Calculations of Magnetic Surfaces

Presented by

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Calculations of Magnetic Surfaces

Outline

- The PIES code: fixed and free boundary equilibria.
- Calculations.
  - Fixed boundary flux surfaces - adequate without flux surface optimization.
  - Optimization to remove residual fixed boundary islands.
  - Optimization of coil design to preserve flux surfaces.

The PIES Code — Background

- PIES code solves 3D equilibrium equation using general representation of magnetic field.
  (VMEC field representation assumes good surfaces.)

- Basic algorithm originally proposed by Spitzer, Grad and Rubin, Shafranov.

- Code developed with international collaborators:

  - A. Salas (TJ-2, Spain)
  - P. Merkel, S. Arndt, M. Drevlak, F. Hertweck (W7X Group, Germany)
    Main code used for flux surface studies for W7X.

- Used by W7AS, W7X, LHD, Heliotron-J, TJ-2 groups.

- 32 PIES publications in journals and conference proceedings.

The PIES Algorithm

- Initial step:

$$\mathbf{j}_\perp = \mathbf{B}_{\rm vmec} \times \nabla p / B^2_{\rm vmec},$$

$$\nabla \cdot \mathbf{j}_{||} = -\nabla \cdot \mathbf{j}_\perp,$$

$$\nabla \times \mathbf{B} = \mathbf{j}.$$

  Updated $\mathbf{B}$ has islands.
  Iterate to convergence (“Picard iteration”).

- Analogous to Picard algorithm used to solve 2D Grad-Shafranov equation:

$$\Delta^* \psi_{n+1} = j_\phi(\psi_n).$$

- Use underrelaxation to extend convergence.

- General representation of field greatly slows algorithm relative to VMEC.

  – Modifications during NCSX study have speeded up code by factor of 5.
    Has allowed routine evaluation of candidate configurations and higher resolution studies.

  – Free-boundary equilibrium takes about 3 days.

Validation of PIES Code

- Testing of individual components.
  - Each piece of code tested as it was built. e.g. For Ampere solver, took $\nabla \times \mathbf{B}$ of several fields, $\mathbf{B}$, to get a $\mathbf{j}$. For each $\mathbf{j}$, verified Ampere solver recovered initial $\mathbf{B}$.
  - Alternative algorithms developed for many pieces of code, have been compared with original. e.g. 2 different ways to calculate magnetic coordinates, new Ampere's Law solver in terms of vector potential, etc.
- Internal checks in code.
  - $\mathbf{B} - (\nabla \times)^{-1} \mathbf{j}(\mathbf{B}) \longrightarrow 0$ as code converges.
  - Also directly check $(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p \approx 0$ to within discretization error. Verify it $\longrightarrow 0$ as resolution increased.
- Comparison with analytic test cases:
  - Soloveev equilibria. (ref. 11)
  - Large aspect ratio stellarator expansions. (ref. 11)

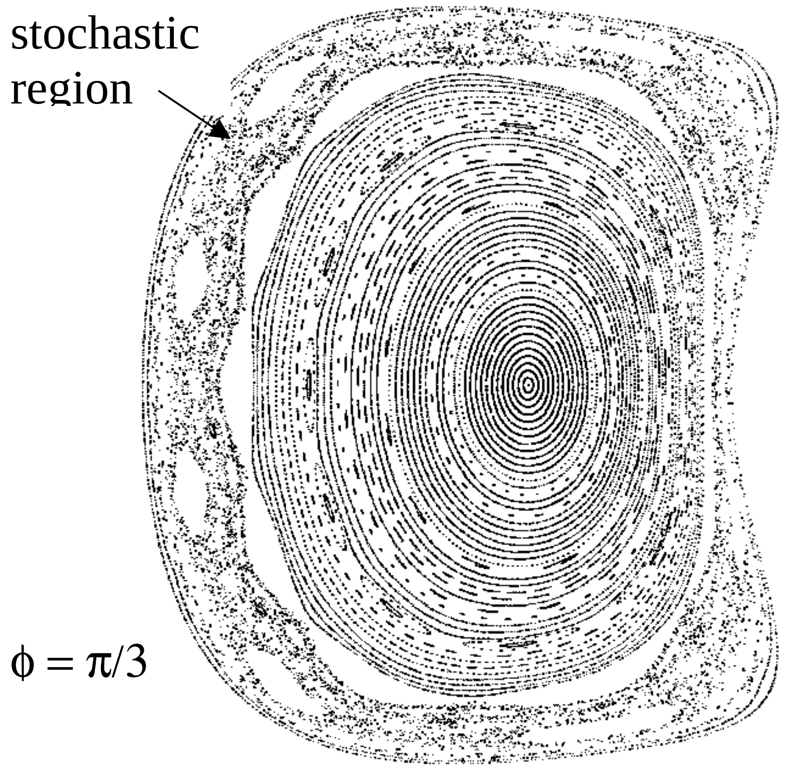
- Helical force-free Bessel function equilibria with islands. (ref. 19)
- Narrow-island, saturated tearing mode solutions of White et al and Zakharov. (Pomphrey and Reiman, Sherwood paper)

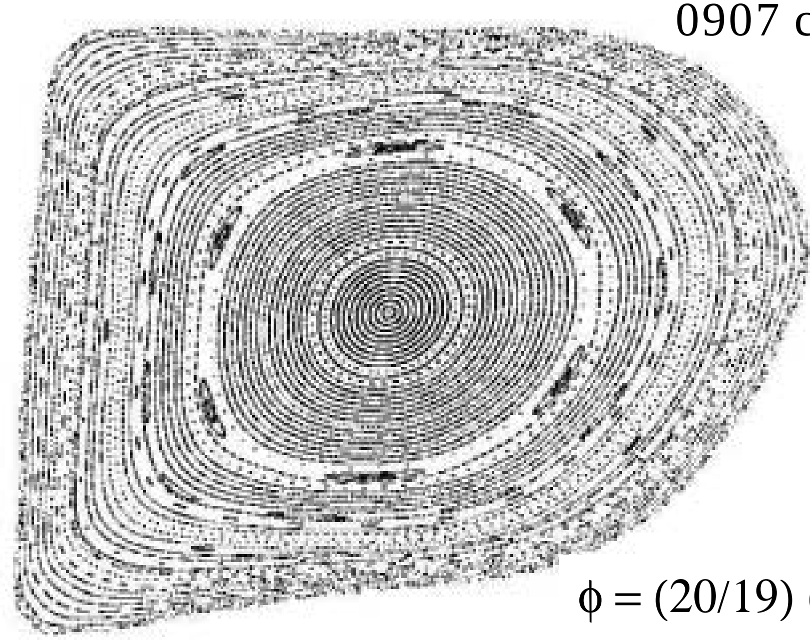
• Comparison with other codes:

- 2D j-solver equilibria for TFTR and DIII-D.
- Biot-Savart vacuum field solvers.
- Hughes linearized resistive time-dependent code (comparison with marginal stability for tearing modes on TFTR).
- VMEC: "Comparison of ATF and TJ-II Stellarator Equilibria as Computed by the 3D VMEC and PIES Codes," Johnson, Monticello, Reiman, Salas, Fraguas and Hirshman, Comp. Phys. Commun. **77**, 1 (1993).
- Routine restarts from VMEC. (Constantly comparing with VMEC.)

