Nonlinear MHD and Energetic Particle Modes in Stellarators

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- Ideal MHD

  For $\beta > \beta_c$, ballooning - like modes, $n \sim 10$.

- Resistive MHD
  - ballooning - like modes occur well below the ideal limit.
  - growth rate scaling with resistivity is similar to tearing modes.
  - Stellarators tend to be resistive interchange unstable, so the resistive modes are not stabilized at small resistivity, as in tokamaks.

- Two fluid
  - resistive modes are stabilized by diamagnetic drifts.
  - realistic value of the Hall parameter, which determines the speed of the diamagnetic drift relative to the Alfvén speed.
  - Ideal modes can also be stabilized, significantly raising β limit.
- Hybrid simulations
  - TAE - like modes can be destabilized in stellarators.
  - TAE growth rate reduced in stellarators

M3D code

- Physics
  - MHD
  - 2 Fluid, neoclassical
  - hybrid
- parallelization
  - shared memory, OMP
  - and distributed memory, MPI
    - Petsci library
    - parallel data structures
    - parallel solvers
- initialization
  - equilibria from the VMEC code
  - used construct the M3D grid
  - initialize the magnetic field and pressure.

- Mesh
  - unstructured mesh of triangles and quadrilaterals in poloidal planes
  - linear finite element discretization
  - higher order finite elements, collaboration with RPI
  - 4th order finite differencing in the toroidal direction

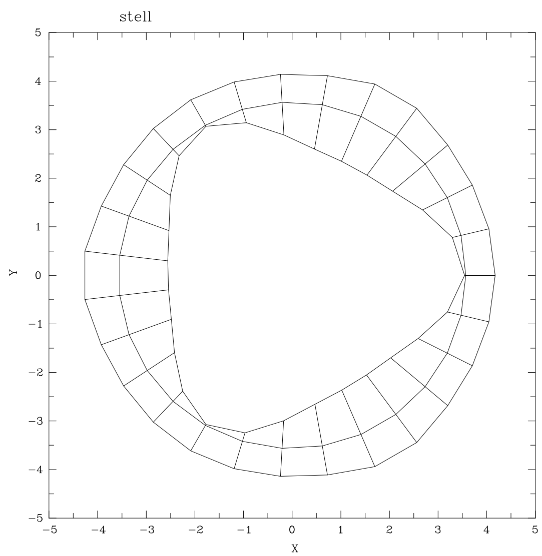


Fig.1. Mesh for stellarator simulations - top view

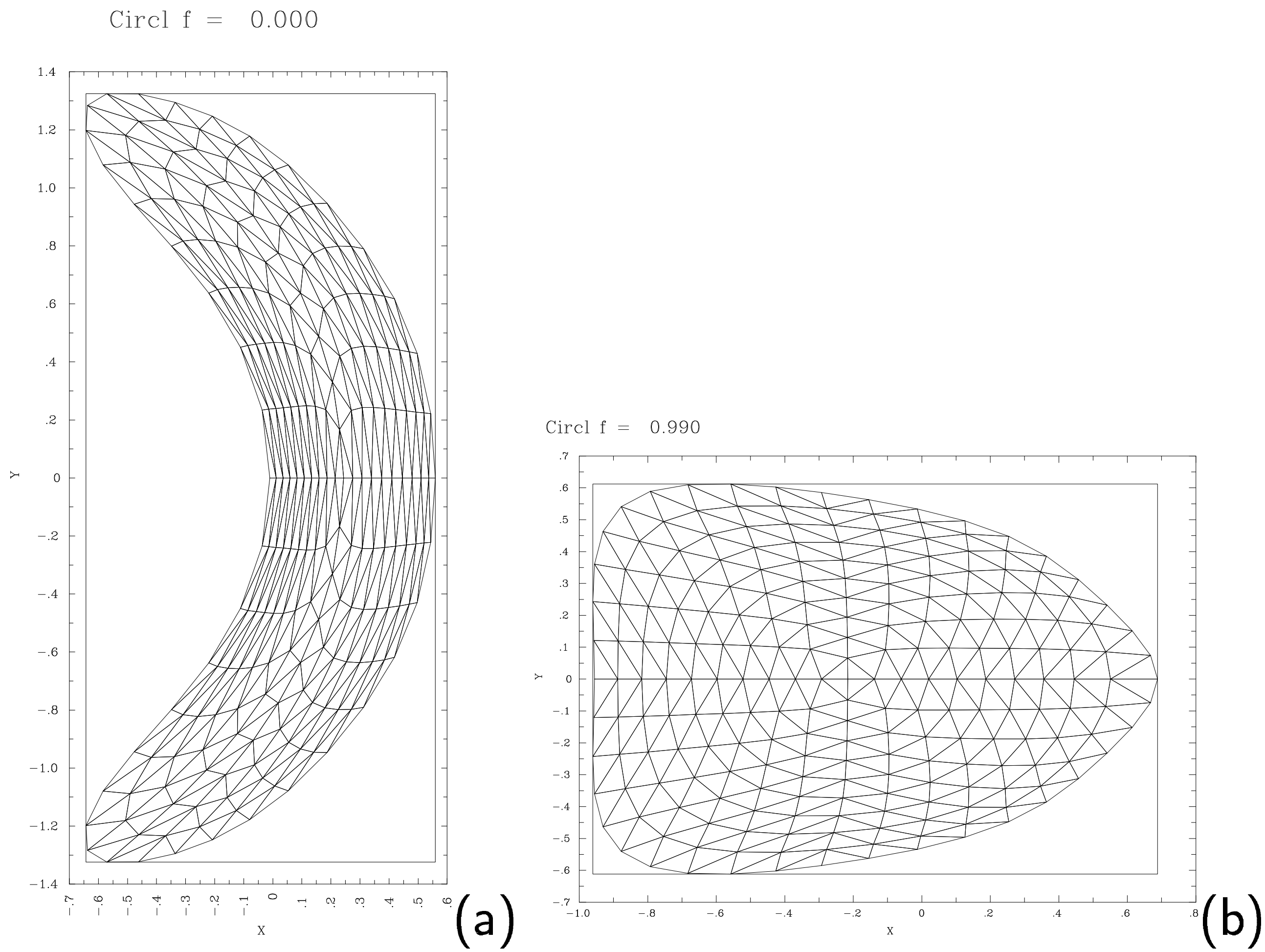


Fig.2. Mesh for stellarator simulations - (a) toroidal angle $\psi = 0$*, (b)* $\psi = 2\pi/6$.

