MHD Equilibrium and Pressure Driven Instabilities in L=1 Heliotron Plasmas

Presented

by

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at

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Outline

1. MHD Equilibrium of a Heliotron J Plasma

1.1 Introduction

Bumpy component (toroidal mirror ratio) can be widely varied by two sets of TF coils (TA & TB coils) in Heliotron J

"Standard" configuration of Heliotron J device ;

$I_{TA} / I_{TB} = 5 / 2$

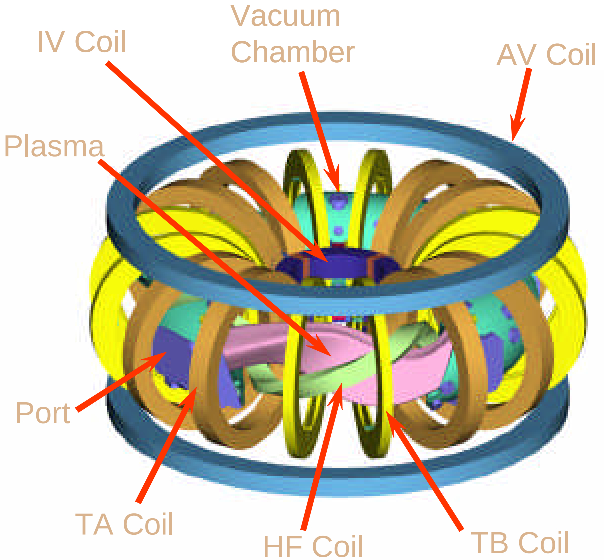
better neoclassical transport
deep magnetic well

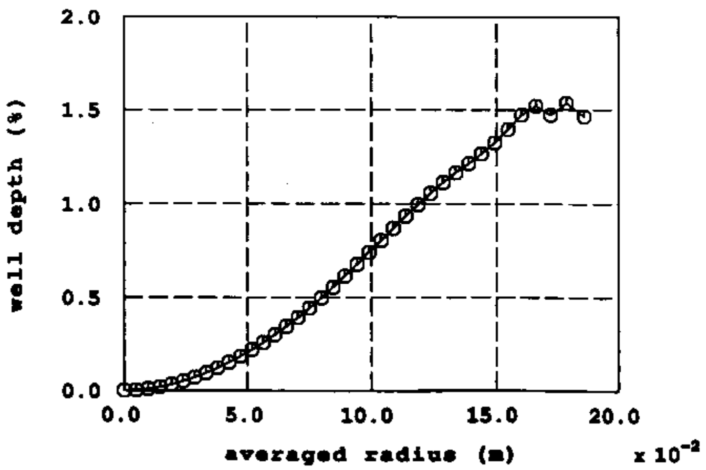
fixed boundary VMEC equilibrium was mainly used for the analysis

magnetic shear is weak
toroidicity is not small

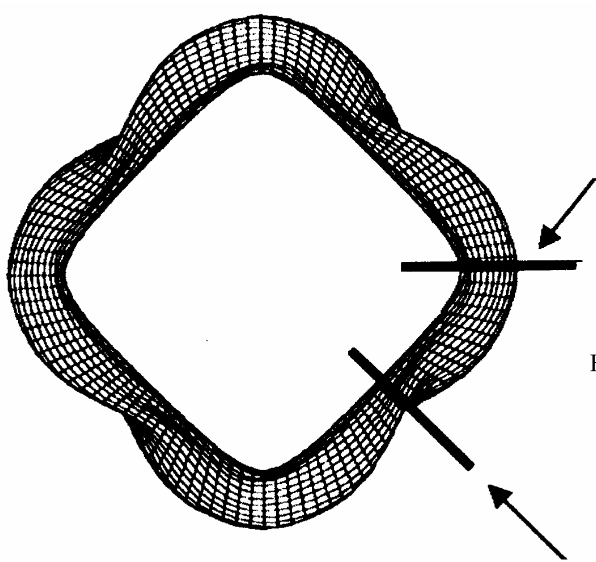
free boundary equilibrium ?
magnetic island ?

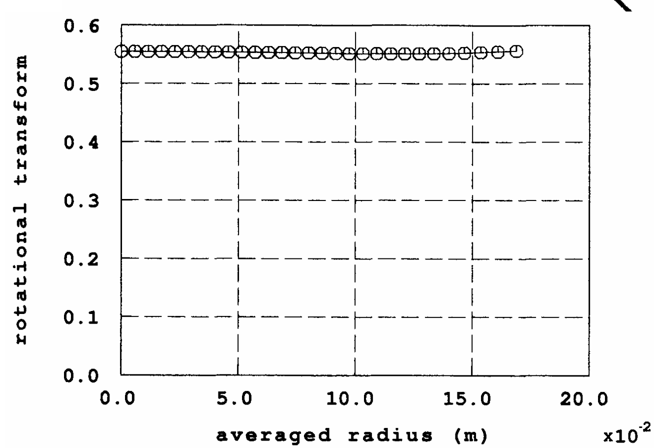
Heliotron J
(L=1/M=4 heliotron)

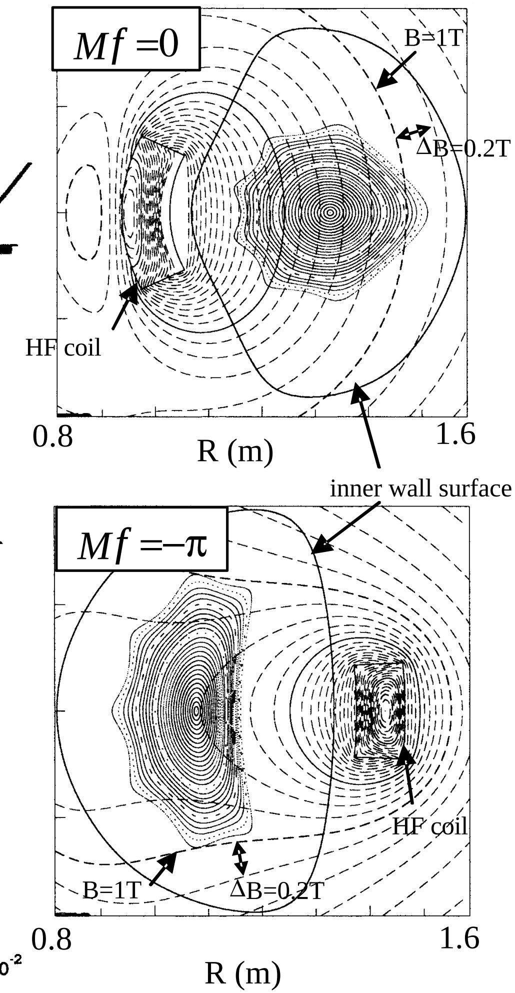




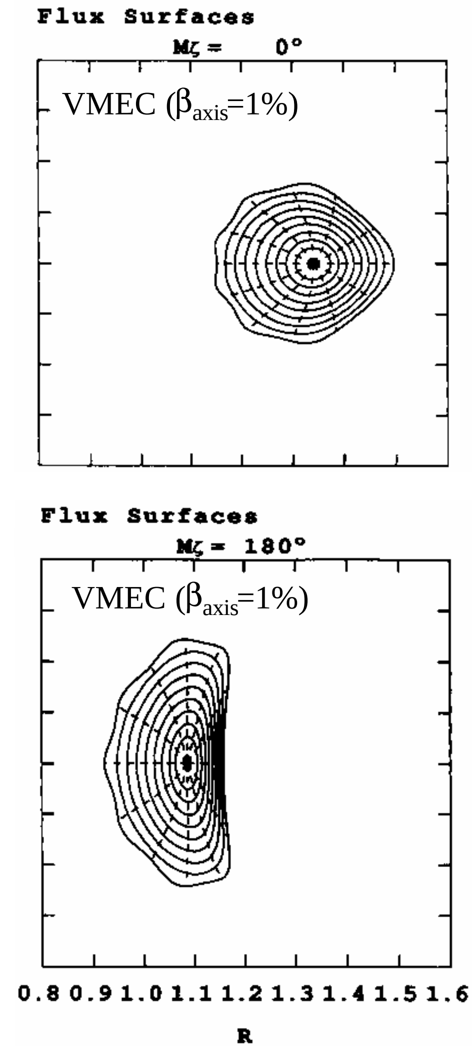
Vacuum flux surfaces
& fixed boundary equilibrium







Vacuum flux surfaces obtained by KMAG code (field line tracing)



Fixed boundary equilibrium obtained by the VMEC (β_{axis} ~ 1%)

1.2 Free boundary calculation by VMEC

Free boundary calculation
- * <u>fixed total toroidal flux</u>
- * combination of the field line tracing code

→ divertor, fast ion orbit calculation,
ECH ray tracing
initial guess for HINT code

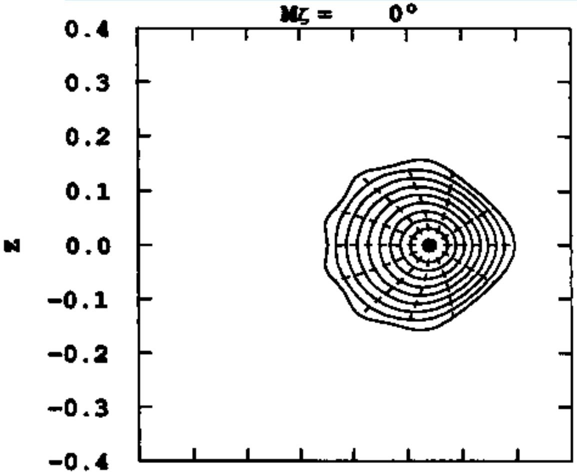
comparison with the fixed boundary equilibium

larger Shafranov shift
larger rotational transform
→ position of resonant rational surface changes.