Progress Since 1999

Poor Flux Surfaces ⟶ Good Flux Surfaces

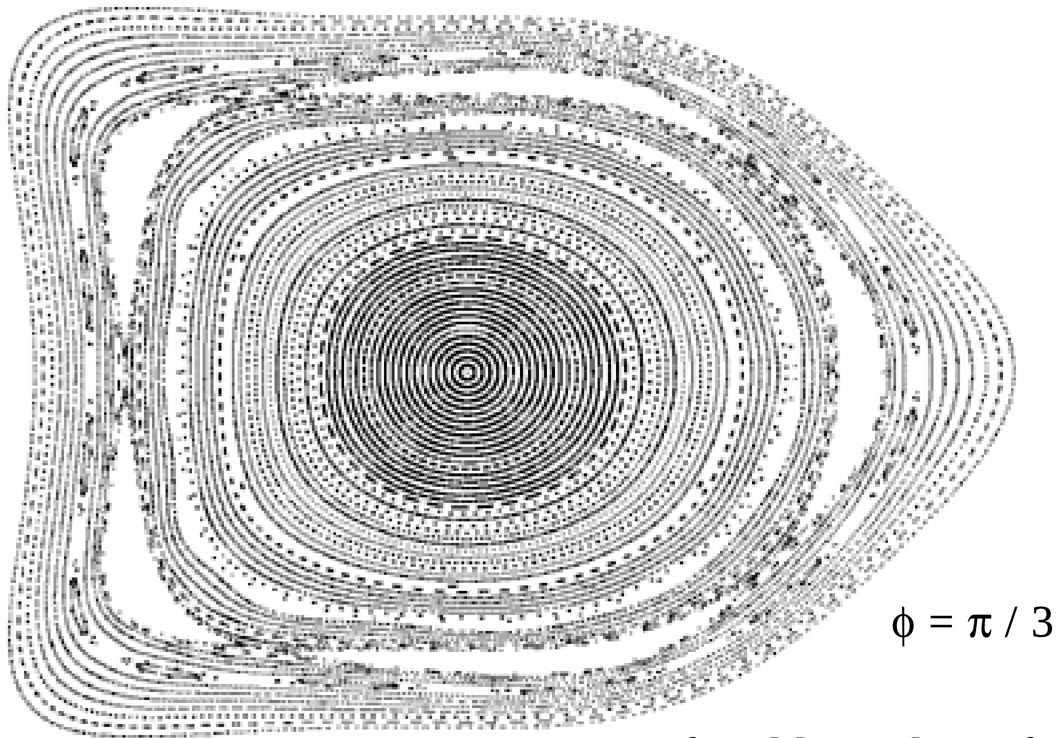




C82 fixed boundary. $\beta = 3\%$.
Continues to deteriorate with iteration.

LI383 free boundary. $\beta = 4\%$.
with surface optimized coils.
PIES calculation completely converged.

Output from Optimizer Has Residual Islands



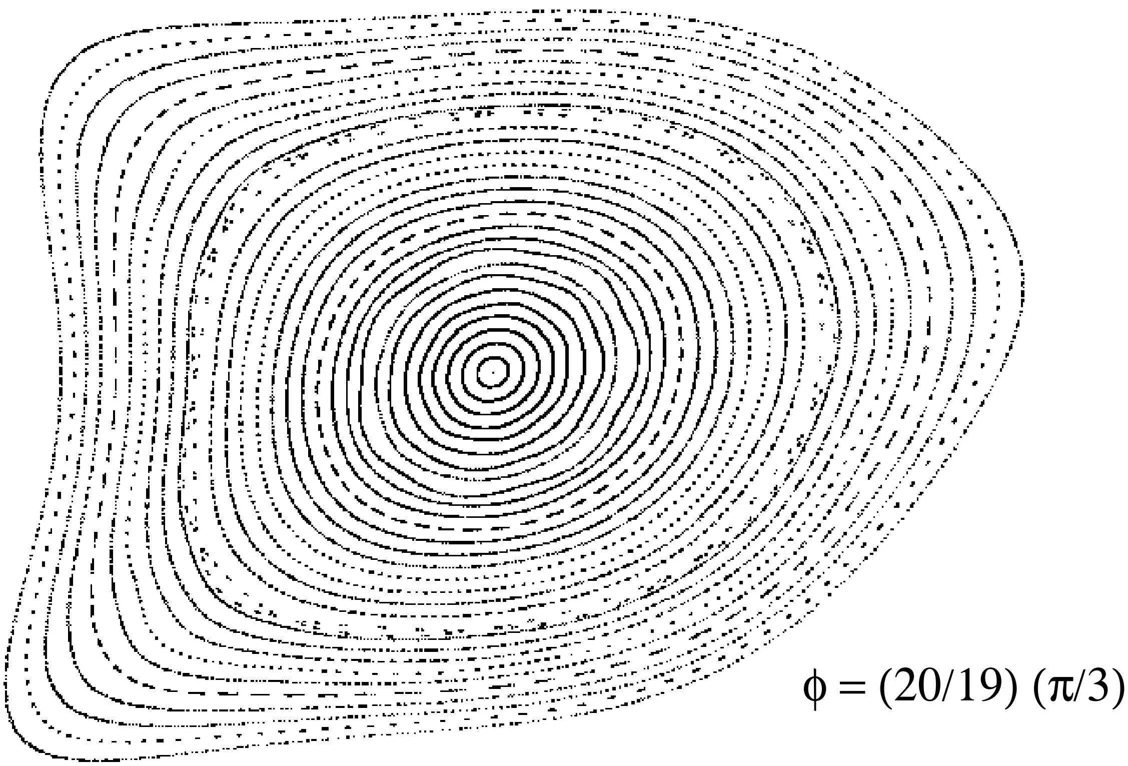
fixed boundary, fully converged

Unhealed LI383 configuration.

Emerged from examination of range of configurations with varying shear profile, D_R.

