Delta-Function Currents in VMEC

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$${}^{OLD}B_s = B^{VMEC} \cdot e_s = \text{OLDBSUBS}$$

where

$$s = \Psi / \Psi_{edge} \quad \text{and} \quad e_s = \frac{\partial \vec{X}}{\partial s}$$

$${}^{NEW}B_s = \text{NEWBSUBS}$$

is defined from the e_s component of the equilibrium equation

$$J \times B = \nabla p$$

$$B \cdot \nabla({}^{NEW}B_s) = \frac{\partial p}{\partial s} + B^{\phi} \frac{\partial B_{\phi}}{\partial s} + B^{\theta} \frac{\partial B_{\theta}}{\partial s}$$

Using ${}^{NEW}B_s$ in the equation $J = \nabla \times B$ the equilibrium equation is satisfied exactly.

IZETA = jJ^{ϕ}

IZETA calculated using OLDBSUBS contains the delta - functions

IZETA calculated using NEWBSUBS does not contain delta - functions

I will look at three NCSX like cases

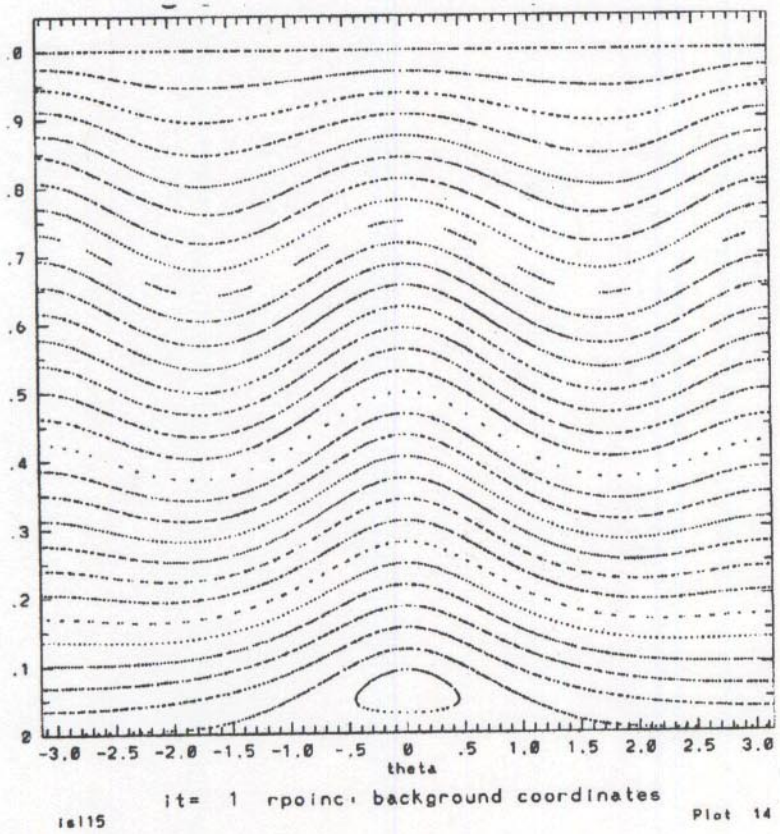
1) zero beta, zero net current

2) zero beta, finite net current

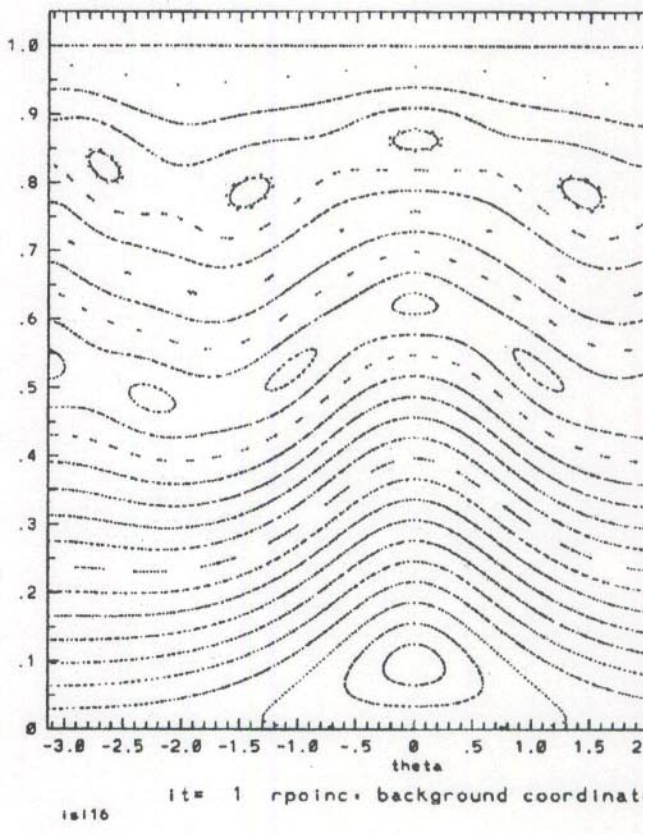
3) finite beta, finite net current

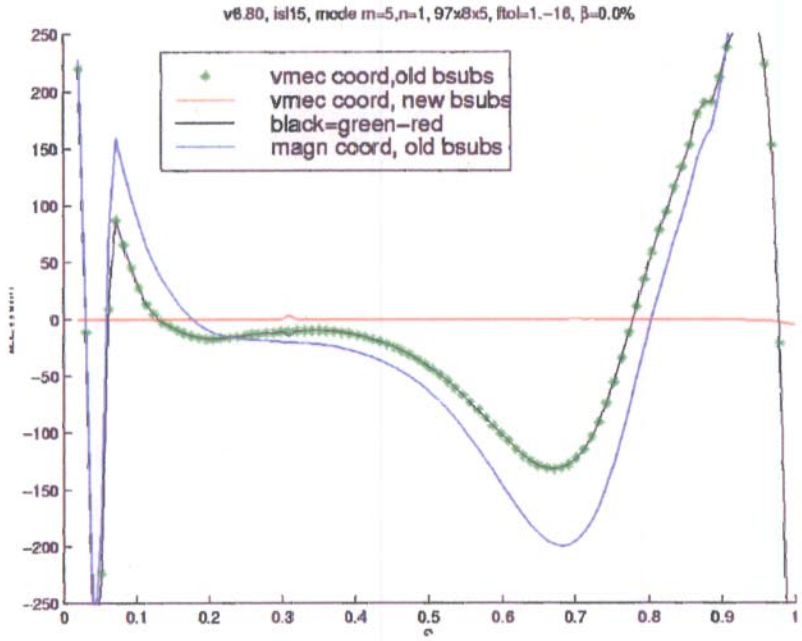
The following plots are for NCSX like cases with zero beta and zero net current.