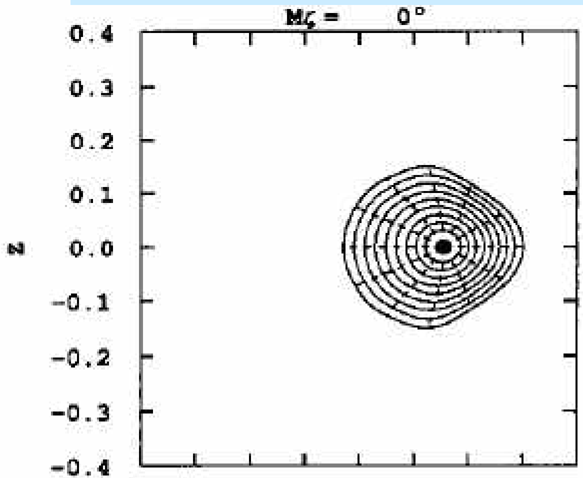
(parallel current calculated from local equilibrium)

wavy plasma boundary structure cannot be reproduced correctly both in fixed and free boundary equilibria by the VMEC

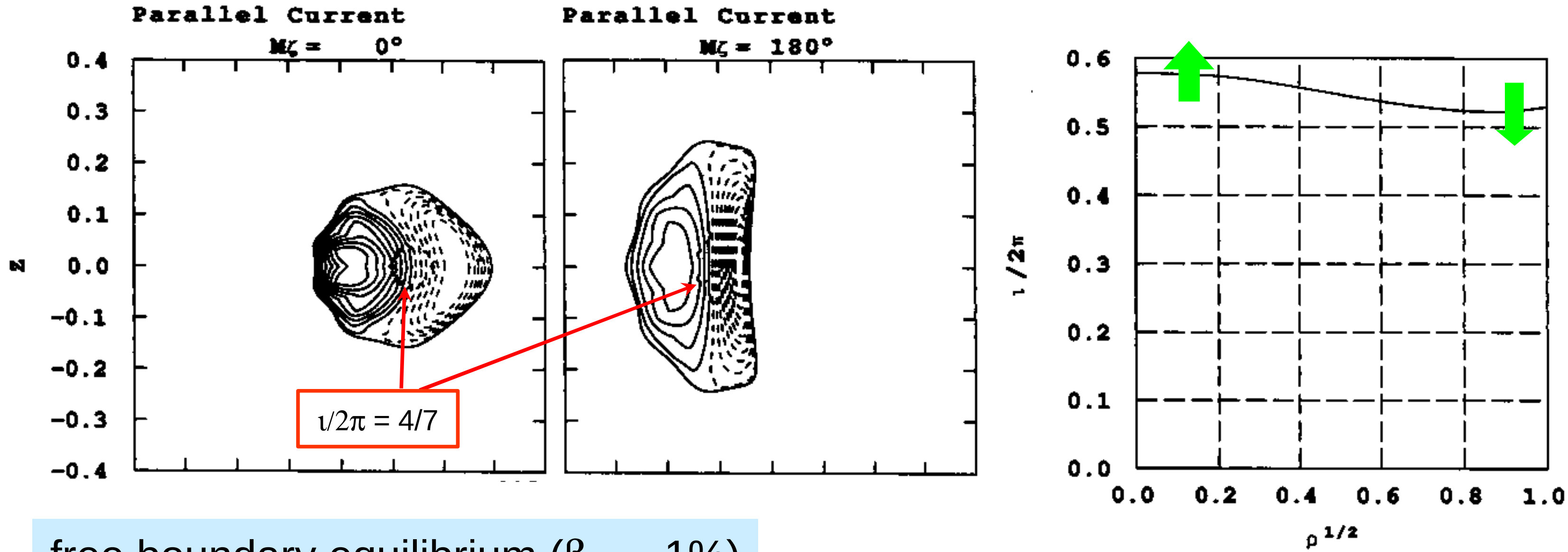
fixed boundary equilibrium (β_{axis}~1%)



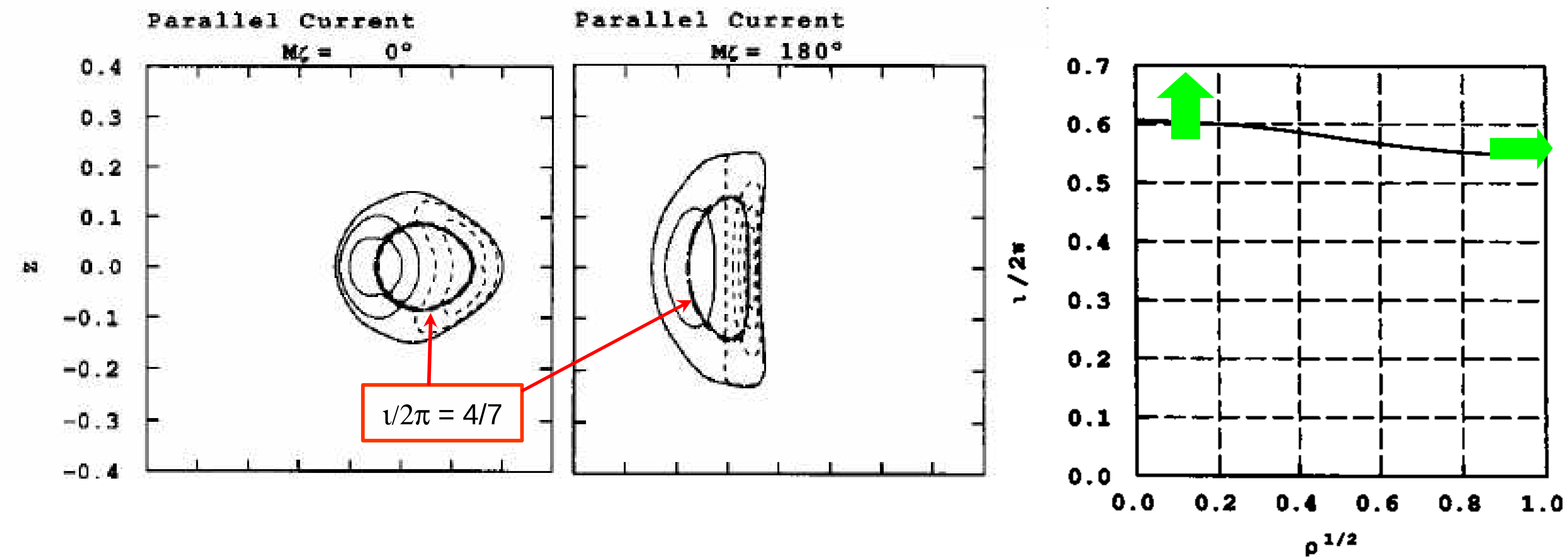
free boundary equilibrium (β_{axis}~1%)



fixed boundary equilibrium (β_{axis}~1%)



free boundary equilibrium (β_{axis}~1%)



1.3 Free boundary calculation by PIES

PIES code (PPPL)

Direct MHD equilibrium calculation by the iterative method

$$\vec{B}^{(n)} \cdot \nabla p(s) = 0$$

update pressure distribution by field line tracing

➔ construction of (quasi) flux coordinates

$$\vec{j}_{\perp} = \vec{B}^{(n)} \times \nabla p \Big/ B^{(n)2}$$

$$\vec{B}^{(n)} \cdot \nabla \left(j_{//} / B^{(n)} \right) = -\nabla \cdot \vec{j}_{\perp}$$

update magnetic field vector

$$\nabla \times \vec{B}^{(n+1)} = \vec{j}(\vec{B}^{(n)})$$

Poisson eq.

Magneto-differential eq. --- (quasi) flux coordinates

Poisson eq. --- **background coordinates**

* separates external field and
the field produced by plasma current

* virtual casing method

➡ free boundary equilibrium

background coordinates

constructed from the VMEC equilibrium

fixed boundary calculation

boundary of background coordinates --- last closed flux surface

In Heliotron J, the shape of the plasma boundary is complicated.

→ fixed boundary calculation is not easy

free boundary calculation

boundary of background coordinates = control surface

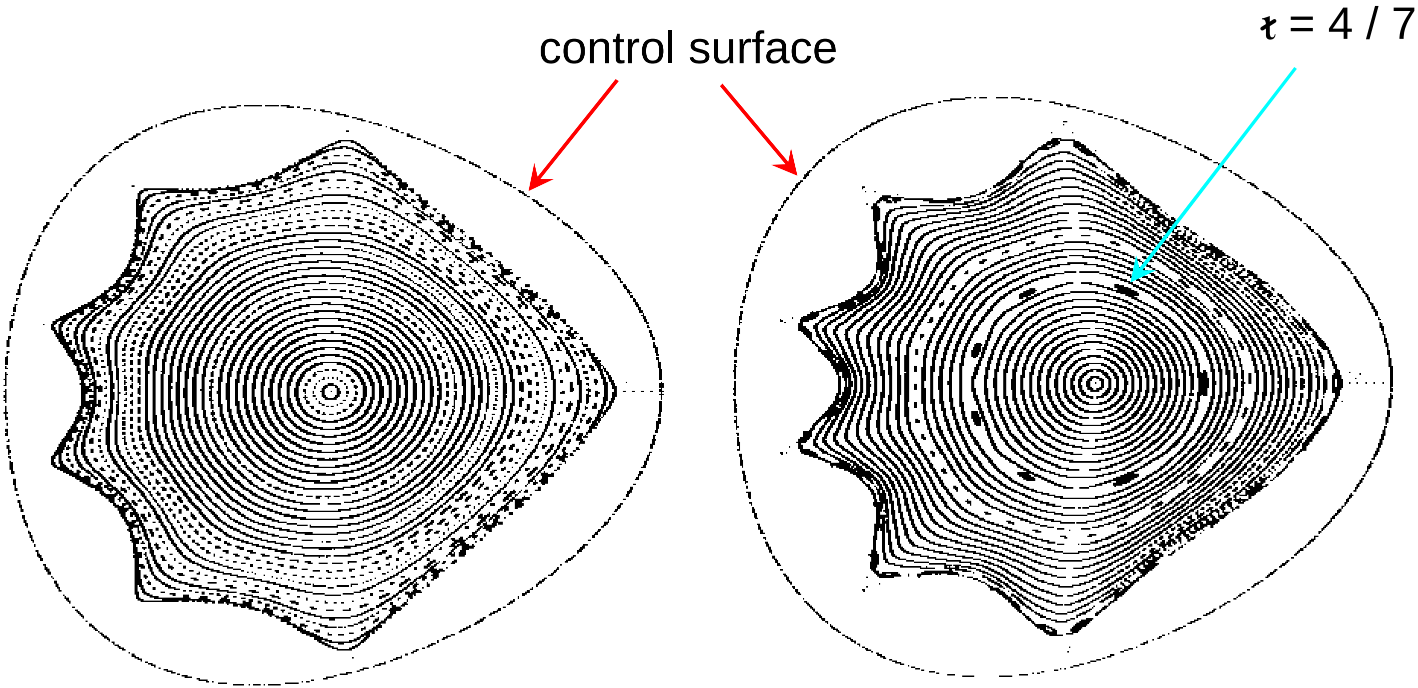
(internal ⟺ external)

(1) background coordinates are constructed from expanded flux surfaces obtained by extrapolating Fourier amplitudes (R_{mn}, Z_{mn})

In Heliotron J, we use VMEC equilibrium calculated by neglecting higher harmonics because of the above reason.

(2) background coordinates are constructed from the "virtual" equilibrium calculated by the VMEC using inner wall surface of the vacuum vessel as a "plasma boundary".