Healing of Fixed Boundary Flux Surfaces (Hudson)

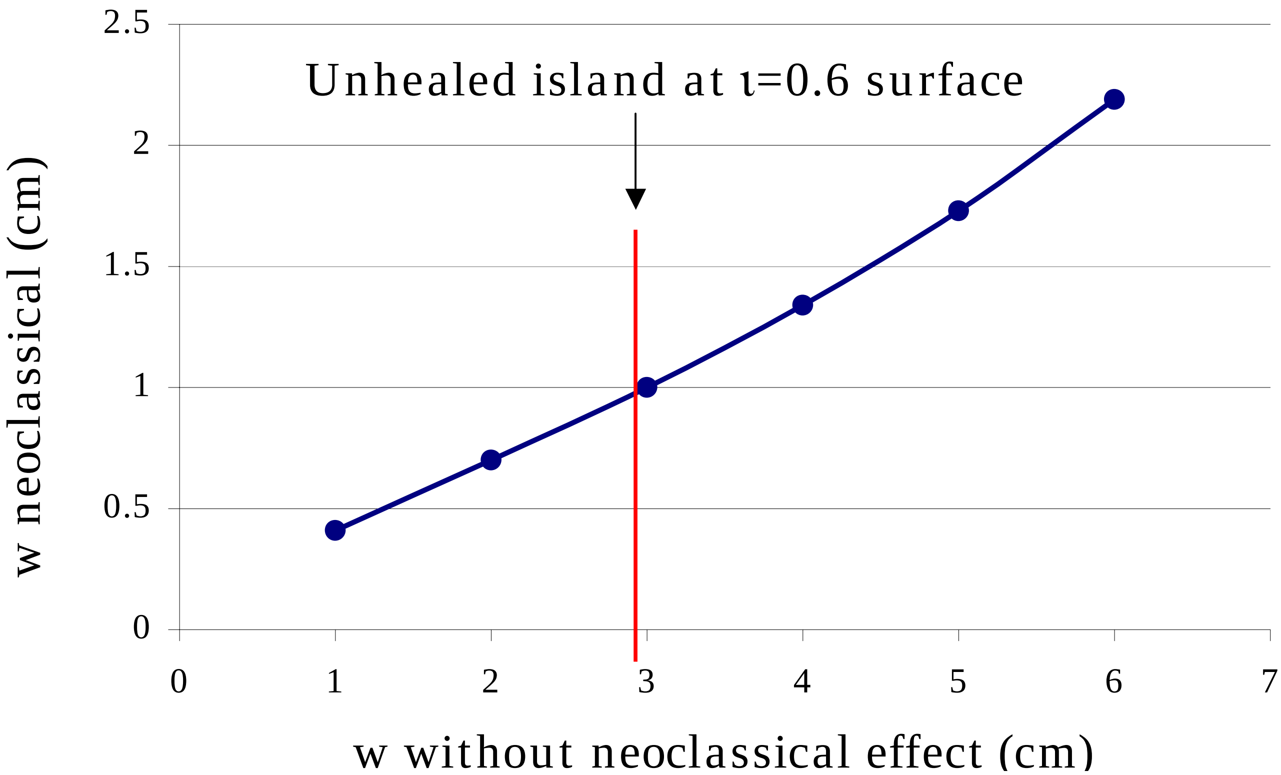
- Series of PIES runs measure effect of a set of boundary perturbations.
- Response matrix determines desirable boundary shape to minimize island widths.
- Quasi-axisymmetry and MHD stability preserved.



fixed boundary, fully converged PIES run

Neoclassical Effect on Island Widths

- Perturbed bootstrap current gives neoclassical tearing mode in tokamaks ($\iota' < 0$).
- For $\iota' > 0$, the effect predicted to reduce NCSX island widths. (Hegna & Callen)
- Effect estimated for island at $\iota = 0.6$ surface in LI383. Analytic calculation in cylinder.
  Estimate $\Delta' = -2\ m / r$.
  Take $\beta = 4.2\%$, $B = 1.2$ T, $\langle n_e \rangle = 5.8 \times 10^{19}\ m^{-3}$.



Reduces island width from 9% to 3% of minor radius.

Small Islands Have Little Effect on Transport

- Large islands short-circuit confinement by allowing heat to flow along separatrix.

- For narrow islands, path length along field line long, and parallel conductivity prevents short-circuiting of confinement.
- Critical island width below which effect on transport is small:

$$w_0 \approx 5\,(\kappa_\perp / \kappa_{//})^{0.25}\,(RL_\iota / m\iota)^{0.5}$$

  where L_ι is shear scale length.

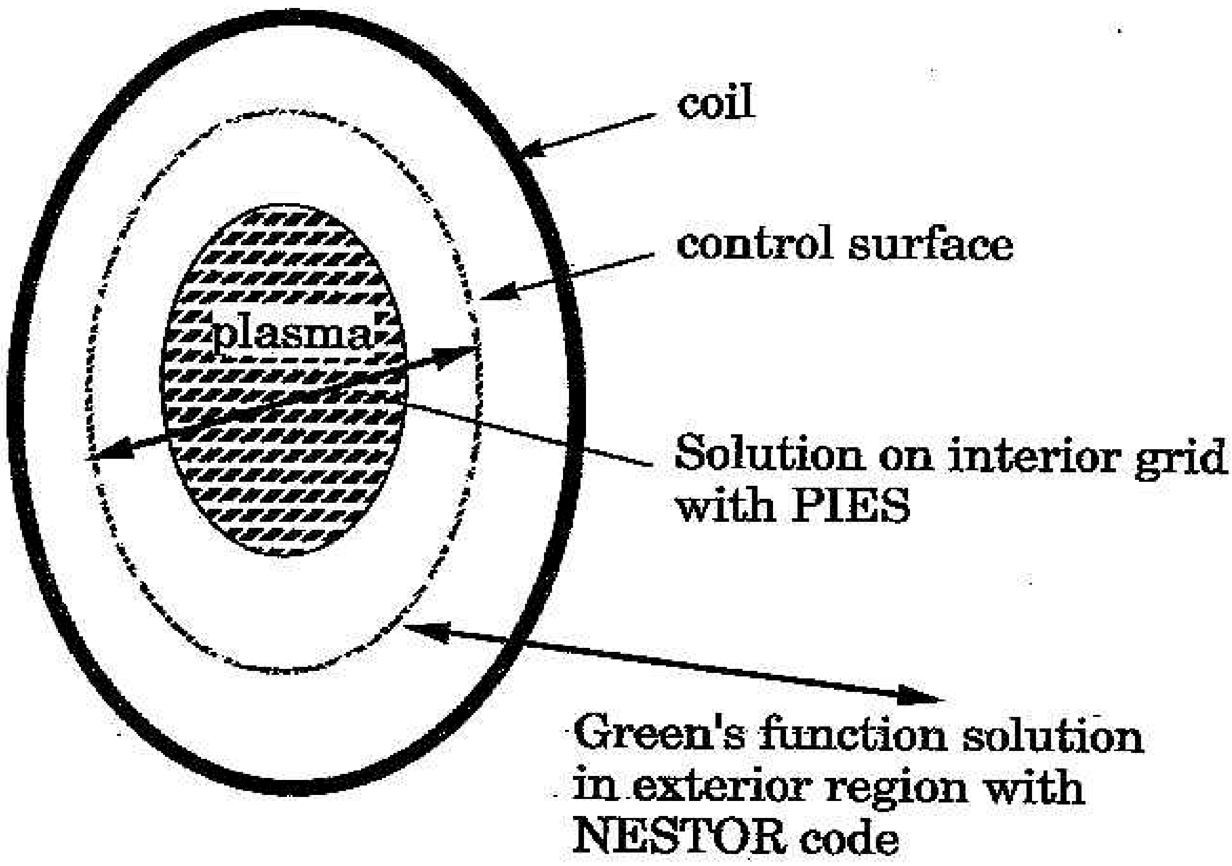
- Estimate $w_0/a \approx$ 3.5% for m=5 island in NCSX. (At $\beta = 4.2\%$, B = 1.2 T, $\langle n_e \rangle = 5.8 \times 10^{19}$ m^{-3})
- Suggests that unhealed islands at rational surfaces other than $\iota = 0.6$ have little effect on transport in LI383 at full current, full beta.