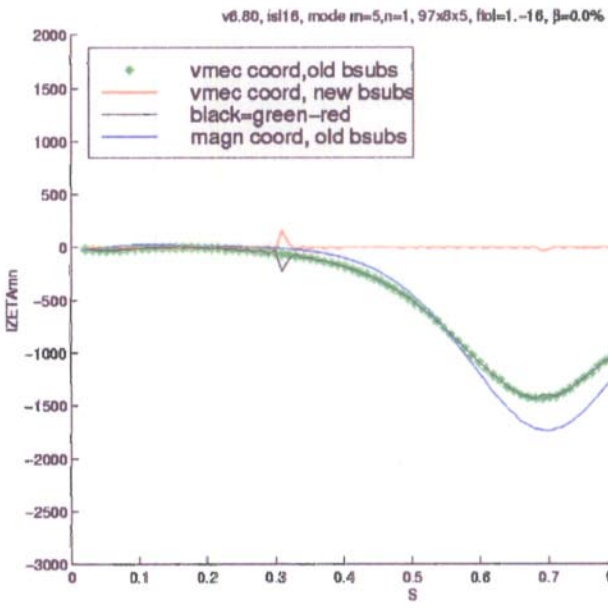
ISL15 column

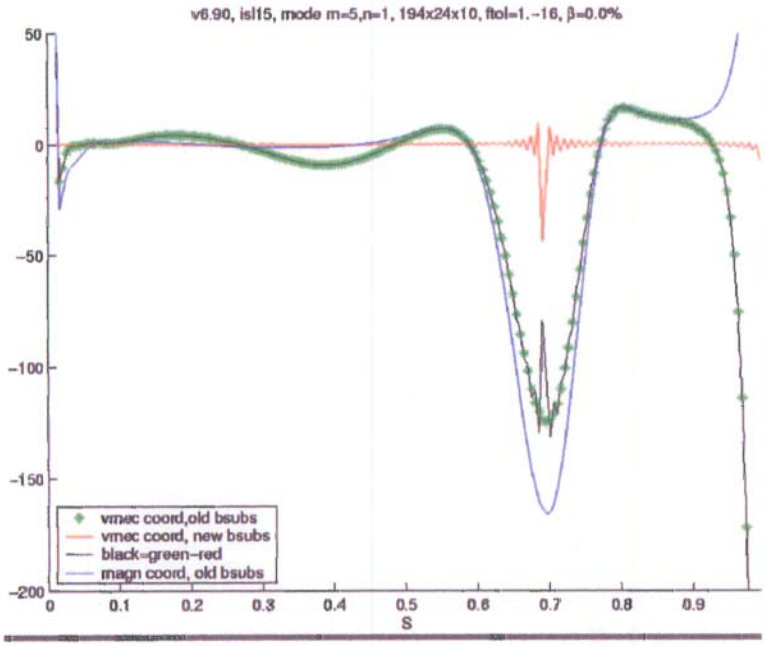


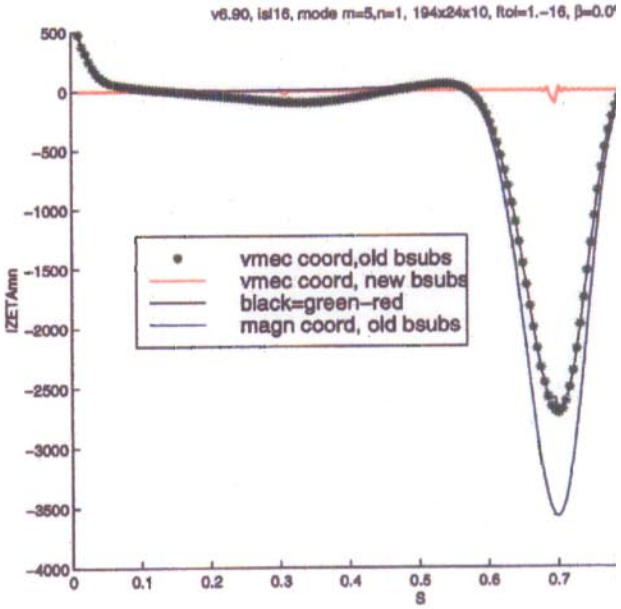
ISL16 column

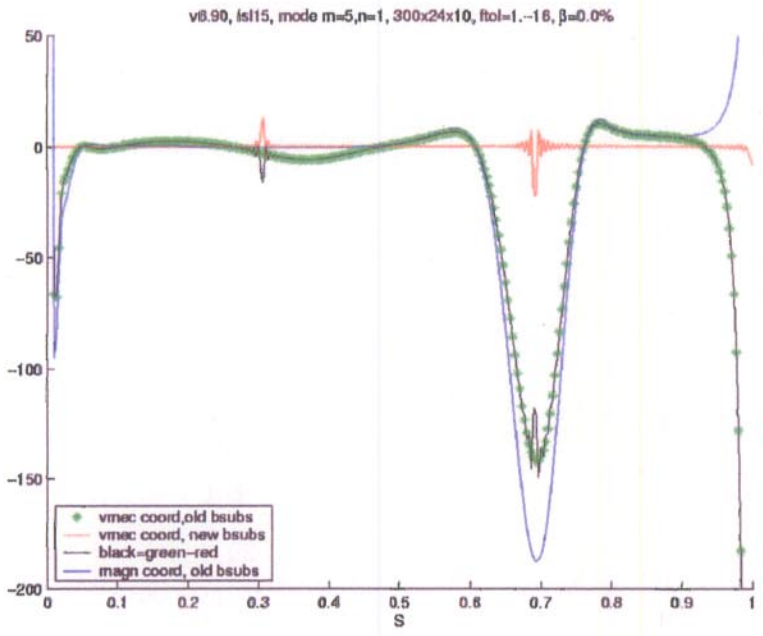


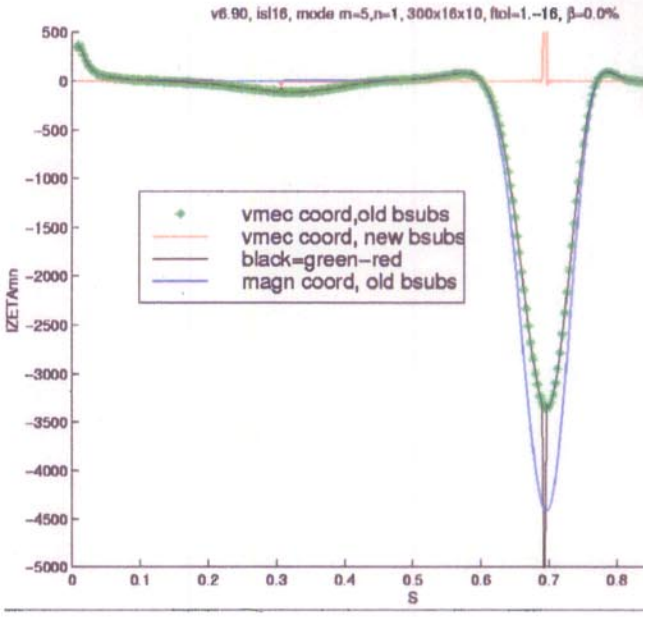

v6.80, isl15, mode m=5,n=1, 97x8x5, ftol=1.-16, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs

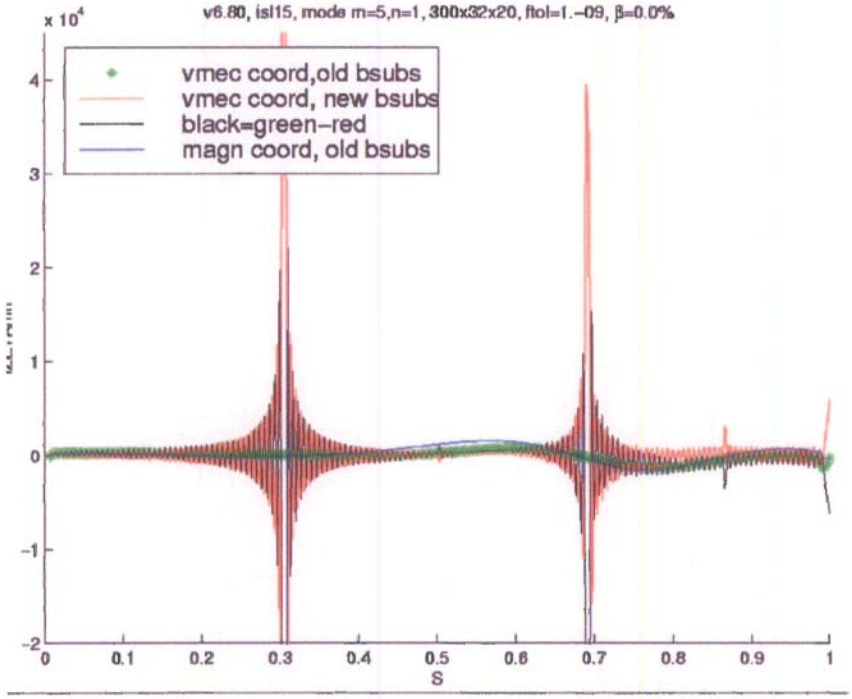

v6.80, isl16, mode m=5,n=1, 97x8x5, ftol=1.-16, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs
IZETAmn
S


v6.90, isl15, mode m=5,n=1, 194x24x10, ftol=1.-16, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs
S


v6.90, isl16, mode m=5,n=1, 194x24x10, ftol=1.-16, β=0.0
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs
IZETAmn
S

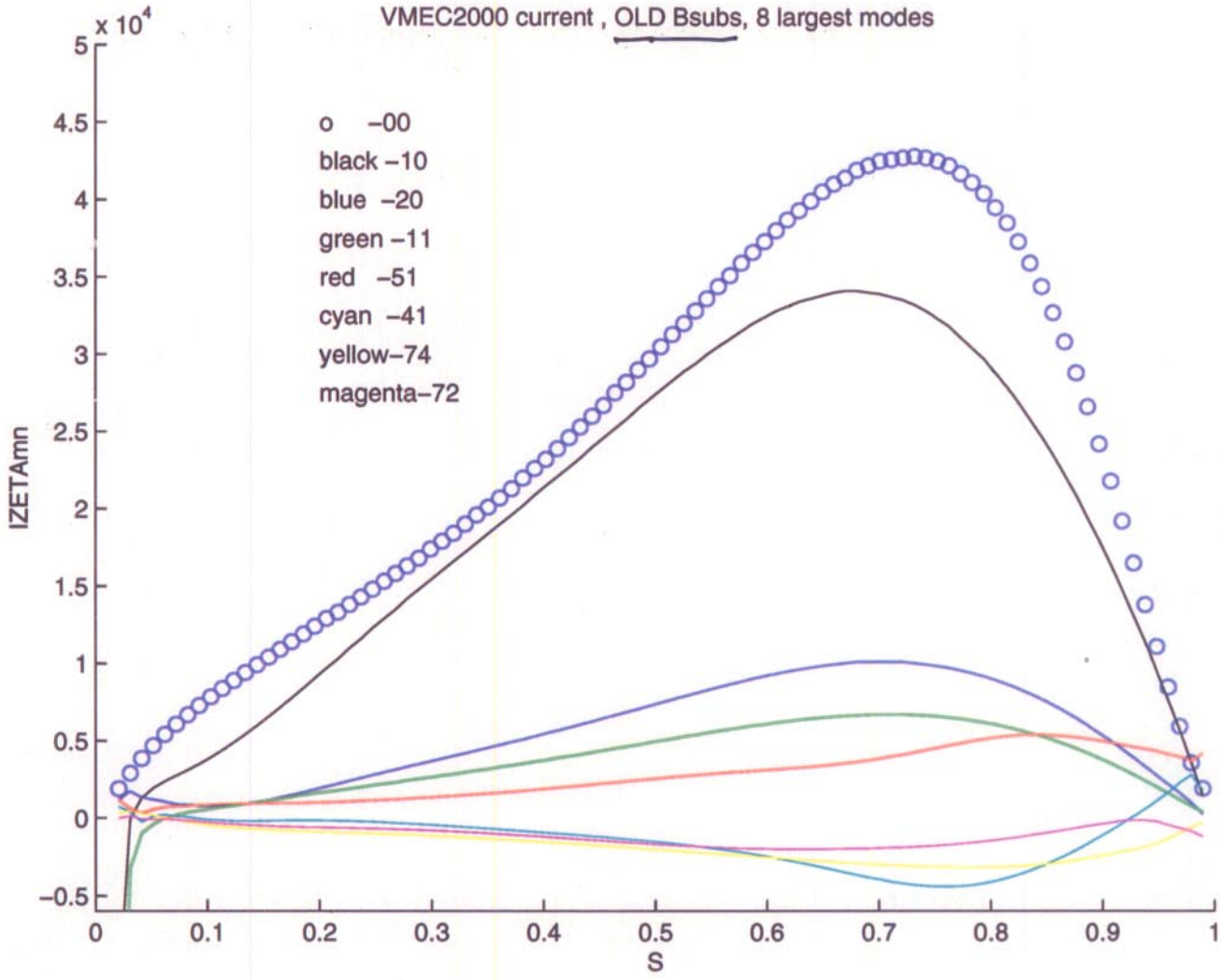

v6.90, isl15, mode m=5,n=1, 300x24x10, ftol=1.-16, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs
S