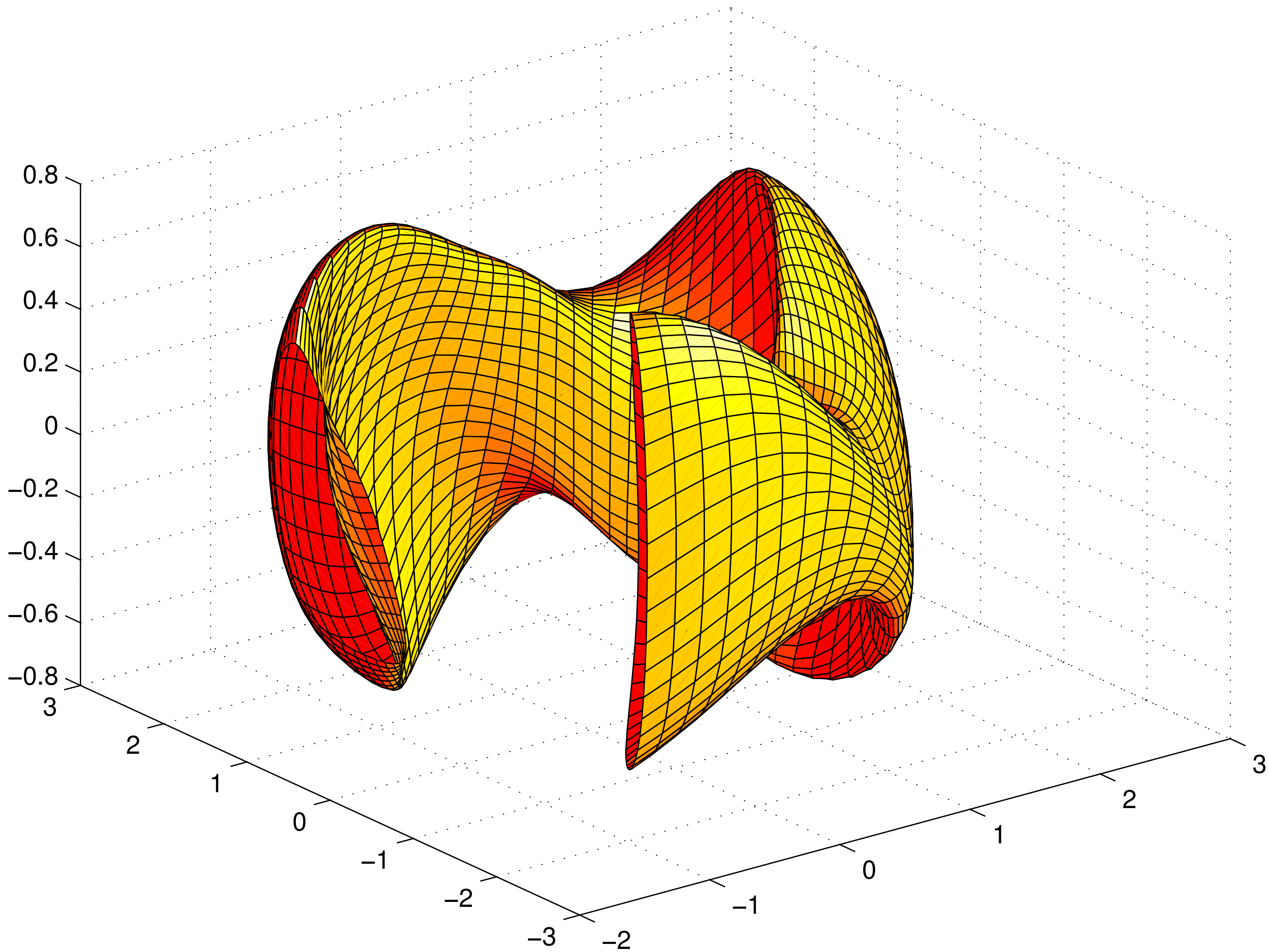


Fig.3. Boundary shape in NCSX stellarator design

Ideal MHD

equilibrium

- use VMEC equilibria
- modify VMEC data and relax,
- viscous dissipation, cross field thermal conduction, and parallel thermal conduction
- "artificial sound" method
- fixed conducting wall boundary condition.
- single period or the whole torus simulations
- M3D equilibria agree well with VMEC, except that M3D magnetic fields can contain islands.
- sequence of equilibria with same current and pressure profiles shape
- vary magnitude of p

stability

- $\beta > \beta_c$ toroidal mode number $(n \sim 10)$ modes grow exponentially
- resolution - $\sim 50 \times 100 \times 32$, poloidal resolution 200 meshpoints on poloidal boundary
- M3D $\beta_c = 6.5\%$
- infinite n ballooning, 4.2%, unstable in thin edge layer
- resolution dependence of β_c consistent with Terpsichore
- edge kink (peeling) $\beta_c = 4\%$, moderate $n \sim 6$,
- For M3D to include kink, vacuum as highly resistive plasma, VMEC equilibria with vacuum.