Free boundary equilibrium calculated by PIES

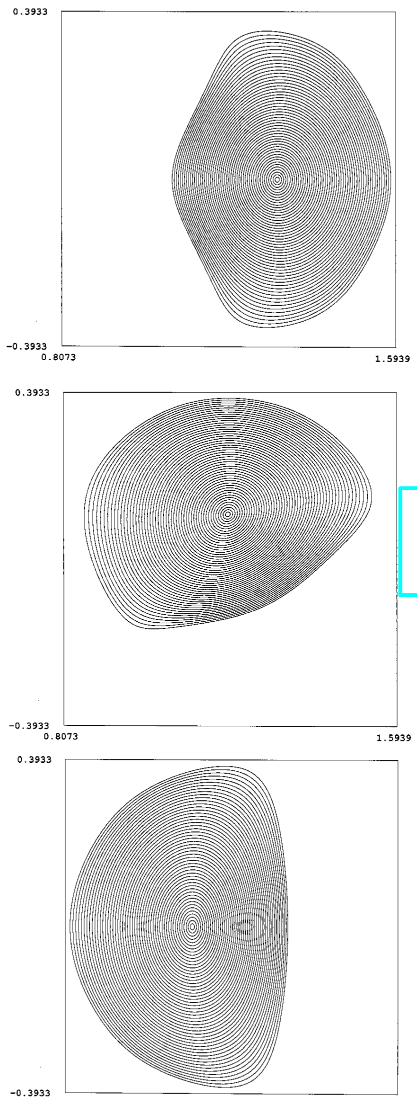


b_0 **~ 0 %** b_0 **~ 1 %**

external magnetic field is calculated by KMAG (Biot-Savart)

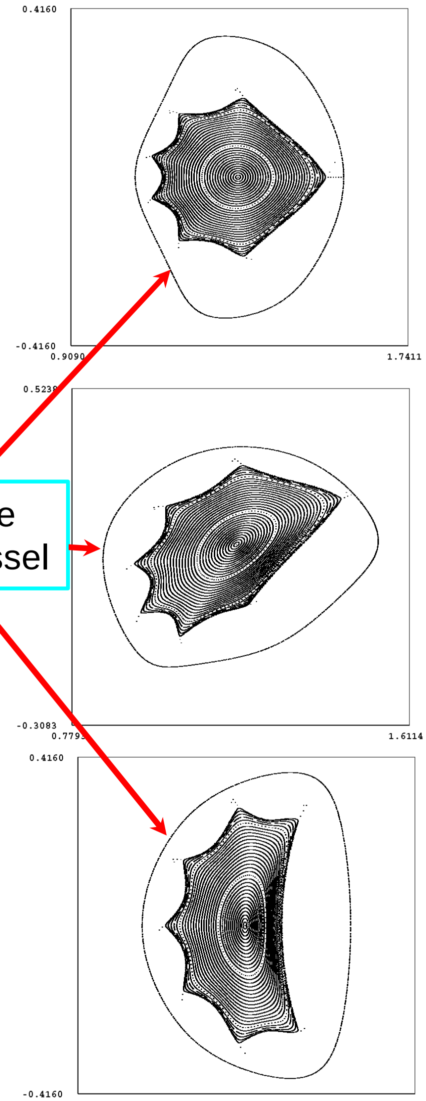
“virtual equilibrium”
by the VMEC

background coordinates

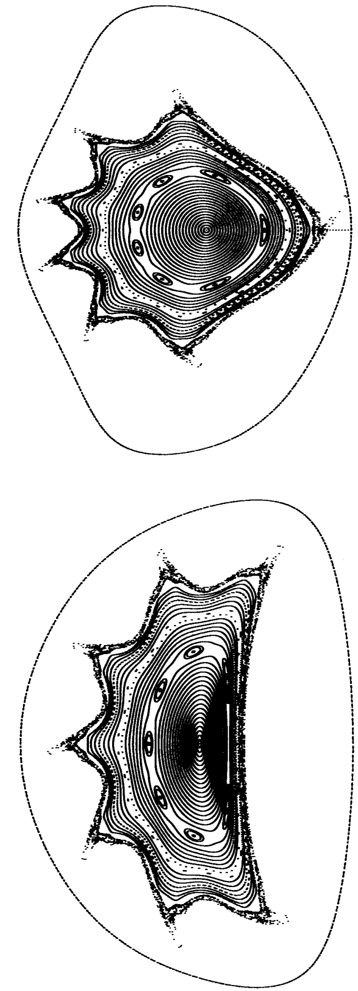


control surface
= vacuum vessel

vacuum flux surfaces
by the KMAG-PIES



β_0 ~ 1.5% equlibrium
by the PIES
(k=51, m=30/34, n=20/24,
niter~100; SX-6: 6.5GB)



~1500 field periods

1.4 "Free boundary" calculation by HINT

HINT code (NIFS)

step A ; distribution of pressure on 3D grid points with fixed magnetic field vector (relaxation method or field line tracing method)

step B; relaxation calculation of magnetic field vector on 3D grid points with fixed pressure distribution (relaxation process using time evolution of the dissipative MHD equations)

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) = -\nabla p + \mathbf{j} \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta(\mathbf{j} - \mathbf{j}_0))$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

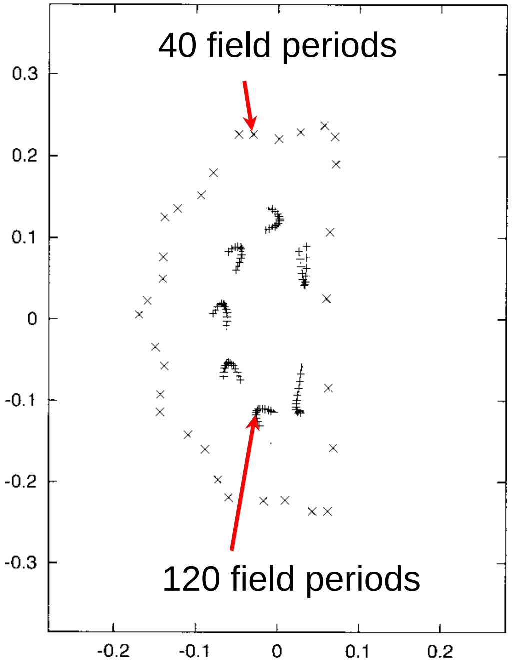
Eulerian coordinates --- rotating helical coordinate system

boundary condition at the computational boundary : fixed boundary

→ we use a large "box" so that the size of the box does not affect the result.

"Free boundary" calculation

step-A : assignment of the pressure

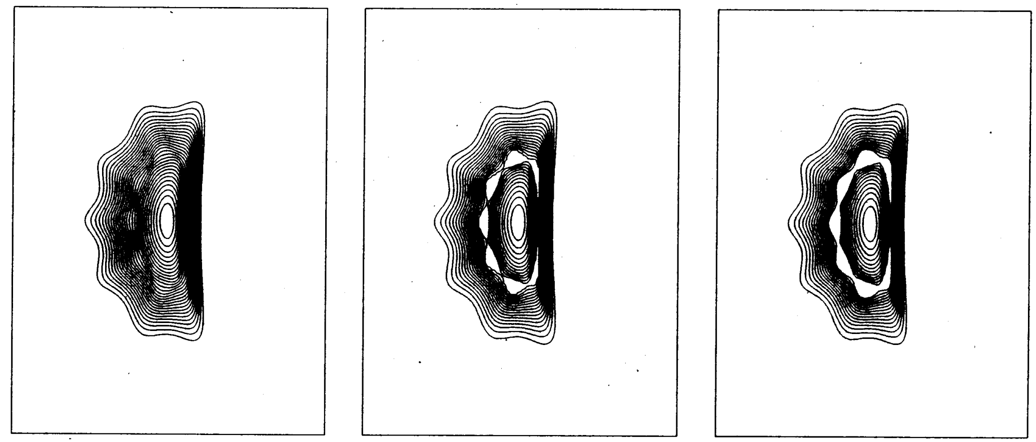


old pressure averaged along the field line running through the grid point

new pressure at the grid point

field line tracing method

pressure distribution in the magnetic islands



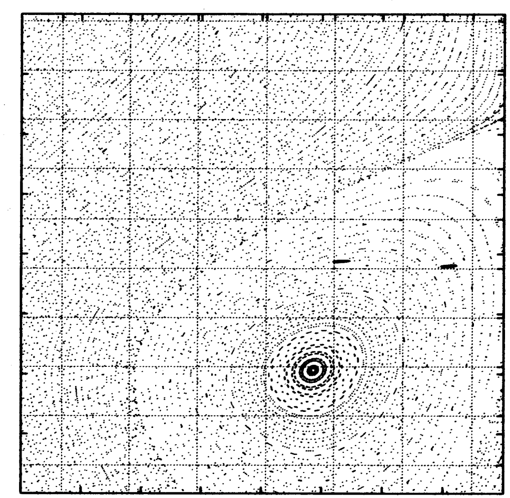
40 field periods | 160 field periods | 240 field periods

time consuming !