Surface Quality with Discrete Coils

- Free-boundary equilibrium calculations include effects of resonant fields produced by discrete coils.
- Two approaches to island healing with coils:
    - Adjust coil shape to reduce resonant fields.
    - Include trim coils in machine design to control low order resonant fields.

Calculations do not include neoclassical effect

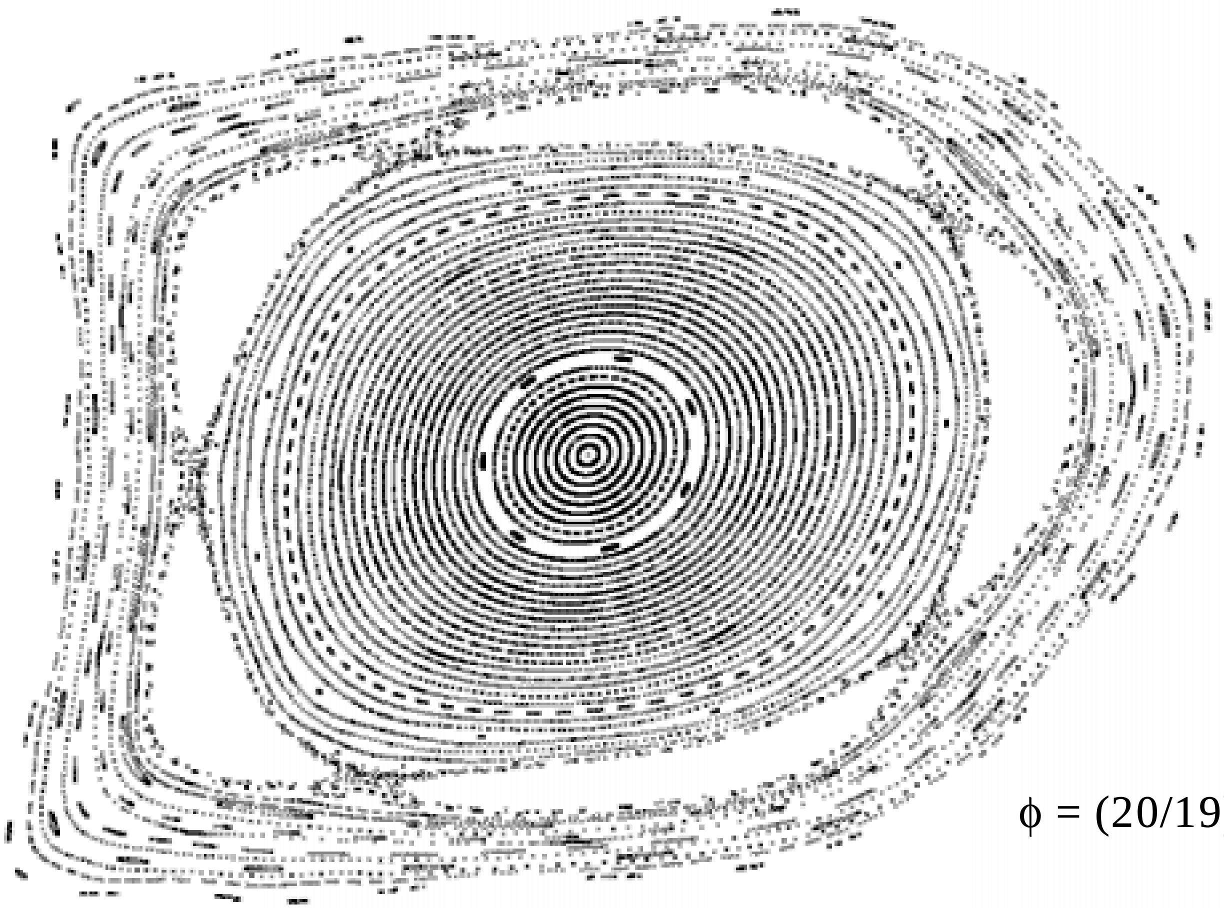
Free-Boundary PIES



Match exterior Green's function solution to interior solution at "control" surface. Want **B** continuous across control surface.

Coil Design Optimizer Does Not Target Islands

- Free-boundary PIES calculation for unhealed 0907 coil set (see S. Hirshman).
- PIES run not converged. Equilibrium continues to deteriorate with iteration