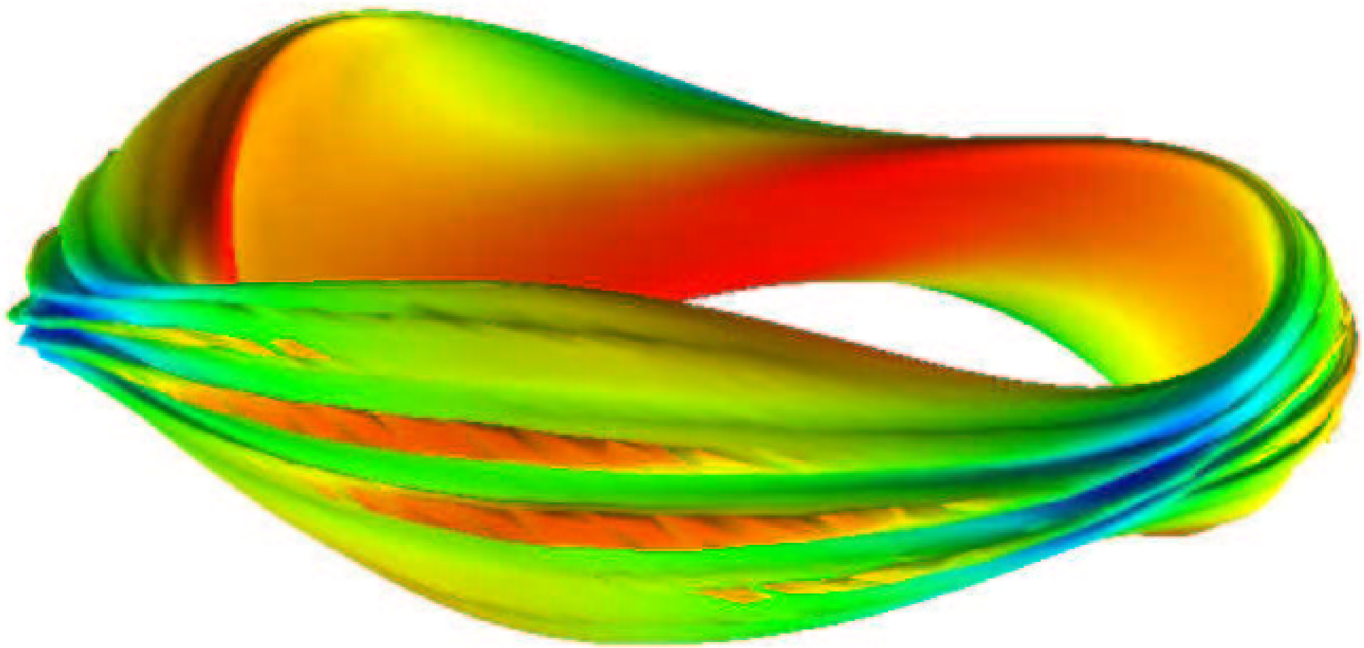


Fig.4. Pressure isosurface in a ballooning unstable NCSX stellarator design

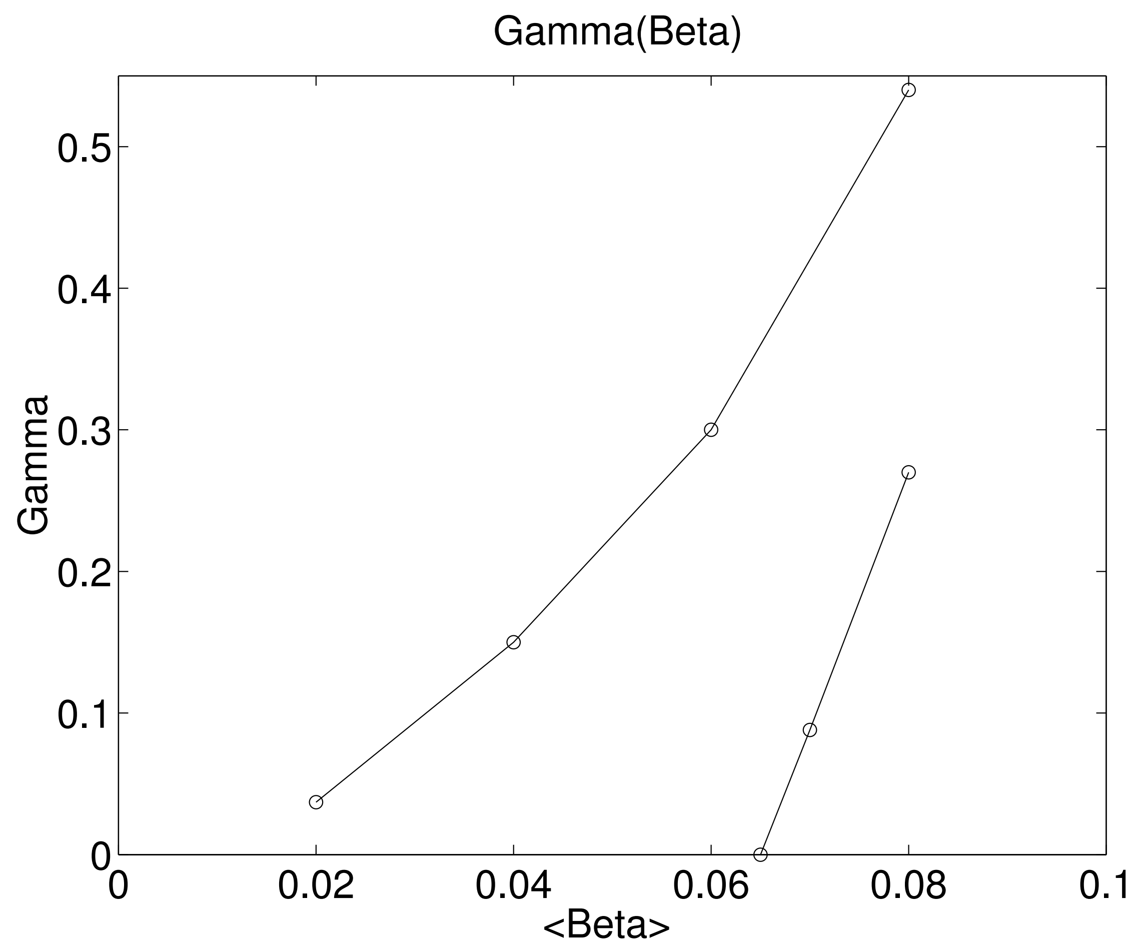


Fig.6. Growth rate γ of Ideal and Resistive Mode vs. β

Resistive MHD

Equilibrium - resistive evolution of islands

Stability - prevents long time equilibrium evolution

- resistive ballooning modes are unstable for almost all values of β.
- dimensionless resistivity $\eta = 1/S = 1.25\ 10^{-4}$
- resistive ballooning modes similar in structure to ideal MHD ballooning modes
- dispersion relation like tearing modes
  - kinematic viscosity $\mu \gtrsim \eta$, the growth rate scales as $\sim \eta^{5/6}$, confirmed by simulations.
  - $\eta \lesssim 10^{-5}$, couple to interchange modes.
  - in most stellarators, resistive interchange modes are unstable.
  - stellarators are resistively unstable

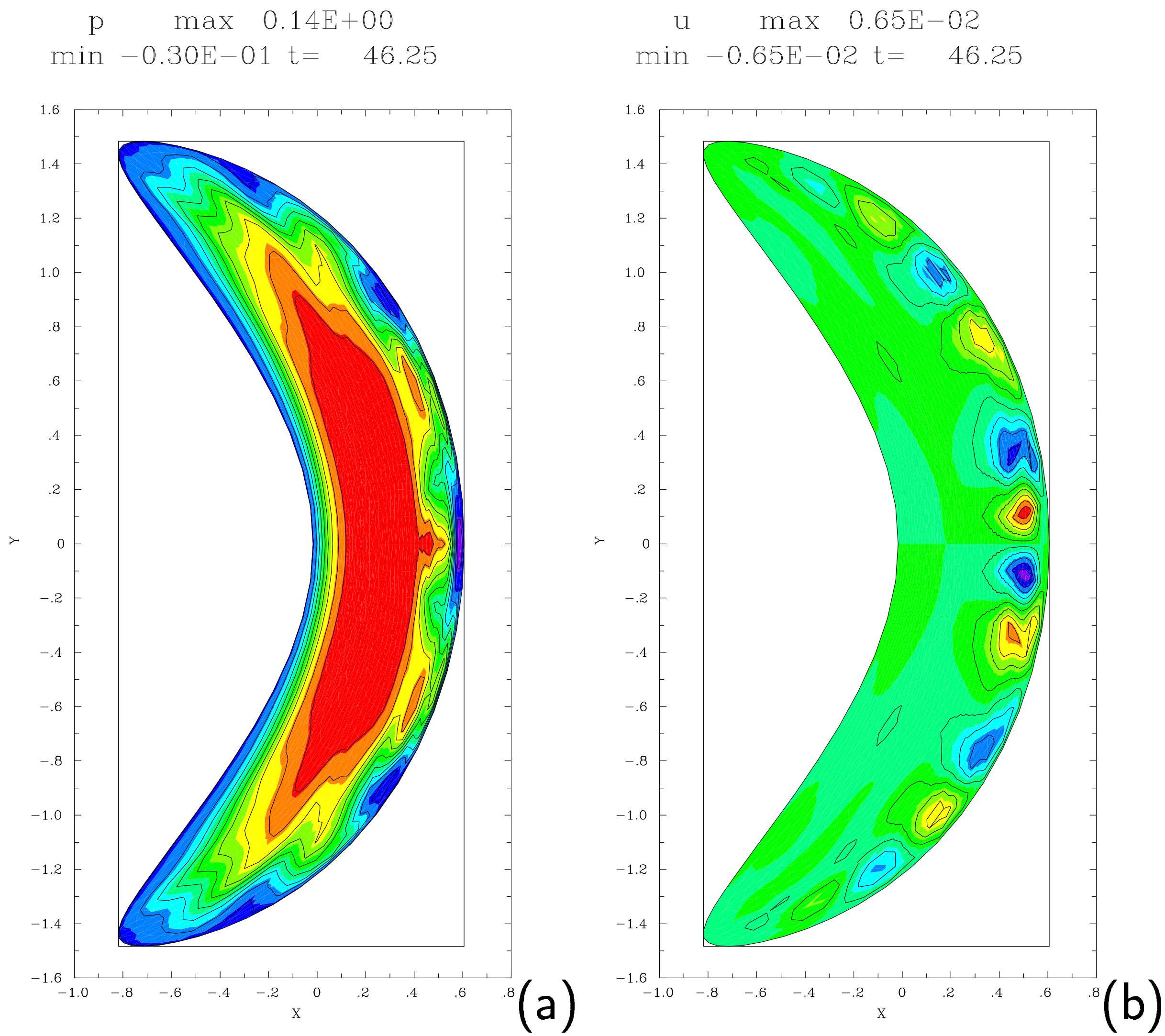


Fig.7. a) pressure b) Electrostatic Potential resistive instability

Two Fluid

Two fluid drifts can stabilize resistive modes.

Hall parameter

$$H = \frac{c}{\omega_{pi} R}$$

where c/ω_{pi} is the ion skin depth and R is the major radius.

$H = 0.01$ in both simulation and experiment

nonlinear drift two fluid equations similar to Braginski equations.