improved method

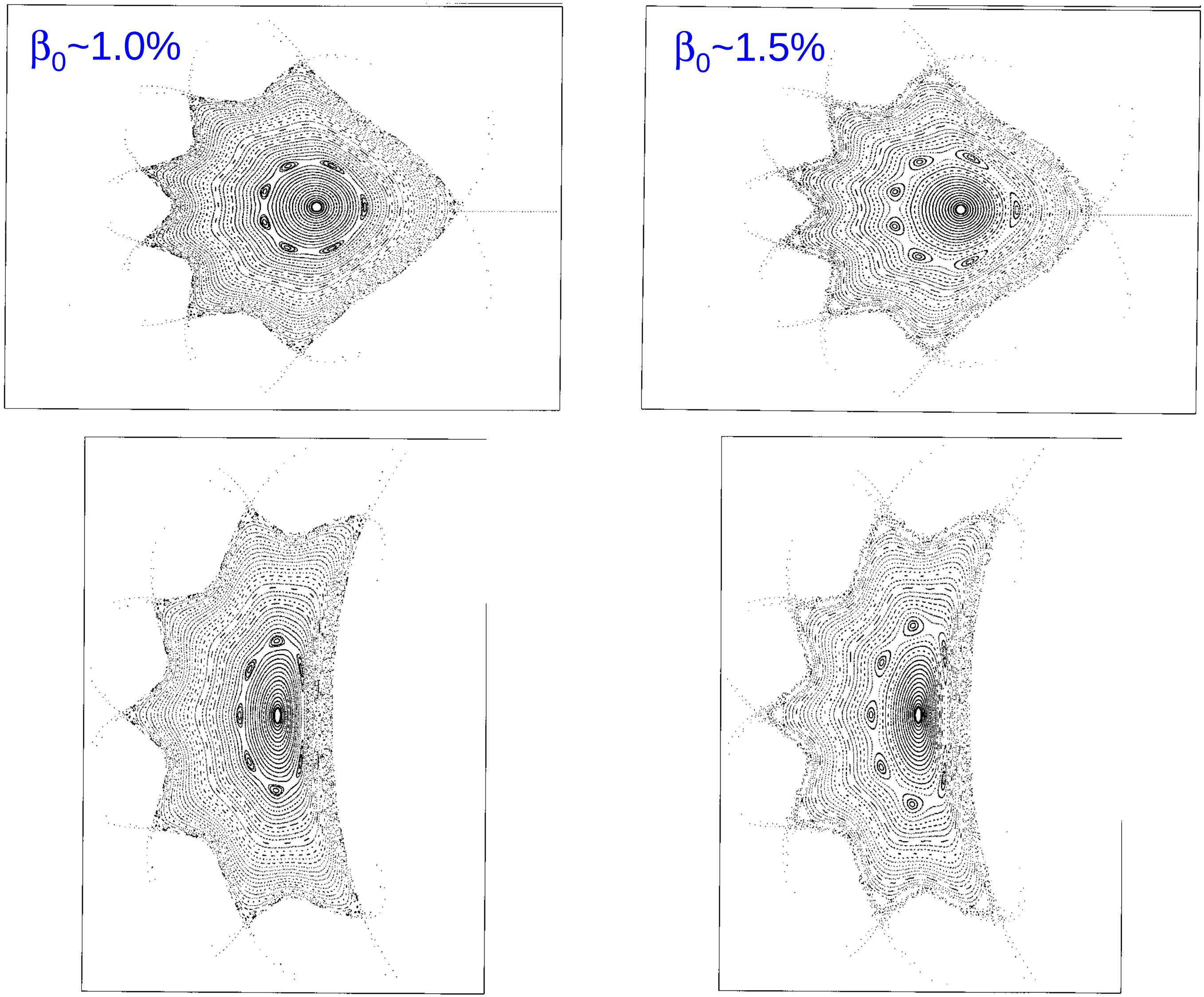
sampling points
→ interpolation

(S.S. Lloids,
H.J. Gardner,
T. Hayashi)



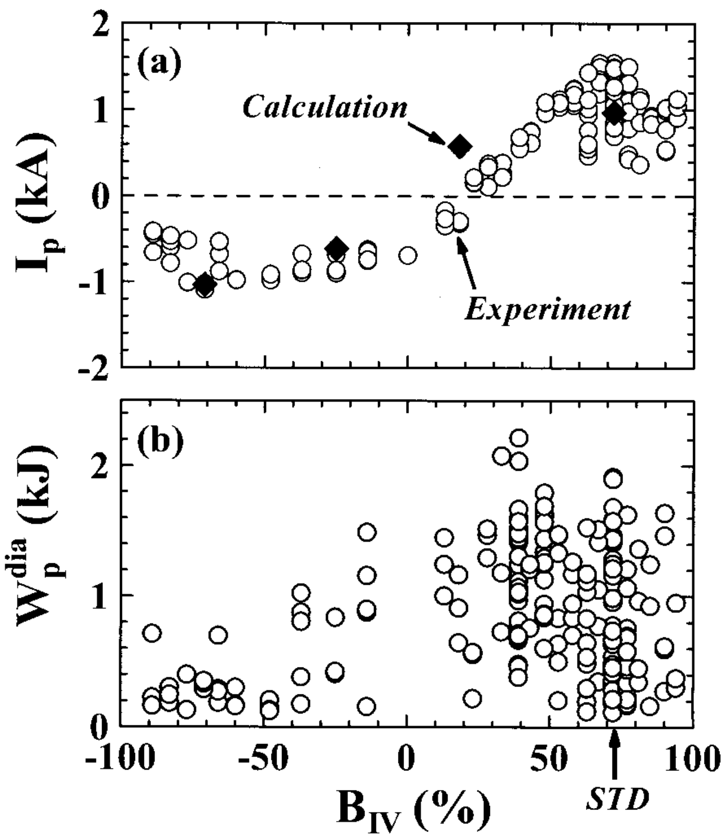
Equilibrium in standard configuration of Heliotron J (HINT)

Initial pressure profile is $p = p_0(1-s)^2$



1.5 Bootstrap current in Heliotron J

* Change of the direction of the bootstrap current was observed in H-J experiments when the IV-coil current is varied.



model calculation with SPBSC (NIFS)

$$n_e = 1.5\times10^{19}(1-r^6)[\mathrm{m}^{-3}]$$

$$T_e = 400(1-r^2)^2[\mathrm{eV}]$$

$$T_i = 150(1-r^2)^2[\mathrm{eV}]$$

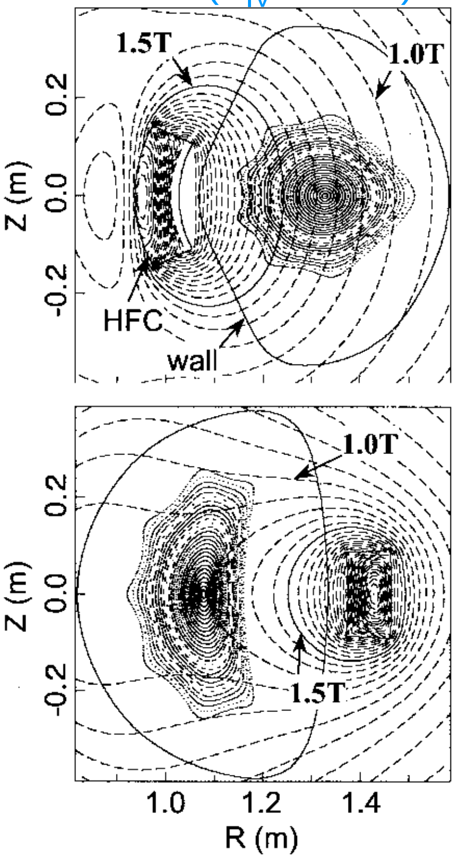
bootstrap current is estimated using fixed boundary VMEC equilibrium without net current.

$\Delta(i/2p) < 0.015$ for $I_p < 2\mathrm{kA}$

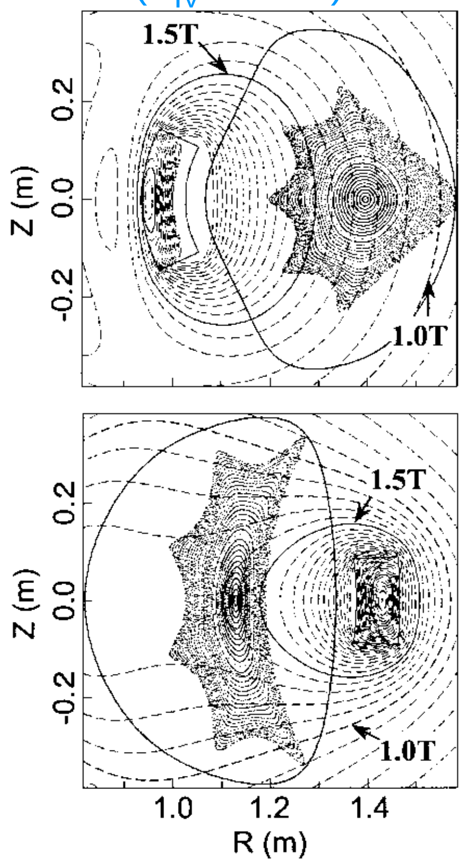
good agreement can be seen

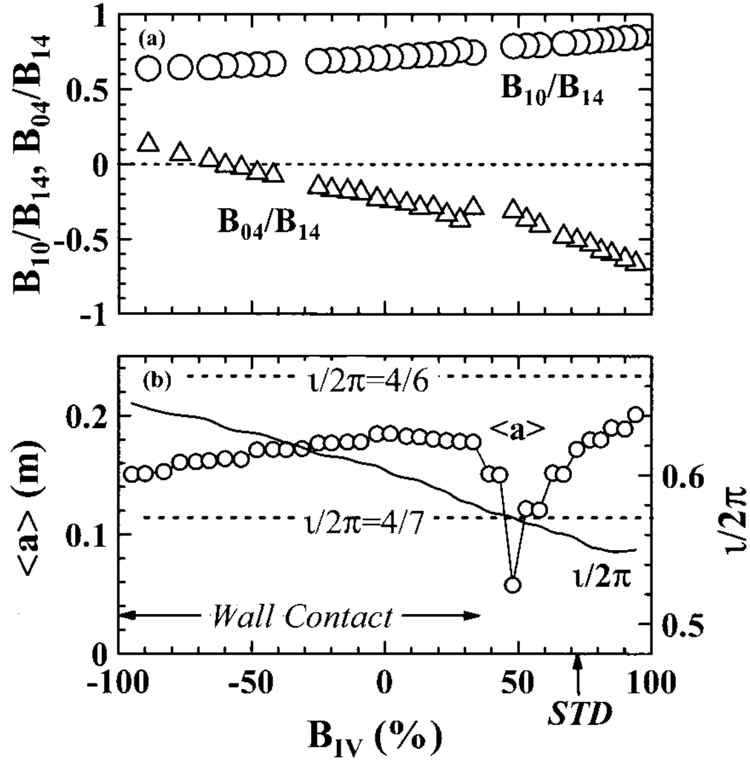
Outward shift by the IV coil current → decrease of bumpy field component (toroidal mirror ratio)
increase of rotational transform
observation of decrease of stored energy

STD(B_{IV}=+72%)



(B_{IV}= -71%)





consistent to the DKES result (→ bumpy field component can control magnitude and direction of BSC)

bumpy field ←→ improved transport ←→ stored energy ?

2. MHD Stability of a Heliotron J Plasma

2.1 Introduction

"Standard" configuration of Heliotron J (fixed boundary equilibrium)
deep magnetic well → interchange mode is stable
(Mercier criterion)

ballooning mode ?
strong bumpiness (toroidal mirror ratio)
→ toroidal dependence ?

local analysis (ballooning mode eq.)

↕

global analysis (CAS3D code)
toroidal mode number dependence ?

effect of the global magnetic shear ?