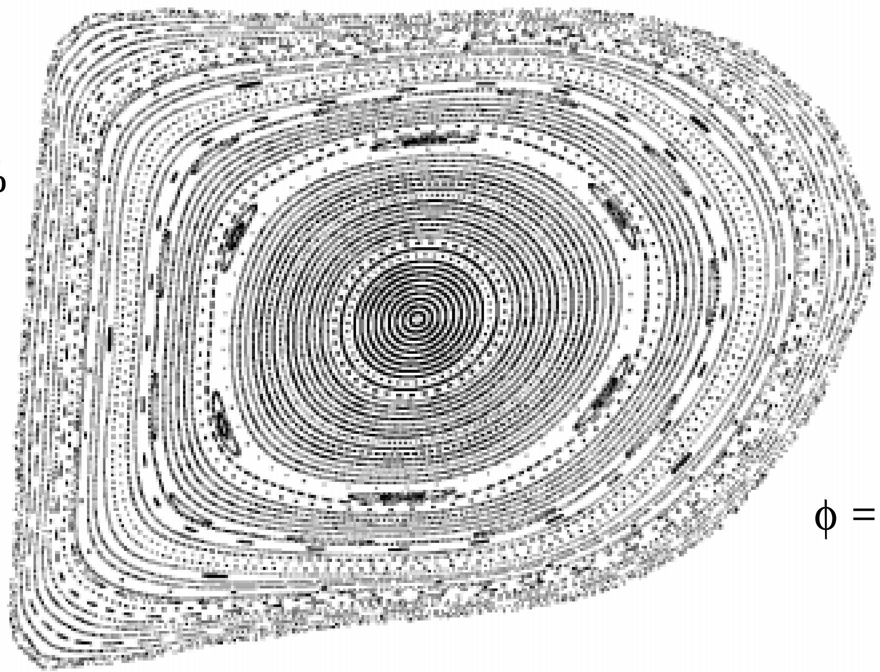


$\phi = (20/19)\ (\pi/3)$

Small Modification of Coils Results in Good Flux Surfaces (Hudson)

- Adjust Fourier coefficients specifying coil winding law, $\phi = \phi(\theta)$.
- Target deviation of plasma boundary, as well as island widths.
- Process took ~ 3 weeks to run on our fastest DEC Alpha workstations.
- Coil perturbations 1.7 cm max.

$I = 125$ kA, $\beta = 4\%$



$\phi = (20/19)\ (\pi/3)$

free boundary PIES calculation, fully converged

Examine a Broad Range of Equilibria for Flux Surface Integrity

Configurations optimized to have good physics properties with unhealed coils. Stable to kink and ballooning modes.
Not optimized for surfaces.

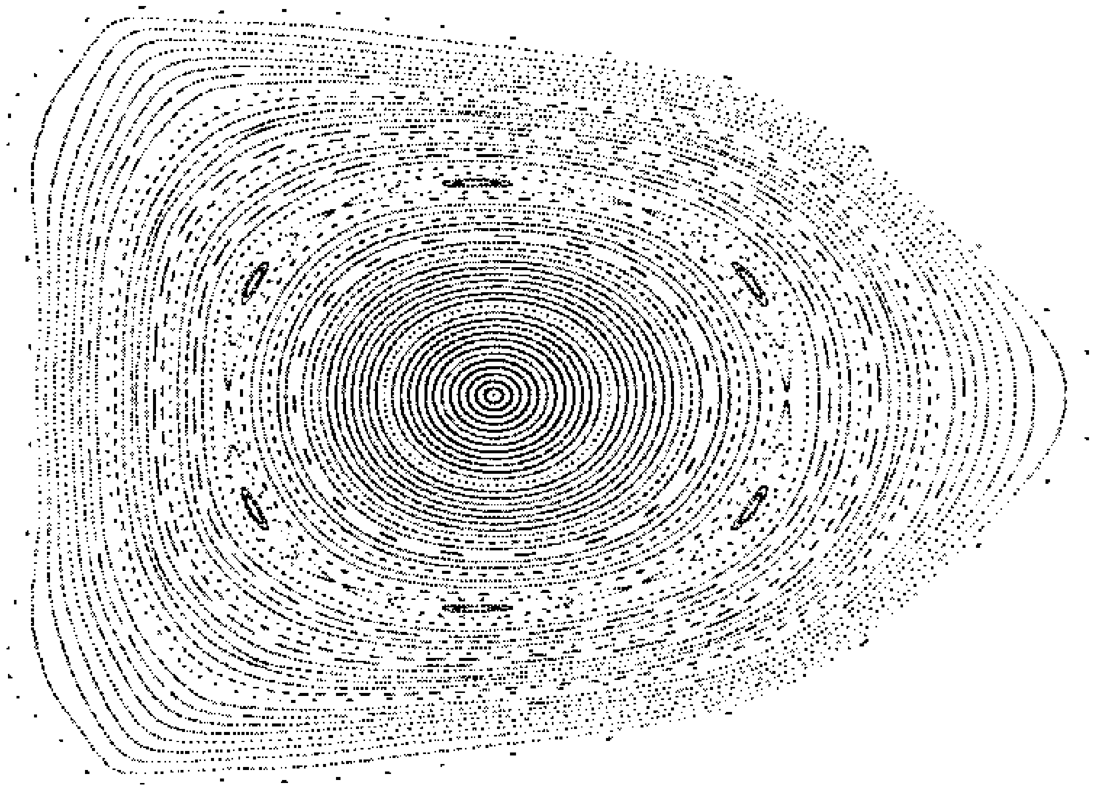
Evaluate surface quality using healed coils:

- Equilibria from β vs. I flexibility scan. (See N. Pomphrey.)
  Reduce β from 4%, I from 125 kA.
  - $\beta = 4\%$, $I = 92$ kA.
  - $\beta = 3\%$, $I = 92$ kA.
  - $\beta = 3\%$, $I = 83$ kA.
  - $\beta = 2\%$, $I = 83$ kA.
- Equilibrium from discharge modeling. (see E. Lazarus)
  60 ms into shot: 40% maximum current, low β.
- Vacuum field.

Equilibrium from β vs. I Flexibility Scan

$\beta = 4\%$, $I = 92$ kA (Reduced from $\beta = 4\%$, $I = 125$ kA)