Simplified equations:

$$\mathbf{v} = \mathbf{v}_i - H\mathbf{v}_{*i}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -H\left(\mathbf{v}_{*i} \cdot \nabla\right)\mathbf{v}_\perp + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p}{\rho}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} - H\nabla_{||} p_e$$

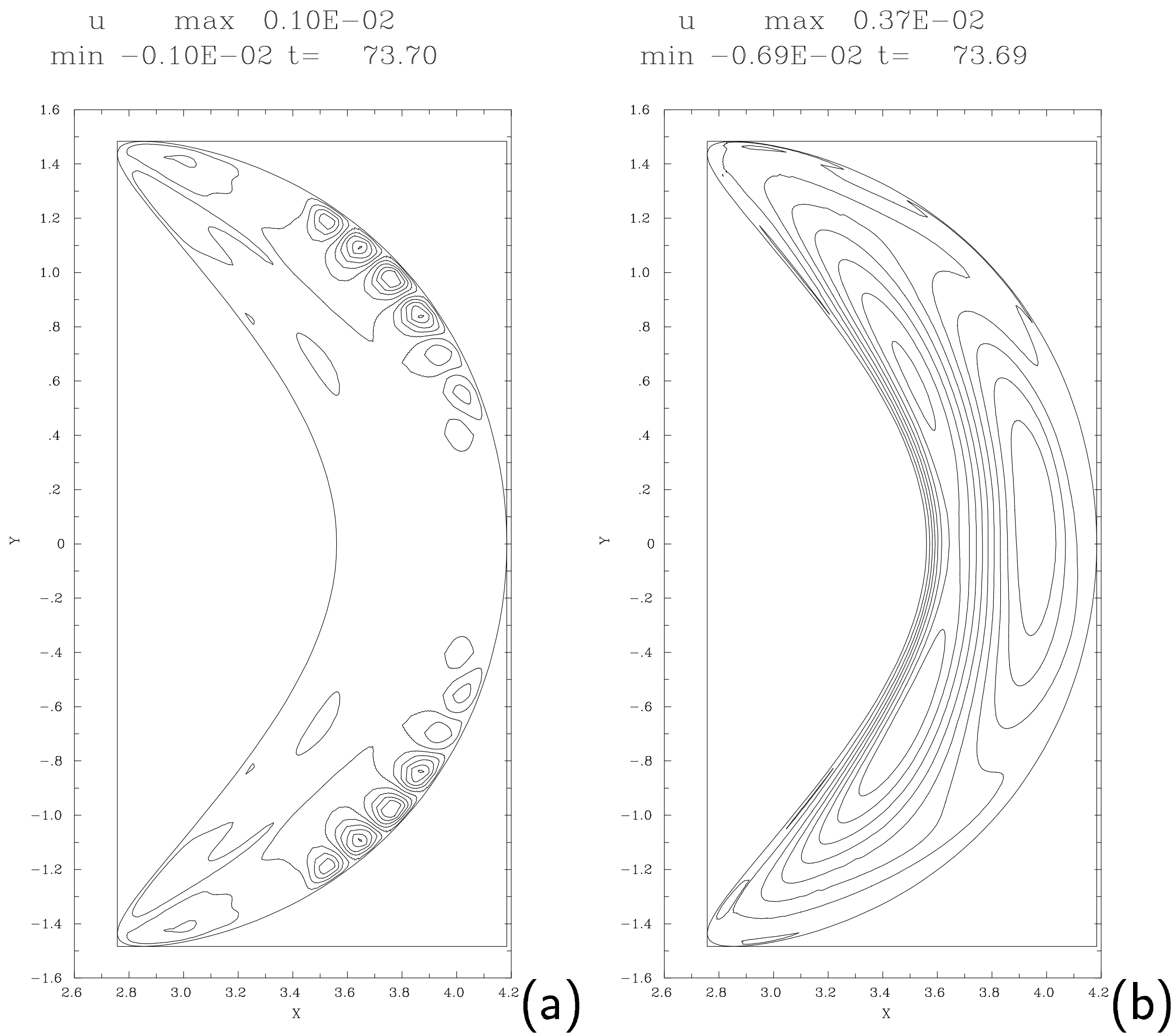


Fig.8. Electrostatic potential for (a) MHD, $H = 0$, *(b) 2 Fluid* $H = 0.01$. *In both cases,* $\beta = 0.04, S = 8000, p_e = 0$, *density = constant.*