2.2 Local analysis

Mercier criterion --- localized Interchange mode

Interchange mode is stable in the “standard” configuration

Ballooning Mode Equation (WKB approximation)

$$\vec{\mathbf{B}}\cdot\nabla\left(\frac{k_\perp{}^2}{B^2}\vec{\mathbf{B}}\cdot\nabla x\right)+2\left(\vec{\mathbf{B}}\times\vec{\mathbf{k}}_\perp\cdot\nabla p\right)\left(\frac{\vec{\mathbf{B}}\times\vec{\mathbf{k}}_\perp\cdot\mathbf{?}}{B^4}\right)x+r_m w^2\left(\frac{k_\perp{}^2}{B^2}\right)x=0$$

$$\vec{\mathbf{B}}=\nabla s\times\nabla(y'q-c'f)=c'\nabla a\times\nabla s$$

$a\equiv f-qq$ → field line label on a flux surface

$$\vec{\mathbf{k}}_\perp/k_a=\nabla a+q_k\nabla q=(\nabla f-q\nabla q)-\underline{(q-q_k)\nabla q}$$

$$q_k\equiv k_q/k_a$$

secular term → non-periodicity → covering space
(flute-like approx.) quasi-mode

non-axisymmetry → α dependence

eigenfunction ; $x(\ell\,|\,s,a,q_k)$ ↔ eigenvalue ; w^2

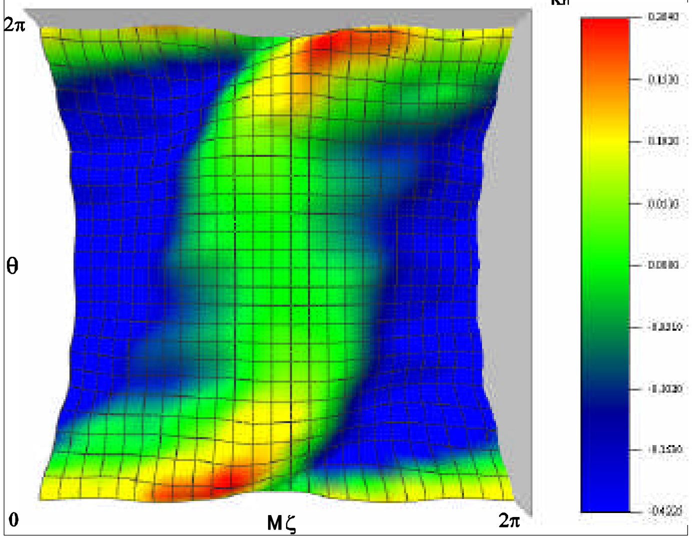
dispersion relation ; $\omega^2=\lambda(s,a,q_k)$

Typical level surface of λ in Heliotron J

→ strong α-dependence
weak θ_k-dependence

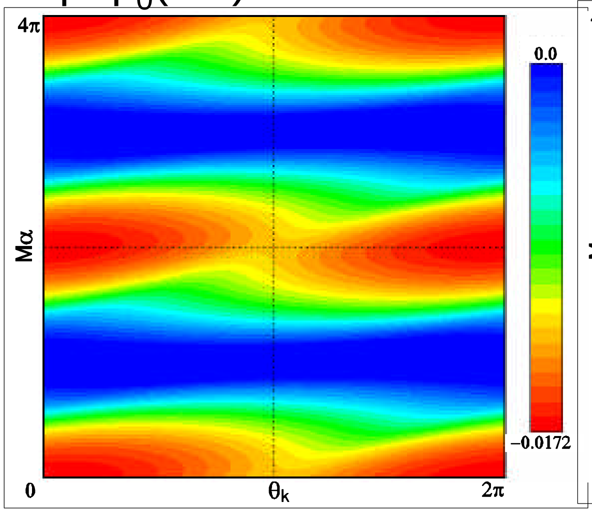
O. Yamagishi, Y.Nakamura, K. Kondo
Phys. Plasma **8** (2001) 2750.

normal curvature

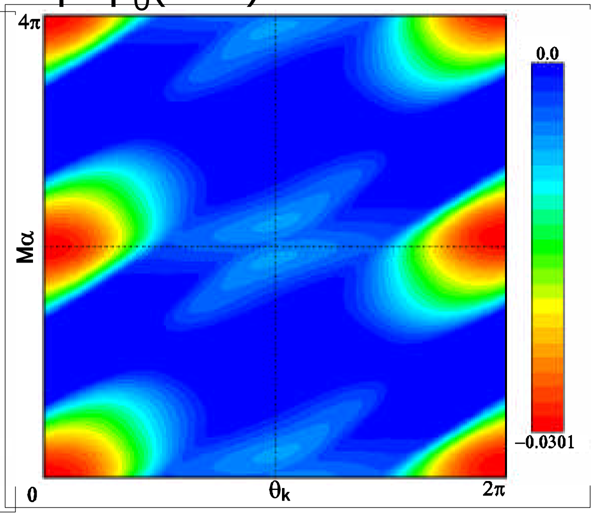


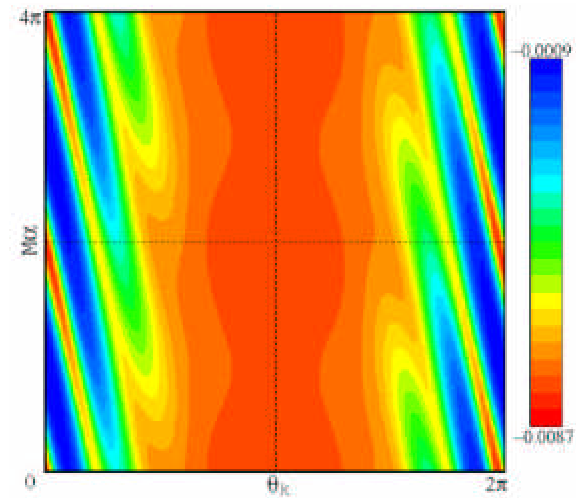
contours of $\lambda(s,\alpha,\theta_k)$

$p=p_0(1-s)^2$



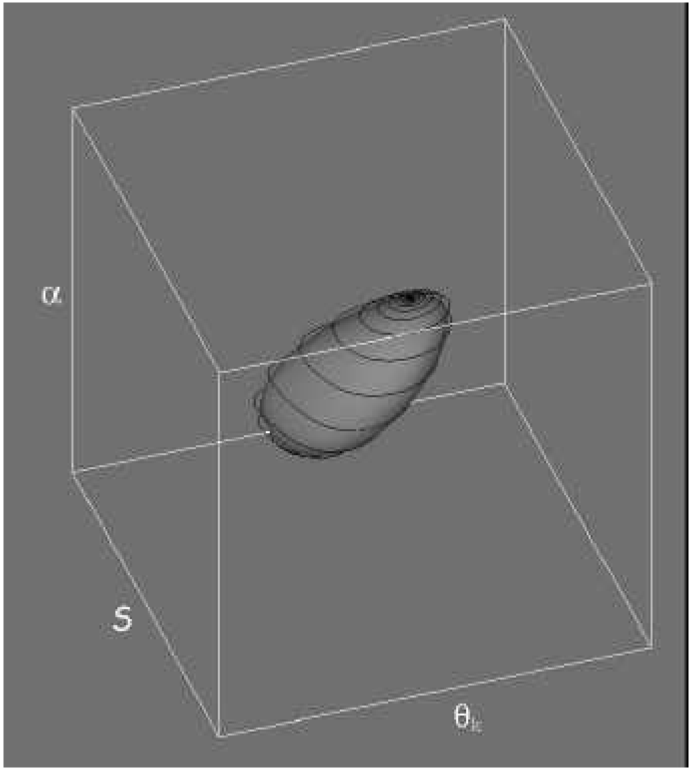
$p=p_0(1-s^2)^2$





Mercier unstable
LHD equilibrium

Quantization
(finite toroidal mode number)



ray equations

$$
\begin{cases}
\dot{a} = -q_k \dfrac{\partial l}{\partial q_k}, \\
\dot{q} = \dfrac{\partial l}{\partial q_k}, \\
q_k = q_k \dfrac{\partial l}{\partial \partial} - \dfrac{\partial l}{\partial q}.
\end{cases}
$$

ray trajectory

type C; periodic in α --- discrete mode → quantization condition;

$$\boxed{\Delta a\,(w^2)/2p = (2N+1)/2n}$$

type S; quantum chaos ?

strong toroidal mode coupling ?
→ single toroidal mode number
cannot dominate the mode structure ?

toroidal mode number ; *n*
number of nodes ; *N*

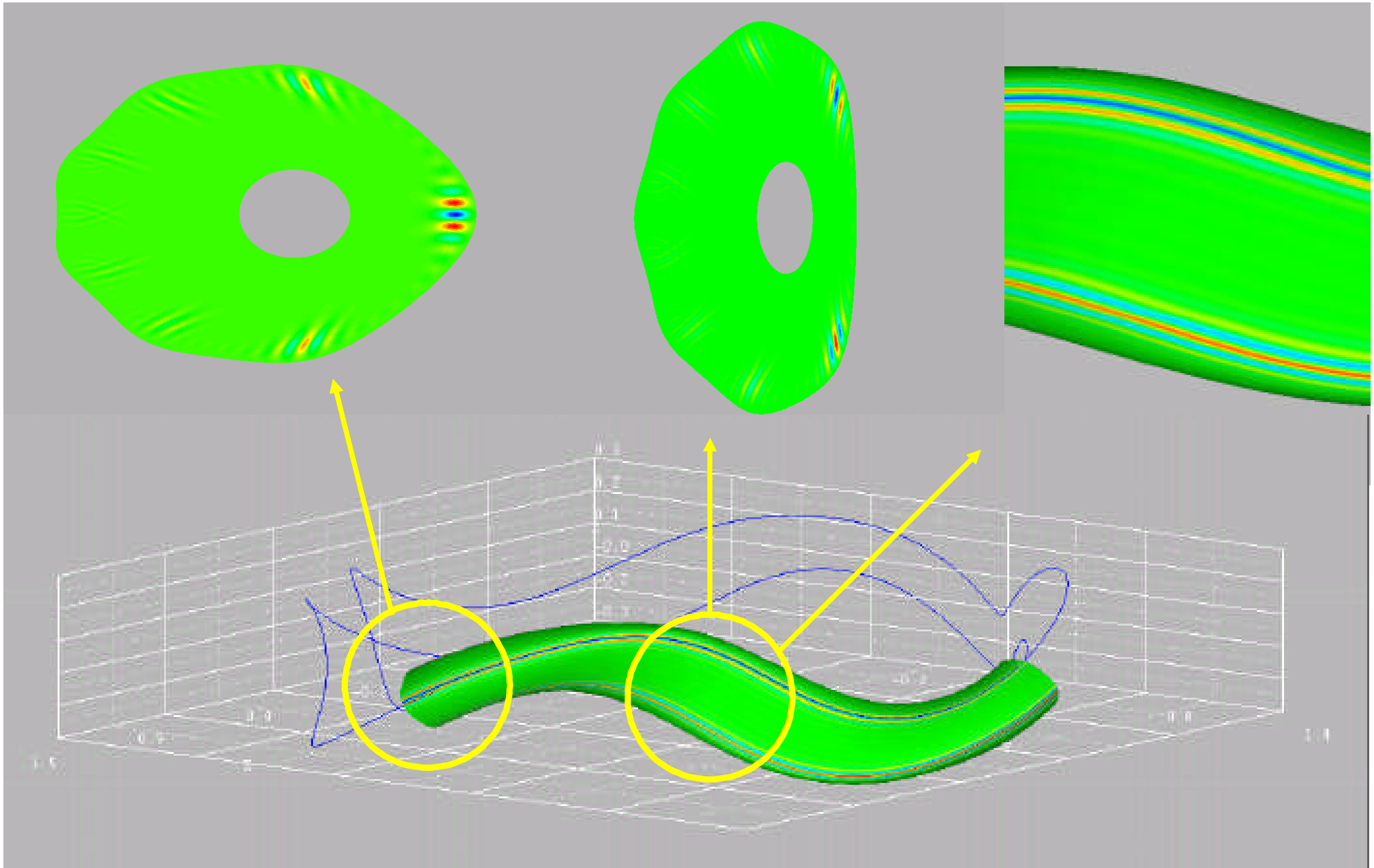
2.2 Global mode analysis by the CAS3D code