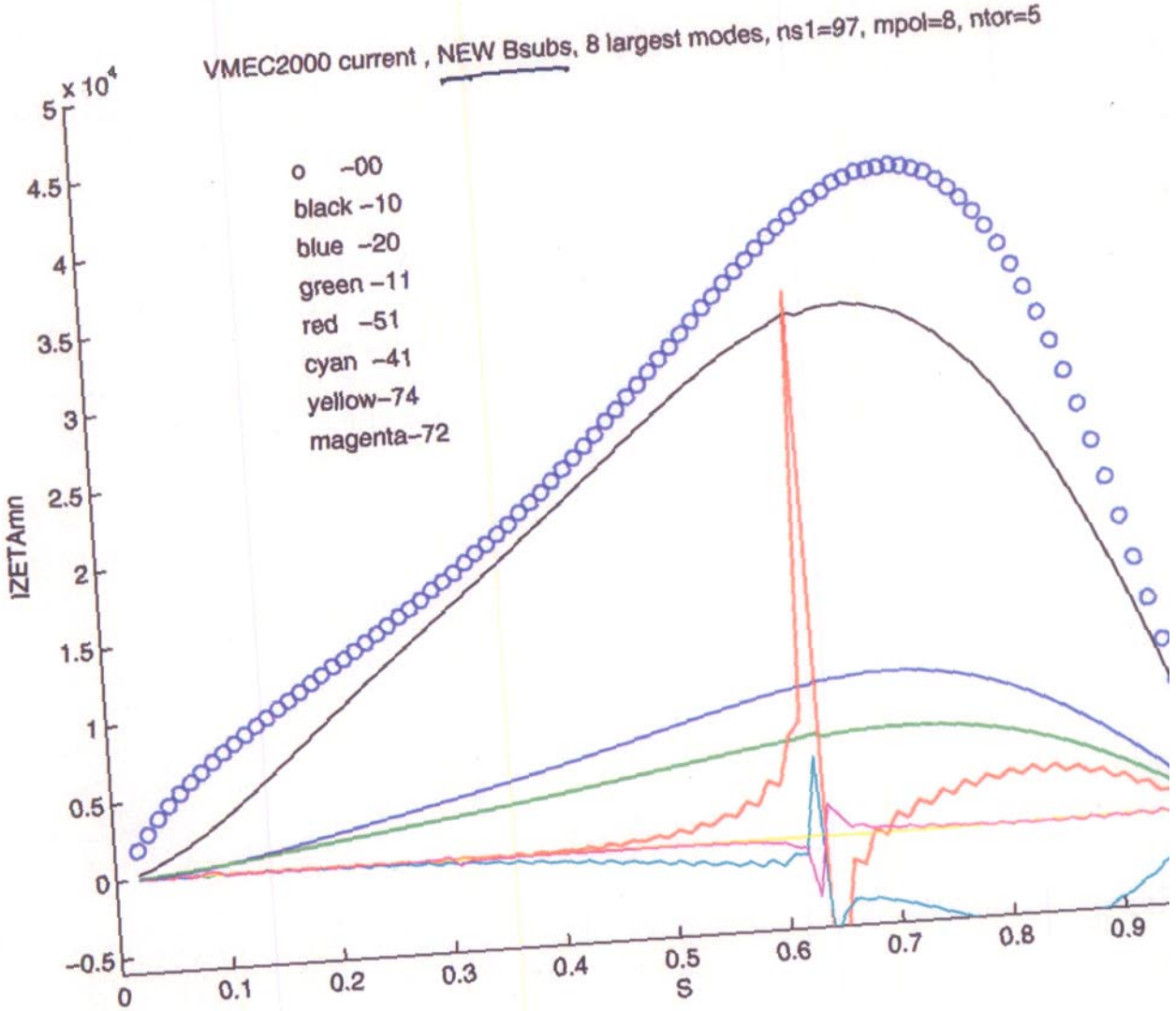

v6.90, isl16, mode m=5,n=1, 300x16x10, ftol=1.-16, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs
|ZETAmn|
S


x 10^4
v6.80, isl15, mode m=5,n=1, 300x32x20, ftol=1.-09, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green-red
magn coord, old bsubs
S

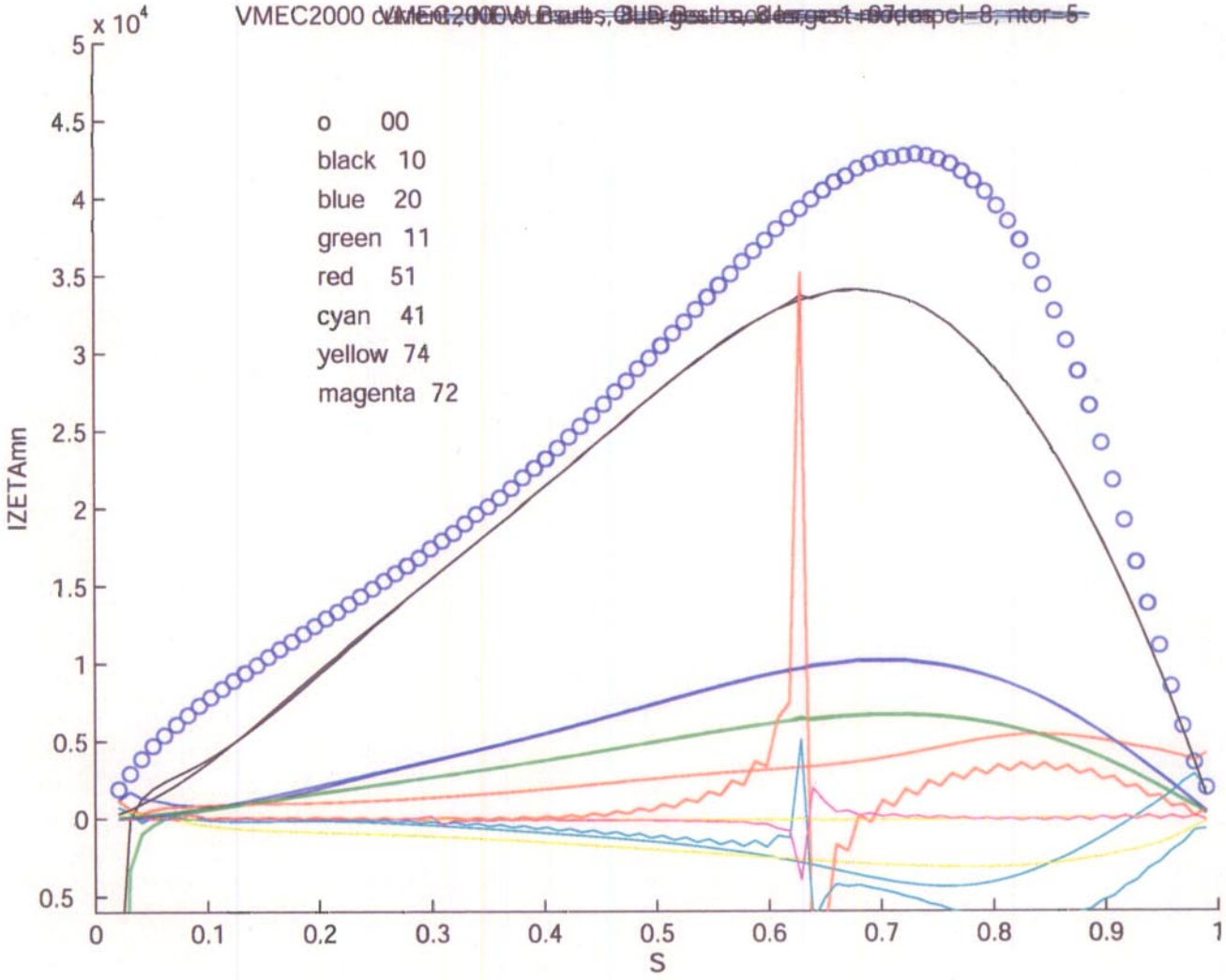
The following plots are for NCSX like cases with zero beta and finite net current.

