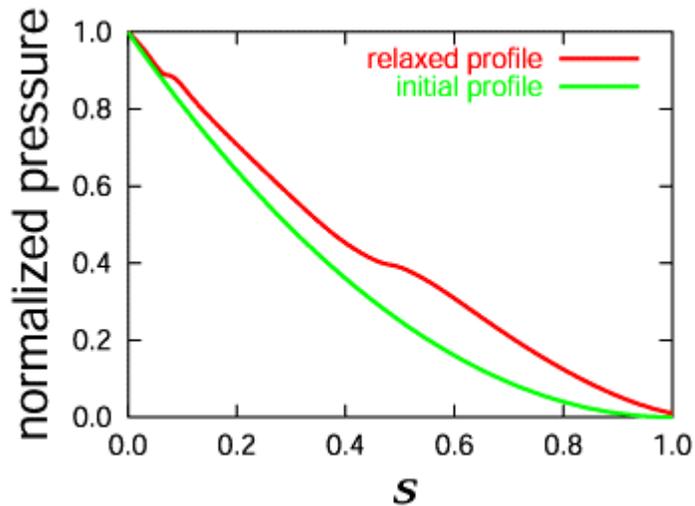
$\beta_{\text{axis}} \sim 3\%$



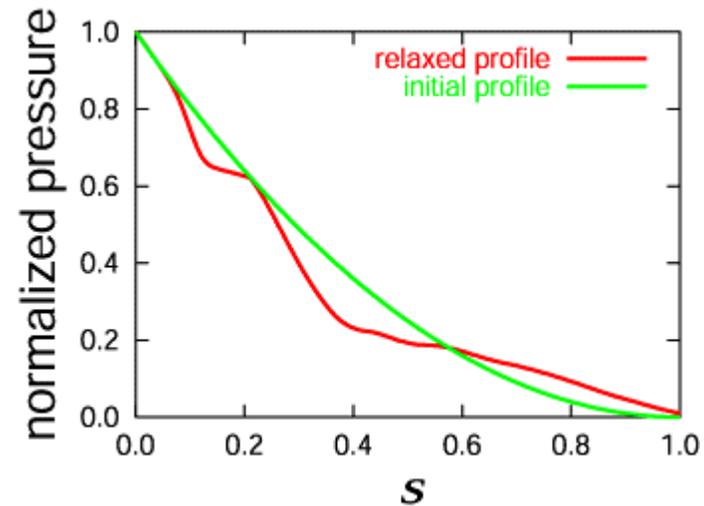
Pressure profiles of HV120 configuration

Initial pressure distribution $p = p_0(1 - s)^2$

β axis~2%



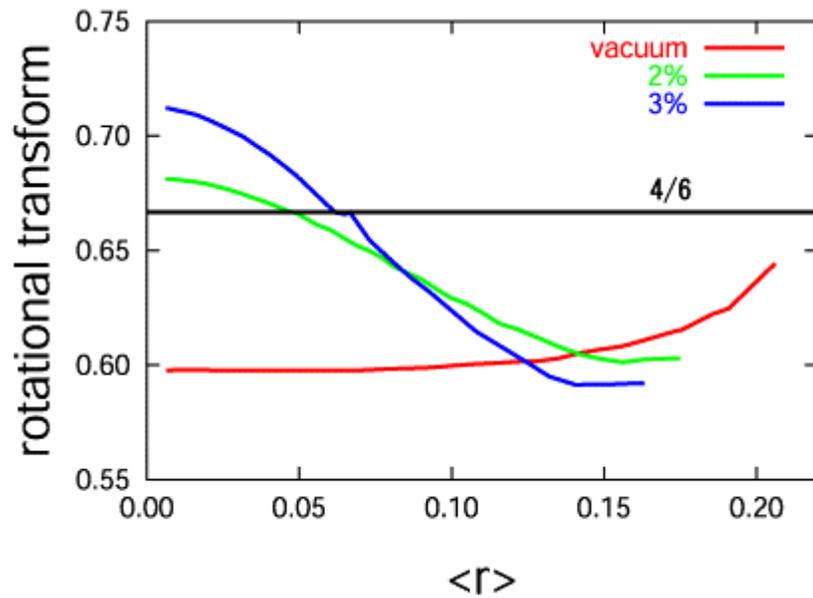
β axis~3%



Island width is small as compared to STD configuration

Radial profiles of HV120 configuration

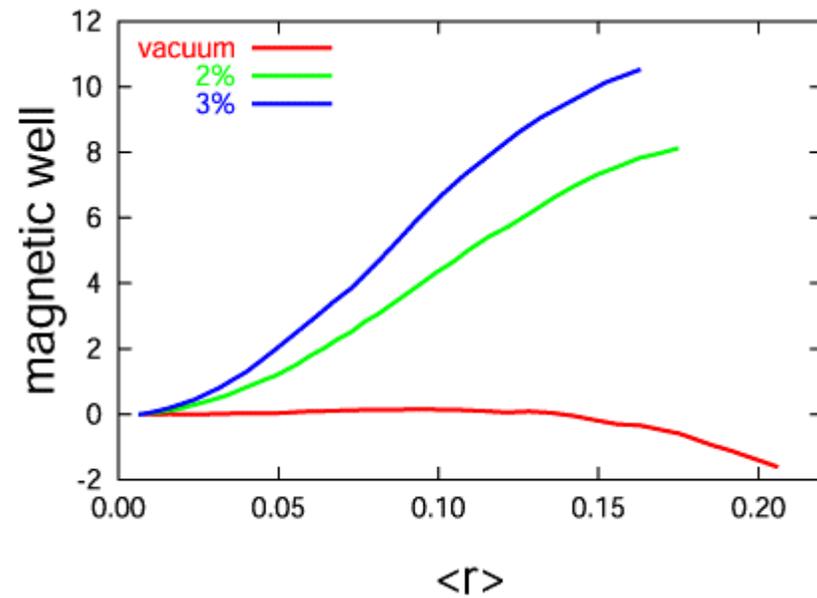
Initial pressure distribution $p = p_0(1 - s)^2$



Magnetic shear is strong at 4/6



Island width is small



Magnetic well is deeply due to beta value

4.Summary

1. Importance of free boundary calculation in Heliotron J plasmas are shown.
2. Improved scheme of HINT code to resolve the convergence of plasma pressure is shown.
3. Free boundary calculation using modified HINT code is in progress.
4. Effects of net-current to equilibrium are discussed.

Future plan

- detailed study of equilibrium with net-current
- study of equilibrium with self-consistent bootstrap current
- study of other configuration for optimization of Heliotron J device