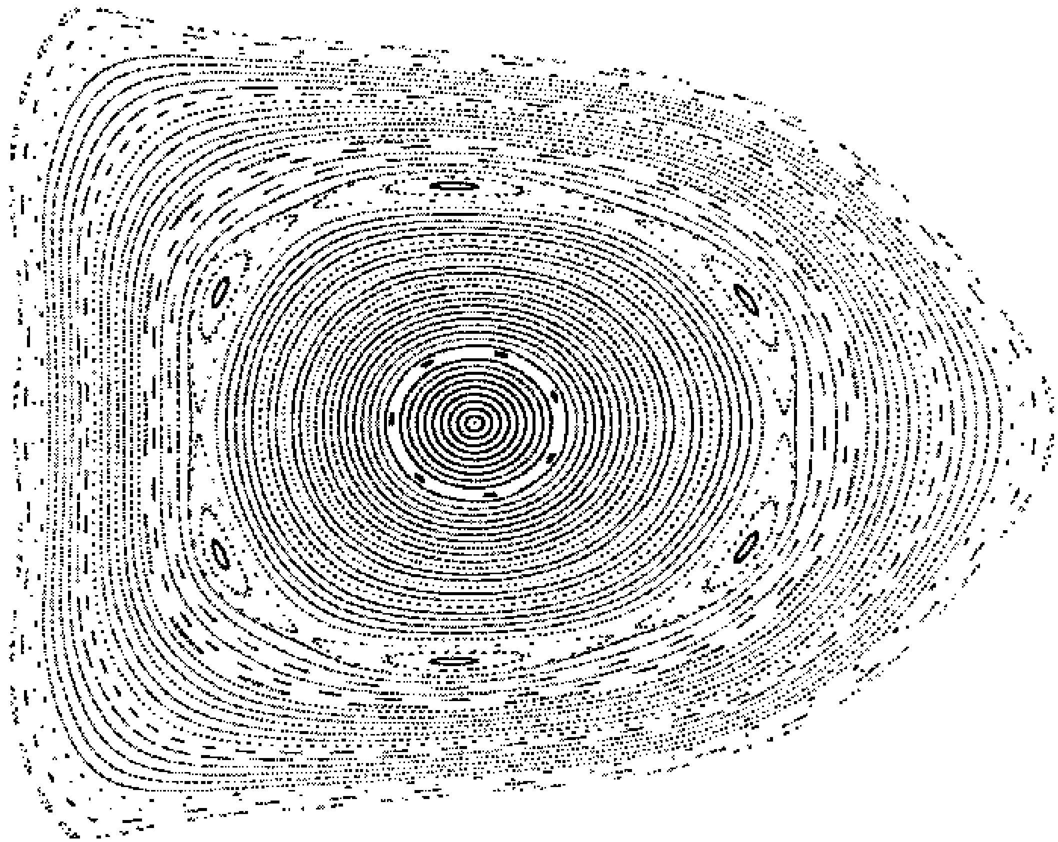
free-boundary,
fully converged
PIES calculation



Equilibrium from β vs. I Flexibility Scan

$\beta = 3\%$, $I = 92$ kA (3/4 full β, 3/4 full current)

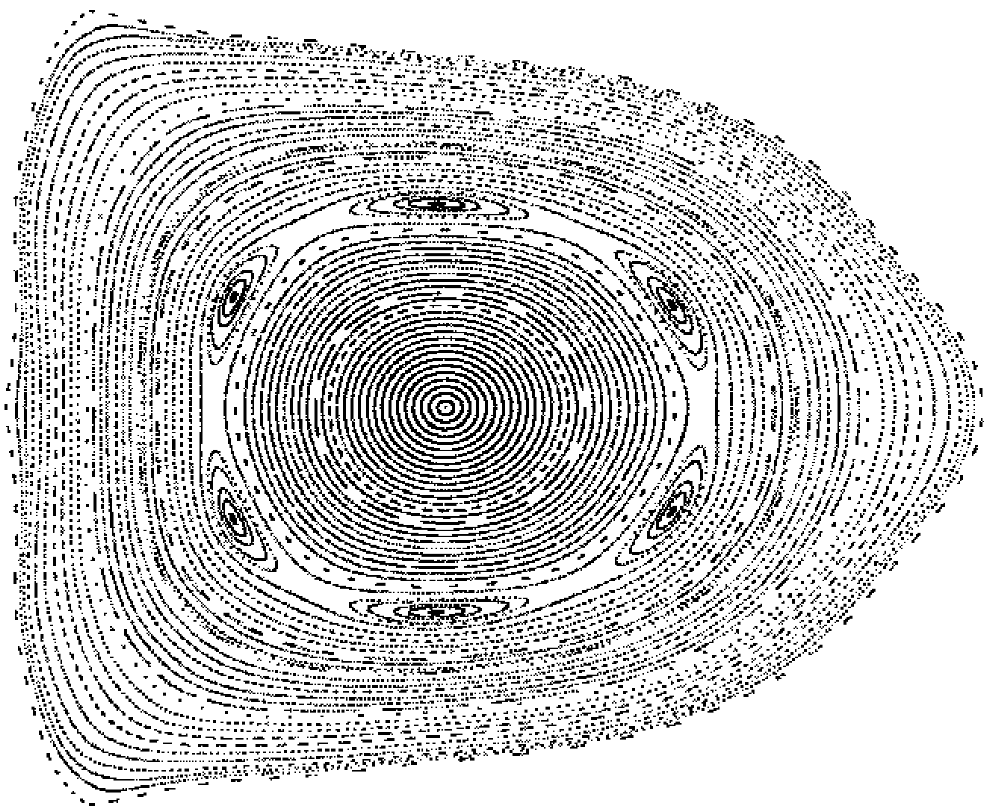
free-boundary,
fully converged
PIES calculation



Equilibrium from β vs. I Flexibility Scan

$\beta = 3\%$, $I = 83$ kA (3/4 full β, 2/3 full current)

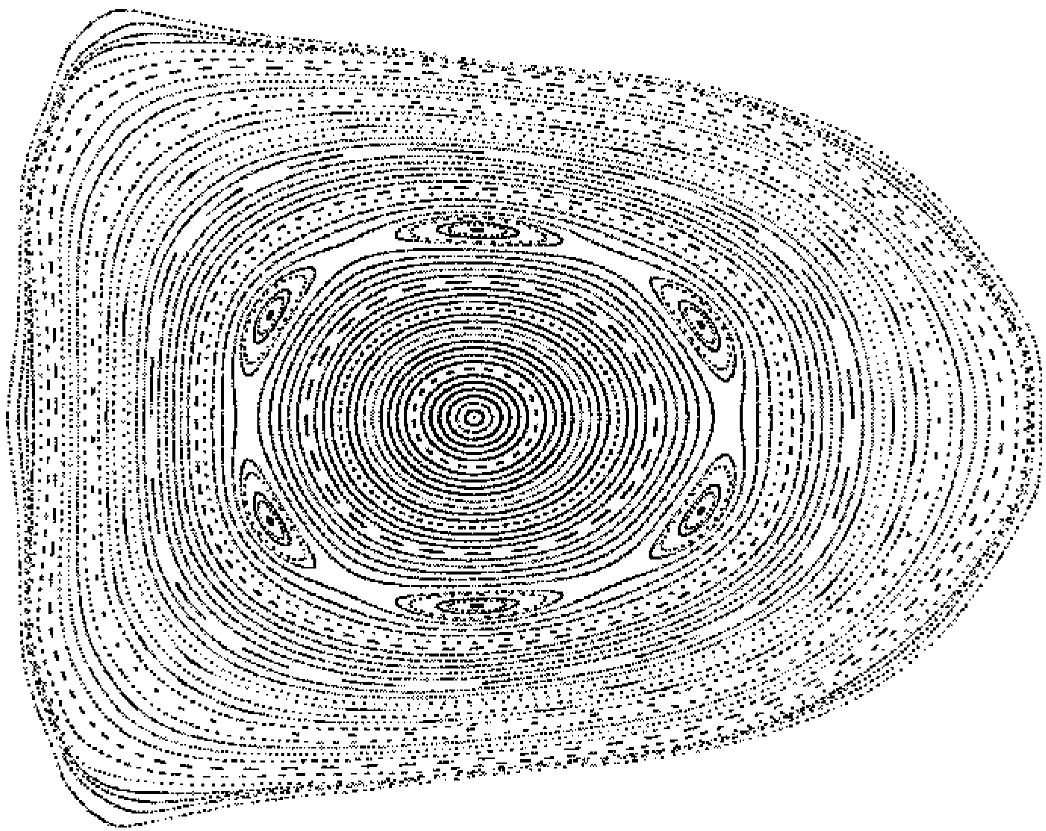
free-boundary,
fully converged
PIES calculation