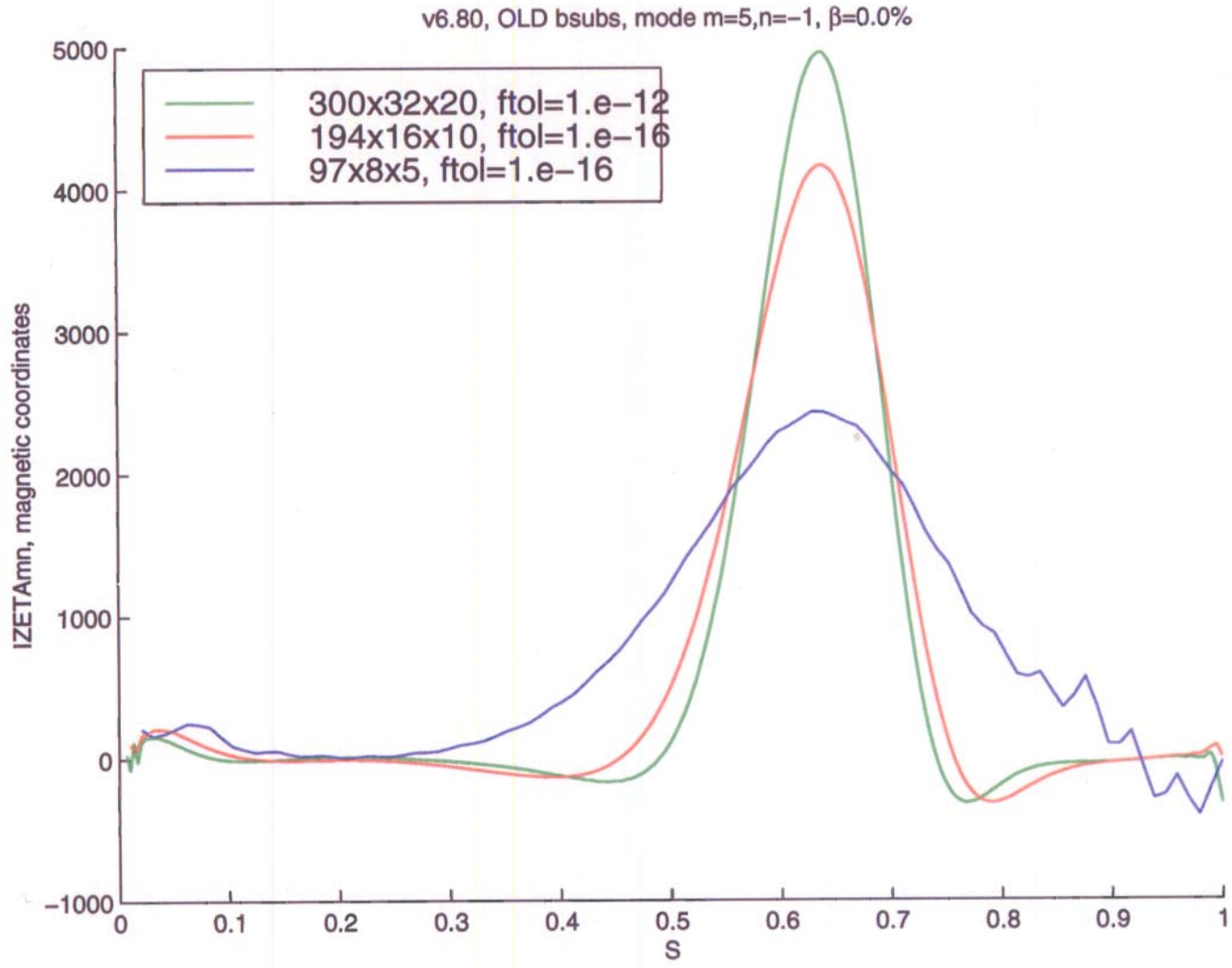
ns = 97, mpol = 8, ntor

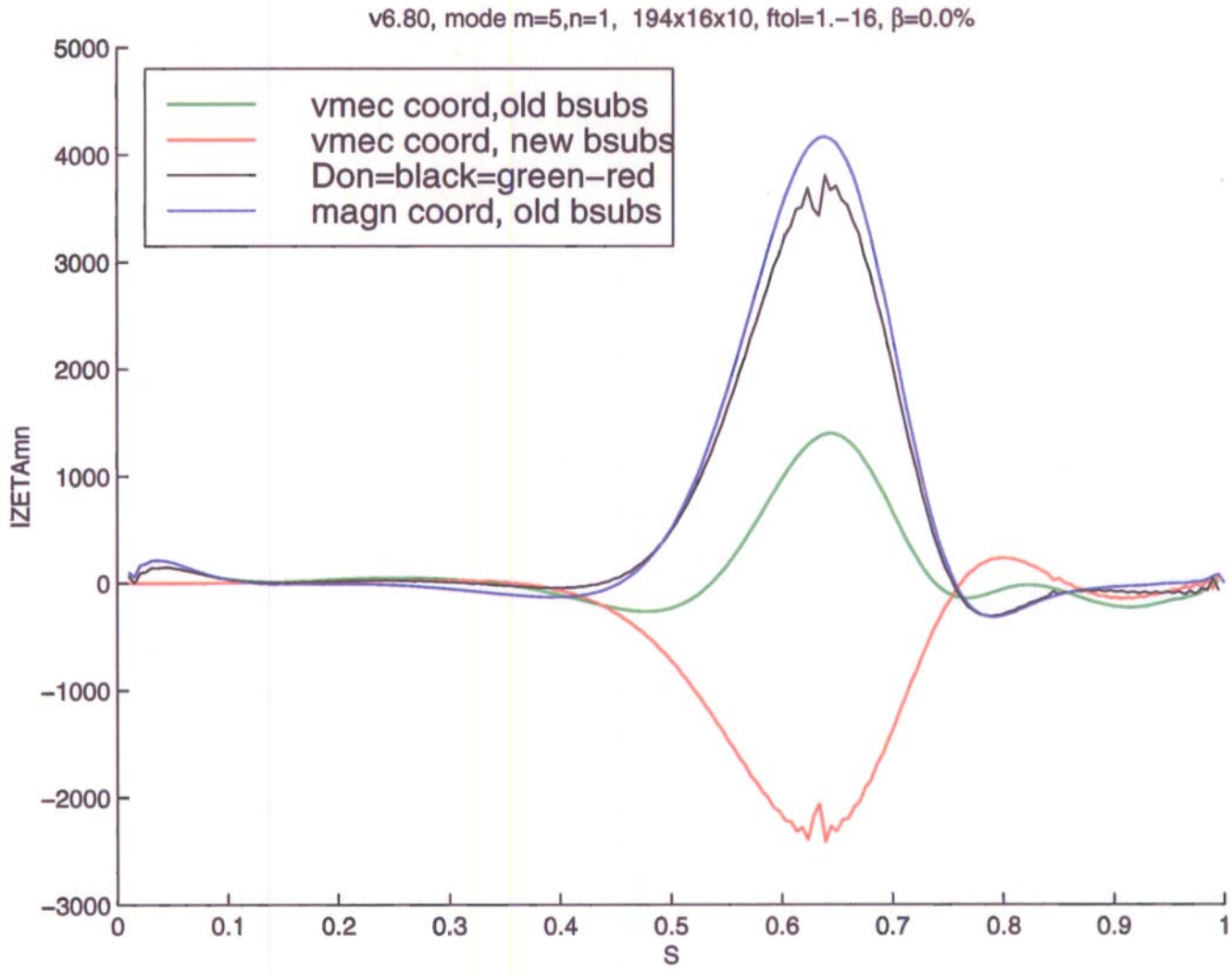


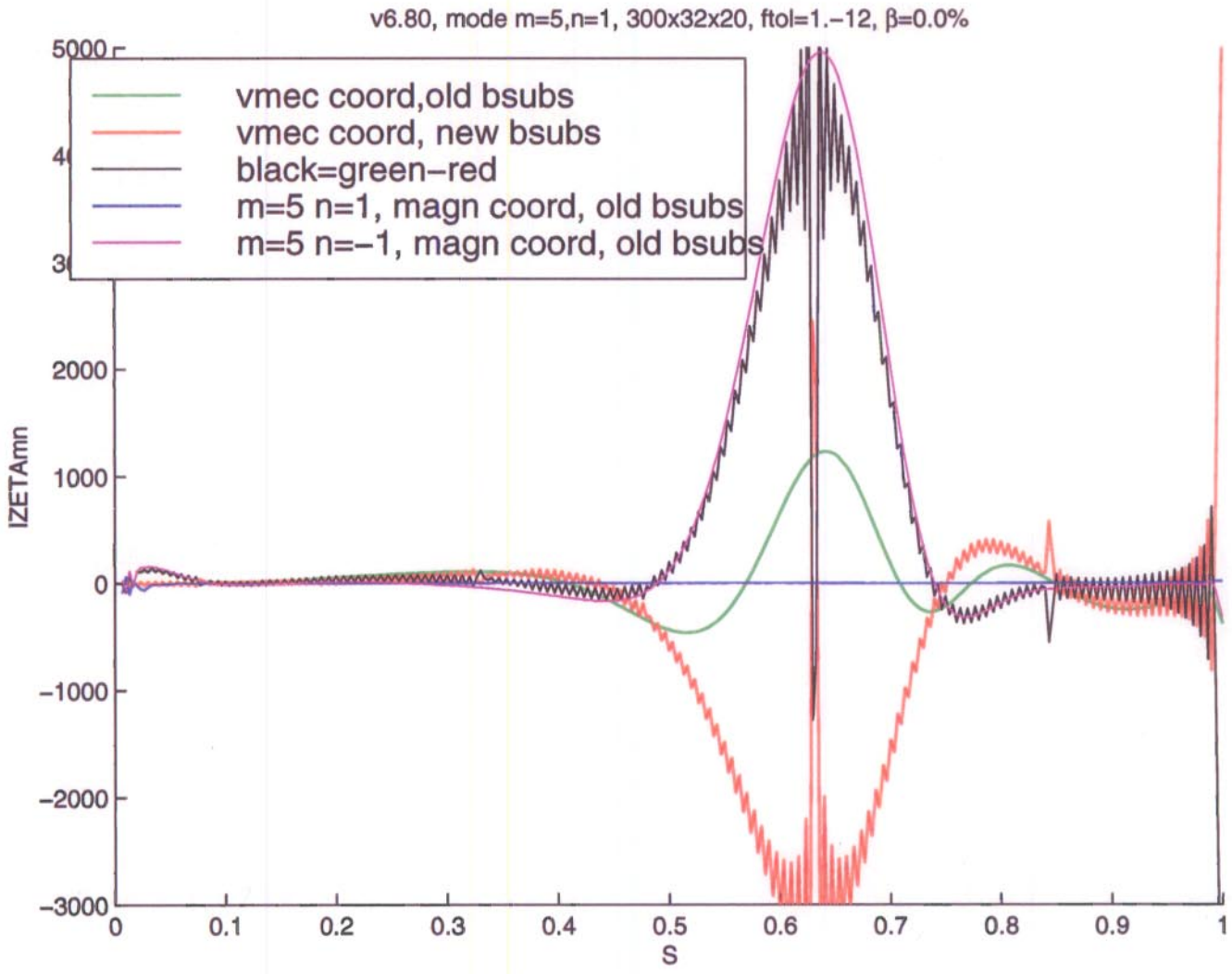

VMEC2000 current , NEW Bsubs, 8 largest modes, ns1=97, mpol=8, ntor=5
x 10^4
o -00
black -10
blue -20
green -11
red -51
cyan -41
yellow-74
magenta-72
|ZETAmn|
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
S