strong toroidal mode coupling
perturbation is extended
along the flux tube

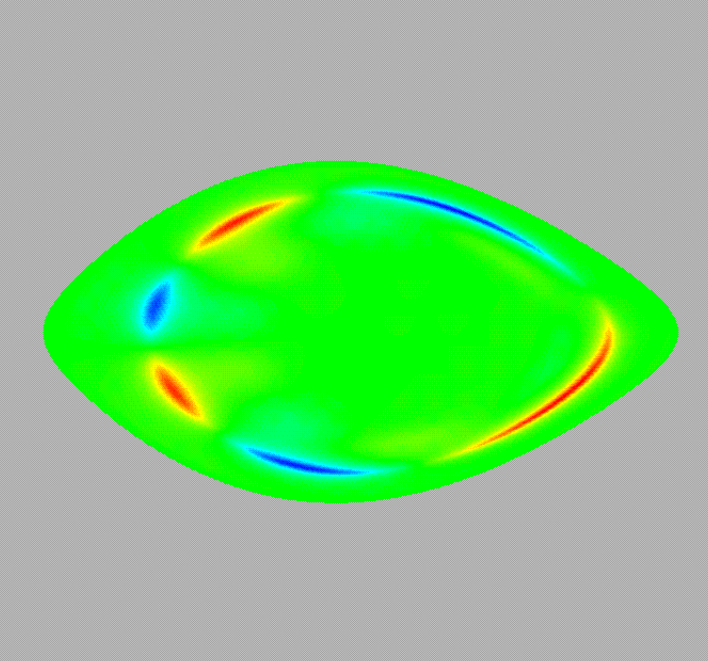
strong α dependence
weak θ_k dependence

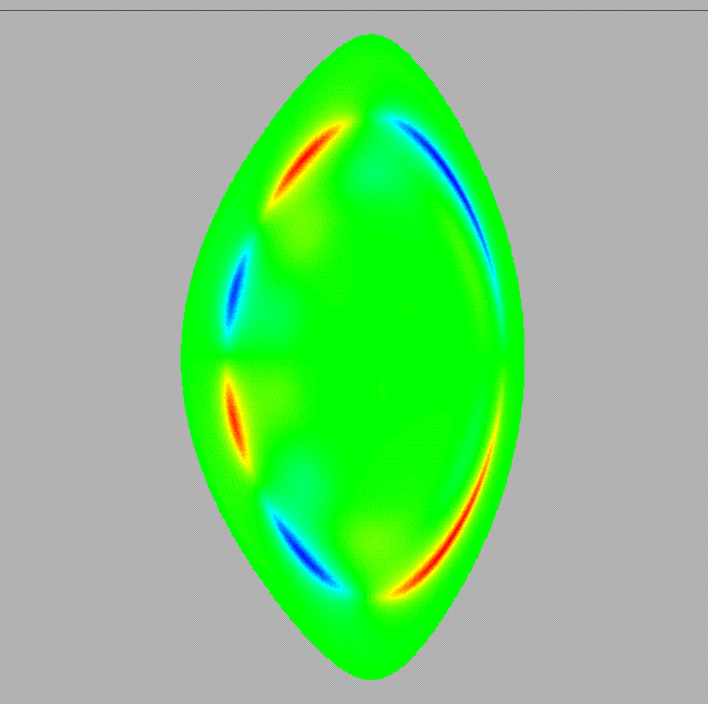
local analysis

helical ballooning



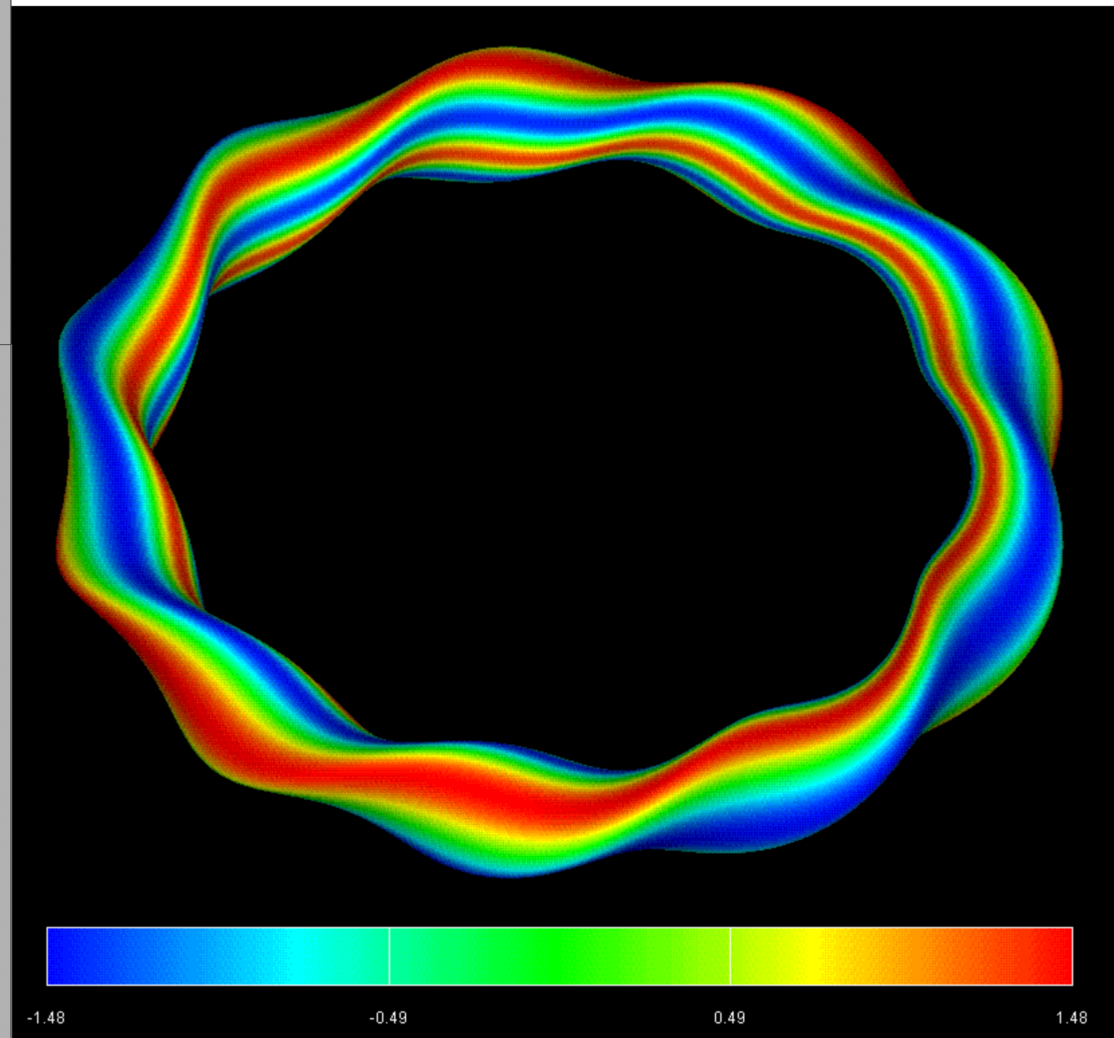
IDEAL Low-n Interchange mode in LHD

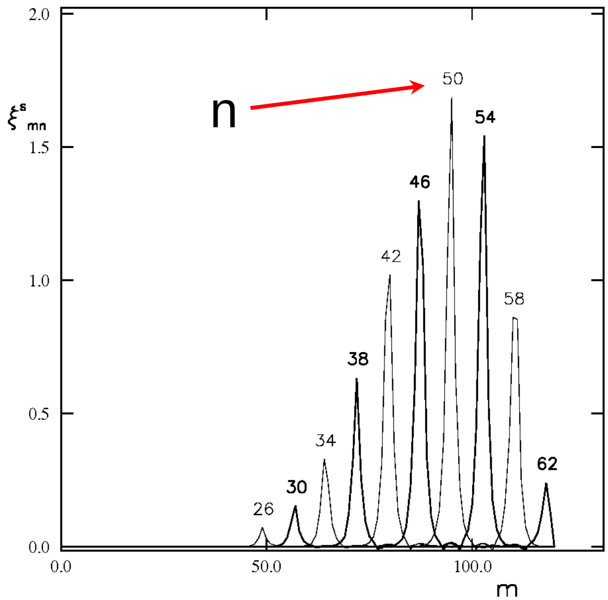


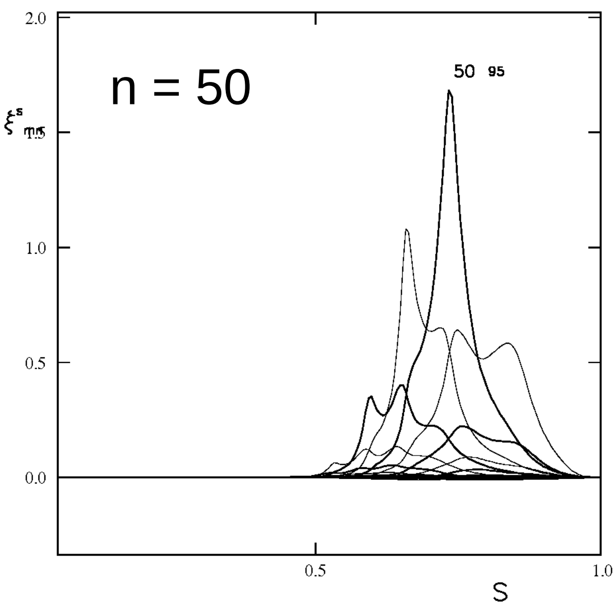


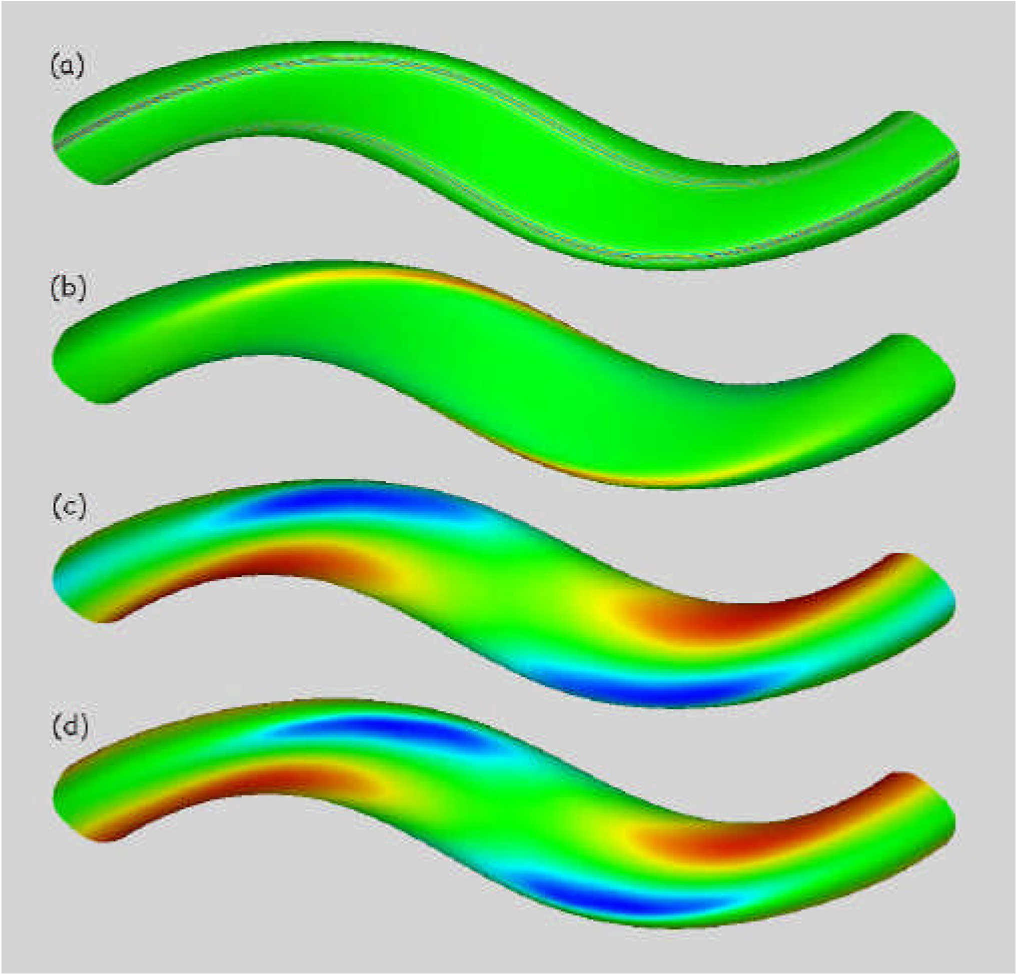
Typical mode structure

GLOBAL: $n=2$, $p=p(0)(1-s^2)^2$









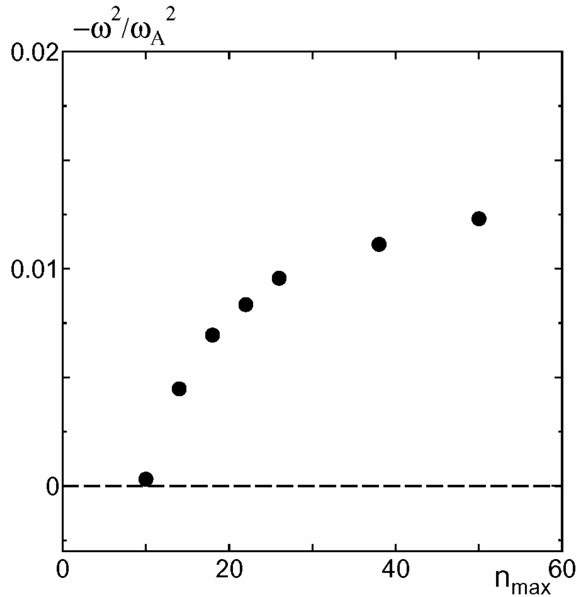
(a) perturbation ξ^s

(b) local shear

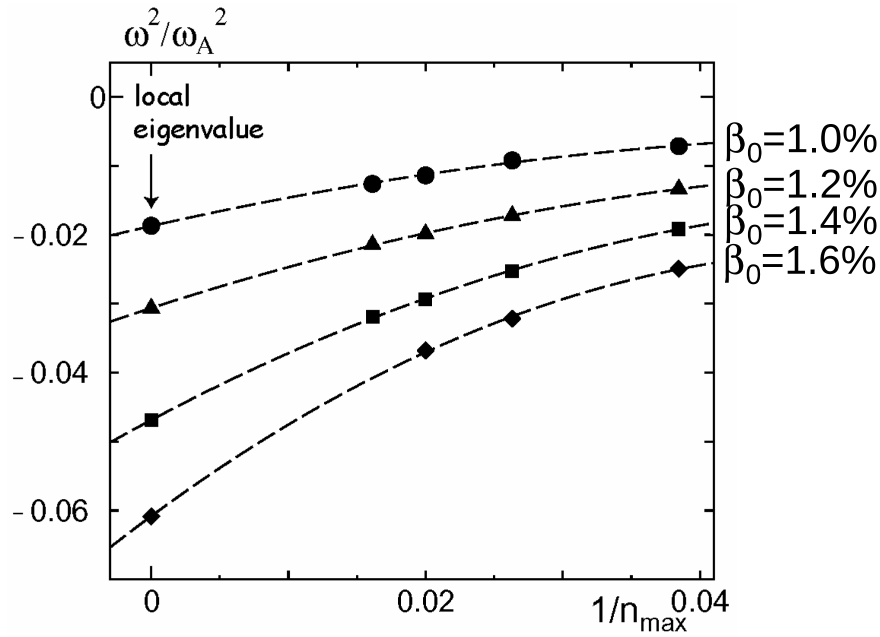
(c) normal curvature

(d) geodesic curvature