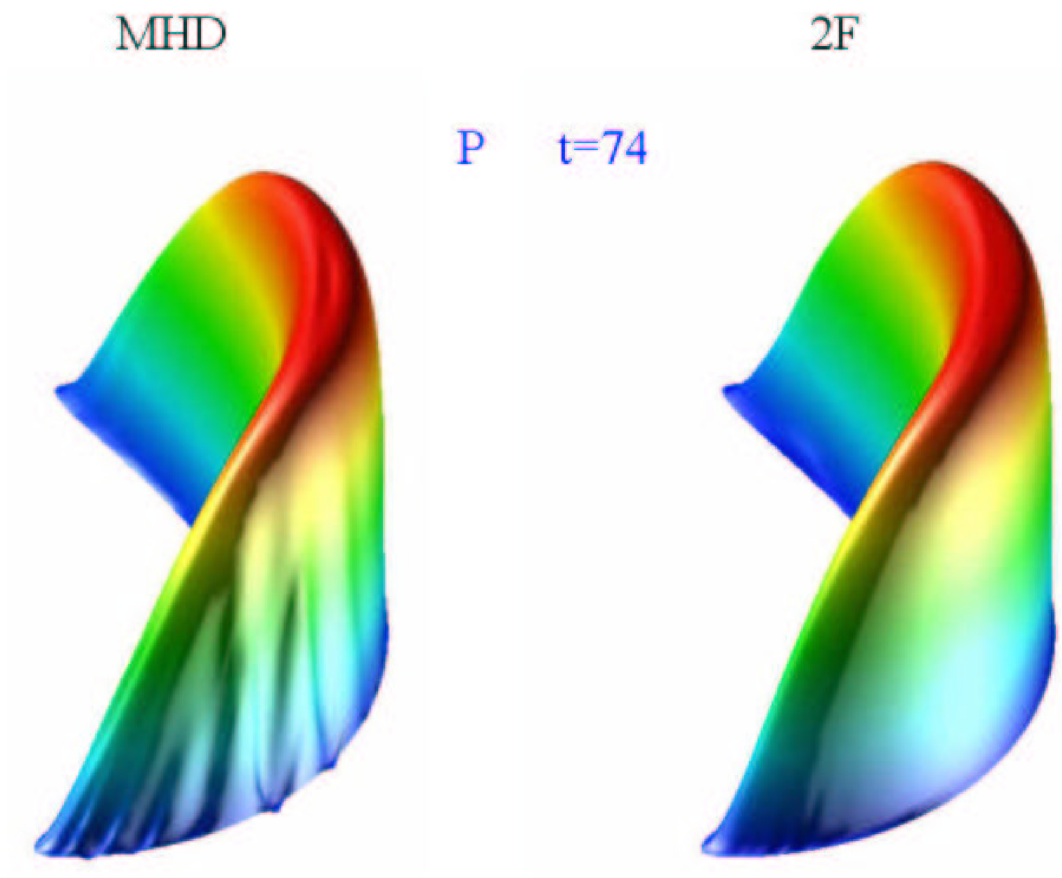


Fig.9. pressure for MHD and 2 Fluid

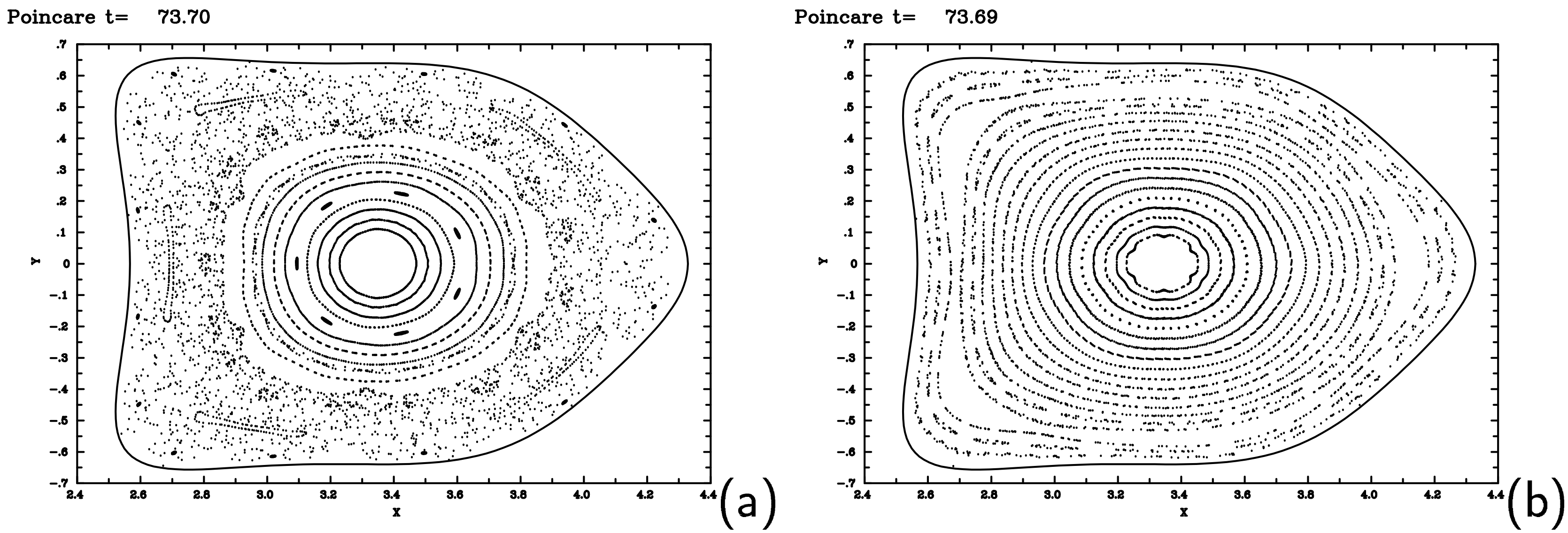


Fig.10 (a). Poincaré plot for MHD. (b). Poincaré plots 2 Fluid. Can examine resistive evolution of islands

2 Fluid Stabilization of ideal MHD mode

2 Fluid stablization can raise β - only internal modes so far

$$H = \frac{c}{\omega_{pi} R} = 0.01$$

$$\omega_* R / v_A = H n \left(\frac{\beta_p a}{2 q L_p}\right)$$

where n is toroidal mode number. Ideal modes are stable if

$$\omega_* > \gamma_{MHD} \sim 0.1 v_A / R$$

$$n \geq 10$$

can be stable, $\beta > 0.07, H = 0.02$

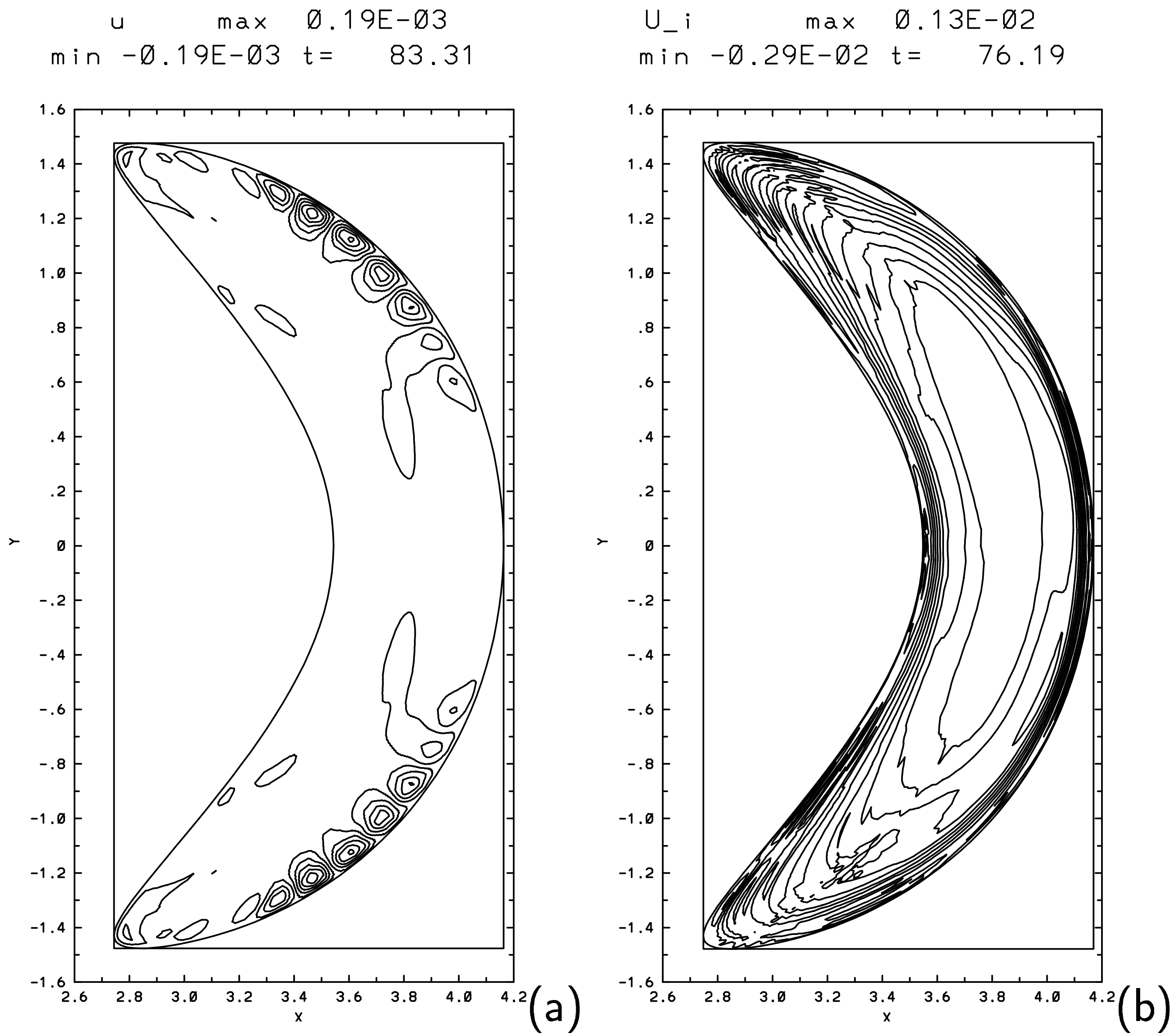


Fig.8. Electrostatic potential for (a) MHD, $H = 0$, (b) 2 Fluid $H = 0.02$. In both cases, $\beta = 0.07, S = 410^5, p_e = p_i$, density = constant.