Equilibrium from β vs. I Flexibility Scan

$\beta = 2\%$, $I = 83$ kA (1/2 full β, 2/3 full current)

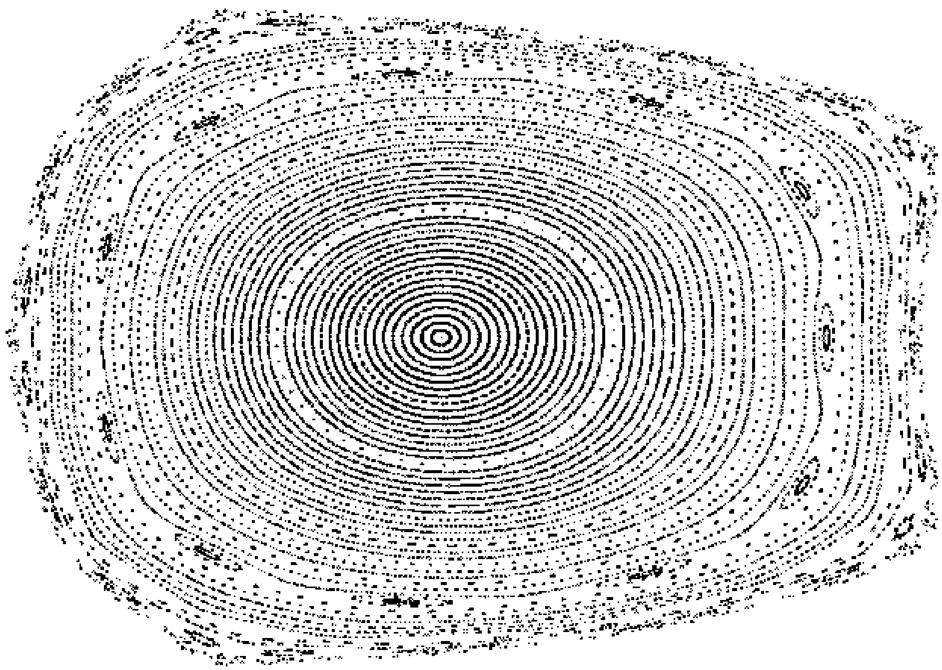
free-boundary,
fully converged
PIES calculation



Equilibrium from Discharge Modeling (see E. Lazarus)

60 ms into shot: I= 43 kA (~40% of maximum), $\beta = 0.3\%$

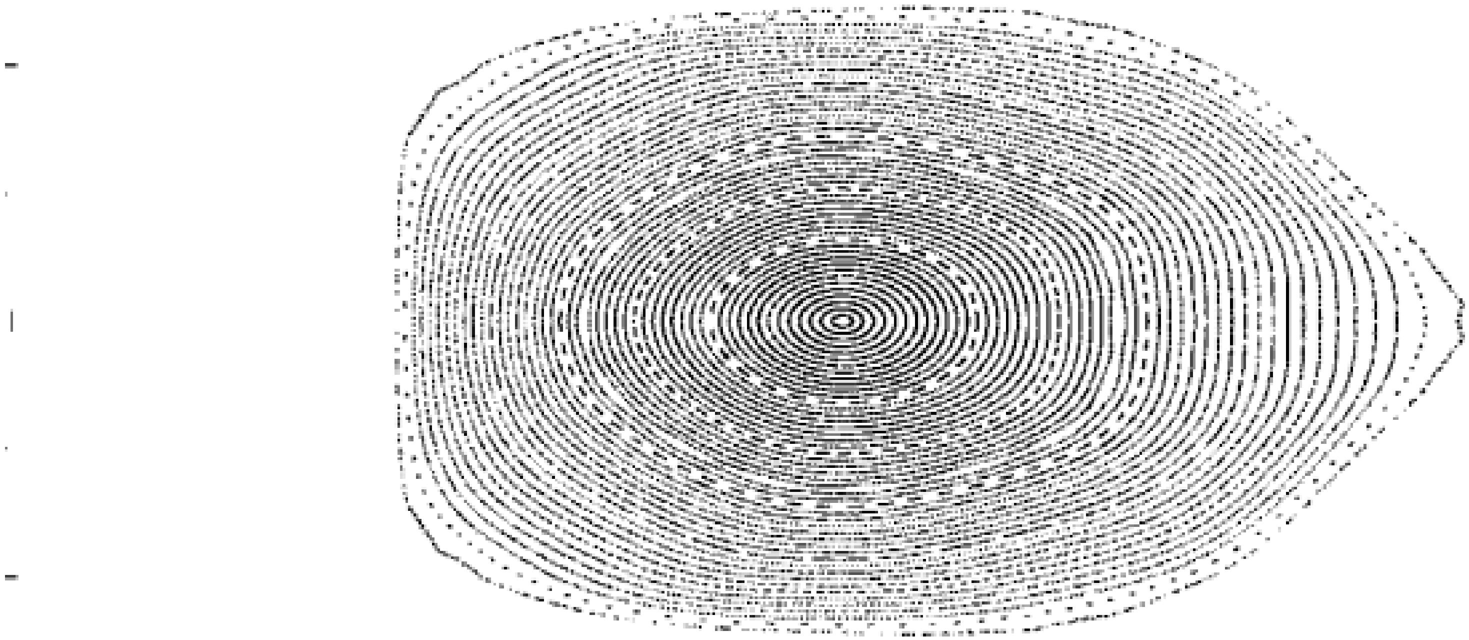
free-boundary,
fully converged
PIES calculation



$\iota = 0.6$ at edge. Loses ~ 20% of minor radius on outside.