OVERLAY OLD & NEW Bsubs
97 x 8 x 5

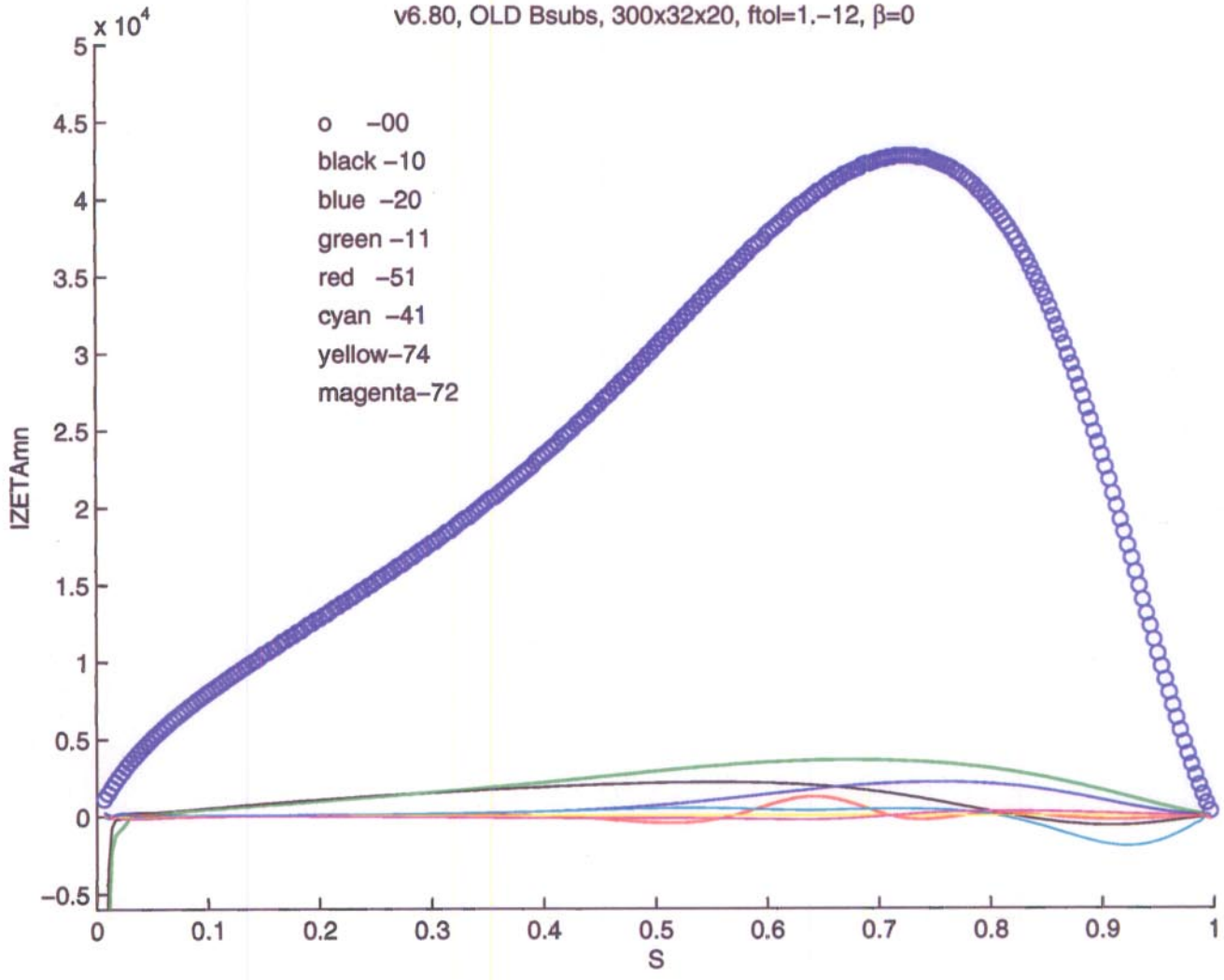


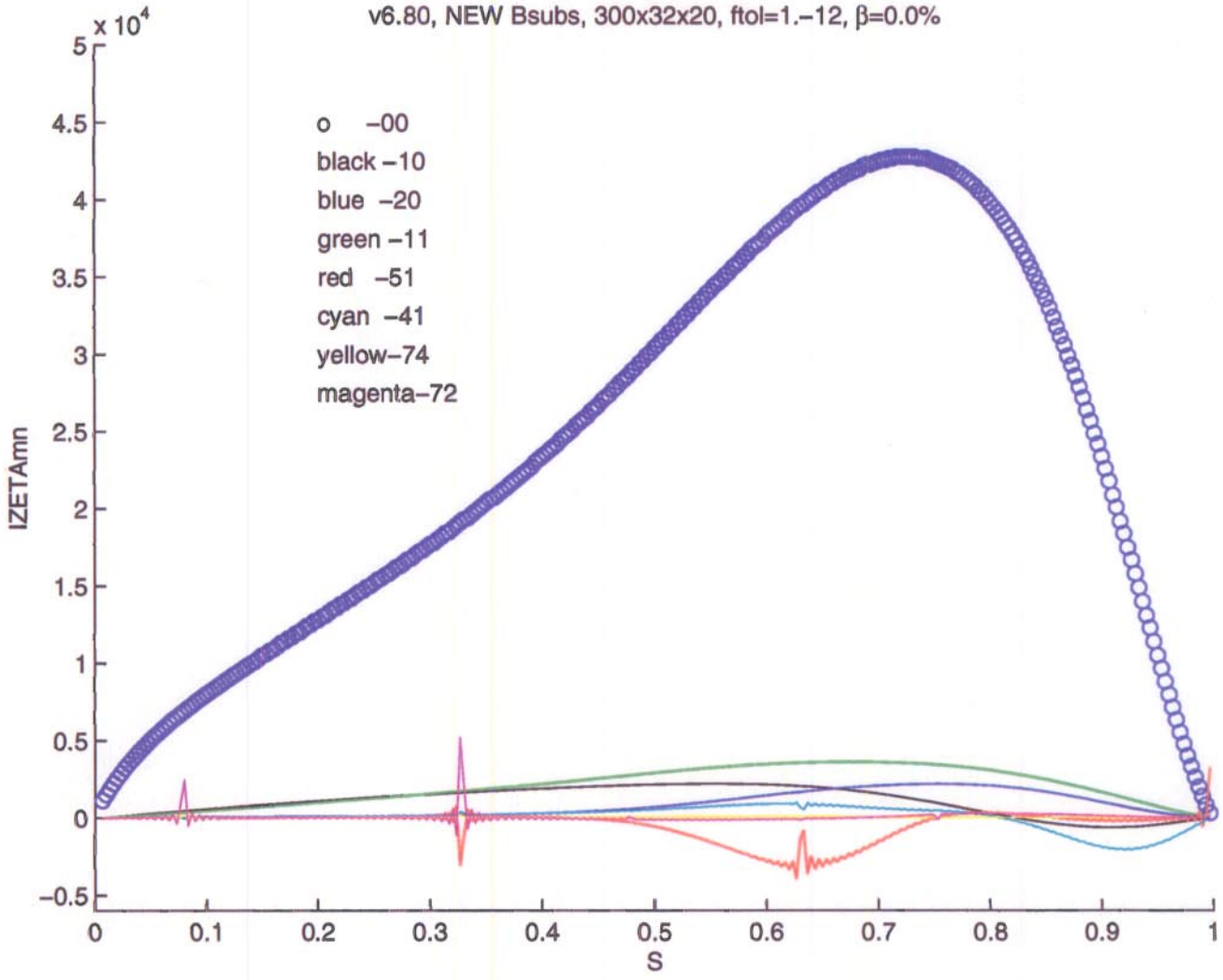

v6.80, OLD bsubs, mode m=5,n=−1, β=0.0%
300x32x20, ftol=1.e−12
194x16x10, ftol=1.e−16
97x8x5, ftol=1.e−16
|ZETAmn, magnetic coordinates
5000
4000
3000
2000
1000
0
−1000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
S

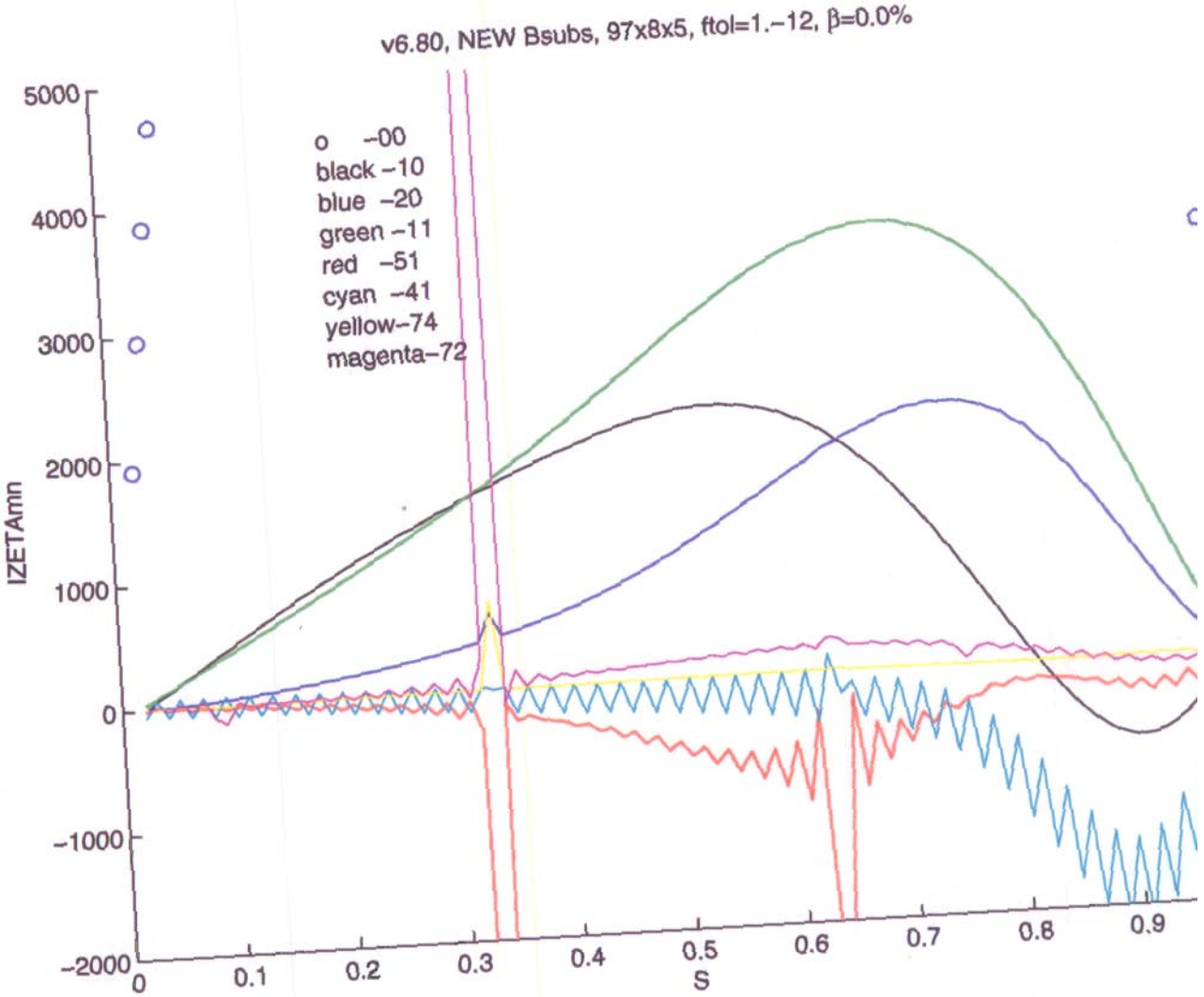

v6.80, mode m=5,n=1, 194x16x10, ftol=1.-16, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
Don=black=green-red
magn coord, old bsubs
IZETAmn
S
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

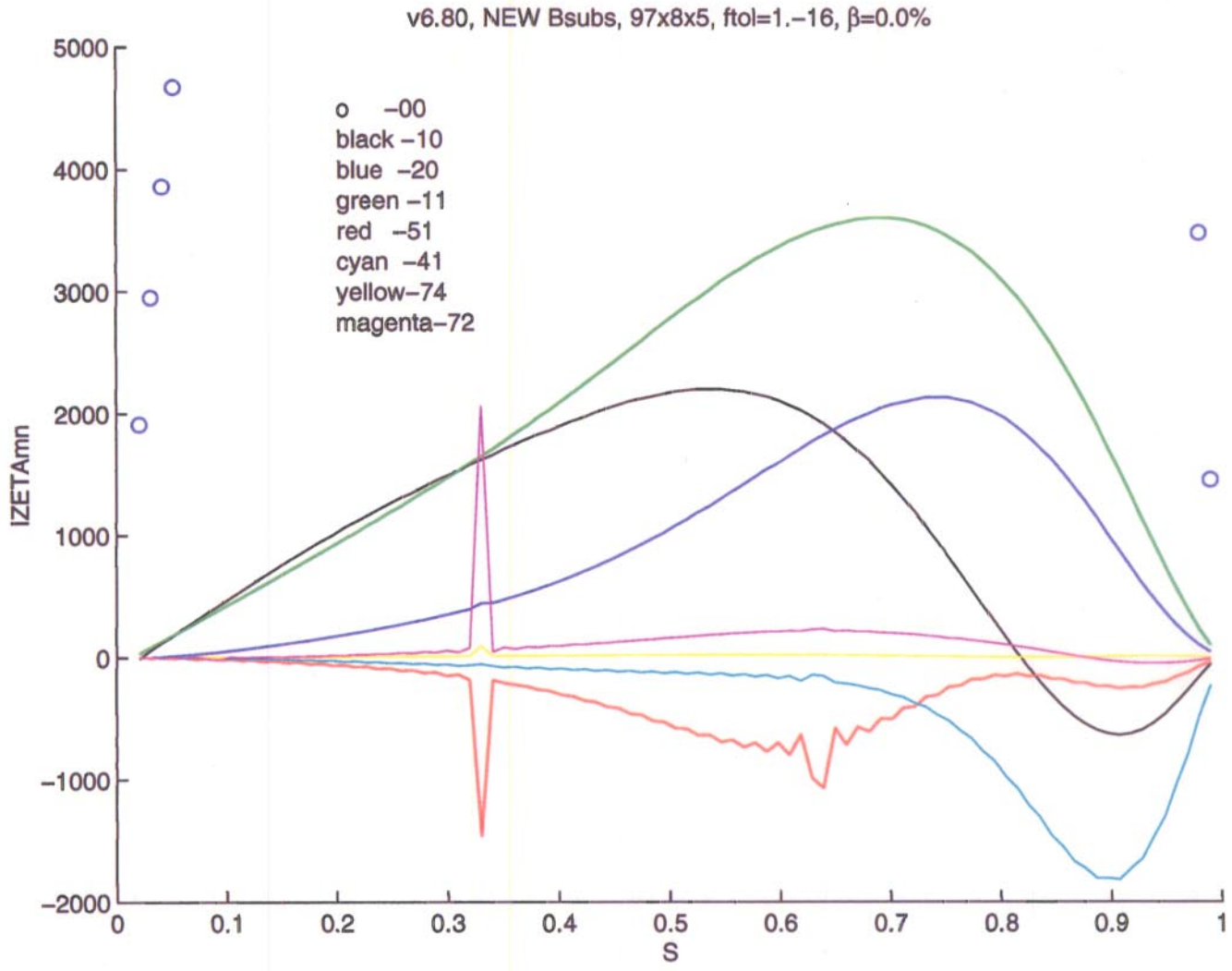

v6.80, mode m=5,n=1, 300x32x20, ftol=1.−12, β=0.0%
vmec coord,old bsubs
vmec coord, new bsubs
black=green−red
m=5 n=1, magn coord, old bsubs
m=5 n=−1, magn coord, old bsubs
|ZETAmn|
S
5000
2000
1000
0
−1000
−2000
−3000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