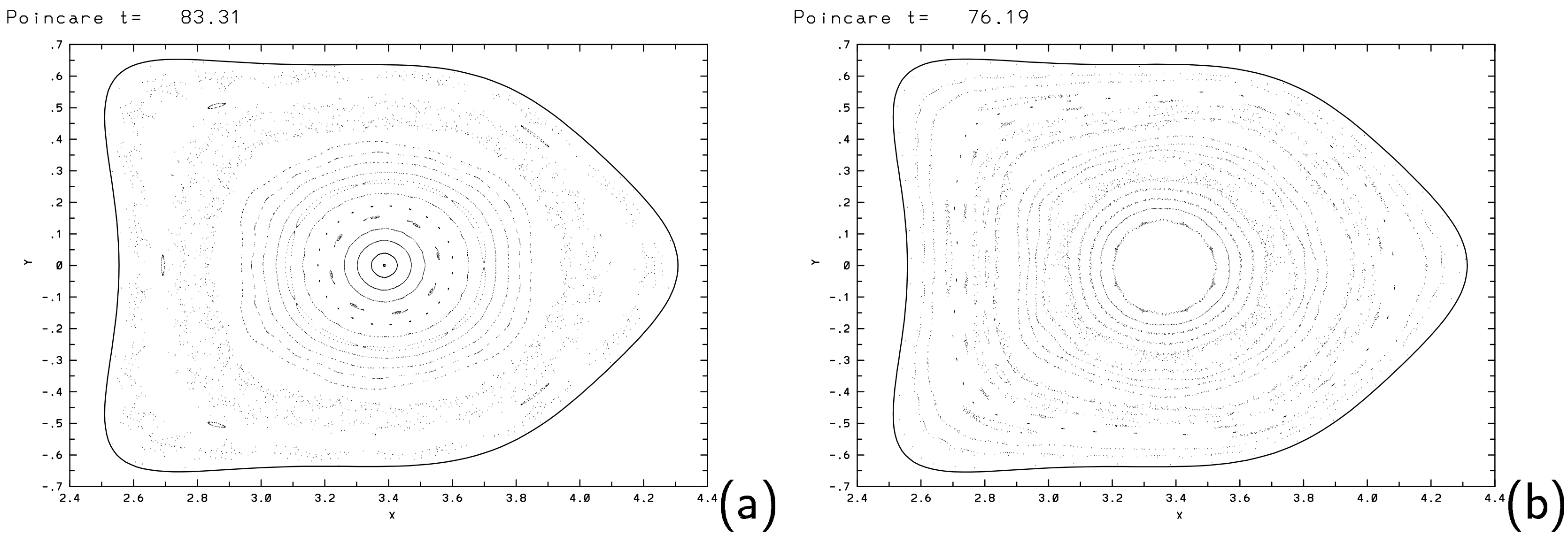
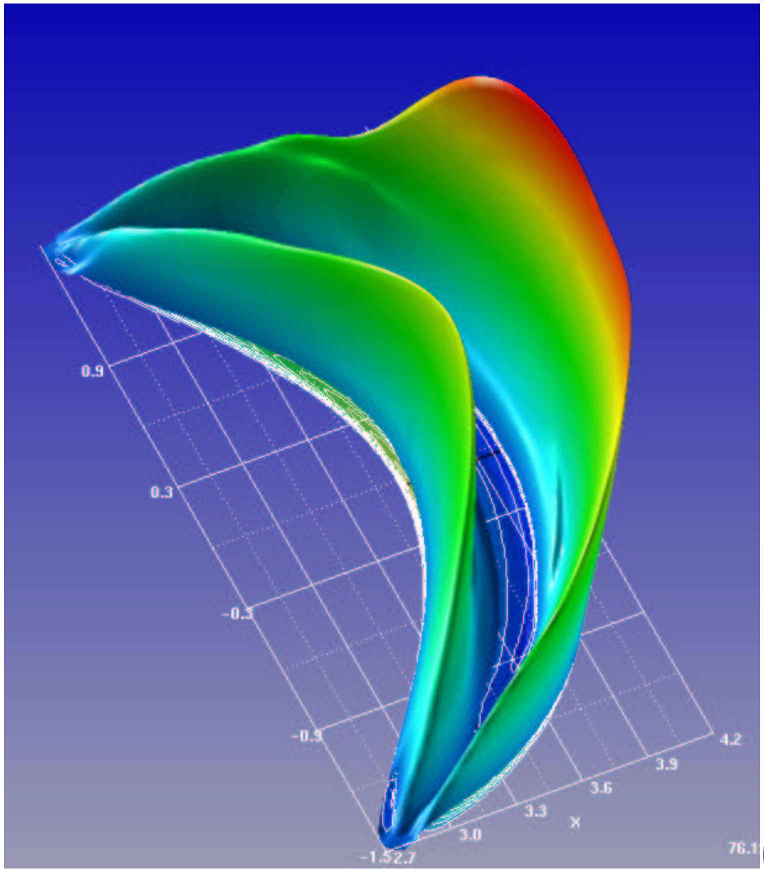
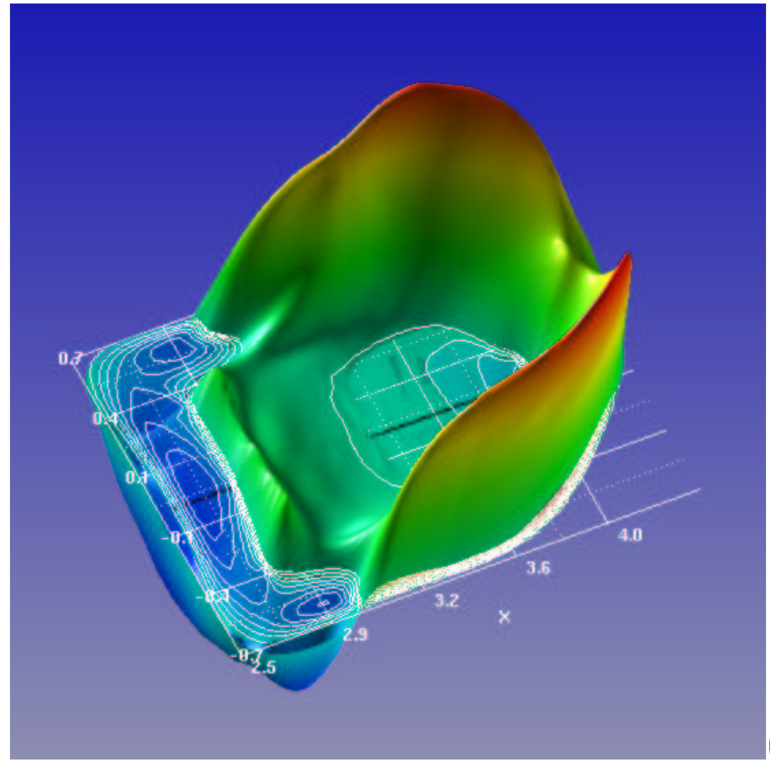


Fig.8. Poincare plots for (a) MHD, H = 0, (b) 2 Fluid H = 0.02. In both cases, $\beta = 0.07, S = 4 10^5, p_e = p_i$*, density = constant.*



(a)



(b)

Fig.8. electron drift v_* *(a)* $\phi = 0$, *(b)* $\phi = \pi/3$ *In both cases,* $H = 0.02, \beta = 0.07, S = 410^5, p_e = p_i$, *density = constant.*

V. Hot Particle Effects on Global Shear Alfvén Modes

- gyrokinetic hot ions, fluid electrons
- pressure tensor coupling to bulk plasma fluid