O. Yamagishi, Y.Nakamura, K. Kondo, N. Nakajima
Phys. Plasma **9** (2002) 3429.



n_{max} dependence of the growth rate



n_{max} dependence of the growth rate

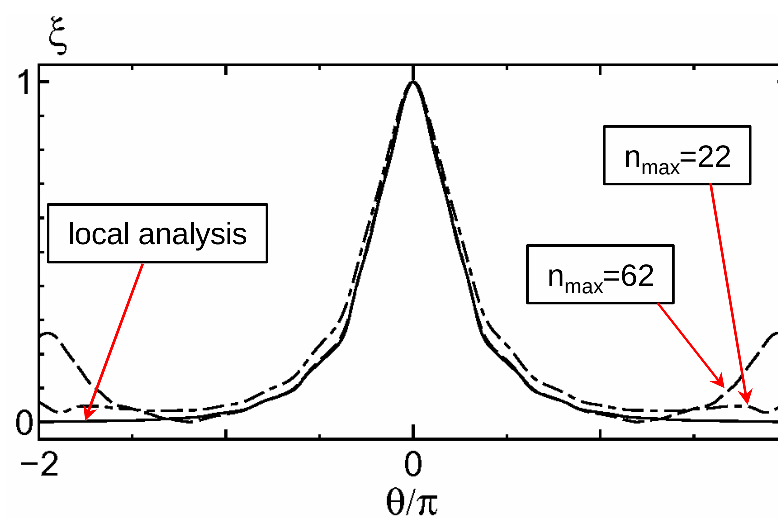
Critical number of n_{max} (maximum toroidal mode number used in a calculation) exists.

$n_{max} < 10$ → stable ↔ ? quantum chaos

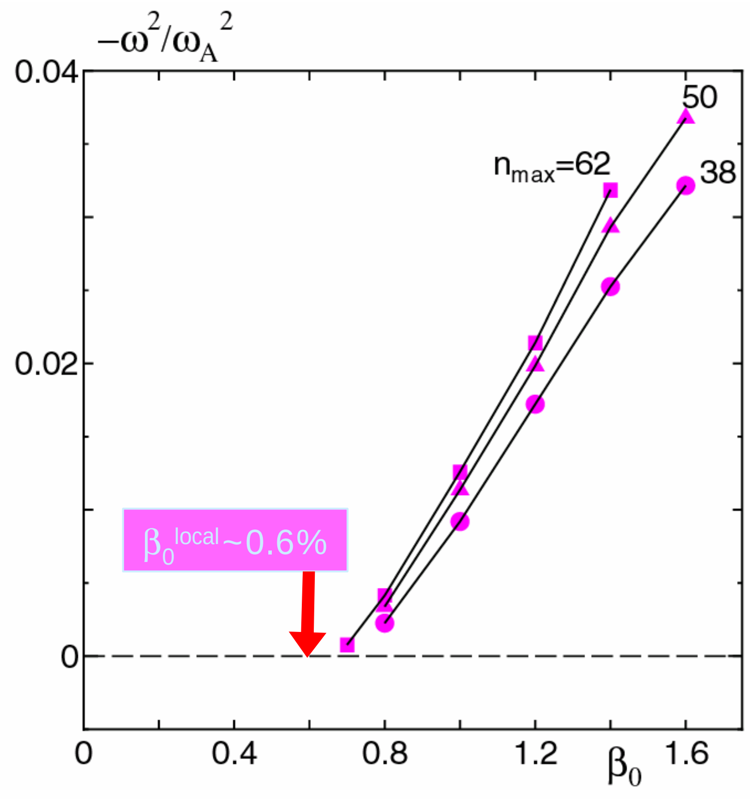
$n_{limit} = 17$

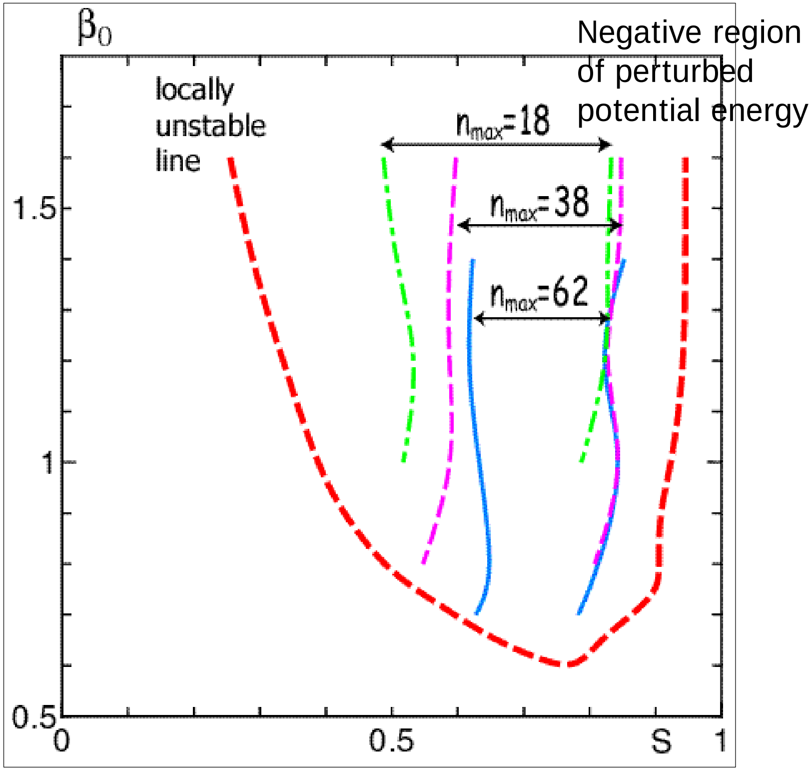
insufficient number of toroidal modes

→ the mode cannot be localized in a flux tube



eigenfunction along the field line





high-n_{max} mode has narrower radial structure
increase n_{max} → critical beta tends to the local value