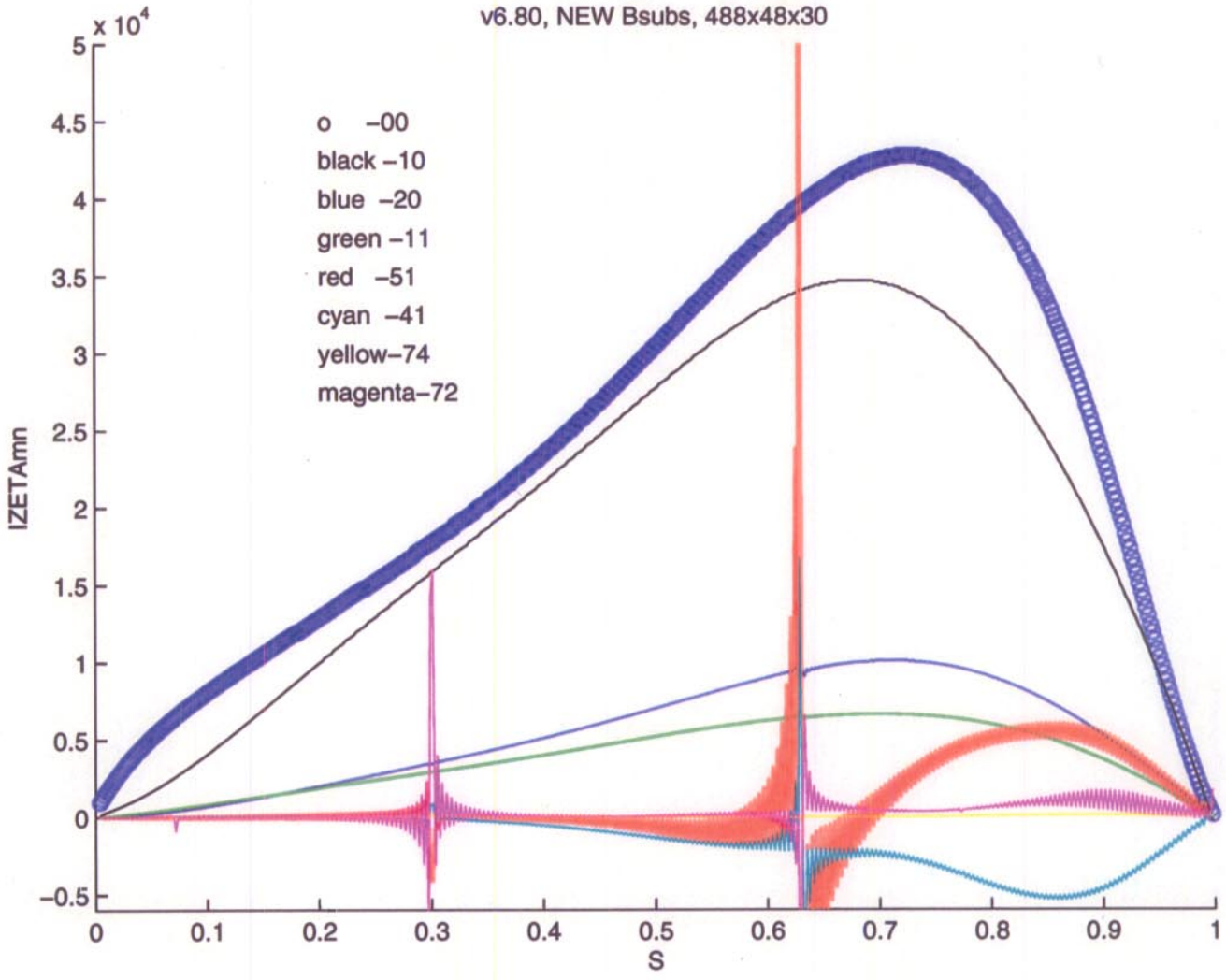
The following plots are for NCSX like cases with finite beta (approximately 4%) and finite net current.

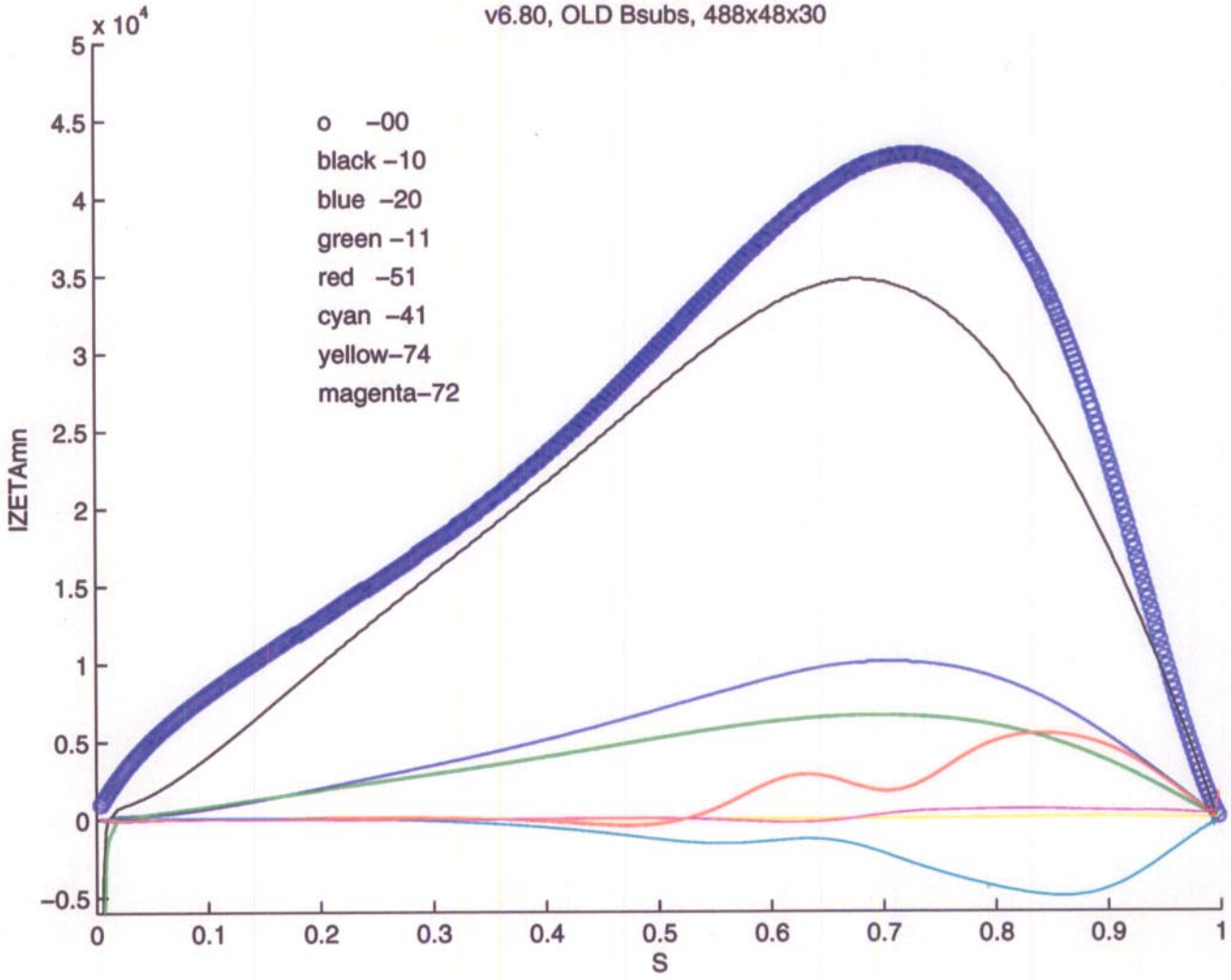

v6.80, OLD Bsubs, 300x32x20, ftol=1.–12, β=0
x 10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
–0.5
IZETAmn
o –00
black –10
blue –20
green –11
red –51
cyan –41
yellow–74
magenta–72
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
S


v6.80, NEW Bsubs, 300x32x20, ftol=1.-12, β=0.0%
x 10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
|ZETAmn
o -00
black -10
blue -20
green -11
red -51
cyan -41
yellow-74
magenta-72
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
S


v6.80, NEW Bsubs, 97x8x5, ftol=1.-12, β=0.0%
o -00
black -10
blue -20
green -11
red -51
cyan -41
yellow-74
magenta-72
IZETAmn
S
5000
4000
3000
2000
1000
0
-1000
-2000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

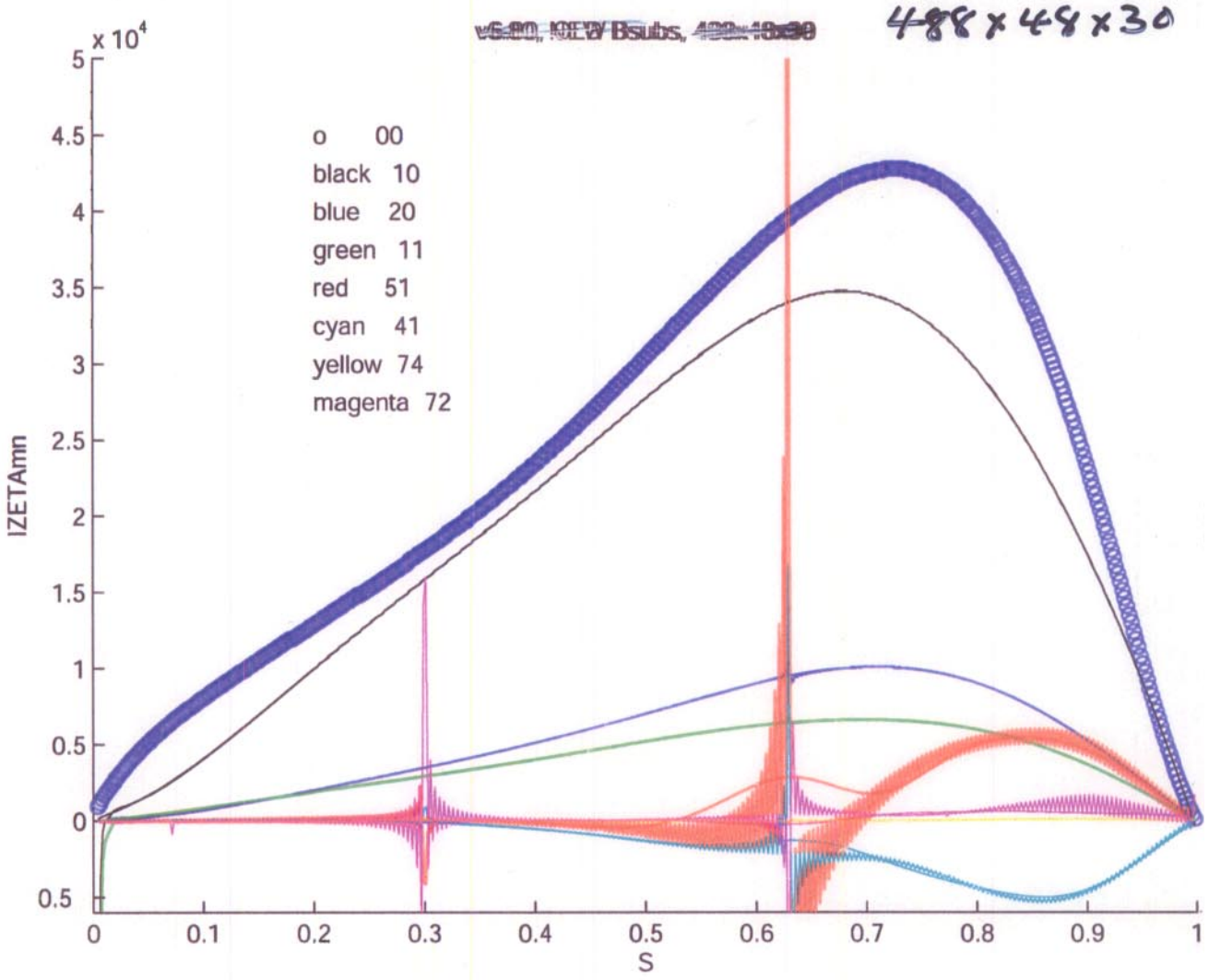

v6.80, NEW Bsubs, 97x8x5, ftol=1.–16, β=0.0%
o –00
black –10
blue –20
green –11
red –51
cyan –41
yellow–74
magenta–72
IZETAmn
S
5000
4000
3000
2000
1000
0
–1000
–2000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