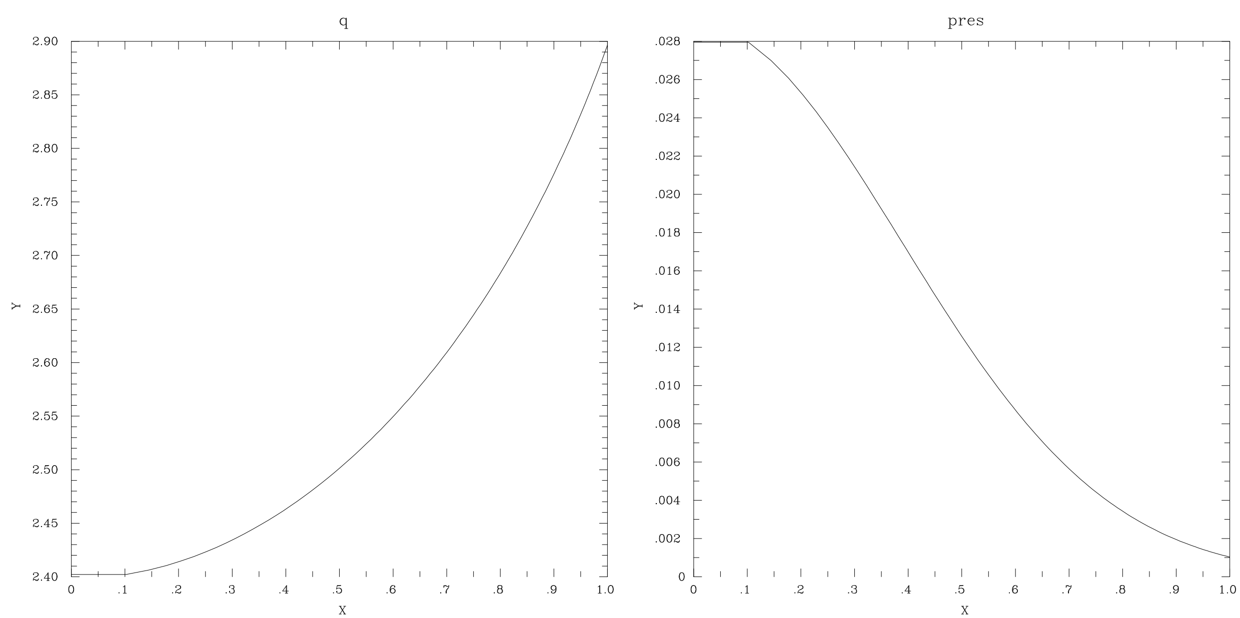
$$\rho_b \frac{d\mathbf{v}_b}{dt} = -\nabla p_b - (\nabla \cdot \mathbf{P}_h)_\perp + \mathbf{J} \times \mathbf{B}$$

- particles pushed on same mesh as MHD fluid
  - main problem is to locate particles on unstructured mesh, already solved for field line trace. search neighborhood of particles' previous location
  - interpolate force to particle location
  - accumulate pressure tensor on mesh
  - in MPP must also move particles between domains
    * particle domains coincide with mesh domain decomposition
    * method used in GTC by Lin et al.

TAE modes in stellarators

- 2 period QAS stellarator equilibrium
  - P. Garabedian, L. P. Ku
  - q profile favorable for TAE in tokamak

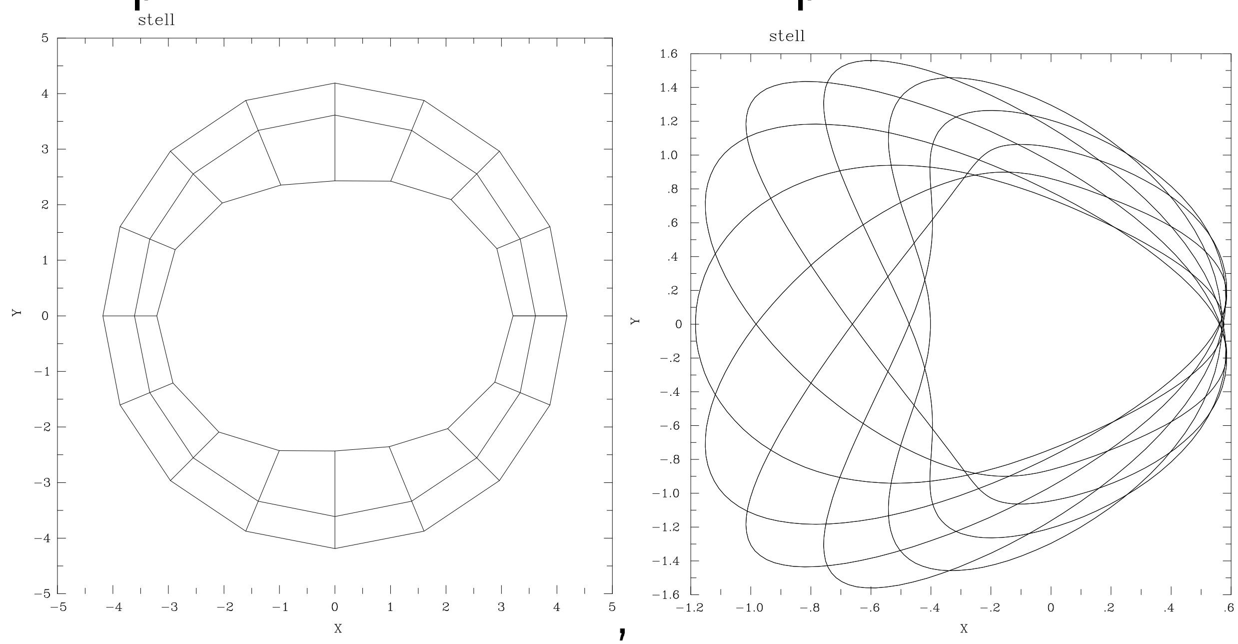
$$2.4 < q < 2.9$$



- linearize
  - $n = 1$ perturbations to break equilibrium symmetry

- $\omega = v_A/(2qR)$
- growth rate is linear in hot particle β_h
- effect of 3 D geometry
  - interpolate between tokamak and stellarator geometry

top view and cross sections of 2 period stellarator

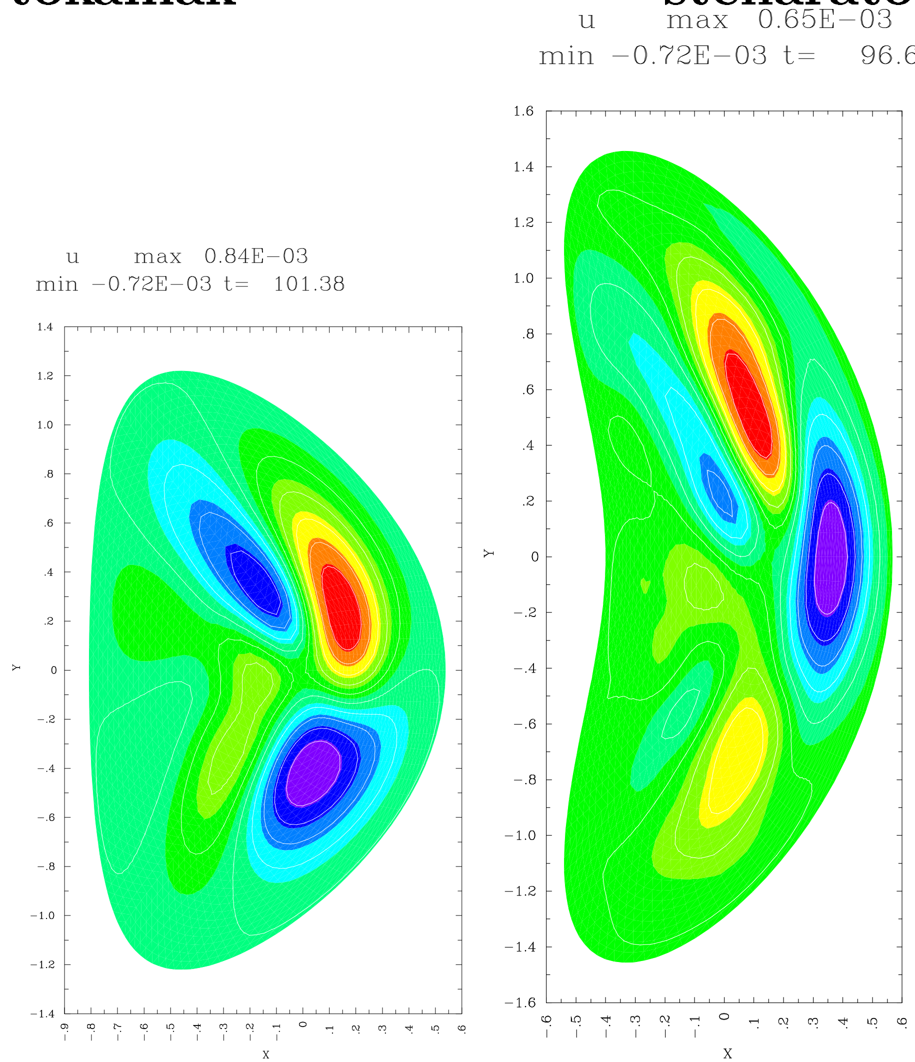


- mode peaks at q = 2.5 continuum gap
- mode frequency close to $\omega_{TAE} = v_A/(2qR) = v_A/(5R)$