Ballooning mode

Pressure driven mode and mode coupling

Cylindrical plasma … no mode coupling
"bad" curvature and local shear
are not localized on a flux surface → interchange

⇩

Axisymmetric torus … poloidal mode coupling
"bad" curvature and local shear
can be localized poloidally on a flux surface → interchange
tokamak ballooning

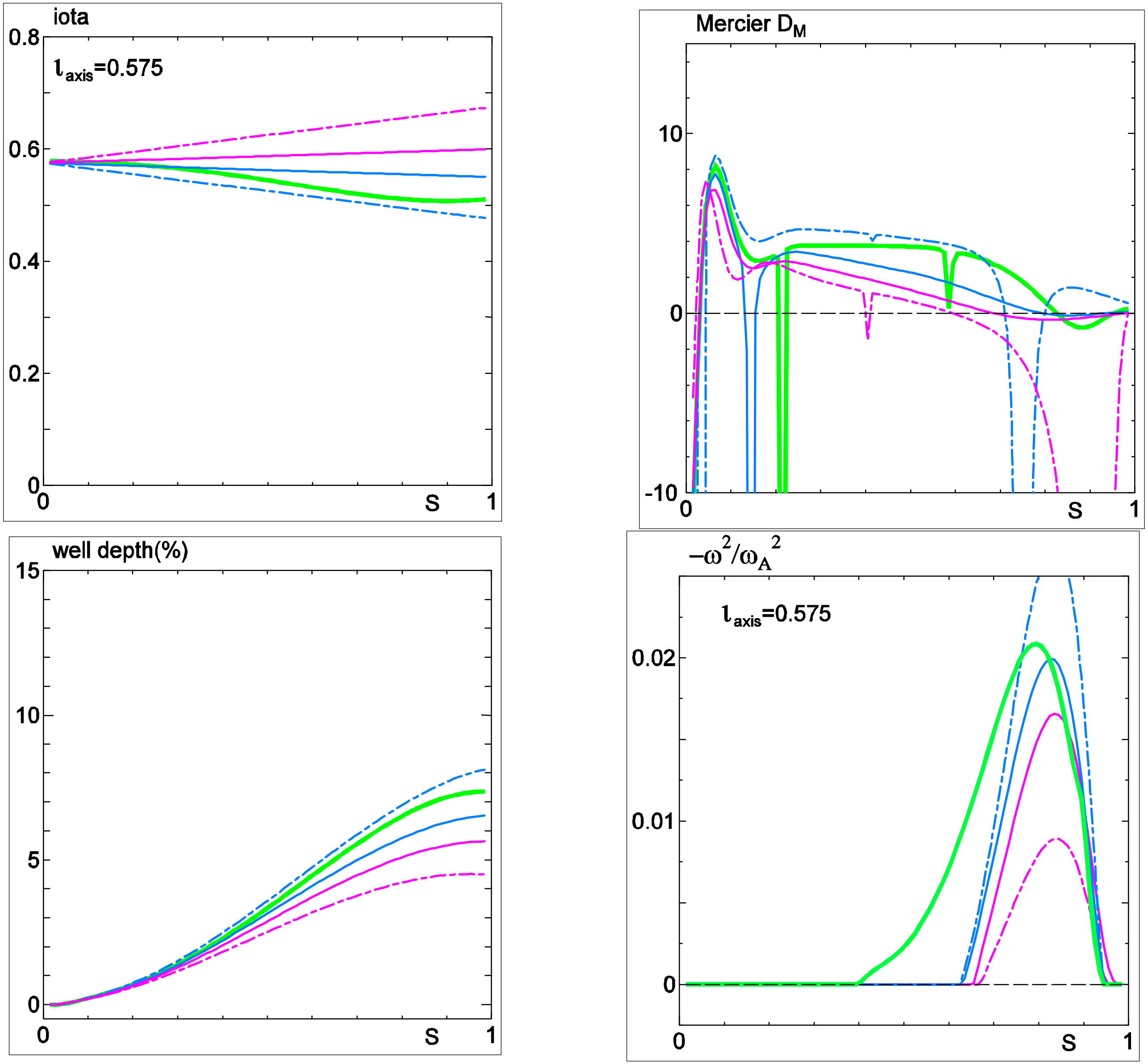
⇩

Non-axisymmetric torus … poloidal & toroidal (helical) mode coupling
"bad" curvature and local shear
can be localized freely on a flux surface → interchange
tokamak ballooning
helical ballooning

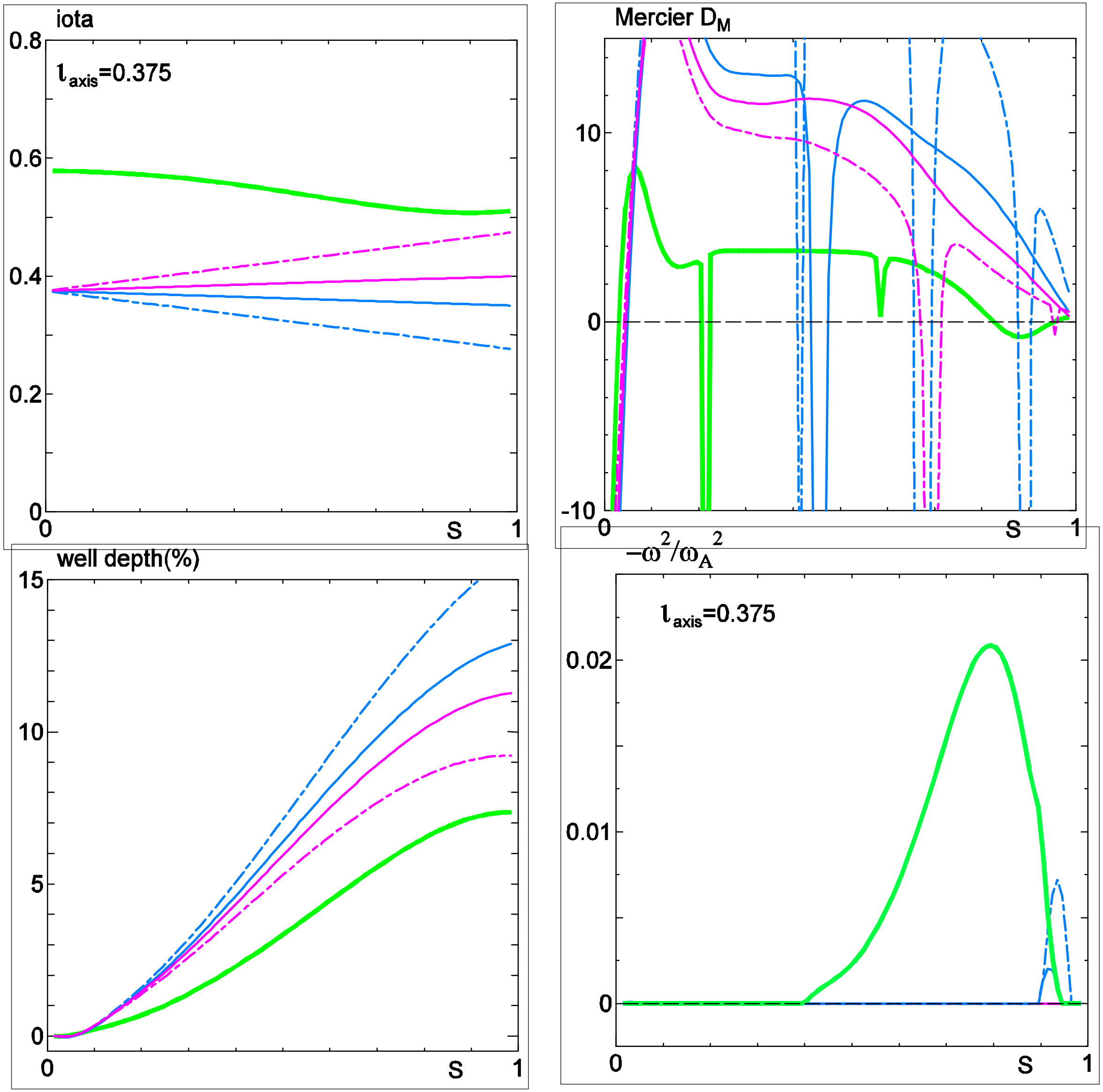
Characteristics of helical ballooning

corresponding to the mode whose level surface of λ is spherical
in a local analysis (ballooning mode eq.)
can be unstable even if a negative shear (q'<0) case
strongly localized in the toroidal direction
➔ strong toroidal mode coupling is essential

Magnetic shear dependence of the pressure driven mode in Heliotron J



Magnetic shear dependence of the pressure driven mode in Heliotron J



positive ι' is favorable against helical ballooning mode in a low shear stellarator.

low ι is favorable against interchange mode and helical ballooning mode.

Summary

MHD equilibrium

1. Importance of free boundary equilibrium calculation in a low shear stellarator is shown.
2. Improved methods to resolve the difficulties in calculating a H-J free boundary equilibrium by PIES and HINT are discussed.
3. Though the free boundary calculation by the VMEC cannot reproduce fine wavy structure near the plasma boundary, it gives reasonable equilibrium properties unless there is no wide magnetic island in the plasma.
4. Good agreement can be seen between bootstrap current observed in H-J and that calculated by the SPBSC code.

Pressure driven instabilities

1. Close relation is found between local and global analysis for the helical ballooning mode.
2. It is shown that the magnetic shear has crutial role on the helical ballooning mode in a low shear stellarator.