v6.80, NEW Bsubs, 488x48x30
x 10^4
IZETAmn
S
o –00
black –10
blue –20
green –11
red –51
cyan –41
yellow–74
magenta–72
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
–0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


v6.80, OLD Bsubs, 488x48x30
x 10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
IZETAmn
o −00
black −10
blue −20
green −11
red −51
cyan −41
yellow−74
magenta−72
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
S

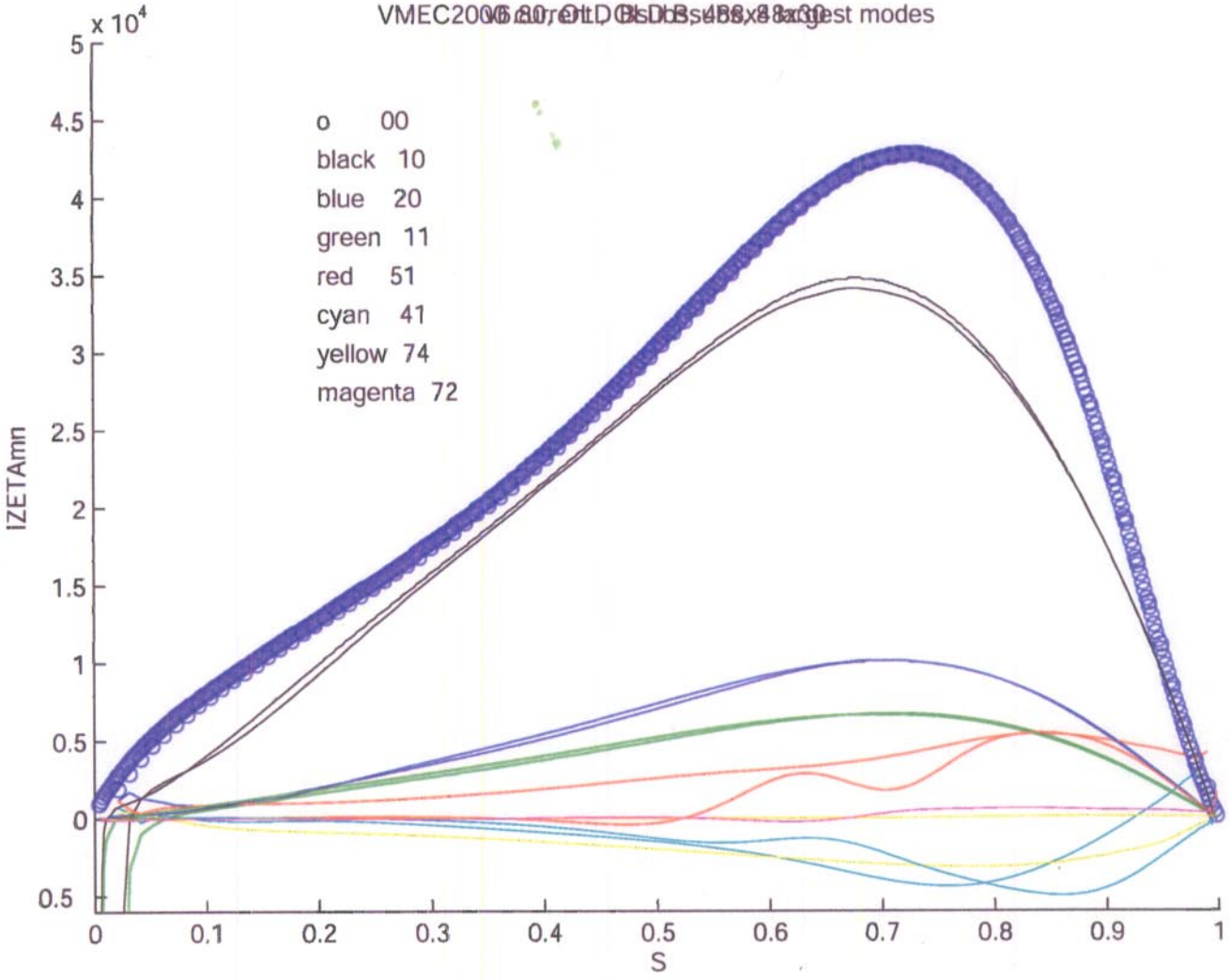
OVERLAY OLD & NEW Bsubs

488 x 48 x 30



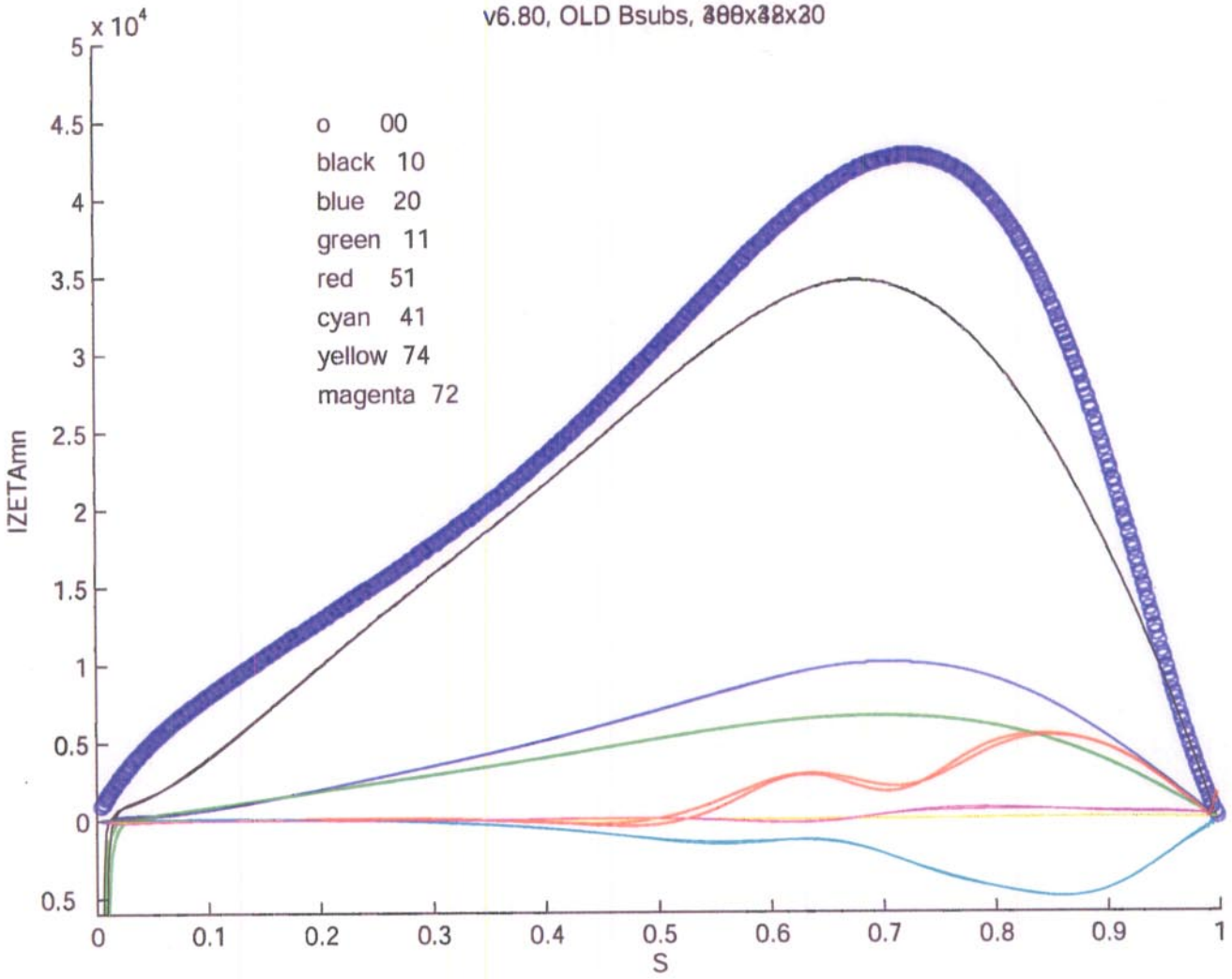
OVERLAY OLD Bsnks

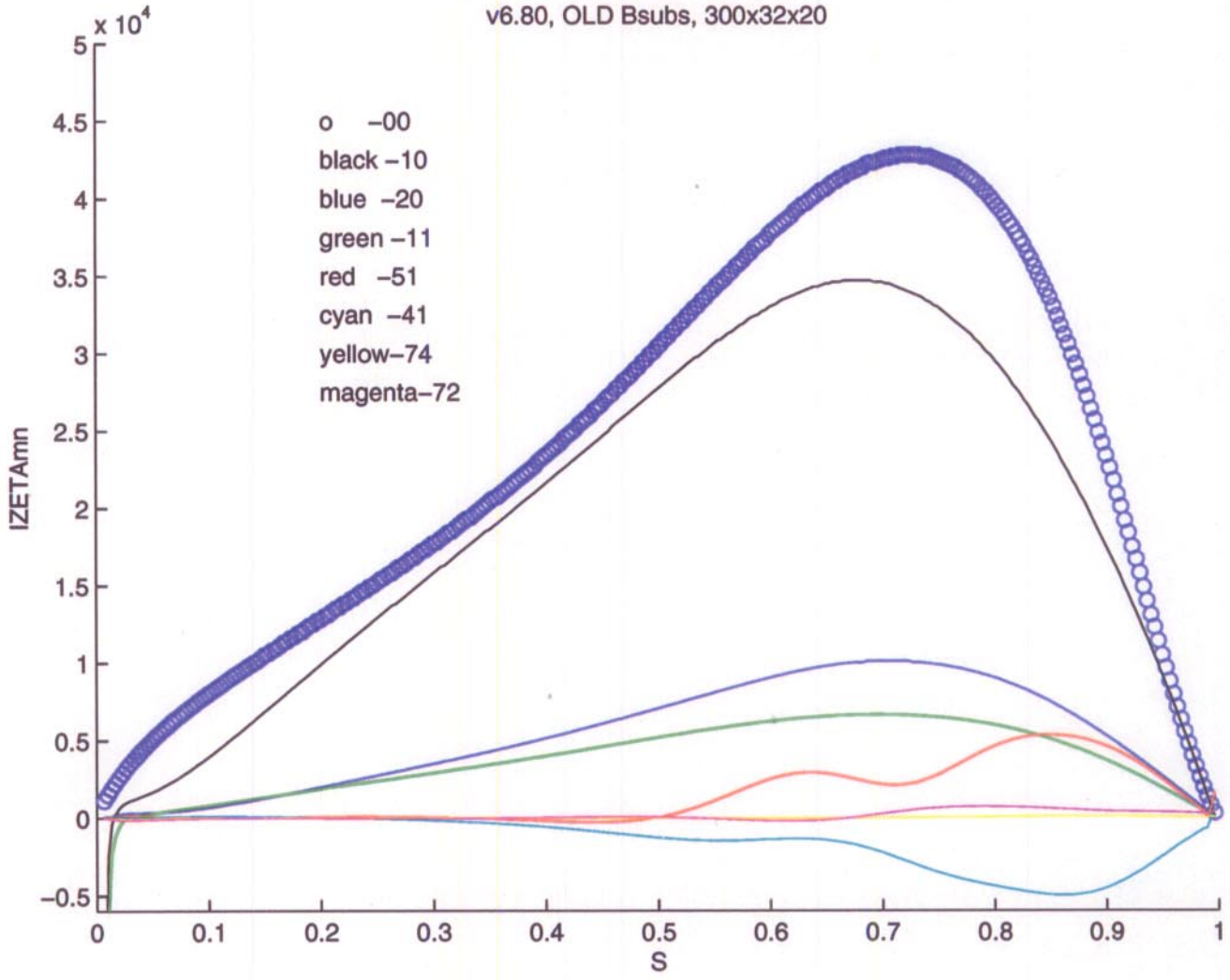
97x8x5 & 488x48x30

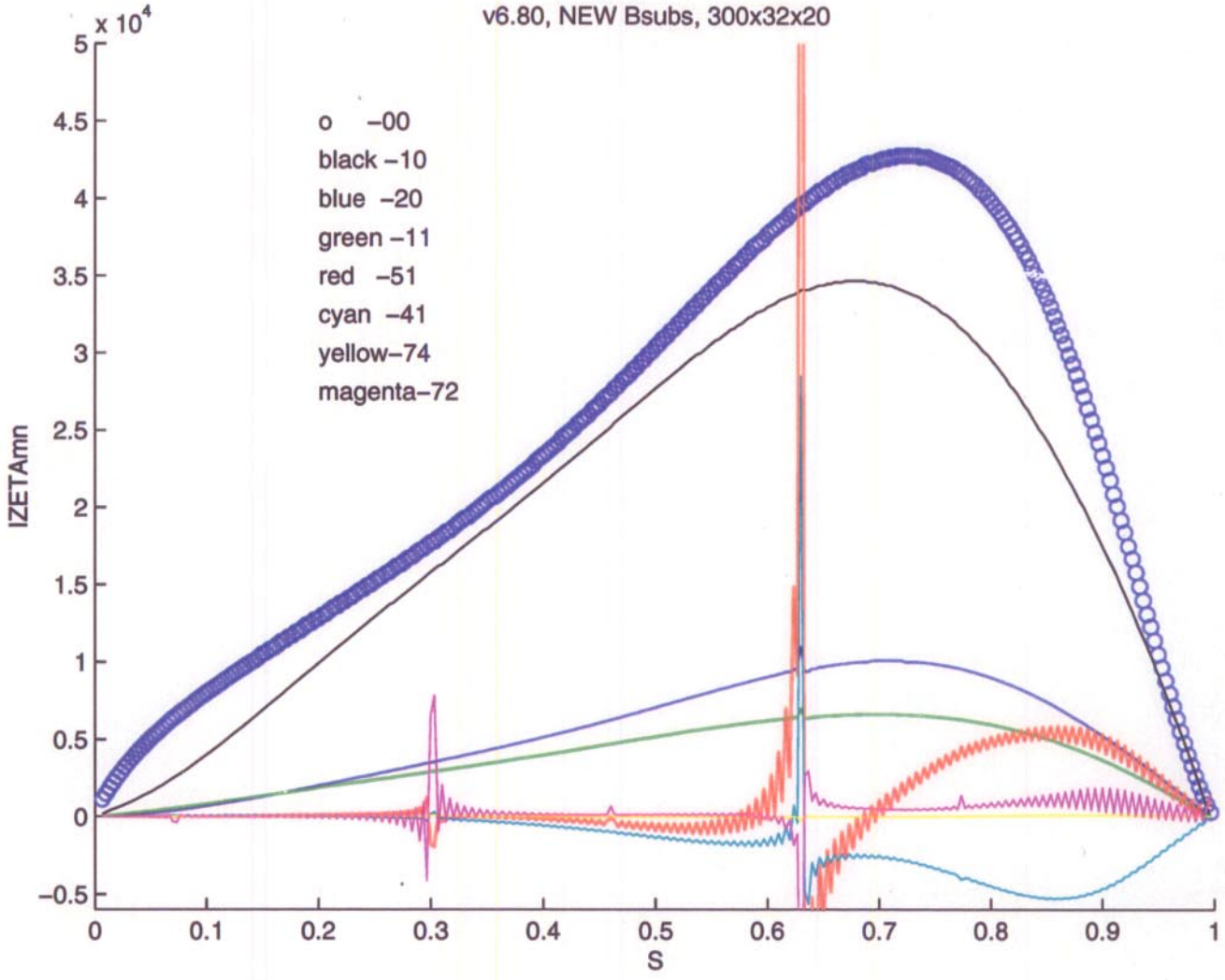


OVERLAY OLD Bsubs

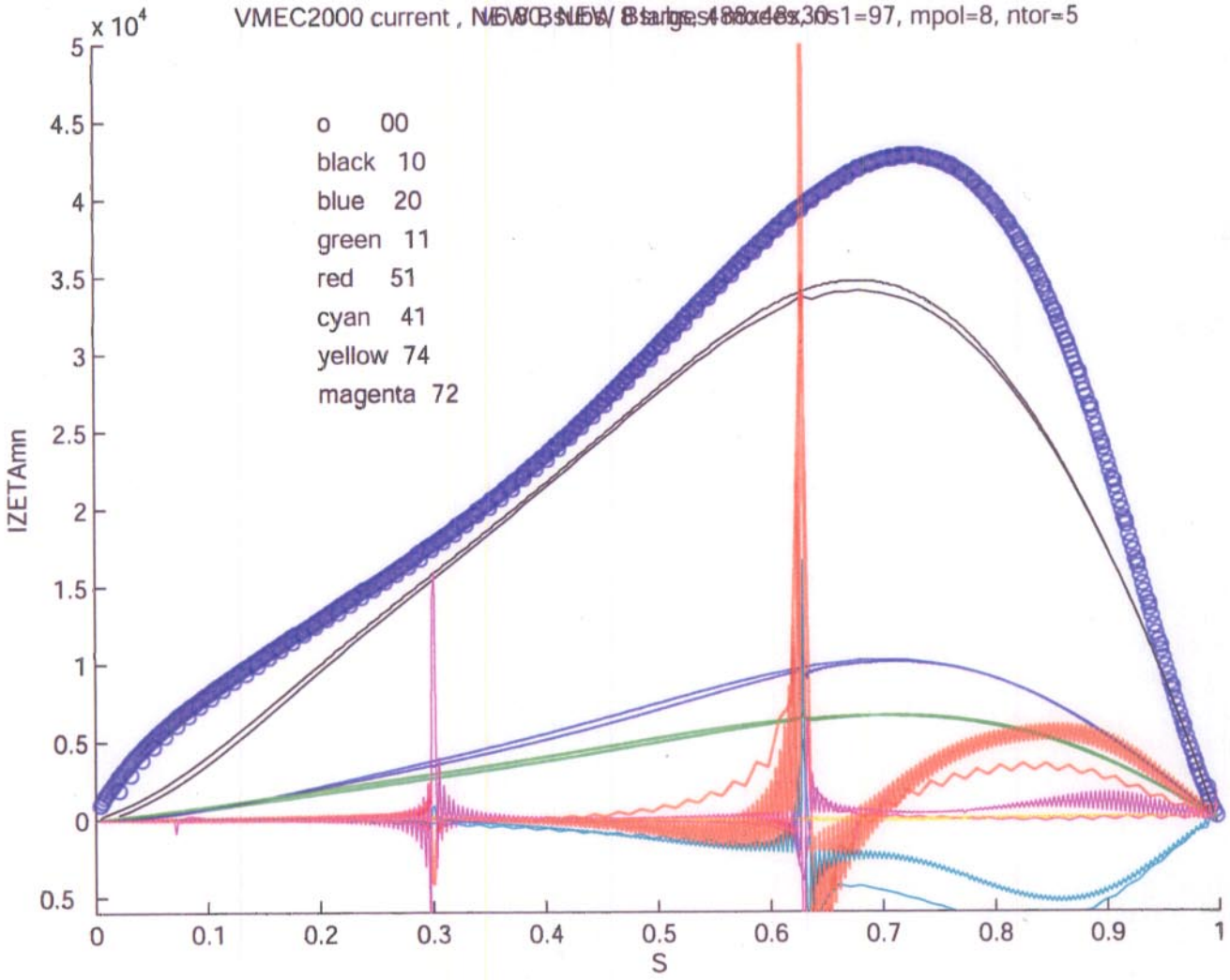
300x32x20 448x48x30




v6.80, OLD Bsubs, 300x32x20
x 10^4
IZETAmn
S
o –00
black –10
blue –20
green –11
red –51
cyan –41
yellow–74
magenta–72
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
–0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


v6.80, NEW Bsubs, 300x32x20
x 10^4
IZETAmn
S
o –00
black –10
blue –20
green –11
red –51
cyan –41
yellow–74
magenta–72
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
–0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

OVERLAY New Bsubs
97x8x5 + 488x48x30



Conclusion:

a) VMEC does include delta-functions in its calculation of the equilibrium B field, but it requires a very large number of radial zones, poloidal and toroidal harmonics to observe them.

b) Convergence of the current spikes to delta-functions is slow. The width of the spike, especially, decreases slowly.

c) The Hamada condition maybe satisfied in the finite beta VMEC equilibrium containing delta-functions.

Motivation

a) Ease the procedure for designing stellarators with good flux surfaces. However, free-boundary VMEC will not converge with the required large number of modes.

b) Verify the procedure of recalculating the equilibrium for use in stability calculations.