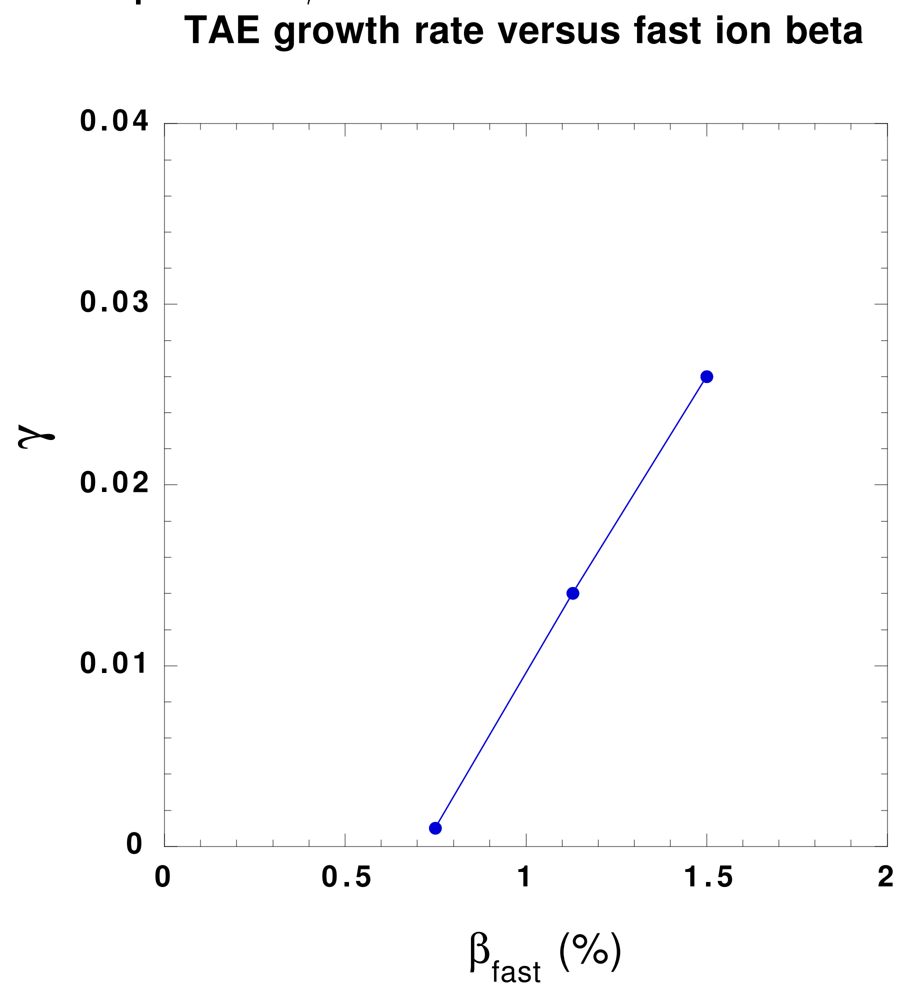
- $R/a = 4, \beta = 0.014, \rho_{fast}/a = 0.12, v_{fast}/v_A = 1.6$
- slowing - down distribution of fast ions
- δ f for noise reduction
- stellarator toroidal coupling can modify the mode

Electrostatic Potential of TAE Mode

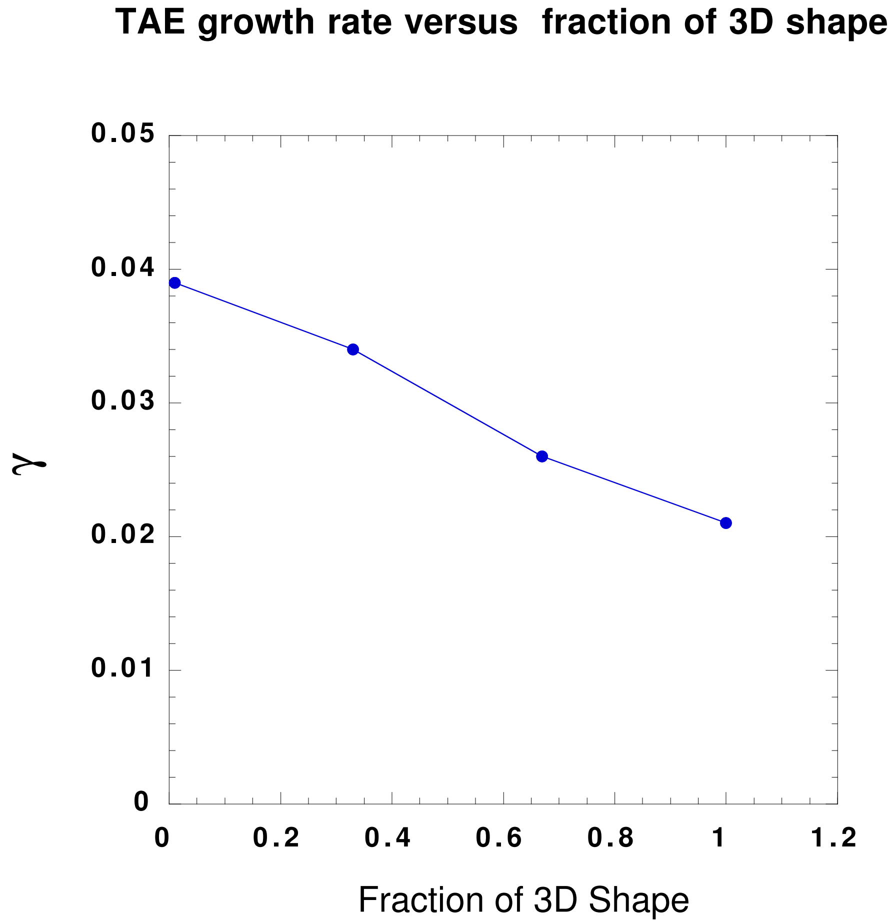
tokamak **stellarator**



Growth rate γ vs. hot particle β. Linear relation is characteristic of TAE.



Growth rate γ vs. 3D shape for Equilibria varying from tokamak to stellarator.



VI. Conclusion

- The M3D code has several levels of physics, which are applied to stellarators.
- Ideal MHD is used to calculate β limits.
- Resistive MHD computations find moderate toroidal mode number unstable resistive ballooning modes, well below the ideal MHD β limit. Stellarators tend to be resistive ballooning mode unstable.
- two fluid diamagnetic drift has a stabilizing effect on the resistive ballooning modes.
- 2 fluid can also stabilize ideal ballooning, raising β limit
- Hot particle kinetic effects can cause TAE modes in stellarators. TAE modes are confirmed by frequency and linear scaling of growth rate with hot particle β. The growth rate in a two period stellarator is lower than in a tokamak with the same profiles and average boundary shaping.