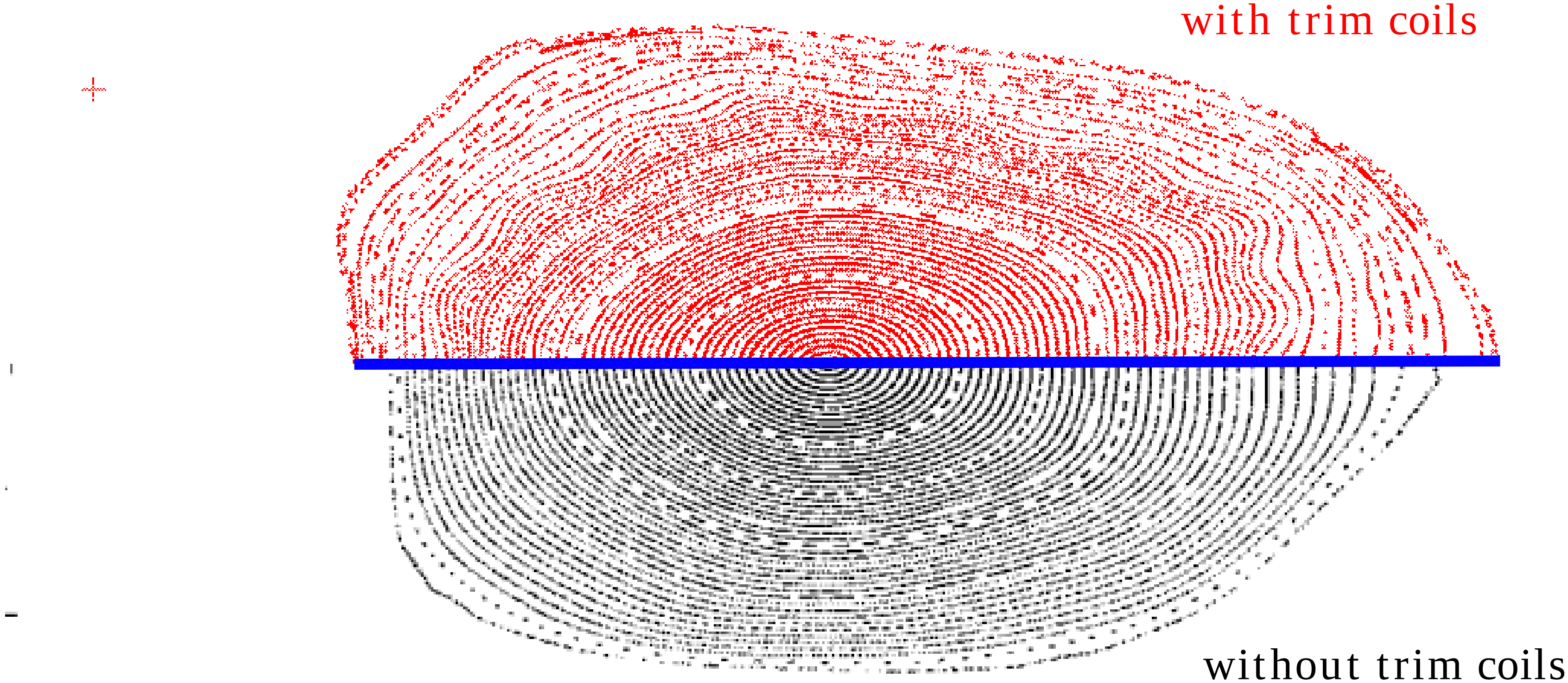
Vacuum Flux Surfaces

Minor radius is 82% that of LI383.



Can restore outer flux surfaces with trim coils.

Trim coils can restore vacuum flux surfaces to full volume.

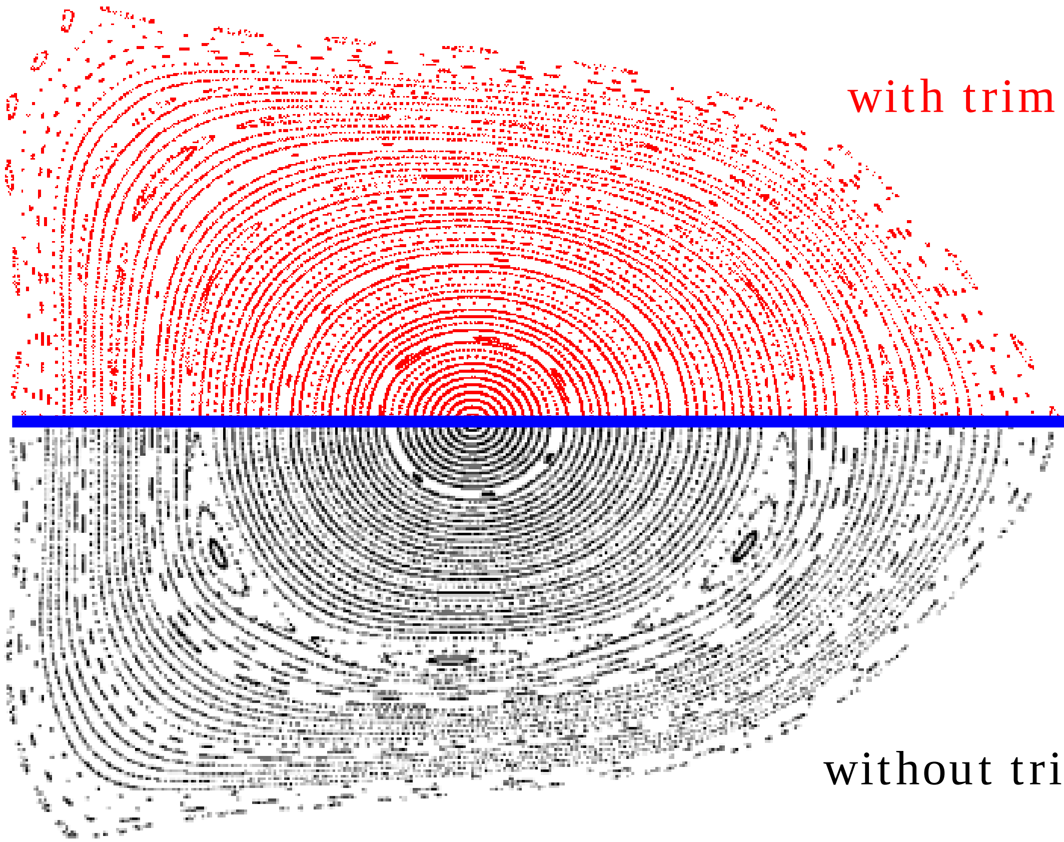


Recovers ≈ 15% loss of minor radius without trim coils.

Trim Coil Removes m=6 Island in Plasma Interior

Equilibrium from flexibility study, $\beta = 3\%$, $I = 92$ kA.

free-boundary,
fully converged
PIES
calculations



Conclusions

We have demonstrated a design process for ensuring adequate flux surface quality for a range of equilibria.

- Healing to remove resonances.
- Trim coils promising.

Neoclassical effects also predicted to substantially reduce island widths.